Primordial Kerr Black Holes

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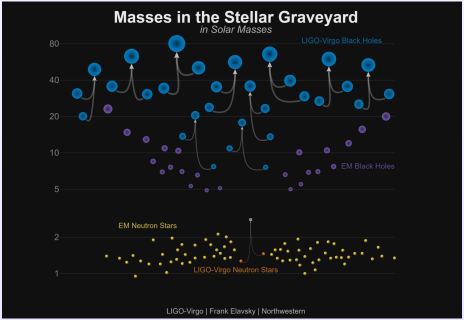
Observed black holes (BHs)

Three types of black holes have been discovered

- Stellar black holes
  BHs originated in the explosion of massive stars/supernovae, $\sim 3 - 100 M_\odot$
- Intermediate mass black holes (IMBHs)
  New class of recently discovered BHs, $\sim 10^3 - 10^6 M_\odot$
- supermassive black holes (SMBHs)
  BHs at the center of galaxies, $\sim 10^6 - 10^9 M_\odot$



Origin of primordial black holes (PBHs)

Multiple inflationary origins

- collapse of large primordial overdensities
- phase transitions
- collapse of cosmic strings, domain walls

Mass predictions

Assuming that one PBH is formed in a Hubble volume in the early Universe, one gets

$$M_{\rm PBH} \sim M_{\rm Planck} \times \frac{t_0}{t_{\rm Planck}} \sim 10^{38}\ {\rm g}\ \times t_0({\rm s})$$

where t_0 is the creation time.

One obtains:

- $M \sim 10^{-5}$ g for $t_0 \sim 10^{-43}$ s $\rightarrow$ Planck black holes
- $M \sim 10^{15}$ g for $t_0 \sim 10^{-23}$ s $\rightarrow$ lightest black holes still (possibly) existing
- $M \sim 10^{5}\ M_\odot$ for $t_0 \sim 1$ s $\rightarrow$ IMHBs? seeds for SMBHs?

Angular momentum of primordial Black Holes

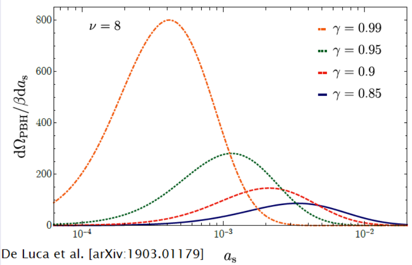
Angular momentum given by dimensionless parameter $a^* \equiv J/M^2$

$a^* \in [0,1]$

$a^* = 0$ for Schwarzschild BHs, $a^* = 1$ for extremal Kerr BHs

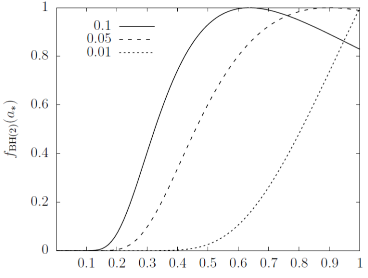
Spin predictions

Standard inflationary model $\Longrightarrow$ low spin



De Luca et al. [arXiv:1903.01179]

Transient matter domination $\Longrightarrow$ high spin



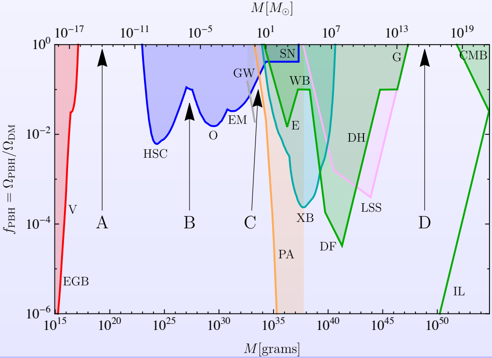
Harada et al. [arXiv:1707.03595]

Primordial Black Holes as dark matter candidates

Plausible dark matter candidates

- no Standard Model / General Relativity extension
- dynamically cold
- BH existence (somehow) proven
- mass ranges still available for BHs to represent all of dark matter

Constraints on PBH – from Carr & Kuhnel, 2006.02838

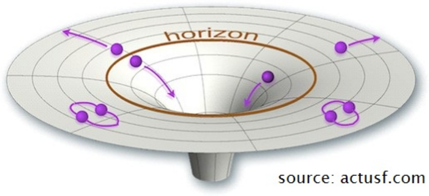


red: evaporation
blue: lensing
gray: gravitational waves
light blue: accretion
orange CMB distortions
green: dynamical effects
purple: large scale structure

A-D: possible open windows

BH Hawking radiation

Black hole horizons are interacting with the (quantum) vacuum.



Fundamental equation for Kerr BHs

Rate of emission of Standard Model particles i at energy E by a BH of mass M and spin parameter a^*:

$$Q_i = \frac{\mathrm{d}^2 N_i}{\mathrm{d}t\mathrm{d}E} = \frac{1}{2\pi} \sum_{\mathrm{dof.}} \frac{\Gamma_i(M, E, a^*)}{e^{E/T(M,a^*)} \pm 1}$$

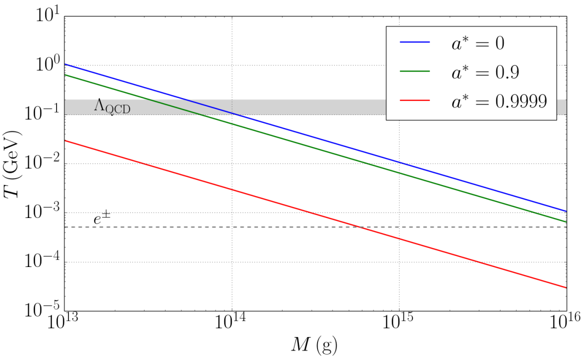
Γ_i is the greybody factor ($\sim$ absorption coefficient in Planck's blackbody radiation law)

Reduced temperature

Hawking temperature for Kerr BHs

$$T(M, a^*) = \frac{1}{4\pi M}\left(\frac{\sqrt{1-(a^*)^2}}{1+\sqrt{1-(a^*)^2}}\right) \xrightarrow[a^*=0]{\text{Schwarzschild}} \frac{1}{8\pi M}$$

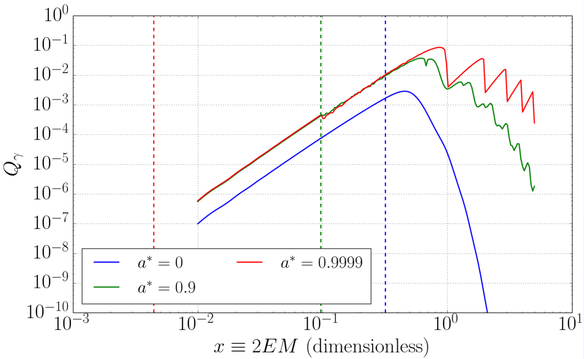
Comparison with the $e^\pm$ rest mass and QCD scale $\Lambda_{\rm QCD}$



Enhanced emission

BH-particle spin coupling $\Rightarrow$ superradiance effects (see e.g. Chandrasekhar & Detweiler papers in the 1970s)
$\rightarrow$ Hawking radiation enhanced for particles of spin 1 or 2

Example of spin 1 massless emissivity (photon)
Dotted lines = Hawking temperature



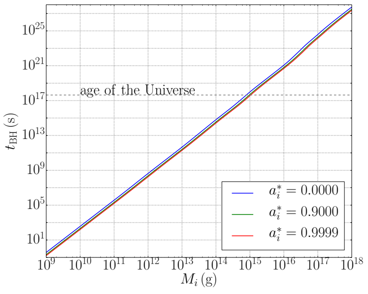
Reduced lifetime

Evolution equations

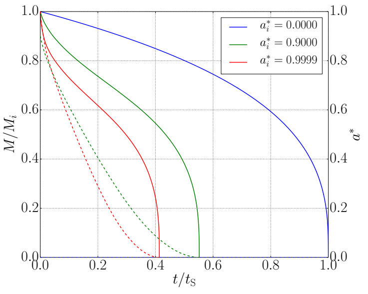
$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M,a^*)}{M^2} \qquad f \sim \int_E ener. \times emiss.$$

$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{a^*(2f(M,a^*) - g(M,a^*))}{M^3} \qquad g \sim \int_E ang.\ mom. \times emiss.$$

BH lifetime



BH mass (solid) and spin (dotted) evolution

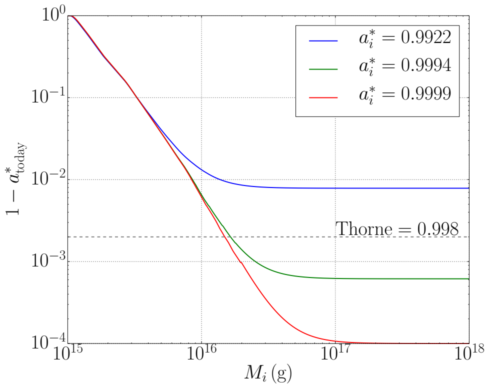


AA, J. Auffinger, J. Silk, MNRAS 494 (2020) 1257

Extremal spin today?

Could high spin BHs exist today? Can we get over Thorne's limit on the spin of rotating BHs from disk accretion?
→ Yes, with sufficiently massive and extremal PBHs

PBH spin today as a function of its initial mass



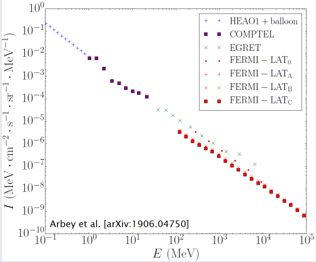
AA, J. Auffinger, J. Silk, MNRAS 494 (2020) 1257

Isotropic gamma ray background (IGRB) constraints

Origin

Diffuse background +

- Active galactic nuclei
- Gamma ray bursts
- DM annihilation/decay?
- Hawking radiation?



Flux estimation for BHs — Arbey *et al.* [PRD 101 (2020) 023010]

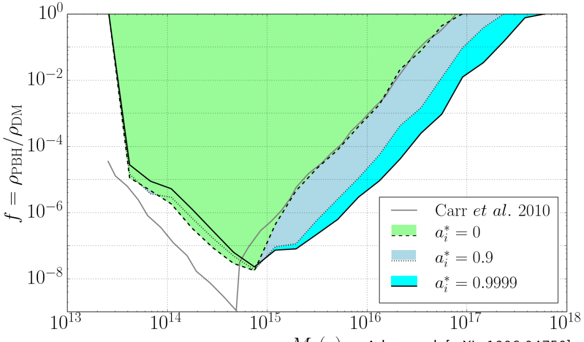
$$I \approx \frac{1}{4\pi} E \int_{t_{\rm CMB}}^{t_{\rm today}} (1 + z(t)) \times \int_M \left[\frac{\mathrm{d}n}{\mathrm{d}M} \frac{\mathrm{d}^2 N}{\mathrm{d}t\mathrm{d}E}(M, (1+z(t))E)\,\mathrm{d}M \right] \mathrm{d}t$$

IGRB and Kerr PBHs: monochromatic mass distributions

Main spin effects

- enhanced luminosity ⇒ stronger constraints
- reduced temperature ⇒ reduced emission energy ⇒ weaker constraints

Monochromatic constraints from the IGRB

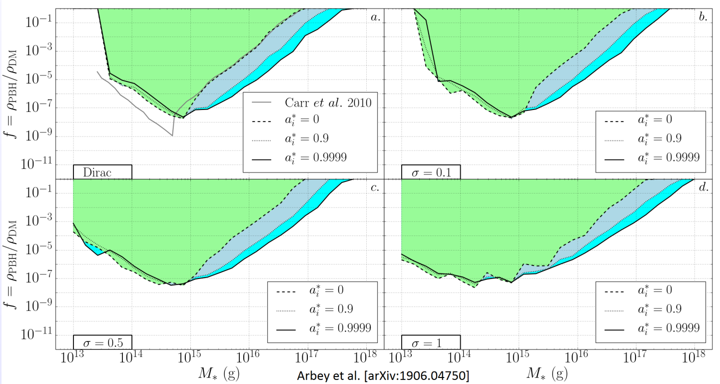


Arbey et al. [arXiv:1906.04750]

IGRB and Kerr PBHs: Extension to broad mass functions

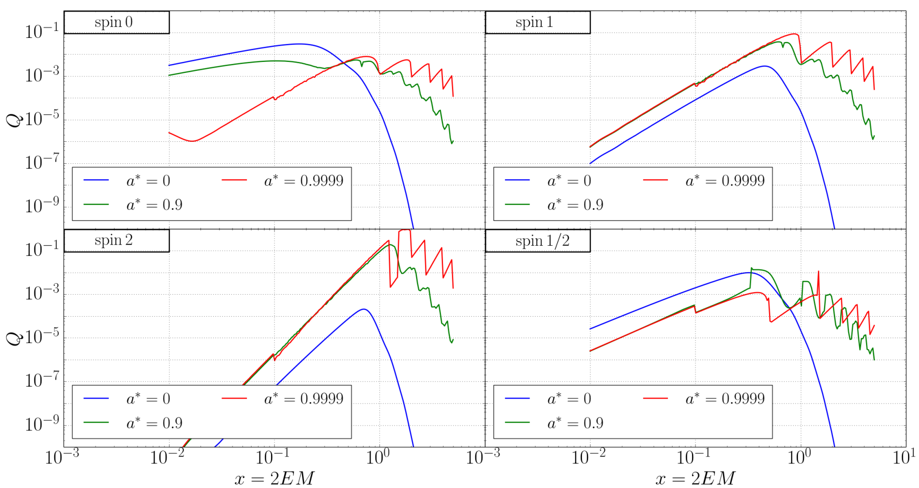
Main width effects — log-normal distribution $M\,\mathrm{d}n/\mathrm{d}M \propto \exp(-\ln(M/M_*)^2/2\sigma^2)$

- broadening of the emission spectrum ⇒ stronger constraint
- broadening of the mass distribution ⇒ larger DM total density ⇒ weaker constraint



Arbey et al. [arXiv:1906.04750]

Hawking radiation of spin-2 gravitons



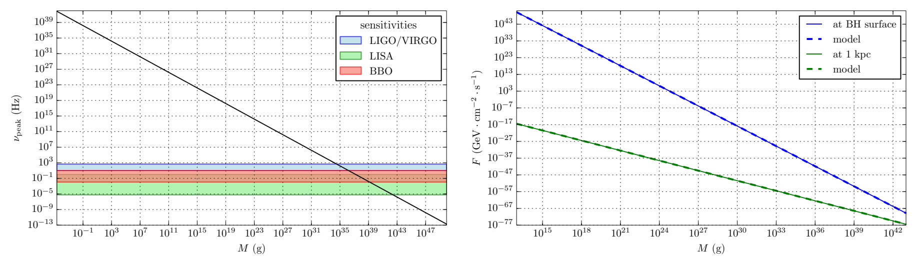
All particles can be emitted by a black hole!

Including gravitons / gravitational waves...

Hawking gravitational waves and detection

Emission of gravitational waves by BHs

Preliminary



Supermassive BHs emit at frequencies of LIGO-VIRGO/LISA/BBO

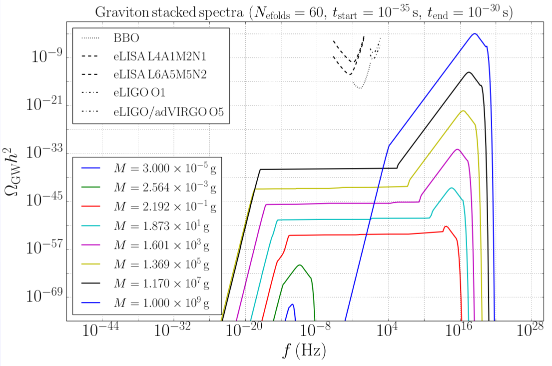
Unfortunately the fluxes of such heavy BHs are too small!

Gravitational waves from very primordial BHs

Gravitational waves emitted by very light PBH which vanished before or after inflation

→ cosmological background of gravitational waves

Preliminary

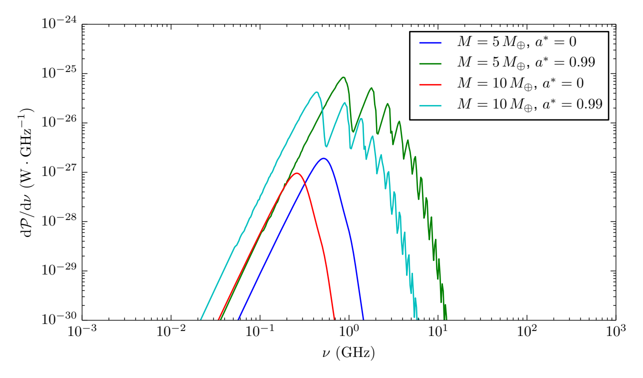


Discovering gravitational waves emitted via Hawking radiation would validate the existence of the graviton!

A black hole in the Solar System?

Anomalous orbits of Trans-Neptunian Objects (TNOs) and excess in microlensing events $\rightarrow$ undiscovered Planet 9 at distance 450 – 700 AU and with mass 5 – 10 $M_\oplus$?

Maybe a primordial black hole (see Scholtz & Unwin 1909.11090)!

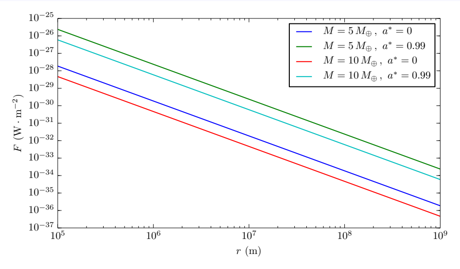


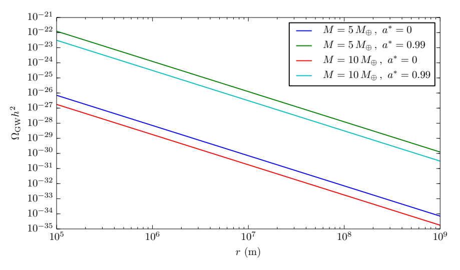
AA, J. Auffinger, 2006.02944

Hawking radiation emitted at the GHz frequency

Towards Planet 9...

Hawking radiation too weak to be seen from Earth





AA, J. Auffinger, 2006.02944

$\rightarrow$ need to send a probe in orbit to study the emitted radio waves (and why not gravitational waves)

($\rightarrow$ Breakthrough Starshot project, proof-of-concept for a fleet of light sail spacecrafts)

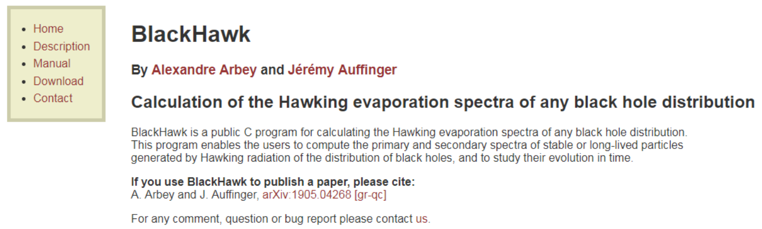
BlackHawk

Public C code computing Hawking radiation:

- Schwarzschild & Kerr PBHs
- primary spectra of all Standard Model fundamental particles
- secondary spectra of stable particles (hadronization with PYTHIA or HERWIG)
- extended mass and spin functions
- time evolution of the PBHs

Download: http://blackhawk.hepforge.org

Manual: arXiv:1905.04268, Eur.Phys.J. C79, 693



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BlackHawk

By Alexandre Arbey and Jérémy Auffinger

Calculation of the Hawking evaporation spectra of any black hole distribution

BlackHawk is a public C program for calculating the Hawking evaporation spectra of any black hole distribution. This program enables the users to compute the primary and secondary spectra of stable or long-lived particles generated by Hawking radiation of the distribution of black holes, and to study their evolution in time.

If you use BlackHawk to publish a paper, please cite:
A. Arbey and J. Auffinger, arXiv:1905.04268 [gr-qc]

For any comment, question or bug report please contact us.

Conclusions

Main results

- Study of the evolution of Kerr PBHs and constraints from IGRB
- Extension to more realistic broad PBH mass functions
- Still open window from planet-mass BHs as dark matter
- Does Planet 9 exist and is it a PBH?
- Public code `BlackHawk` to compute Hawking radiation

Perspectives

- Closing the remaining PBH mass windows for all dark matter into PBHs?
- Primordial BH / Astrophysical BH discrimination using GW events?
- Constraints from extrasolar planet searches?
- Other constraints...

Backup

(in the natural unit system with $c = \hbar = k_B = 1$)

Schwarzschild metric for a static compact object of mass M

$$d\tau^2 = \left(1 - \frac{2GM}{r}\right) dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

One defines the Schwarzschild radius: $R_s = 2GM$.
If the mass M is completely within $r < R_s$, the radius $r = R_s$ constitutes a horizon.

$\longrightarrow$ Black Hole!

Kerr metric for a static compact object of mass M and angular momentum J

$$\begin{aligned} d\tau^2 &= (dt - a\sin^2\theta d\phi)^2 \frac{\Delta}{\Sigma} - \left(\frac{dr^2}{\Delta} + d\theta^2\right)\Sigma \\ &\quad - ((r^2 + a^2)d\phi - a dt)^2 \frac{\sin^2\theta}{\Sigma} \end{aligned}$$

$a = J/M$, $\Sigma = r^2 + a^2\cos^2\theta$, $\Delta = r^2 - R_s r + a^2$, $R_s = 2GM$

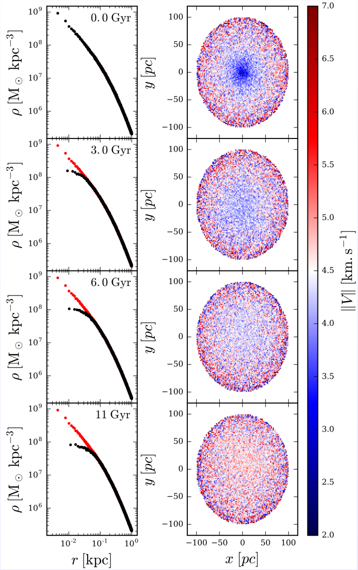
The horizon exists but is deformed and flattened $\longrightarrow$ Kerr (Rotating) Black Hole!

In presence of heavy PBHs, possible transition from cusp to core

On the right: N-body simulation of dwarf galaxy with 10^7 $M_\odot$ halo made of 50% of dark matter in the form of 100 $M_\odot$ PBHs and 50% of 1 $M_\odot$ DM particles.
From Boldrini et al. [1909.07395, MNRAS 492 (2020) 5218].

Gravitational heating by heavy PBHs:

- Dynamical friction of DM particles on PBHs
- Two body relaxation between PBHs



Kerr Hawking radiation equations

Kerr metric

$$\mathrm{d}s^2 = \left(1 - \frac{2Mr}{\Sigma^2}\right)\mathrm{d}t^2 + \frac{4a^* M^2 r \sin(\theta)^2}{\Sigma^2}\mathrm{d}t\mathrm{d}\phi - \frac{\Sigma^2}{\Delta}\mathrm{d}r^2$$
$$- \Sigma^2 \mathrm{d}\theta^2 - \left(r^2 + (a^*)^2 M^2 + \frac{2(a^*)^2 M^3 r \sin(\theta)^2}{\Sigma^2}\right)\sin(\theta)^2 \mathrm{d}\phi^2$$

$\Sigma \equiv r^2 + (a^*)^2 M^2 \cos(\theta)^2$ and $\Delta \equiv r^2 - 2Mr + (a^*)^2 M^2$

Equations of motion in free space

$$\mathrm{Dirac:}\ (i\partial\!\!\!/ - \mu)\psi = 0\ \mathrm{(fermions)}$$
$$\mathrm{Proca:}\ (\Box + \mu^2)\phi = 0\ \mathrm{(bosons)}$$

μ = rest mass

Kerr Hawking radiation equations

Teukolsky radial equation

$$\frac{1}{\Delta^s}\frac{\mathrm{d}}{\mathrm{d}r}\left(\Delta^{s+1}\frac{\mathrm{d}R}{\mathrm{d}r}\right)+\left(\frac{K^2+2is(r-M)K}{\Delta}-4isEr-\lambda_{slm}-\mu^2r^2\right)R=0$$

R radial component of ψ/ϕ

$K\equiv(r^2+a^2)E+a\,m$, $s=$ spin, $l=$ angular momentum and $m=$ projection

Transformation into a Schödinger equation

Change $\psi/\phi\longrightarrow Z$ and $r\longrightarrow r^*$ (generalized Eddington - Finkelstein coordinate)
(Chandrasekhar & Detweiler 1970s)

$$\frac{\mathrm{d}^2Z}{\mathrm{d}r^{*2}}+(E^2-V(r^*))Z=0 \tag{1}$$

Solved with purely outgoing solution $Z \underset{r^*\to-\infty}{\longrightarrow} e^{-iEr^*}$

Transmission coefficient $\Gamma\equiv|Z_{\rm out}^{+\infty}/Z_{\rm out}^{\rm horizon}|^2$

Chandrasekhar potentials

$$V_0(r) = \frac{\Delta}{\rho^4}\left(\lambda_{0\,lm} + \frac{\Delta + 2r(r-M)}{\rho^2} - \frac{3r^2\Delta}{\rho^4}\right)$$

$$V_{1/2,\pm}(r) = (\lambda_{1/2\,lm}+1)\frac{\Delta}{\rho^4} \mp \frac{\sqrt{(\lambda_{1/2,l,m}+1)\Delta}}{\rho^4}\left((r-M) - \frac{2r\Delta}{\rho^2}\right)$$

$$V_{1,\pm}(r) = \frac{\Delta}{\rho^4}\left((\lambda_{1\,lm}+2) - \alpha^2\frac{\Delta}{\rho^4} \mp i\alpha\rho^2\frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{\Delta}{\rho^4}\right)\right)$$

$$V_2(r) = \frac{\Delta}{\rho^8}\left(q - \frac{\rho^2}{(q-\beta\Delta)^2}\left((q-\beta\Delta)\left(\rho^2\Delta q'' - 2\rho^2 q - 2r(q'\Delta - q\Delta')\right) + \rho^2(\kappa\rho^2 - q' + \beta\Delta')(q'\Delta - q\Delta')\right)\right)$$

$\rho^2 \equiv r^2 + \alpha^2$ and $\alpha^2 \equiv a^2 + am/E$

$$q(r) = \nu\rho^4 + 3\rho^2(r^2 - a^2) - 3r^2\Delta$$

$$q'(r) = r\left((4\nu+6)\rho^2 - 6(r^2 - 3Mr + 2a^2)\right)$$

$$q''(r) = (4\nu+6)\rho^2 + 8\nu r^2 - 6r^2 + 36Mr - 12a^2$$

$$\beta_\pm = \pm 3\alpha^2$$

$$\kappa_\pm = \pm\sqrt{36M^2 - 2\nu(\alpha^2(5\nu+6) - 12a^2) + 2\beta\nu(\nu+2)}$$

Evolution parameters

Page parameters (Page 1976)

$$f(M,a^*) \equiv -M^2\frac{\mathrm{d}M}{\mathrm{d}t} = M^2\int_0^{+\infty}\sum_{\mathrm{dof.}}\frac{E}{2\pi}\frac{\Gamma(E,M,a^*)}{e^{E'/T}\pm 1}\mathrm{d}E$$

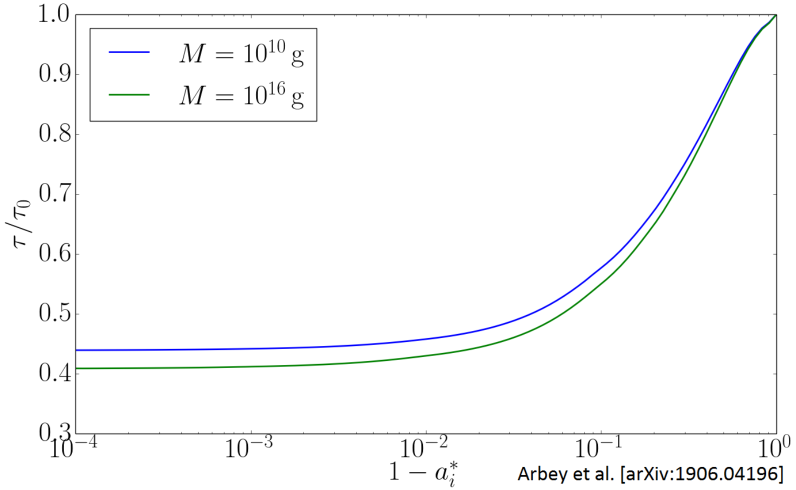
$$g(M,a^*) \equiv -\frac{M}{a^*}\frac{\mathrm{d}J}{\mathrm{d}t} = \frac{M}{a^*}\int_0^{+\infty}\sum_{\mathrm{dof.}}\frac{m}{2\pi}\frac{\Gamma(E,M,a^*)}{e^{E'/T}\pm 1}\mathrm{d}E$$

Evolution equations (Page 1976)

$$\frac{\mathrm{d}M}{\mathrm{d}t} = -\frac{f(M,a^*)}{M^2}$$

$$\frac{\mathrm{d}a^*}{\mathrm{d}t} = \frac{a^*(2f(M,a^*)-g(M,a^*))}{M^3}$$

Decrease of BH lifetime τ for increasing initial spin a_i^*, compared to the Schwarzschild case (τ_0)



Definition

$$\frac{\mathrm{d}n}{\mathrm{d}M} = \frac{A}{\sqrt{2\pi}\sigma M} \exp\left(-\frac{(\log(M/M_*))^2}{2\sigma^2}\right)$$

M^* = central mass, σ = width (dimensionless)

Log-normal distributions (normalized to unity, $M^* = 3 \times 10^{15}$ g)