Electroweak monopoles and the electroweak baryogenesis

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What is the next energy scale to be probed?

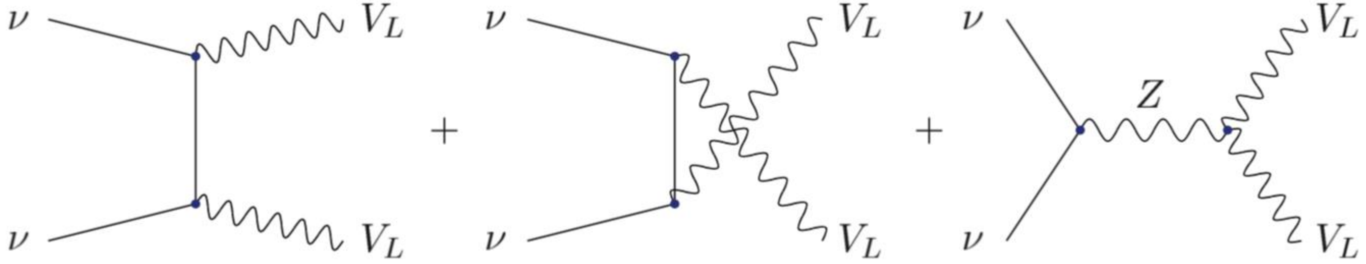
- Neutrino masses - robust evidence from particle physics (neutrino oscillation) experiments

Add neutrino mass to the SM Lagrangian (EW gauge invariance is still OK, but nonlinear):

$$\mathcal{L}_\nu = -\frac{1}{2} m_\nu \nu_L^T C \nu_L + h.c \equiv -\frac{1}{2} m_\nu \left[L^T \epsilon \Sigma\right] C \left[\Sigma^T \epsilon L\right]$$

$$L = (\nu, \ell),\ \Sigma = \exp\{i\sigma^a \pi^a(x)\}(0,1)$$

Consider in this theory neutrino scattering off longitudinal EW bosons:



Perturbative unitarity implies:

$$\boxed{\Lambda \lesssim \frac{4}{\alpha_2} \cdot \frac{M_W^2}{m_\nu} \sim 10^{11}\ \text{GeV}}$$

Maltoni, Niczyporuk, and Willenbrock, 01’

What is the next energy scale to be probed?

- Dark Matter – robust, but only observed in gravitational interactions

Assuming non-relativistic DM is produced thermally via weak-strength scatterings with SM particles, we arrive at the 'WIMP miracle':

$$\Omega_X h^2 \sim \frac{3 \cdot 10^{-27} \mathrm{cm}^3/\mathrm{sec}}{\langle \sigma v_{\mathrm{rel}} \rangle}$$

Cross section is constrained from perturbative unitarity:

$$\sigma_J \leq \pi(2J+1)/p_i^2 \approx 16\pi(2J+1)/(m_X^2 v_{\mathrm{rel}}) \Longrightarrow m_X^2 \leq 16\pi/(\sigma_{J=0} v_{\mathrm{rel}}), \ [v_{\mathrm{rel}} \approx 1/4]$$

$$\boxed{\Lambda \sim m_X \lesssim 100 \ \mathrm{TeV}}$$

Griest and Kamionkowski, 90'

What is the next energy scale to be probed?

- EW vacuum has topologically non-trivial structure [SU(2) sector].

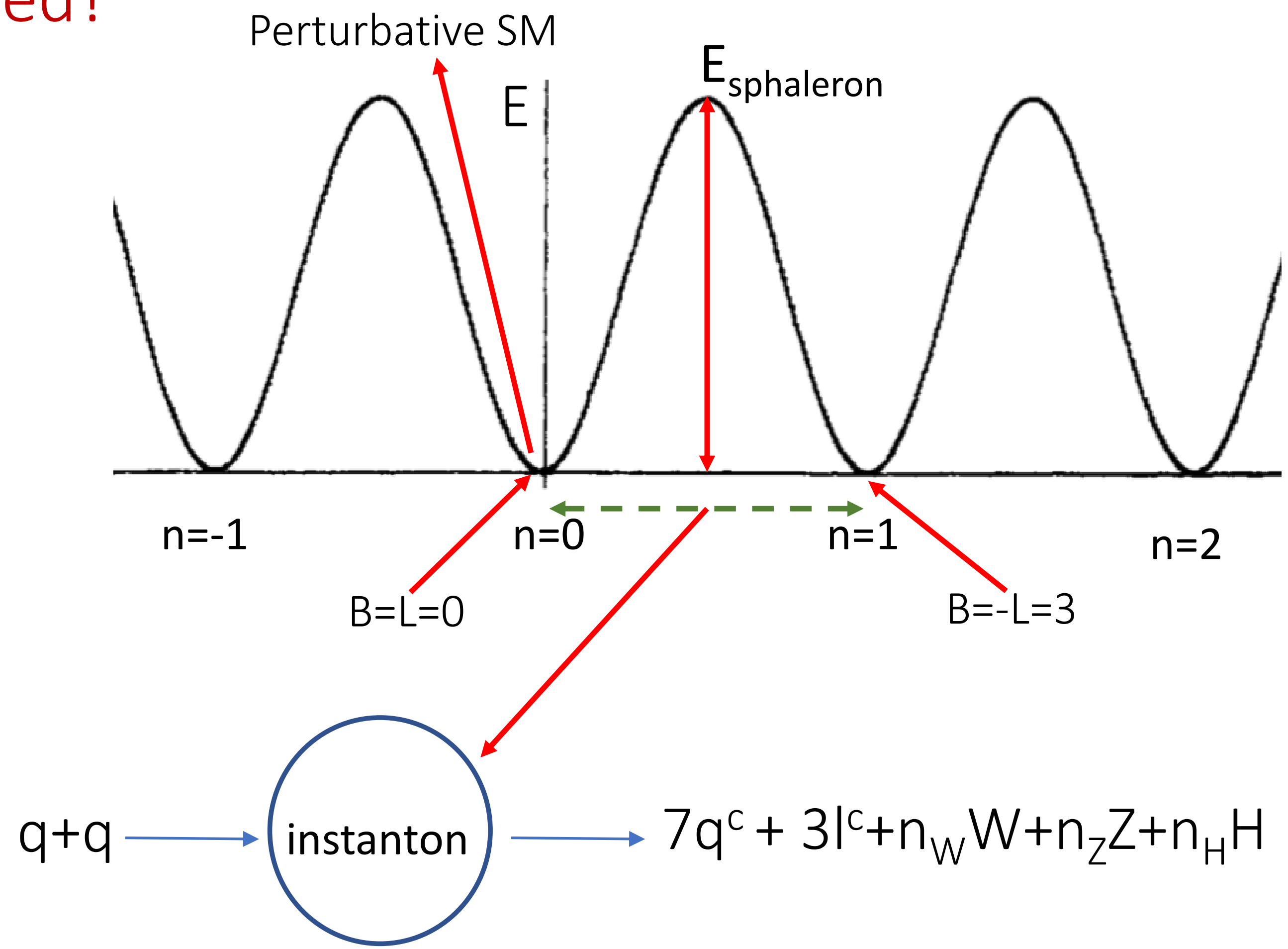
- Transition between vacua change B and L by 3 units: $\Delta B=-\Delta L=3\Delta n$ (quantum anomaly); $\Delta(B-L)=0$.

- EW instantons are classical solution of Euclidean e.o.m., with action, e.g., for $\Delta n=1$,

$$S_{\text{inst.}} = \frac{2\pi}{\alpha_2}$$

(multiple of W,Z, H particles in a coherent state)

- describe vacuum-to-vacuum transitions)



$$\mathcal{A}_{\text{inst.}} \sim \exp\{-S_{\text{inst}}\} \approx 10^{-80}$$

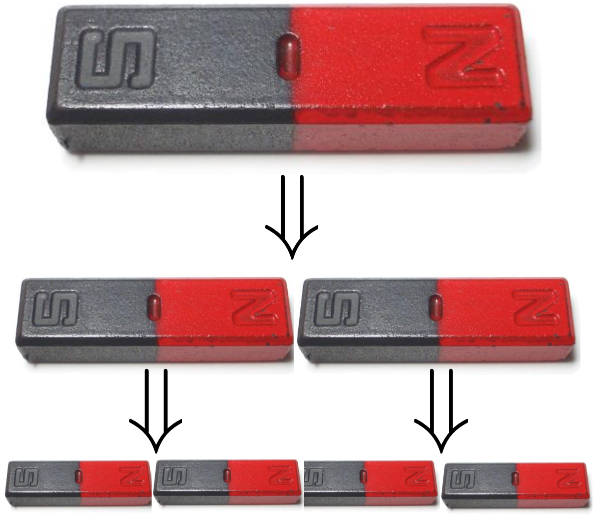
$$\boxed{\Lambda \sim E_{\text{sphaleron}} \sim \frac{4M_W}{\alpha_2} \approx 10 \ TeV}$$

Maxwell's Electromagnetism – the triumph of symmetry in physics

$$\vec{\nabla}\cdot\vec{E} = \rho_{\mathrm{E}}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

There are no isolated magnetic charges



$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\times\vec{B} = \frac{\partial\vec{E}}{\partial t} + \vec{j}_{\mathrm{E}}$$

Maxwell added this term relying on mathematical consistency and aesthetic considerations (dual-symmetric to the Faraday's (magnetic) induction law) => extraordinary progress in fundamental physics (special/general relativity,...) and technologies (electromagnetic radiation,...)

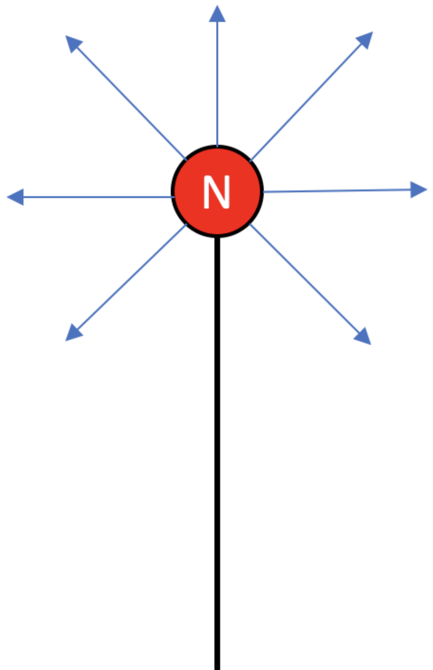
The Dirac magnetic monopole (1931)

$$\vec{\nabla} \cdot \vec{E} = \rho_{\mathrm{E}}$$
$$\vec{\nabla} \cdot \vec{B} = \rho_{\mathrm{M}}$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \vec{j}_{\mathrm{M}}$$
$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{j}_{\mathrm{E}}$$

The equations become invariant under the duality map:

$$\vec{E} \to \vec{B},\ \ \vec{B} \to -\vec{E},$$
$$\rho_{\mathrm{E}} \to \rho_{\mathrm{M}},\ \ \rho_{\mathrm{M}} \to -\rho_{\mathrm{E}},$$
$$\vec{j}_{\mathrm{E}} \to -\vec{j}_{\mathrm{M}},\ \ \vec{j}_{\mathrm{M}} \to \vec{j}_{\mathrm{E}}.$$

Montonen-Olive duality 77’

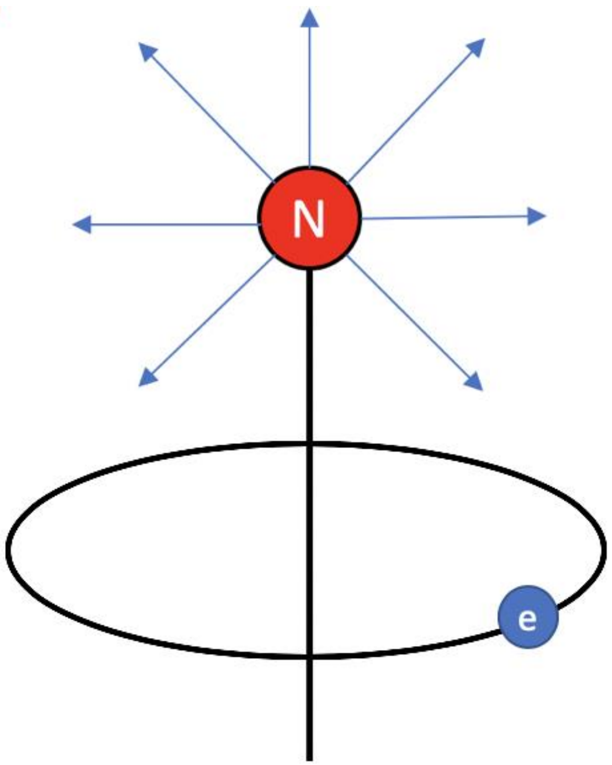


$$\vec{B} = \nabla \times \vec{A},\ \ \vec{\nabla} \cdot \vec{B} = \vec{\nabla} \cdot \left(\vec{\nabla} \times \vec{A}\right) \neq 0 \implies \vec{A}(r) = \frac{g}{4\pi\, r}\, \frac{\vec{r} \times \vec{n}}{r - \vec{r} \cdot \vec{n}}$$

- Dirac string – singularity along $\vec{n}$.
- Classically the singularity is not physical, because of gauge invariance

$$A'_\mu = A_\mu + \partial_\mu \alpha$$

The Dirac magnetic monopole (1931)



The Dirac quantisation condition: In quantum mechanics, to make the Dirac string undetectable (aka unphysical) an extra condition must be satisfied:

$$eg = 2\pi n, \;\; n \in \mathbb{Z}$$

“The quantization of electricity is one of the most fundamental and striking features of atomic physics, and there seems to be no explanation for it apart from the theory of poles. This provides some grounds for believing in the existence of these poles.” P. Dirac, 1948

The static energy (aka the rest mass) of the Dirac monopole is divergent:

- Magnetic Coulomb field $\vec{B} = \frac{g}{4\pi}\frac{\vec{r}}{r^3}$, monopole charge is localized at a point.

$$E_{\mathrm{monopole}} = \frac{1}{2}\int \vec{B}^2 \, \mathrm{d}^3\mathrm{x} \sim g^2\Lambda \sim \frac{\Lambda}{\alpha}$$

The Maxwell theory has to be modified at small distance scales $\sim 1/\Lambda$!

‘t Hooft-Polyakov monopole (1974)

Assume the Maxwell electromagnetism (U(1) gauge theory) at small distance scales is described by more complicated SU(2) gauge theory which is spontaneously broken by the triplet scalar field:

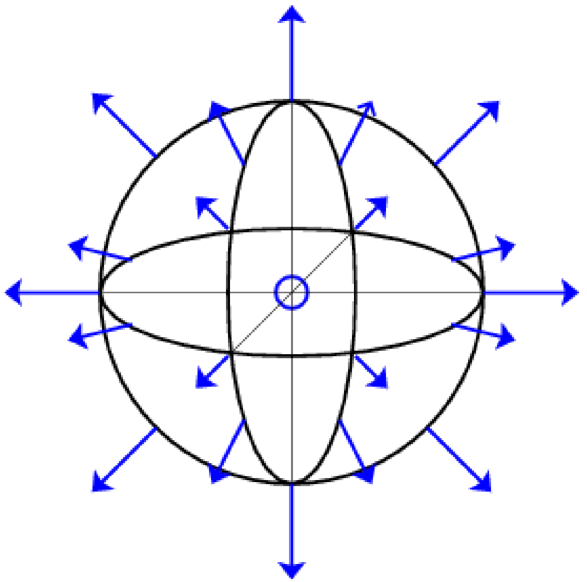
$$\phi_1^2 + \phi_2^2 + \phi_3^2 = v^2 \implies SU(2) \to U(1)$$

- Two out of three SU(2) gauge fields develop masses $M_W = g_2 v/2$. The corresponding charges at distances $>> 1/M_W$ are screened and the observer probes only U(1) charges

- Smooth solutions to field equations

$$\phi_i = \rho(r)\frac{\vec{r}}{r} \,,$$

$$A^i_\mu = \frac{f(r)-1}{g_2\ r^2}\,(\vec{r}\times\partial_\mu\vec{r})^i$$



- Finite monopole mass:

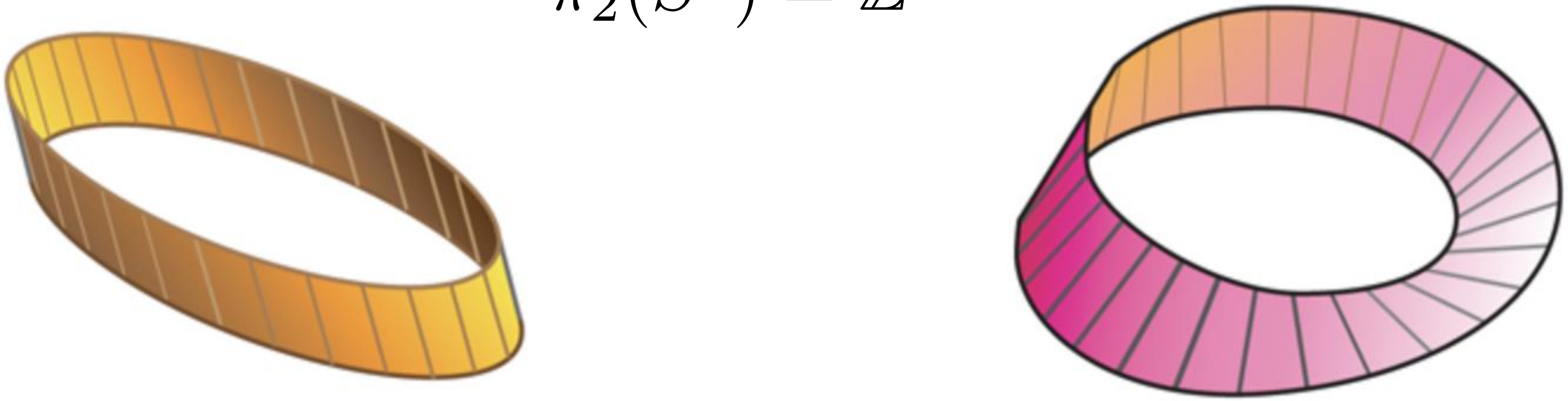
$$E_{\text{monopole}} \simeq \frac{4\pi M_W}{g_2^2}$$

‘t Hooft-Polyakov monopole (1974)

‘t Hooft-Polyakov monopole is stable because of the topological properties:

- Vacuum manifold $\phi_1^2 + \phi_2^2 + \phi_3^2 = v^2$ describes 2-sphere;
- The solutions approach their vacuum configurations at spatial infinity, which is also 2-sphere.
- The map S^2 -> S^2 provided by the monopole solution is topologically non-trivial:

$$\pi_2(S^2) = \mathbb{Z}$$



- Topological monopoles are not optional, they are mandatory!

The electroweak monopole

- The Standard Model is the established physics down the distance scales ~ 10^{-17}cm.
- Maxwell's electromagnetism is unified together with weak forces into the electroweak theory. The theory exhibits spontaneous symmetry breaking via the Higgs mechanism:

$$SU(2) \times U(1)_Y \to U(1)_{\text{EM}}$$

"The first is the electroweak theory, with SU(2) × U(1) broken to U(1). There are no topologically nontrivial configurations of the Higgs field, and hence no topologically stable monopole solutions." - D. Mealsted & E.J. Weinberg, PDG 2017.

This is plain wrong!

The electroweak monopole

[Arunasalam, Collison, AK, 18’]

- Standard (and incorrect) argument against electroweak monopoles:

$$H^\dagger H \equiv \phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 \overset{r\to\infty}{=} \rho_0^2$$

Map of S^2 (boundary at spatial infinity) onto the vacuum manifold S^3 . The map is trivial, hence topological (‘t Hooft-Polyakov) monopoles do not exist.

- However, ϕ_i can be singular (gauge d.o.f.). In that case the vacuum manifold may not be S^3.

- Consider an ansatz:

Cho and Maison, 96’

$$H = \frac{1}{\sqrt{2}}\rho(r)\zeta, \quad \zeta = i\begin{pmatrix}\sin(\theta/2)e^{-i\phi} \\ -\cos(\theta/2)\end{pmatrix},$$

$$\mathbf{A}_\mu = -\frac{1}{g_2}A(r)\partial_\mu t\hat{r} + \frac{1}{g_2}(f(r)-1)\hat{r}\times\partial_\mu\hat{r},$$

$$B_\mu = -\frac{1}{g_1}B(r)\partial_\mu t - \frac{1}{g_1}(1-\cos\theta)\partial_\mu\phi.$$

Singular at $\theta = \pi/2$

The electroweak monopole

- Denote the components of doublet Higgs as: $z_1 \equiv \phi_1 + i\phi_2,\, z_2 \equiv \phi_3 + i\phi_4$

- Are defined up to hypercharge gauge transformations: $(z_1, z_2)^T \equiv (\lambda z_1, \lambda z_2)^T,\ \lambda \in U(1)_Y$:

Hence could be viewed as coordinates on a complex plane C^2 (modulo singularities).

- Remove singularities by using the gauge freedom and defining two monopole solutions on two different patches of space:

$$H_N = i\frac{\rho(r)}{\sqrt{2}}\begin{pmatrix}\sin(\theta/2)e^{-i\phi}\\ -\cos(\theta/2)\end{pmatrix},\quad B_\phi^N = -\frac{1}{g'}\frac{1-\cos\theta}{r\sin\theta}\quad \text{for } 0 \le \theta \le \pi/2, \text{ and}$$

$$H_S = i\frac{\rho(r)}{\sqrt{2}}\begin{pmatrix}\sin(\theta/2)\\ -\cos(\theta/2)e^{i\phi}\end{pmatrix},\quad B_\phi^S = \frac{1}{g'}\frac{1+\cos\theta}{r\sin\theta}\quad \text{for } \pi/2 \le \theta \le \pi.$$

- At the equator ($\theta = \pi/2$) the transition function $e^{i\phi}$ is a holomorphic function => (z_1, z_2) actually span a projective complex plane CP^1 .
- Hence, monopole solution is topologically nontrivial: $\pi_2(CP^1) = \pi_2(S^2) = \mathbb{Z}$

The electroweak monopole

- Considering, two monopole solutions on the whole space (with opposite magnetic charges), one gets monopole-antimonopole bound state, which actually is a sphaleron!

- Monopole – particle scattering is known unsuppressed (Rubakov 81'; Callan 82'). By crossing symmetry the process of production of monopole-antimonopole pair in two-particle collision must not be suppressed either. Monopole-antimonopole pair then can form sphaleron:

$$q + q \to M + M^c \to 7q^c + 3l^c + n_W W + n_Z Z + n_H H$$

- EW monopoles inevitably introduce new CP violating phase (Witten effect):

$$\mathcal{L}_\theta = \theta_2 F^a_{\mu\nu}\tilde{F}^{a\mu\nu} + \theta_1 B_{\mu\nu}\tilde{B}^{\mu\nu} \implies \mathcal{L}_\theta = \theta_{ew} F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\ , \quad \theta_{ew} = \theta_2 - \theta_1$$

- Contribute to EDM of known particles
- Successful electroweak baryogenesis scenario [Arunasalam and AK, 17']

The electroweak monopole

- The mass of EW monopoles is divergent. Within the string theory inspired Born-Infeld-type extension of the Standard Model [Arunasalam, AK, 17’]:

$$E_{\rm monopole} \simeq 77.1\sqrt{\beta} + 2.8 \text{ TeV}, \quad \sqrt{\beta} \text{ is the Born-Infeld mass parameter}$$

- PVLAS measurements of nonlinearity in light propagation:

$$\sqrt{\beta} \gtrsim 5.0 \cdot 10^{-4} \text{ GeV} \implies E_{\rm monopole} \gtrsim 2.8 \text{ TeV}$$

- Constraints from the light-by-light scattering data extracted from heavy ion collisions at LHC:

$$\sqrt{\beta} \gtrsim 88 \text{ GeV} \implies E_{\rm monopole} \gtrsim 9.6 \text{ TeV}$$ Ellis, Mavromatos, You, 17’

- LHC is not capable to produce EW monopoles. Higher energy collider or search in cosmic rays!

- ‘Sweet spot’ value for EW monopole mass ~ 10^7 GeV (baryogenesis)

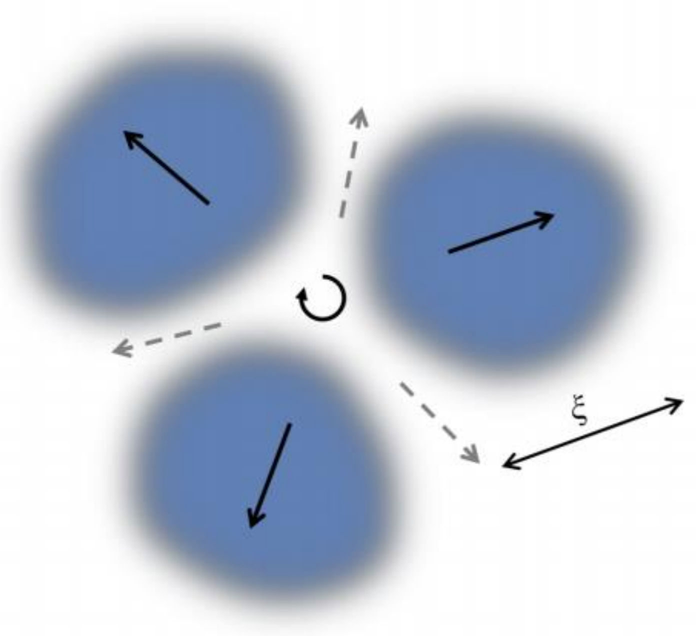
EW baryogenesis with EW monopoles

- In 1967, Andrei Sakharov proposed three conditions for baryogenesis to occur:
  1. Baryon number violation
  2. C and CP violation
  3. Departure from thermal equilibrium- 1st order EWPT.

EW baryogenesis with EW monopoles

- The EW monopoles are produced during the electroweak phase transition when the temperature of the universe was T_{EW}~ 100 GeV (~10^{-32} s after Big Bang) via the Kibble mechanism.

- The estimated monopole number density today:



$$n_M \sim \frac{1}{\alpha_g^3}\left(\frac{M}{M_{\rm Pl}}\right) T_{\rm CMB}^3 \sim 10^{-26}\left(\frac{M}{10^7\ {\rm GeV}}\right)\ {\rm cm}^{-3}.$$

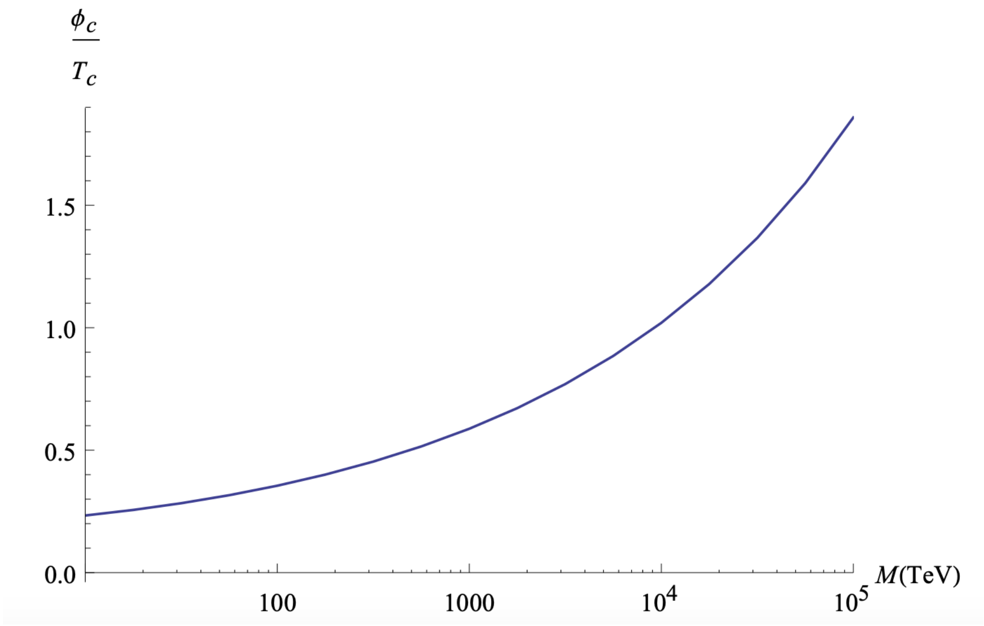
$\alpha_g = 1/2\alpha \simeq 68.5$ - the magnetic ‘fine structure’ constant

EW baryogenesis with EW monopoles

- Production of EW monopoles during the EW phase transition may drive it to be 1st order;

- Energetics at the critical temperature:

$$V(0, T_c) = V(\phi_c, T_c) + E_{monopole}$$



- For monopoles with mass $M > 0.9 \cdot 10^4$ TeV $\phi_c/T_c > 1$, hence sphalerons are ineffective in the broken phase (no wash out of generated baryon asymmetry)

- Constraints from BBN: $M < 2.3 \cdot 10^4$ TeV

New source of CP violation in the Standard Model

- Consider the θ- terms:

$$\mathcal{L}_\theta = \theta_2 F^a_{\mu\nu}\tilde{F}^{a\mu\nu} + \theta_1 B_{\mu\nu}\tilde{B}^{\mu\nu}$$

- In the usual case:
  - hypercharge sector is topologically trivial, and hence, θ_1 is unphysical
  - θ_2 can be rotated away by a $B+L$-rotation of quarks and leptons.
  - no CP-violation

- With electroweak monopoles:
  - Monopoles gain an electric charge proportional to θ_{EM} through the WItten effect
  - Supports θ_1
  - Only one can be rotated away
  - a new source of CP violation

New source of B violation in the Standard Model

$$\mathcal{L}_\theta = \theta_{ew} F^a_{\mu\nu} \tilde{F}^{a\mu\nu} \ ,$$

- Topologically inequivalent vacuum configurations related by large gauge transformations $g \in SU(2)_L$ give rise to the θ_{ew}-vacuum structure.

$$|M, \theta_{ew}\rangle = \sum_{n=-\infty}^{n=+\infty} e^{in\theta_{ew}} (U[g])^n |M, 0\rangle \ .$$

- monopole-antimopole pair that carries $\Delta n = 1$ topological charge, would annihilate into 9 quarks and 3 leptons, giving rise to $\Delta B = \Delta L = 3$.
- not suppressed even at zero temperature (Callan, 1982) (Rubakov, 1981)

Baryon asymmetry

$$\frac{d\bar{n}_B}{dt} = -\kappa\theta \frac{dn_M}{dt}$$

- $\bar{n}_B$ is the difference in the number densities of matter and antimatter
- κ describes the asymmetry generated in each collision
- for monopoles, $n_{M0} >> n_{Mf}$.
- Hence,

$$\bar{n}_B \approx \kappa\theta n_0 = \kappa\theta\alpha_{EM}^3 T_c^3$$

Baryon asymmetry

- The asymmetry parameter, η_B, can now be evaluated:

$$\eta_B = \frac{\bar{n}_B}{s} = \kappa\theta \frac{45\alpha_{\text{EM}}^3 T_c^3}{2\pi^2 g_\star T_f^3}$$

- $1.6 \times 10^{-8}\kappa\theta \leq \eta_B \leq 2.5 \times 10^{-7}\kappa\theta$.
- Empirical values for the asymmetry parameter $\eta_B \approx 10^{-10}$ can be accommodated for with $\kappa\theta_{ew} \sim 10^{-3} - 10^{-2}$.

Conclusion

- Electroweak monopoles are inescapable prediction of the Standard Model;

- There could be extra, yet unaccounted, electroweak CP-violating θ-term

- EW monopoles could be as light as ~ 10 TeV; a collider with E > 20 TeV is needed; spectacular multiparticle signature with B+L – violation

- EW monopoles M ~ 10^7 GeV drive 1st order electroweak phase transition and successful baryogenesis; could be search for in astrophysical experiments

Extra slides

Flux of EW monopoles

- While produced non-relativistic, (not very heavy) EW monopoles are easily accelerated in a galactic magnetic field B ~ 3μG:

$$v_{\rm mag} \sim \begin{cases} c, & M \lesssim 10^{11}\mathrm{GeV} \;, \\ 10^{-3}c\left(\frac{10^{17}\,\mathrm{GeV}}{M}\right)^{1/2}, & M \gtrsim 10^{11}\mathrm{GeV} \;. \end{cases}$$

- The flux of relativistic monopoles:

$$F = \frac{c n_M}{4\pi} \approx 2.3 \cdot 10^{-19} \left(\frac{M}{10^7 \ GeV}\right) \ \mathrm{cm}^{-2} \ \mathrm{sr}^{-1} \ \mathrm{s}^{-1}$$

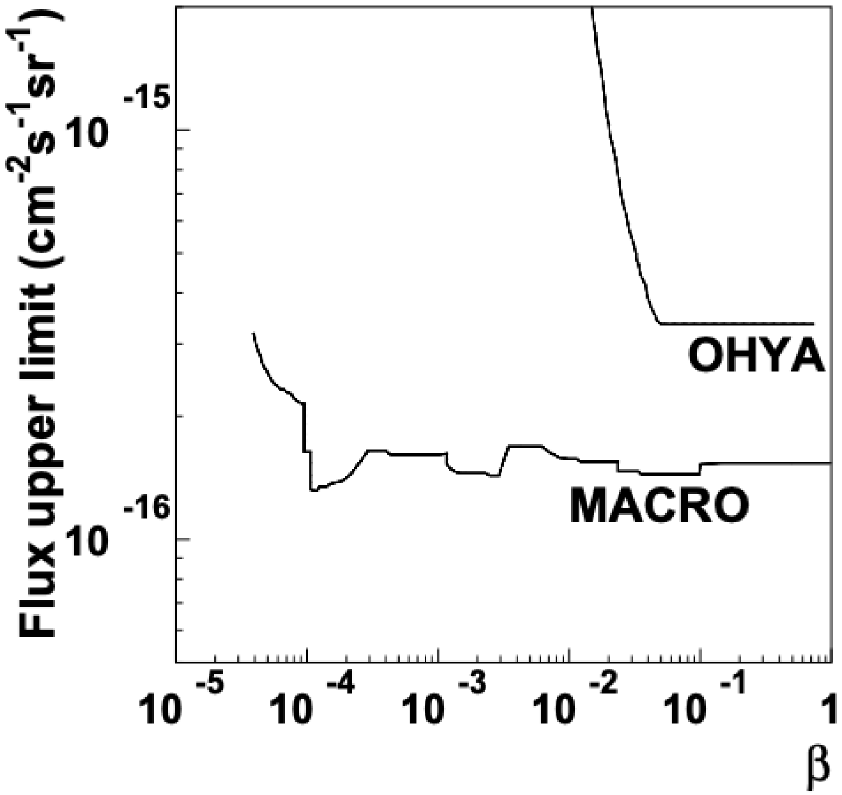
Astrophysic constraints of EW monopoles

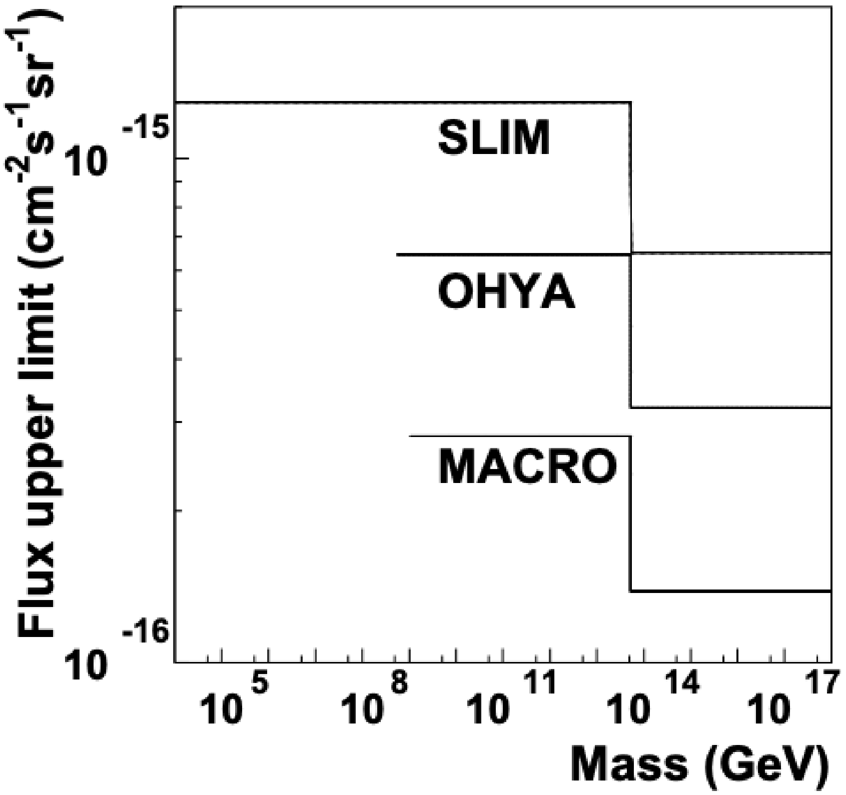
- Constraints from the survival of galactic magnetic field (the Parker bound):

$$F < \begin{cases} 10^{-15}\,\mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{sec}^{-1}\,, & M \lesssim 10^{17}\,\mathrm{GeV}\ , \\ 10^{-15}\left(\dfrac{M}{10^{17}\,\mathrm{GeV}}\right)\mathrm{cm}^{-2}\,\mathrm{sr}^{-1}\,\mathrm{sec}^{-1}\,, & M \gtrsim 10^{17}\,\mathrm{GeV}\ . \end{cases}$$

- EW monopoles do not catalise proton decay (like GUT monopoles). Therefore, bounds from the heating of compact objects does not directly apply. However, they mediate different B+L violating processes, requires careful study.

Summary of constraints





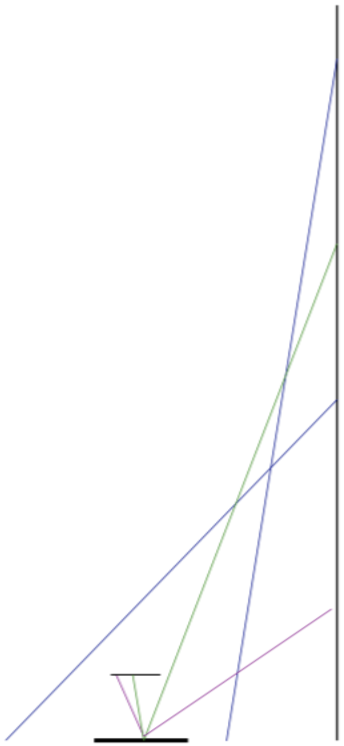
D. Mealsted & E.J. Weinberg, PDG 2017

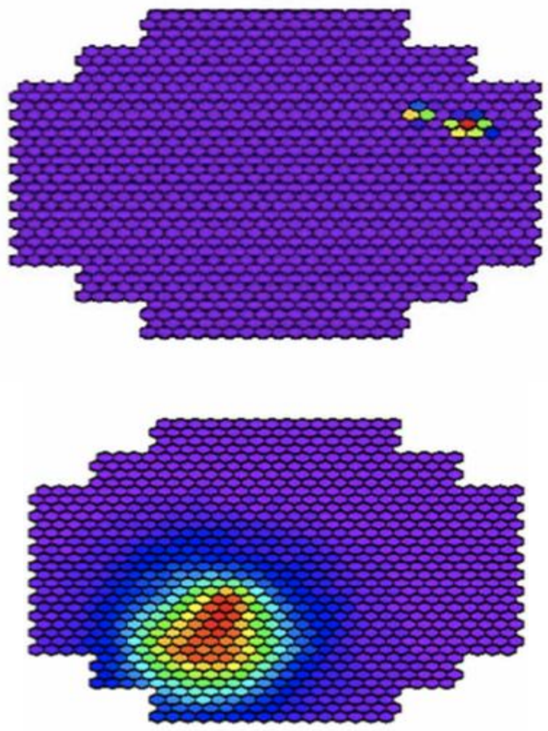
Cherenkov light from relativistic monopoles

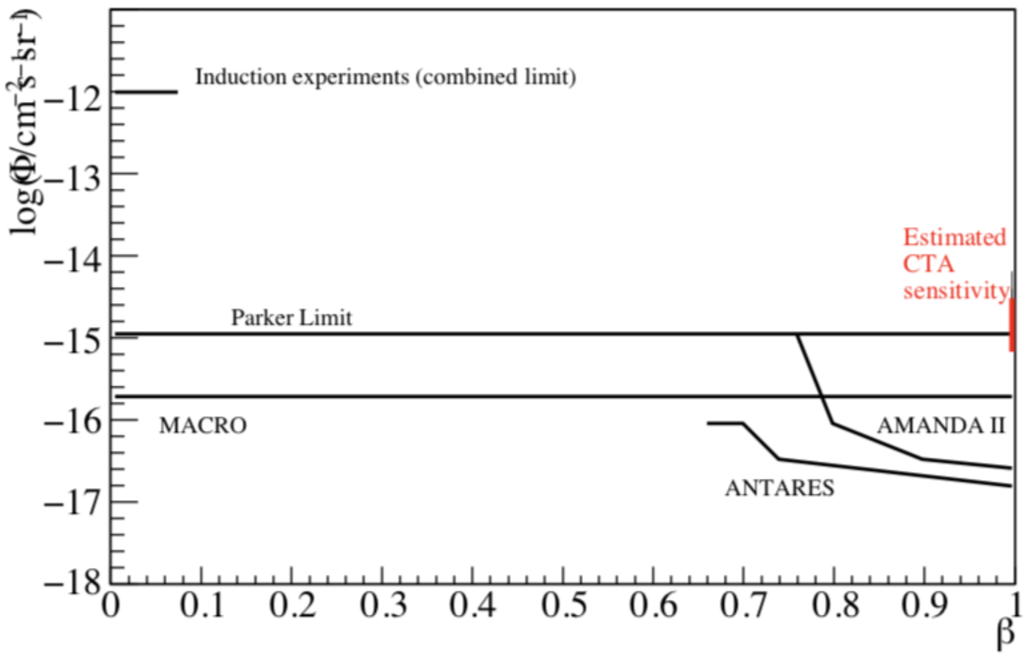
- Because much stronger electromagnetic interactions a relativistic monopole produces

$\sim n^2 \left(\frac{g}{e}\right)^2 = 2n^2\alpha_g^2 \approx 9400$ more photons than e.g., a relativistic muon.

- If no other interactions, very distinct image [Spengler, Schwanke, 11']:







M. Doro et al, 2012

More exotic signatures from EW monopoles

- B+L-violating electroweak scatterings:

$$M + N \to M + 6q^c + 3l^c + \ \text{[Higgses, W's, Z's]}$$

Much more brighter showers than usual hadronic ones.

- More theoretical work required
- Are we throwing them out as a ‘background’?
- A dedicated image cleaning? A dedicated analysis?