Density Dependent B-parameter model of Compact object with Strange Quark Matter

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Motivation

- Einstein field equations are the relations between geometry of the space-time and energy momentum tensor of the interior matter content of the compact objects.
- Eqation of State (EOS) of the matter content interior to the compact objects is essential to study its properties.
- If the interior matter content is supposed to be strange quark matter following equation of state as precribed by MIT bag model given below,

$$p = \frac{1}{3}(\rho - 4B) \tag{1}$$

  then the Bag Constant $'B'$ may possess density dependence i.e $B = B(\rho)$.
- In my work I have tried to find out the density dependence of B from the consideration of space-time geometry which is relavent for the EoS of strange matter.

Introduction

- The analysis of very compact astrophysical objects has been a key issue in relativistic astrophysics for the last few decades.
- The estimated masses and radii of many compact objects such as X-ray pulsar Her X-1, X-ray burster 4U 1820-30, millisecond pulsar SAX J 1808.4-3658, X-ray sources 4U 1728-34, PSR 0943+10 and RXJ185635-3754 are not compatible with the standard neutron star models.
- Strange quark matter may be useful to understand the observed physical features of some of these compact objects [1, 2, 3, 4, 5]
- The matter densities of such compact objects are normally above the nuclear matter density. It has both maximum mass and radius less than those for neutron stars, with higher compactification factor (ratio of mass to radius).

Introduction

- It is physically realistic to consider an ultra-compact star with two different pressures inside [6], namely, the radial pressure and the tangential pressure incorporating the anisotropy.
- It is considered [7, 8, 9] that compact stars may be made up of quark matter which may be formed in two different ways:
  - (i) quark-hadron phase transition in the early universe, and
  - (ii) conversion of neutron star into strange star ones at ultrahigh densities.
- The theories of strong interaction with quark bag model, strange quark matter may be useful in order to obtain a relevant equation of state (EOS).
- Here we assume that the quarks are mass less and non-interacting giving the quark pressure

$$p_q = \frac{1}{3}\rho_q$$

where ρ_q is the quark energy density.

- The total energy density $\rho = \rho_q + B$ and total pressure $p = p_q - B$.
  where B is the Bag constant.
- From the above discussion the equation of state (EOS) for the strange quark matter [1] is given by:

$$p = \frac{1}{3}(\rho - 4B) \tag{2}$$

Introduction

- In the MIT Bag model [2] and in the original version of fuzzy Bag model the non-perterbative QCD vacuum is parameterized by a constant B in the Lagrangian density.
- For these stars quark confinement is important which is described by the energy term proportional to the volume [3].
- In this model, the constituent quark matter is considered to be mass less u, d quarks and massive s quarks and electrons. The quarks are considered to be degenerate Fermi gases, which may exist only in a region of space with a vacuum energy density B.
- In this work, I have investigated the density dependence of 'B' and role of pressure anisotropy of a super dense star relating it with the value of Bag parameter in the framework of Vaidya-Tikekar model[4, 5].

Introduction

- In this approach the model of a super dense star is obtained by stipulating a law for variation of density of its matter content which follows from prescribing a geometry characterized by two curvature parameters for the physical space of the configuration which is a departure from the spherical geometry of a uniform density configuration.
- The equation of state of matter content follows from the system of associated Einstein field equations which in a number of specific cases is found to approximate to linear EOS connecting pressure and density [3].

Einstein Field Equations

- The space time in the interior of a spherically symmetric,cold compact star in equilibrium is described by

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\mu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (3)$$

Where $\nu(r)$ and $\mu(r)$ are the two unknown metric functions.

- The interior matter content of the star is prescribed to be that of a fluid with anisotropic pressures with the energy momentum tensor

$$T_{ij} = \text{diag}\ (-\rho,\ p_r,\ p_\perp,\ p_\perp) \qquad (4)$$

where ρ is the energy-density, p_r is the radial pressure and $p_\perp$ is the tangential pressure and $\Delta = p_\perp - p_r$ is the measure of pressure anisotropy [4, 6, 7, 8, 1] in this model, which depends on metric potentials $\nu(r)$ and $\mu(r)$.

Einstein Field Equations

- The Einstein Field Equation

$$\mathbf{R}_{ij} - \frac{1}{2} g_{ij} \mathbf{R} = 8\pi G \ \mathbf{T}_{ij} \tag{5}$$

where $\mathbf{R}_{ij}$ is Ricci tensor and $\mathbf{R}$ is the Ricci scalar.

- The Einstein field equation relates the metric parameters $\mu(r)$ and $\nu(r)$ and of the space timewith the dynamical variables of its physical content which reduces to the following system of three equations :

$$\rho = \frac{\left(1 - e^{-2\mu}\right)}{r^2} + \frac{2\mu' e^{-2\mu}}{r} \tag{6}$$

$$p_r = \frac{2\nu' e^{-2\mu}}{r} - \frac{\left(1 - e^{-2\mu}\right)}{r^2} \tag{7}$$

$$p_\perp = e^{-2\mu} \left[\nu'' + \nu'^2 - \nu'\mu' + \frac{\nu'}{r} - \frac{\mu'}{r} \right] \tag{8}$$

where we have used that $8\pi G = 1$.

Einstein Field Equations

- The geometry of a more realistic star with variable matter density is expected to be departure from 3-spherical geometry. TheVaidya -Tikekar models are obtained by prescribing [5, 2]

$$e^{2\mu} = \frac{1 + \lambda r^2/R^2}{1 + r^2/R^2} \tag{9}$$

where $'\lambda'$ and $'R'$ are two different parameters, $'\lambda'$ being the spheroidicity parameter and the geometrical parameter $'R'$ is expressed in Km. The geometry of the physical 3-space of the star is that of a 3-Pseudo spheroid and the parameter λ is related with the eccentricity of the 3-Pseudo spheroid.

- In view of the field equation (6) this is equivalent to prescribing the law for variation of matter density at the centre of the star for the physical content of the star

$$\rho_0 = \frac{3(\lambda - 1)}{R^2} \tag{10}$$

The variation of ρ is governed by two parameters $'\lambda'$ and $'R'$. The matter density has maximum value at the center from which it decreases radialy outward.

Einstein Field Equations

- Equations (7) and (8) determine the pressures along radial and transverse directions in terms of $\nu(r)$ and these parameters at all points of the star. If the nature of anisotropy is known these equations determine the metric potential $\nu(r)$.
- Now using equations (7) and (8), ones obtains a second order differential equation in $'x'$

$$(1-\lambda+\lambda x^2)\Psi_{xx} - \lambda x \Psi_x + (\lambda(\lambda-1)-\alpha)\Psi = 0 \qquad (11)$$

where $\Psi = e^{\nu(r)}$, with $x^2 = 1 + \frac{r^2}{R^2}$.
For simplicity, we choose the pressure anisotropy Δ as follows,
$\Delta = \frac{\alpha\lambda^2(x^2-1)}{R^2(1-\lambda+\lambda x^2)^2}$

- The above relation is chosen so that the regularity at the center is ensured and to obtain relativistic solution similar to that obtained by R.Tikekar *et al.*[3] for the field equations (6)-(8). Using the transformation $z = \sqrt{\frac{\lambda}{\lambda-1}}\, x$ eq. (11) can be written as

$$(1 - z^2)\Psi_{zz} + z\Psi_z + (\beta^2 - 1)\Psi = 0 \tag{12}$$

where $|\beta|^2 = (2 - \lambda + \lambda\alpha)$ is a constant.

General Solution

The general solution of eq. (12) [3] is given below in two cases
Case-I: In this case the value of $'\lambda'$ and $'\alpha'$ are such that $\beta(=\sqrt{2-\lambda(1-\alpha)})$ is positive and the solution is

$$\Psi = C[\beta\sqrt{z^2-1}\,cosh(\beta\eta) - z\,sinh(\beta\eta)]$$

$$+D[\beta\sqrt{z^2-1}\,sinh(\beta\eta) - z\,cosh(\beta\eta)] \qquad (13)$$

Case-II: In this case the values of $'\lambda'$ and $'\alpha'$ are such that $\beta(=\sqrt{\lambda(1-\alpha)-2})$ is positive and the corresponding solution is

$$\Psi = C[\beta\sqrt{z^2-1}\,cos(\beta\eta) - z\,sin(\beta\eta)]$$

$$+D[\beta\sqrt{z^2-1}\,sin(\beta\eta) + z\,cos(\beta\eta)] \qquad (14)$$

where C and D are two constants to be determined from boundary condition and $z = cosh(\eta)$.

Thermodynamics of density dependence of B parameter

From Thermodynamic point of view one can obtains the following relations.

$$n_i^{Bag} = -\left(\frac{\partial \Xi}{\partial \mu_i}\right), \tag{15}$$

$$p^{Bag} = -(\Xi + B) + n_b \frac{\partial B}{\partial n_b}, \tag{16}$$

$$E^{Bag} = (\Xi + B) + \sum_i \mu_i n_i. \tag{17}$$

Where Ξ is the thermodynamic potential which is a funtion of temparature T, μ_i = Chemical potential, B = Bag constant, n_b=Baryon number density, n_i = number density of particles of type i. The last term in middle equation leads to a density dependent B parameter. For a constant B, the additional terms which arises in the above three equations vanishes leading to the MIT Bag model EOS

$$p = \frac{1}{3}(\rho - 4B) \tag{18}$$

Physical Parameters

- The physical parameters of a general relativistic star are given by

$$\rho = \frac{1}{R^2(z^2-1)}\left[1 + \frac{2}{(\lambda-1)(z^2-1)}\right] \quad (19)$$

$$p_r = -\frac{1}{R^2(z^2-1)}\left[1 - \frac{2z}{(\lambda-1)}\frac{\Psi_z}{\Psi}\right] \quad (20)$$

$$\Delta = \frac{\alpha\lambda[(\lambda-1)(z^2-1)-1]}{R^2(\lambda-1)^2(z^2-1)^2} \quad (21)$$

- The total mass of a star of radius b is given by

$$M = \frac{(\lambda-1)b^3}{2R^2\left(1+\lambda\frac{b^2}{R^2}\right)} \quad (22)$$

Physical Parameters

- The compactness factor $'u'$ (the ratio of mass to radius) is given by

$$u = \frac{M}{b} = \frac{(\lambda - 1)y^2}{2R^2(1 + \lambda y^2)} \tag{23}$$

where $y = \frac{b}{R}$.

- In the case of a compact star,we impose the following conditions:
  (i) At the boundary of the star, the Schwarzschild exterior solution is matched with the interior solution *i.e.*,

$$e^{2\nu(r=b)} = e^{-2\mu(r=b)} = \left(1 - \frac{2M}{b}\right) \tag{24}$$

  (ii) At $r = b$, which defines the boundary of the star, the radial pressure p_r should vanish, which yields,

$$\frac{\Psi_z(z_b)}{\Psi(z_b)} = \frac{(\lambda - 1)}{2z_b} \tag{25}$$

  where $z_b^2 = \frac{\lambda(1+b^2/R^2)}{(\lambda-1)}$.

  (iii) The pressure $p_r \geq 0$ inside the star, which leads to an inequality, is given by $\frac{\Psi_z}{\Psi} \leq \frac{(\lambda-1)}{2z}$.

Physical Parameters

- We first use eq. (24) to determine $'R'$, which can be evaluated for a given set of values $'\lambda'$, $'M'$ and $'b'$.
- Using equations (24) & (25), we can determine the constants C and D.
- Onec the constant C & D are known, we can evaluate the value of $\frac{\Psi_z}{\Psi}$ from eqs.(13) or (14) as required at any inside points of the compact objects for particular choice of λ, α, $'M'$, $'b'$ etc.
- Now we can evaluate energy density(ρ) and pressure(p) at any inside points of compact objects from equations (19) & (20) respectively.

- Using the values of energy density (ρ) and corresponding pressure(p_r) at different points $'r'$ inside the star, one can express p_r as a polynomial function of ρ. I have choosen $p_r = \sum_{i=0}^{6} a_i \rho^i$, where a_is are constants to be determined from fitting.
- Again if the matter content interior to the star is supposed to be strange matter having equation of state $p_r = \frac{1}{3}(\rho - 4B)$, then p_r can be eliminated from these two expressions. Which leads to the expression of B as

$$\mathrm{B} = \mathrm{B}(\rho) = \frac{1}{4}\left[\left(\frac{1}{3} - a_1\right) - \sum_{i=0,2}^{6} a_i \rho^i\right].$$

Numerical Result

- We study the interior of a compact star in two distinguished regions
  (i) near the center of the star and
  (ii) away from the center up to the surface.
- Two different cases I have studied.

Case-1: X Ray Pulsar HER X-1 [4] Mass $M = 0.88M_\odot$, where $M_\odot$ is the solar mass. Radius $b = 7.7Km$. which leads to Compactness $u = 0.1686$ and we choose the parameter λ=10, for which $R = 18.2243Km$. from eq. (24).

λ	Constants (a_i)	$\alpha = 0$	$\alpha = 0.1$
10	a_0	56.77	49.96
	a_1	0.2112	0.2109
	a_2	-4.34×10^{-4}	-3.67×10^{-4}
	a_3	6.79×10^{-7}	5.92×10^{-7}
	a_4	-5.81×10^{-10}	-5.15×10^{-10}
	a_5	2.70×10^{-13}	2.43×10^{-13}
	a_6	-5.30×10^{-17}	-4.83×10^{-17}

Table 1: Values of parametric constants a_i for different α for HER X-1.

Graphical Result

Using the parametric constants given in the table 1 for HER X-1 [4] Mass $M = 0.88 M_{\odot}$, Radius $b = 7.7Km$. and the parameter λ=10, I have calculated the variation of $B(\rho)$ at different points inside the compact object which are given in the following figures.

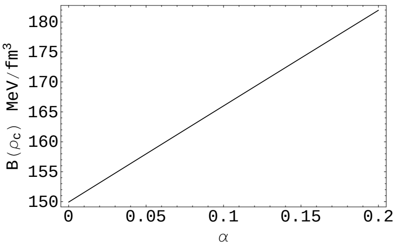

Figure 1: Variations of $B(\rho_c)$ with anisotropy parameter (α) with $\lambda = 10$ for HER X-1

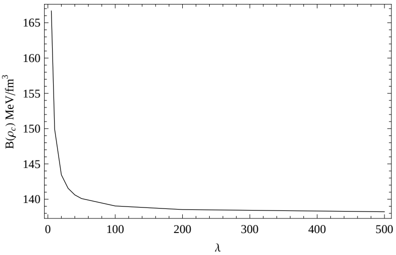

Figure 2: Variations of $B(\rho_c)$ with spheroidicity parameter (λ) for isotropic $(\alpha = 0)$ HER X-1.

- From the Fig. (1) we note that at the centre of the star $B(\rho_c)$ almost linearly increases with anisotropy parameter α when λ=constant.
- From the Fig. (2) we also note that at the centre of the star $B(\rho_c)$ decreases with increase of λ when anisotropy parameter α = constant (0.05) and at large value of λ, $B(\rho_c)$ is independent of λ.

Numerical Result

Set of parametric values of a_is for HER X-1 [4] Mass $M = 0.88M_\odot$, Radius $b = 7.7Km$. and parameter λ=500 is given in table 2

λ	Constants (a_i)	$\alpha = 0$	$\alpha = 0.1$
500	a_0	61.2	55.7
	a_1	0.239	0.238
	a_2	-5.7×10^{-4}	-5.2×10^{-4}
	a_3	9.1×10^{-7}	8.7×10^{-7}
	a_4	-7.98×10^{-10}	-7.85×10^{-10}
	a_5	3.82×10^{-13}	3.88×10^{-13}
	a_6	-7.76×10^{-17}	-8.12×10^{-17}

Table 2: Values of parametric constants a_i for different α for HER X-1 for large λ.

Graphical Result

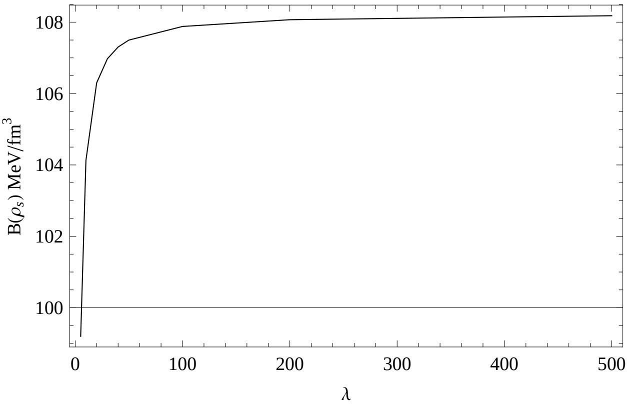

Figure 3: Variations of $B(\rho_s)$ with spheroidicity parameter (λ) for isotropic $(\alpha = 0)$ HER X-1.

Graphical Result

- From the Fig. (3) we note that at the surface of the star $B(\rho_s)$ increases with increase of λ when anisotropy parameter $\alpha =$ constant and at large value of λ, $B(\rho_s)$ tends to constant value.

Graphical Result

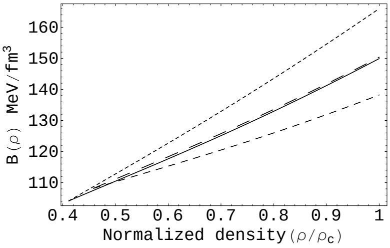

Figure 4: Variations of $B(\rho)$ with normalized energy density (ρ/ρ_c) in HER X-1. Solid and dotted line for $\alpha = 0$ and 0.1 with $\lambda = 10$ respectively. Dashed and long dashed line for $\alpha = 0$ and 0.1 with $\lambda = 500$ respectively.

Graphical Result

- Variation of $B(\rho)$ with normalized density is shown in the Fig. (4) for different values of α and λ.

Numerical Result

Case-II: SAX J 1808.4-3658 [4] Mass $M = 1.435 M_{\odot}$, where $M_{\odot}$ is the solar mass. Radius $b = 7.07 Km$. which leads to Compactness $u = 0.2994$ and we choose the parameter $\lambda = 5.1$, for which $R = Km$. from eq. (24).

a_i	$\alpha = 0$	$\alpha = 0.0256$	$\alpha = 0.072$
a_0	119.49	115.21	107.41
a_1	0.125	0.127	0.132
a_2	-2.393×10^{-4}	-2.264×10^{-4}	-2.027×10^{-4}
a_3	1.388×10^{-7}	1.324×10^{-7}	1.207×10^{-7}
a_4	-4.108×10^{-11}	-3.935×10^{-11}	-3.613×10^{-11}
a_5	6.278×10^{-15}	6.028×10^{-15}	5.557×10^{-15}
a_6	-3.888×10^{-19}	-3.737×10^{-19}	-3.455×10^{-19}

Table 3: Value of parametric constants a_i for different α with λ=5.1 for the SAX J 1808.4-3658.

Graphical Result

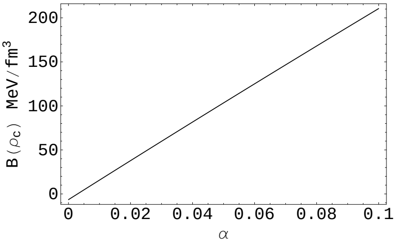

Figure 5: Variations of $B(\rho_c)$ with anisotropy parameter (α) when $\lambda = 5.1$ for SAX J 1808.4-3658.

Graphical Result

- From the Fig. (5) we note that in case of SAX J 1808.4-3658 at the centre of the star $B(\rho_c)$ almost linearly increases with anisotropy parameter α when λ=constant.

Graphical Result

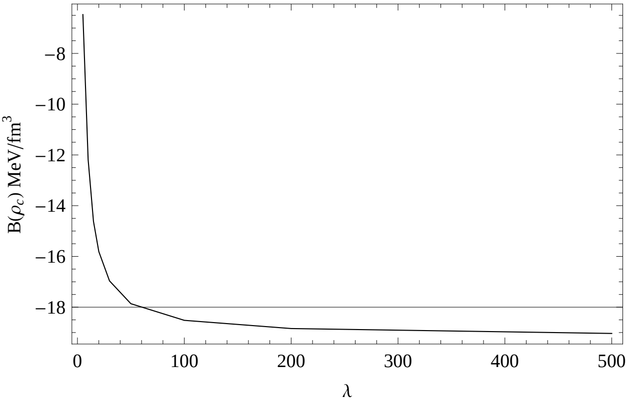

Figure 6: Variations of $B(\rho_c)$ with spheroidicity parameter (λ) for isotropic $(\alpha = 0)$ SAX J 1808.4-3658.

- From the Fig. (6) we also note that at the centre of the star $B(\rho_c)$ decreases with increase of λ when anisotropy parameter $\alpha =$ constant (0.05) and at large value of λ, $B(\rho_c)$ is independent of λ.

a_i	$\alpha = 0$	$\alpha = 0.0495$	$\alpha = 0.1295$	$\alpha = 0.15$
a_0	129.2	123.5	114.3	111.9
a_1	0.2	0.2	0.2	0.2
a_2	-3.4×10^{-4}	-3.1×10^{-4}	-2.7×10^{-4}	-2.5×10^{-4}
a_3	2.1×10^{-7}	2.0×10^{-7}	1.8×10^{-7}	1.71×10^{-7}
a_4	-7.3×10^{-11}	-6.8×10^{-11}	-6.1×10^{-11}	-5.9×10^{-11}
a_5	1.3×10^{-14}	1.2×10^{-14}	1.1×10^{-14}	1.1×10^{-14}
a_6	-9.6×10^{-19}	-9.1×10^{-19}	-8.2×10^{-19}	-8.0×10^{-19}

Table 4: Values of parametric constants a_i for different α when λ=10 for SAX J 1808.4-3658.

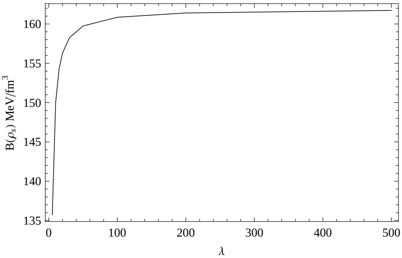

Figure 7: Variations of $B(\rho_s)$ with spheroidicity parameter (λ) if isotropic $(\alpha = 0)$ SAX J 1808.4-3658.

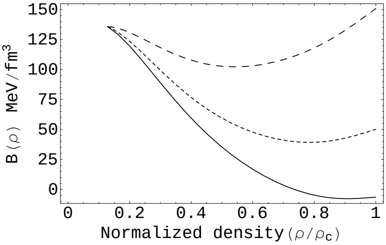

Figure 8: Variations of $B(\rho)$ with normalized energy density (ρ/ρ_c) for λ=5.1 in the SAX J 1808.4-3658. Lines from top to bottom are for α=0.0472, 0.0256 and 0 respectively

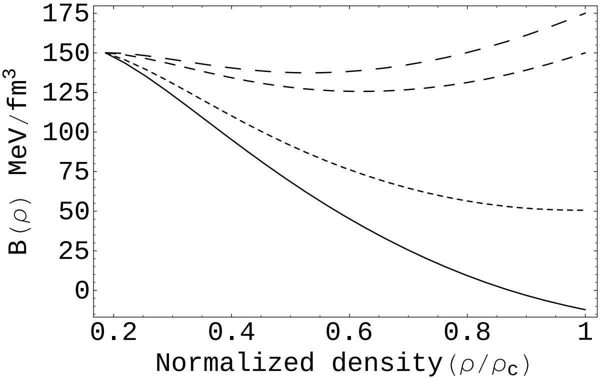

Figure 9: Variation of $B(\rho)$ with normalized energy density (ρ/ρ_c) for λ=10 for SAX J 1808.4-3658. Lines from top to bottom are for α=0.15, 0.1295, 0.0495, 0 respectively

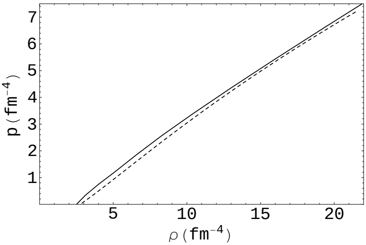

Figure 10: Variations of energy density (ρ) with pressure (p). Solid line represents equation of state of SQM as obtained in [6, 5] and dotted line represents equation of state of the matter in our model with $\lambda = 5.1$ for SAX J 1808.4-3658 ($Mass = 1.435\ M_{\odot}$, $Radius = 7.07\ km$.)

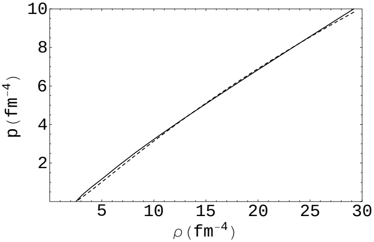

Figure 11: Variations of energy density (ρ) with pressure (p). Solid line represents equation of state of SQM as obtained in [6, 5] and dotted line represents equation of state of the matter in our model with $\lambda = 5.8$ for SAX J 1808.4-3658 ($Mass = 1.435\ M_{\odot}$, $Radius = 7.07\ km$)

Equation of State

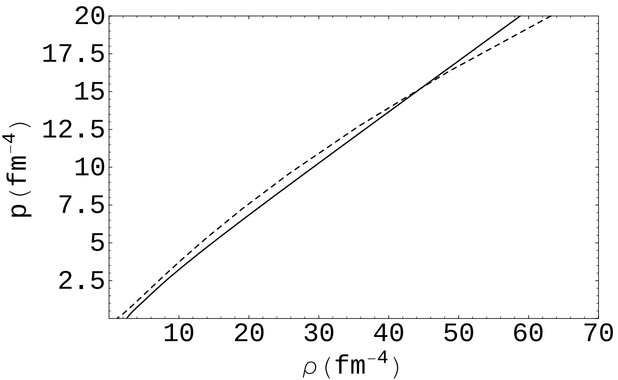

Figure 12: Variations of energy density (ρ) with pressure (p). Solid line represents equation of state of SQM as obtained in [6, 5] and dotted line represents equation of state of the matter content in our model with $\lambda = 5.4$ for EXO 1745-248 ($Mass = 1.7\ M_{\odot}$, $Radius = 9\ km$)

Conclusion

- Physically viable relativistic models of a compact objects having anisotropic matter content with strange-matter EOS $p = \frac{1}{3}(\rho - 4B)$, can be obtained by following the procedure suggested by Mukherjee *et al.*[8].
- In this model parameter B acquired a density dependence.
- At the center of the star Parameter Bincreases linearly with anisotropy parameter for given compactness and spheroidicity parameter.
- At the surface of the star there is practically no effect of spheroidicity parameter (λ) on the value of parameter B as it tends to contant value.

Conclusion

- For specific configuration of the compact object, Parameter B_0 picks up negative value, indicating the repulsive nature of core region for such configuration or existence of exotic kind of matter.
- Equation of state of matter content inside the star in this model is similar to that obtained from the experimental data.
- Equation of state of matter content of compact objects may be described from the geometry of the space-time.

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