Interpretation of LHCb Hidden-Charm Pentaquarks within the Compact Diquark Model

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40th International Conference on High Energy Physics (ICHEP-2020)
Prague, Czech Republic, July 28 – August 4, 2020

based on the papers PLB 793 (2019) 365 [arXiv:1904.00446] & JHEP 10 (2019) 256 [arXiv:1907.06507]

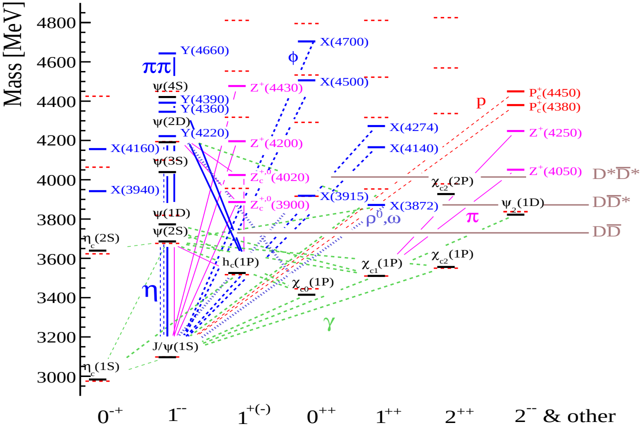
Outline

Introduction

- In 2003, the first exotic hidden-charm state $X(3872)$ was observed by the Belle Collaboration
- This state was confirmed by BaBar, CDF, D0, BESII, and LHCb collaborations
- Soon after, many new other mesons with masses above the $D\bar{D}$ and $B\bar{B}$ thresholds have been observed
- In 2015 LHCb Collaboration found two resonances which have got the interpretation as hidden-charm pentaquarks

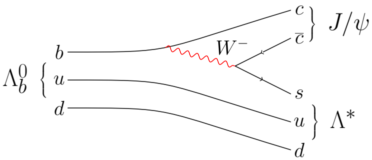
X, Y, Z, P_c and Charmonium States

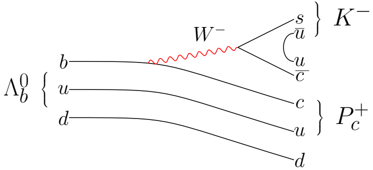
[S. L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Mod. Phys. 90 (2018) 015003]



$\Lambda_b \to p + J/\psi + K^-$ Decay: 2015 Results by LHCb

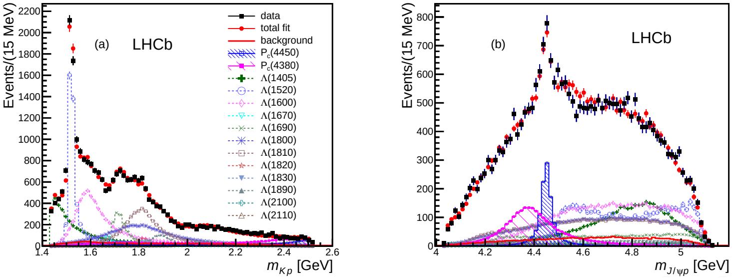
- Pentaquarks were found in three-body weak decays of Λ_b-baryons $\Lambda_b \to p + J/\psi + K^-$ [LHCb, PRL, 2015]
- In addition to non-resonant channel, there are two quasi-two-body decay modes of Λ_b-baryon:
  1. $\Lambda_b \to \Lambda^{(*)} + J/\psi$ decay, where Λ-hyperon or its excitations are produced with the following decay $\Lambda^{(*)} \to p + K^-$
  2. $\Lambda_b \to P_c^+ + K^-$ decay, where P_c^+-pentaquarks were found in the decay mode $P_c^+ \to p + J/\psi$
- Heavy-quark symmetry preserves the spin of light diquark; $\Lambda_b \to \Sigma_c + X$ are highly suppressed [PDG, 2018]





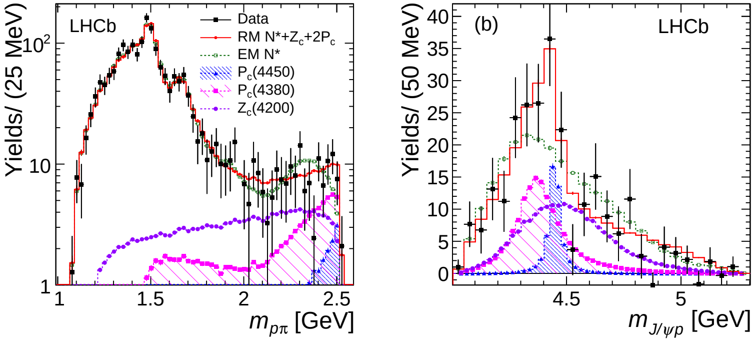
$\Lambda_b \to p + K^- + J/\psi$ Decay: 2015 Results by LHCb

- Two peaks in invariant-mass, $m_{p\,J/\psi} = \sqrt{(p_P + p_{J/\psi})^2}$, distribution were interpreted as hidden-charm pentaquarks
  1. $P_c^+(4380)$: spin-parity $J^P = 3/2^-$ (preferred)
     $M = (4380 \pm 8 \pm 29)$ MeV, $\Gamma = (205 \pm 18 \pm 86)$ MeV
  2. $P_c^+(4450)$: spin-parity $J^P = 5/2^+$ (preferred)
     $M = (4449.8 \pm 1.7 \pm 2.5)$ MeV, $\Gamma = (39 \pm 5 \pm 19)$ MeV
- Assignments $(3/2^+, 5/2^-)$ and $(5/2^+, 3/2^-)$ are possible



LHCb Results on $\Lambda_b \to p + J/\psi + \pi^-$ Decay

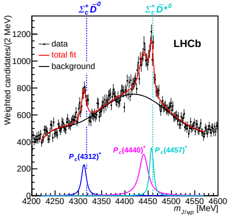
- Evidence of these resonances was also pointed out in the other decay $\Lambda_b \to p + J/\psi + \pi^-$ [LHCb, PRL, 2016]
- Combined significance is calculated to be 3.1σ
- Contributions from pentaquarks are shown as shaded



$\Lambda_b \to p + J/\psi + K^-$ Decay: 2019 Results by LHCb

- Λ_b-baryon decay $\Lambda_b \to p + J/\psi + K^-$ was studied on 9 times more data based on Run 1 and 2 than on Run 1
- Three narrow peaks were observed in $m_{J/\psi\, p}$ distribution

State	Mass [MeV]	Width [MeV]	(95% CL)	$\mathcal{R}$ [%]
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	(< 27)	$0.30 \pm 0.07^{+0.34}_{-0.09}$
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	(< 49)	$1.11 \pm 0.33^{+0.22}_{-0.10}$
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	(< 20)	$0.53 \pm 0.16^{+0.15}_{-0.13}$



- $P_c(4312)$ is a new resonance
- $P_c(4450)$ splits into $P_c(4440)$ and $P_c(4457)$
- $P_c(4380)$ under question
- Spin-parities are unknown yet

Results by D0 & ATLAS Collaborations

D0 Collab. [V. M. Abasov *et al.*, arXiv:1910.11676]

- Analysis is based on 10.4 fb^{-1} of data
- Enhancement in $J/\psi\, p$ invariant mass distribution originated by decays of b-flavored hadrons
- Consistent with a sum of $P_c(4440)^+$ and $P_c(4457)^+$
- Significance is 3.0σ
- No evidence of $P_c(4312)^+$ state
- $R = N(4312)/[N(4440) + N(4457)] < 0.6$ at 95% C.L.

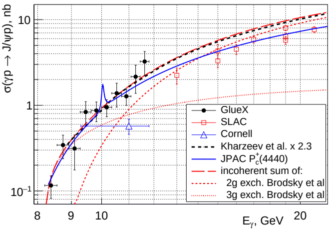
ATLAS Collab. [I. Eletskikh, ATL-PHYS-PROC-2020-007]

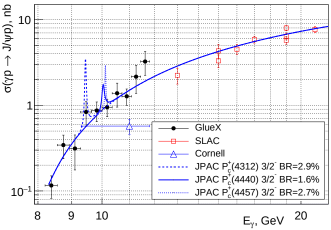
- Based on 4.9 fb^{-1} at 7 TeV and 20.6 fb^{-1} at 8 TeV
- $\Lambda_b \to J/\psi\, p\, K^-$ with large $m_{p\,K^-}$ invariant mass
- Model without pentaquarks is not excluded
- Data prefer model with two or more pentaquarks
- Masses and widths of two $P_c(4380)^+$ and $P_c(4450)^+$ pentaquarks are consistent with those from LHCb
- Data are also compatible with the three narrow LHCb pentaquarks

$\gamma + p \to J/\psi + p$ Scattering: 2019 Results by GlueX

GlueX Collab. [A. Ali *et al.*, PRL 123 (2019) 072001]

- Hall D of Jafferson Lab., data of 2016–2017
- Photon energy $E_\gamma \in [8.2\ \mathrm{GeV}, 11.8\ \mathrm{GeV}]$
- For $J^P = 3/2^-$ $\mathcal{B}(P_c^+ \to J/\psi\, p) < 2.0\%$; consistent with LHCb
- Upper limits on BF do not exclude the molecular model of P_c^+ but are an order of magnitude lower than predictions in hadrocharmonium model





Theoretical Interpretations of 3 Narrow Pentaquarks

- Molecular Picture:
  Open charm-meson and charm-baryon bound states
  - Masses slightly below the $\Sigma_c^+ \bar{D}^{(*)0}$ thresholds:
    - $\Sigma_c^+ \bar{D}^0$: $E_{\text{thr}} = (4317.73 \pm 0.41)$ MeV
    - $\Sigma_c^+ \bar{D}^{*0}$: $E_{\text{thr}} = (4459.9 \pm 0.9)$ MeV
  - S-wave molecular-like states
  - Negative parity $P = (-1)^{L+1}$
- Hadrocharmonium Picture:
  Compact charmonium state inside the proton interior
- Compact Multiquark Picture:
  - Quarks and antiquarks are tightly bound into colorless state
  - Introduction of point-like diquarks and antidiquarks simplifies consideration drastically

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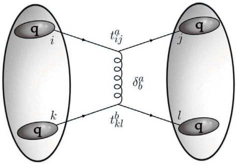
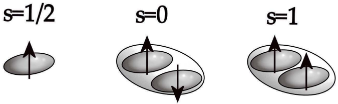
Diquark Model of Hadrons

- Quarks q_i^α and diquarks $\mathcal{Q}_{i\alpha}$ are building blocks of baryons and exotic hadrons
- α is the $SU(3)_C$ index and i is the $SU(3)_F$ index
- Color repres.: $3 \otimes 3 = \bar{3} \oplus 6$; only $\bar{3}$ is attractive

$$t^a_{ij} t^a_{kl} = -\frac{2}{3} \underbrace{(\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj})/2}_{\text{antisymmetric: projects } \bar{3}} + \frac{1}{3} \underbrace{(\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj})/2}_{\text{symmetric: projects } 6}$$





- Interpolating diquark operators for the two spin states

$$\text{Scalar:} \quad 0^+ \quad \mathcal{Q}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}_c^\beta \gamma_5 q_i^\gamma - \bar{q}_{ic}^\beta \gamma_5 c^\gamma\right)$$

$$\text{Axial-Vector:} \quad 1^+ \quad \vec{\mathcal{Q}}_{i\alpha} = \epsilon_{\alpha\beta\gamma}\left(\bar{c}_c^\beta \vec{\gamma} q_i^\gamma + \bar{q}_{ic}^\beta \vec{\gamma} c^\gamma\right)$$

- Colorless combination with the quark results into the baryon

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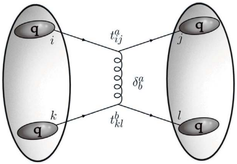

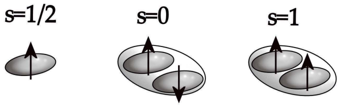

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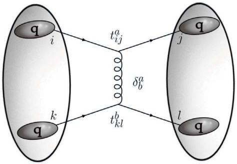

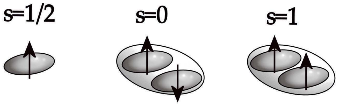

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Diquark Model of Pentaquarks

- Antiquark $\bar{q}_k^{\gamma}$ and two diquarks $\mathcal{Q}_{i\alpha}$ and $\mathcal{Q}'_{j\beta}$ are the building blocks of pentaquarks
- At least, three approaches are suggested for hidden-charm pentaquarks in the compact diquark model
- Heavy triquark — heavy diquark model within the "Dynamical Diquark Model" [R. Lebed, PLB 749 (2015) 454]
- Heavy tetraquark — heavy antiquark model [A. Ali, I. Ahmed, M. J. Aslam, and A. Rehman, PRD 94 (2016) 054001]
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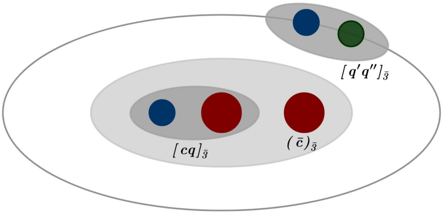
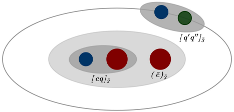
Doubly-Heavy Triquark — Light Diquark Model

- Heavy diquark couples with c-antiquark in the color-triplet doubly-heavy triquark (DHT)
- Light diquark being a color antitriplet makes pentaquark colorless
- DHT is practically static
- Light diquark is “rotating” around triquark
- Light diquark is easier to excite orbitally than constituents inside the DHT



State Vector of Pentaquark

$|S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J$



- Charmed antiquark spin $S_{\bar{c}} = 1/2$; implicitly assumed
- Heavy diquark spin $S_{hd} = 0, 1$
- Heavy triquark spin $S_t = 1/2, 3/2$
- Orbital angular momentum of heavy triquark L_t; assumed *S*-wave for triquark
- Light diquark spin $S_{ld} = 0, 1$
- Orbital angular momentum of light diquark L_{ld}
- Pentaquark spin $S = 1/2, 3/2, 5/2$
- Orbital angular momentum of pentaquark L
- Total angular momentum of pentaquark J

Set of Pentaquark State Vectors

- Spin-parity J^P and state vectors of pentaquarks with the ground-state triquark and light diquark with $S_{ld} = 0$

J^P	$\lvert S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J$
$1/2^-$	$\lvert 0, 1/2, 0; 0, 0; 1/2, 0\rangle_{1/2}$
$1/2^-$	$\lvert 1, 1/2, 0; 0, 0; 1/2, 0\rangle_{1/2}$
$3/2^-$	$\lvert 1, 3/2, 0; 0, 0; 3/2, 0\rangle_{3/2}$

J^P	$\lvert S_{hd}, S_t, L_t; S_{ld}, L_{ld}; S, L\rangle_J$
$1/2^+$	$\lvert 0, 1/2, 0; 0, 1; 1/2, 1\rangle_{1/2}$
$3/2^+$	$\lvert 0, 1/2, 0; 0, 1; 1/2, 1\rangle_{3/2}$
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$5/2^+$	$\lvert 1, 3/2, 0; 0, 1; 3/2, 1\rangle_{5/2}$

- Number of state vectors with the ground-state triquark and light diquark with $S_{ld} = 1$ are larger and not presented

Hamiltonian for pentaquarks with hidden charm

- Involves constituent diquarks' and quark masses and spin-spin, spin-orbit, orbital and tensor interactions

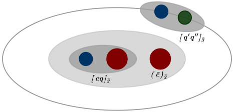
$$H^{(L=0)} = H_t + H_{ld}$$

- Doubly-heavy triquark Hamiltonian

$$H_t = m_c + m_{hd} + 2\,(\mathcal{K}_{cq})_{\bar 3}\,(\mathbf{S}_c \cdot \mathbf{S}_q) + 2\,\mathcal{K}_{\bar c q}\,(\mathbf{S}_{\bar c} \cdot \mathbf{S}_q) + 2\,\mathcal{K}_{\bar c c}\,(\mathbf{S}_{\bar c} \cdot \mathbf{S}_c)$$

- Light diquark Hamiltonian

$$H_{ld} = m_{ld} + 2\,(\mathcal{K}_{q'q''})_{\bar 3}\,(\mathbf{S}_{q'} \cdot \mathbf{S}_{q''}) + H_{SS}^{t-ld}$$



- Triquark-diquark spin-spin interactions

$$H_{SS}^{t-ld} = 2\,(\tilde{\mathcal{K}}_{cq'})_{\bar 3}\,(\mathbf{S}_c \cdot \mathbf{S}_{q'}) + 2\,(\tilde{\mathcal{K}}_{qq'})_{\bar 3}\,(\mathbf{S}_q \cdot \mathbf{S}_{q'}) + 2\,\tilde{\mathcal{K}}_{\bar c q'}\,(\mathbf{S}_{\bar c} \cdot \mathbf{S}_{q'})$$
$$+ 2\,(\tilde{\mathcal{K}}_{cq''})_{\bar 3}\,(\mathbf{S}_c \cdot \mathbf{S}_{q''}) + 2\,(\tilde{\mathcal{K}}_{qq''})_{\bar 3}\,(\mathbf{S}_q \cdot \mathbf{S}_{q''}) + 2\,\tilde{\mathcal{K}}_{\bar c q''}\,(\mathbf{S}_{\bar c} \cdot \mathbf{S}_{q''})$$

- Anticipate that spin-spin couplings in H_{SS}^{t-ld} are strongly suppressed and neglected

Hamiltonian for pentaquarks with hidden charm

- Involves constituent diquarks' and quark masses and spin-spin, spin-orbit, orbital and tensor interactions

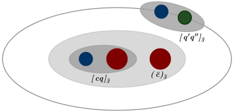
$$H^{(L=0)} = H_t + H_{ld}$$

- Doubly-heavy triquark Hamiltonian

$$H_t = m_c + m_{hd} + 2\,(\mathcal{K}_{cq})_{\bar{3}}\,(\mathbf{S}_c \cdot \mathbf{S}_q) + 2\,\mathcal{K}_{\bar{c}q}\,(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_q) + 2\,\mathcal{K}_{\bar{c}c}\,(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_c)$$

- Light diquark Hamiltonian

$$H_{ld} = m_{ld} + 2\,(\mathcal{K}_{q'q''})_{\bar{3}}\,(\mathbf{S}_{q'} \cdot \mathbf{S}_{q''}) + H_{SS}^{t-ld}$$



- Triquark-diquark spin-spin interactions

$$H_{SS}^{t-ld} = 2\,(\tilde{\mathcal{K}}_{cq'})_{\bar{3}}\,(\mathbf{S}_c \cdot \mathbf{S}_{q'}) + 2\,(\tilde{\mathcal{K}}_{qq'})_{\bar{3}}\,(\mathbf{S}_q \cdot \mathbf{S}_{q'}) + 2\,\tilde{\mathcal{K}}_{\bar{c}q'}\,(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{q'})$$
$$+ 2\,(\tilde{\mathcal{K}}_{cq''})_{\bar{3}}\,(\mathbf{S}_c \cdot \mathbf{S}_{q''}) + 2\,(\tilde{\mathcal{K}}_{qq''})_{\bar{3}}\,(\mathbf{S}_q \cdot \mathbf{S}_{q''}) + 2\,\tilde{\mathcal{K}}_{\bar{c}q''}\,(\mathbf{S}_{\bar{c}} \cdot \mathbf{S}_{q''})$$

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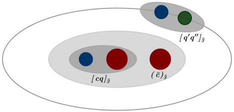

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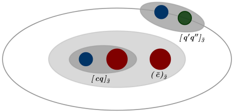

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Hamiltonian for orbitally-excited pentaquarks

- Should be extended by including orbital momentum $L = L_{ld}$

$$H = H^{(L=0)} + H_L + H_T$$

- Spin-orbit and orbital interactions

$$H_L = 2A_t\,(\mathbf{S}_t \cdot \mathbf{L}) + 2A_{ld}\,(\mathbf{S}_{ld} \cdot \mathbf{L}) + \frac{1}{2}\,B\,\mathbf{L}^2$$

- Tensor interaction

$$H_T = b\,\frac{S_{12}}{4} = b\left[3\,\frac{(\mathbf{S}_t \cdot \mathbf{R})\,(\mathbf{S}_{ld} \cdot \mathbf{R})}{R^2} - (\mathbf{S}_t \cdot \mathbf{S}_{ld})\right]$$

- $\mathbf{R}$ determines position of light diquark relative to heavy triquark
- Non-vanishing for light diquark with $S_{ld} = 1$
- Remember that orbital excitation in DHT is absent ($L_t = 0$)
- Hence, $\Delta H_L = 0$ and $\Delta H_T = 0$

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Tensor Interaction in Orbitally Excited Pentaquark

- Tensor interaction operator in the term H_T

$$\frac{S_{12}}{4} = Q(\mathbf{S}_1, \mathbf{S}_2) = 3\,(\mathbf{S}_1 \cdot \mathbf{n})\,(\mathbf{S}_2 \cdot \mathbf{n}) - (\mathbf{S}_1 \cdot \mathbf{S_2}) = 3\, S_1^i\, S_2^j\, N_{ij}$$

- $\mathbf{S}_1 = \mathbf{S}_t$ and $\mathbf{S}_2 = \mathbf{S}_{ld}$ are spins of triquark and light diquark
- $\mathbf{n} = \mathbf{R}/R$ is the unit vector along the vector $\mathbf{R}$
- The scalar operator above written as the convolution contains

$$N_{ij} = n_i\, n_j - \frac{1}{3}\,\delta_{ij}$$

- Need its transform which can be easily evaluated between states with the same fixed value L of the angular momentum operator $\mathbf{L}$
- Required identity can be found in the Landau and Lifshitz book

$$\langle N_{ij} \rangle = -\frac{1}{(2L-1)\,(2L+3)}\left[L_i L_j + L_j L_i - \frac{2}{3}\, L\,(L+1)\,\delta_{ij}\right]$$

$$\langle Q(\mathbf{S}_1, \mathbf{S}_2) \rangle_{L=1} = -\frac{3}{5}\left[(\mathbf{L} \cdot \mathbf{S}_1)\,(\mathbf{L} \cdot \mathbf{S}_2) + (\mathbf{L} \cdot \mathbf{S}_2)\,(\mathbf{L} \cdot \mathbf{S}_1) - \frac{4}{3}\,(\mathbf{S}_1 \cdot \mathbf{S}_2)\right]$$

Mass Formulas for Ground-State Pentaquarks — 1

- From three states, two states with $J^P = 1/2^-$ mix due to spin-spin interaction of charm antiquark and heavy diquark, and the third one with $J^P = 3/2^-$ remains unmixed
- For the later state, the mass m_3^{S0} is the average of the effective Hamiltonian over this state:

$$m_3^{S0} = M_0 + \frac{1}{2}(\mathcal{K}_{cq})_{\bar{3}} - \frac{3}{2}(\mathcal{K}_{q'q''})_{\bar{3}} + \frac{1}{2}(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c})$$

- Superscript on mass denotes the S-wave pentaquark with "good" light diquark, having $S_{ld} = 0$
- Two states with $J^P = 1/2^-$, after sandwiching the effective Hamiltonian, yield the following (2×2) mass matrix:

$$M_{J=1/2}^{S0} = M_0 - \frac{1}{2}(\mathcal{K}_{cq})_{\bar{3}} - \frac{3}{2}(\mathcal{K}_{q'q''})_{\bar{3}} - (\mathcal{K}_{cq})_{\bar{3}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \frac{1}{2}(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}) \begin{pmatrix} 0 & \sqrt{3} \\ \sqrt{3} & -2 \end{pmatrix}$$

Mass Formulas for Ground-State Pentaquarks — 2

- Masses of ground-state pentaquarks with the spin-0 light diquark

$$m_1^{S0} = M_0 - \frac{1}{4}\left(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}\right)\left[2 + r_{hd} + 3r_{ld} + 2\sqrt{3 + (1 - r_{hd})^2}\right]$$

$$m_2^{S0} = M_0 - \frac{1}{4}\left(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}\right)\left[2 + r_{hd} + 3r_{ld} - 2\sqrt{3 + (1 - r_{hd})^2}\right]$$

$$m_3^{S0} = M_0 + \frac{1}{4}\left(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}\right)\left(2 + r_{hd} - 3r_{ld}\right).$$

- Universal contribution $M_0 \equiv m_{hd} + m_{ld} + m_c$
- Introduced two ratios of the spin-spin couplings

$$r_{hd} \equiv \frac{2(\mathcal{K}_{cq})_{\bar{3}}}{\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}}, \qquad r_{ld} \equiv \frac{2(\mathcal{K}_{q'q''})_{\bar{3}}}{\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}}$$

Mass Formulas for Orbitally-Excited Pentaquarks

- Masses of orbitally-excited pentaquarks with spin-0 light diquark

$$
\begin{aligned}
m_1^{P0} &= M_0 - \frac{1}{4}\left(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}\right)\left[2 + r_{hd} + 3r_{ld} + 2\sqrt{3 + (1 - r_{hd})^2}\right] + B - 2A_t \\
m_2^{P0} &= m_1^{P0} + \left(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}\right)\sqrt{3 + (1 - r_{hd})^2} \\
m_{3,4}^{P0} &= m_{1,2}^{P0} + 3A_t \\
m_5^{P0} &= M_0 + \frac{1}{4}\left(\mathcal{K}_{\bar{c}q} + \mathcal{K}_{\bar{c}c}\right)(2 + r_{hd} - 3r_{ld}) + B - 5A_t \\
m_6^{P0} &= m_5^{P0} + 3A_t, \qquad m_7^{P0} = m_5^{P0} + 8A_t
\end{aligned}
$$

- Other pentaquark masses are also obtained from matrix elements of the effective Hamilltonian after diagonalization
- For all the masses of ground-state pentaquarks and their first orbital excitations were derived analytical expressions

Input for Masses and Spin-Spin Couplings

- Obtained from phenomenological analysis of ordinary hadrons
- Charm quark mass: $m_c^b = (1710 \pm 10)$ MeV
- Diquark masses: $m_{[ud]} = (576 \pm 15)$ MeV
  $m_{[cq]} = (1976 \pm 15)$ MeV
- Spin-spin couplings: $(\mathcal{K}_{cq})_{\bar{3}} = 67$ MeV, $(\mathcal{K}_{qq'})_{\bar{3}} = 98$ MeV
  $\mathcal{K}_{\bar{c}q} = 70$ MeV, $\mathcal{K}_{\bar{c}c} = 113$ MeV
- Assign a 10% error of each value of spin-spin coupling; the same as in Ω_c^*-baryons
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Pentaquark Assignment

- Former $P_c(4450)$ splits into $P_c(4440)$ and $P_c(4457)$
- Both of them are P-wave pentaquarks: $P_c(4457)$ with $J^P = 5/2^+$
  $P_c(4440)$ with $J^P = 3/2^+$
- Third pentaquark $P_c(4312)$ is S-wave state with $J^P = 3/2^-$
- Keeping spin-spin couplings fixed, we performed χ^2-fit

$$M_0 = (4333.9 \pm 3.9)\ \mathrm{MeV}, \quad B = (135.2 \pm 4.6)\ \mathrm{MeV}, \quad A_t = (3.4 \pm 1.1)\ \mathrm{MeV}$$

- The best-fit value of M_0 comes out about 72 MeV higher than the sum of the diquarks' and charm quark masses, $M_0 = 4262$ MeV
- Value of B is too small in comparison with the strengths of the orbital excitations in other hadrons, and, in particular, $B(\Omega_c) = 325$ MeV, obtained from the Ω_c^*-baryons

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- This gives the value of orbital coupling $B \simeq 207$ MeV; closer to estimates in the hidden and open charm hadrons
- Mass of the third pentaquark $M = (4240 \pm 29)$ MeV with $J^P = 3/2^-$ is somewhat lower than the mass of the observed $P_c(4312)$ peak, but is still in the right ball-park

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Mass Predictions for Unflavored Pentaquarks

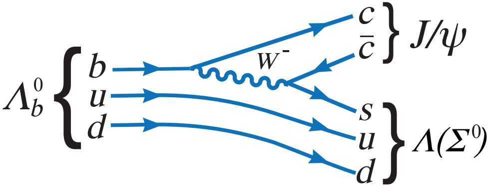
J^P	This work	AAAR
	$S_{ld}=0,\ L=0$	
$1/2^-$	3830 ± 34	4086 ± 42
	4150 ± 29	4162 ± 38
$3/2^-$	4240 ± 29	4133 ± 55
	$S_{ld}=1,\ L=0$	
$1/2^-$	4026 ± 31	4119 ± 42
	4346 ± 25	4166 ± 38
	4436 ± 25	4264 ± 41
$3/2^-$	4026 ± 31	4072 ± 40
	4346 ± 25	4300 ± 40
	4436 ± 25	4342 ± 40
$5/2^-$	4436 ± 25	4409 ± 40
	$S_{ld}=0,\ L=1$	
$1/2^+$	4030 ± 39	4030 ± 62
	4351 ± 35	4141 ± 44
	4430 ± 35	4217 ± 40
$3/2^+$	4040 ± 39	
	4361 ± 35	
	4440 ± 35	
$5/2^+$	4457 ± 35	4510 ± 57

J^P	This work	AAAR
	$S_{ld}=1,\ L=1$	
$1/2^+$	4144 ± 37	3970 ± 50
	4209 ± 37	4174 ± 44
	4465 ± 32	4198 ± 50
	4530 ± 32	4221 ± 40
	4564 ± 33	4240 ± 50
	4663 ± 32	4319 ± 43
$3/2^+$	4187 ± 37	
	4250 ± 37	
	4508 ± 32	
	4570 ± 32	
	4511 ± 33	
	4566 ± 32	
	4656 ± 32	
$5/2^+$	4260 ± 37	4450 ± 44
	4581 ± 32	4524 ± 41
	4601 ± 32	4678 ± 44
	4656 ± 32	4720 ± 44
$7/2^+$	4672 ± 32	

Isospin Violation in Λ_b-Decays

LHCb Collab. [R. Aaij *et al.*, arXiv:1912.02110]

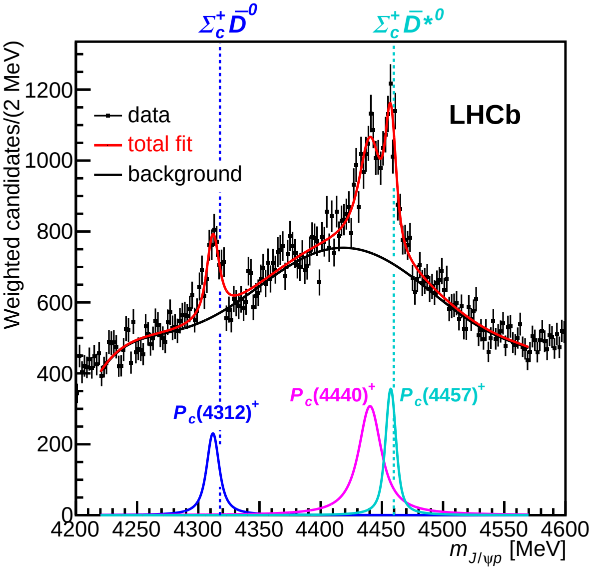
- Data: 1.0 fb^{-1} at 7 TeV , 2.0 fb^{-1} at 8 TeV, and 5.5 fb^{-1} at 13 TeV
- Isospin-0 FS: $\Lambda_b^0 \to J/\psi\, \Lambda^0$
  Isospin-1 FS: $\Lambda_b^0 \to J/\psi\, \Sigma^0$
- Decays through the $\Delta I = 0$ transition $b \to cs\bar{s}$
- Amplitide's ratio: $|A_1/A_0| < 1/20.9$ at 95% C.L.
- Rules out isospin violation at 1% rate



Mass Predictions for Unflavored Pentaquarks

J^P	This work	AAAR	J^P	This work	AAAR
	$S_{ld}=0, L=0$			$S_{ld}=1, L=1$	
$1/2^-$	3830 ± 34	4086 ± 42	$1/2^+$	4144 ± 37	3970 ± 50
	4150 ± 29	4162 ± 38		4209 ± 37	4174 ± 44
$3/2^-$	4240 ± 29	4133 ± 55		4465 ± 32	4198 ± 50
	$S_{ld}=1, L=0$			4530 ± 32	4221 ± 40
$1/2^-$	4026 ± 31	4119 ± 42		4564 ± 33	4240 ± 50
	4346 ± 25	4166 ± 38		4663 ± 32	4319 ± 43
	4436 ± 25	4264 ± 41	$3/2^+$	4187 ± 37	
$3/2^-$	4026 ± 31	4072 ± 40		4250 ± 37	
	4346 ± 25	4300 ± 40		4508 ± 32	
	4436 ± 25	4342 ± 40		4570 ± 32	
$5/2^-$	4436 ± 25	4409 ± 40		4511 ± 33	
	$S_{ld}=0, L=1$			4566 ± 32	
$1/2^+$	4030 ± 39	4030 ± 62		4656 ± 32	
	4351 ± 35	4141 ± 44	$5/2^+$	4260 ± 37	4450 ± 44
	4430 ± 35	4217 ± 40		4581 ± 32	4524 ± 41
$3/2^+$	4040 ± 39			4601 ± 32	4678 ± 44
	4361 ± 35			4656 ± 32	4720 ± 44
	4440 ± 35		$7/2^+$	4672 ± 32	
$5/2^+$	4457 ± 35	4510 ± 57			

$\Lambda_b \to p + J/\psi + K^-$ Decay: 2019 Results by LHCb



Comments about Mass Predictions

- $M^{\text{thr}}_{J/\psi\, p} = M_{J/\psi} + m_p = 4035.17$ MeV is the threshold for observed pentaquarks and some states are below it or very close
- Other possibility is to assign $P_c(4312)$ with the state having $J^P = 3/2^+$ and $M = 4361$ MeV or having $J^P = 1/2^+$ and $M = 4351$ MeV
- Despite masses are close, input parameters have unphysical values
- Alternative assignment is given in the Dynamical Diquark Model [J. F. Giron, R. F. Lebed & C. T. Peterson, JHEP 1905 (2019) 061]
- Mass predictions for hidden-charm strange pentaquarks are also obtained

Mass Predictions for Strange Pentaquarks

- Inclusion of strange quark(s) into the content makes spectrum of hidden-charm pentaquarks very rich
- They can be classsified according to their strangeness and color connection of four quarks
  - Singly-strange: $(\bar{c}_{\bar{3}}\,[cs]_{\bar{3}}\,[qq']_{\bar{3}})$ and $(\bar{c}_{\bar{3}}\,[cq]_{\bar{3}}\,[sq']_{\bar{3}})$
  - Doubly-strange: $(\bar{c}_{\bar{3}}\,[cs]_{\bar{3}}\,[sq]_{\bar{3}})$ and $(\bar{c}_{\bar{3}}\,[cq]_{\bar{3}}\,\{ss\}_{\bar{3}})$
  - Triple-strange: $(\bar{c}_{\bar{3}}\,[cs]_{\bar{3}}\,\{ss\}_{\bar{3}})$
- Can be produced in weak decays of Ξ_b- and Ω_b-baryons at LHC
  - $\Xi_b^- \to P_\Lambda^0 + K^- \to J/\psi + \Lambda^0 + K^-$
  - $\Xi_b^{-,0} \to P_\Sigma^{0,+} + K^- \to J/\psi + \Sigma^{0,+} + K^-$
  - $\Omega_b^- \to P_{\Xi_{10}}^0 + K^- \to J/\psi + \Xi'^0 + K^-$
  - $\Omega_b^- \to P_{\Omega_{10}}^- + \phi \to J/\psi + \Omega^- + \phi$
- Ω_b-decays gives a new avenue to study pentaquarks with “bad” light diquarks

Masses of Singly-Strange ($\bar{c}_{\bar{3}}\,[cq]_{\bar{3}}\,[sq']_{\bar{3}}$) Pentaquarks

J^P	This work	AAAR	J^P	This work	AAAR
	$S_{ld}=0,\ L=0$			$S_{ld}=1,\ L=1$	
$1/2^-$	4112 ± 32	4094 ± 44	$1/2^+$	4348 ± 36	3929 ± 53
	4433 ± 26	4132 ± 43		4414 ± 36	4183 ± 45
$3/2^-$	**4523 ± 26**	4172 ± 47		4669 ± 32	4159 ± 53
	$S_{ld}=1,\ L=0$			4735 ± 32	4189 ± 44
$1/2^-$	4230 ± 30	4128 ± 44		4768 ± 32	4201 ± 53
	4551 ± 25	4134 ± 42		4867 ± 32	4275 ± 45
	4641 ± 25	4220 ± 43	$3/2^+$	4392 ± 36	
$3/2^-$	4230 ± 30	4031 ± 43		4454 ± 36	
	4551 ± 25	4262 ± 43		4713 ± 32	
	4641 ± 25	4303 ± 43		4775 ± 32	
$5/2^-$	4641 ± 25	4370 ± 43		4716 ± 32	
	$S_{ld}=0,\ L=1$			4770 ± 32	
$1/2^+$	4312 ± 37	4069 ± 56		4861 ± 32	
	4633 ± 33	4149 ± 45	$5/2^+$	4465 ± 36	4409 ± 47
	4713 ± 33	4187 ± 44		4786 ± 32	4486 ± 45
$3/2^+$	4323 ± 37			4806 ± 32	4639 ± 47
	4643 ± 33			4860 ± 32	4681 ± 47
	4723 ± 33		$7/2^+$	4877 ± 32	
$5/2^+$	**4740 ± 33**	4549 ± 51			

Masses of Triple-Strange ($\bar{c}_3\,[cs]_{\bar{3}}\,\{ss\}_{\bar{3}}$) Pentaquarks

J^P	Mass	J^P	Mass
$S_{ld}=1,\,L=0$		$S_{ld}=1,\,L=1$	
$1/2^-$	4642 ± 31	$3/2^+$	4804 ± 37
	4974 ± 25		4866 ± 37
	5043 ± 25		5136 ± 32
$3/2^-$	4642 ± 31		5198 ± 32
	4974 ± 25		5118 ± 32
	5043 ± 25		5173 ± 32
$5/2^-$	5043 ± 25		5263 ± 32
$S_{ld}=1,\,L=1$		$5/2^+$	4877 ± 37
$1/2^+$	4761 ± 37		5209 ± 32
	4826 ± 37		5208 ± 32
	5092 ± 32		5263 ± 32
	5158 ± 32	$7/2^+$	5279 ± 32
	5171 ± 32		
	5270 ± 32		

- All of them are decaying strongly

Summary

- Effective Hamiltonian based on the doubly-heavy triquark — light diquark picture of pentaquark is worked out for ground states and orbital excitations
- All the masses are obtained analytically under assumption that spin-spin couplings between constituents of triquark and light diquark can be neglected
- Newly observed three narrow resonances are interpreted as follows: $P_c(4312)$ is S-wave state with $J^P = 3/2^-$ while $P_c(4440)$ and $P_c(4457)$ are P-wave states with $J^P = 3/2^+$ and $J^P = 5/2^+$, respectively
- Complete spectrum of hidden-charm unflavored pentaquarks is presented; spectrum of strange pentaquarks is calculated and partially presented
- We wait for spin-parity analysis by LHCb
- $\mathcal{B}(\Lambda_b \to P_c^+ \pi^-)/\mathcal{B}(\Lambda_b \to P_c^+ K^-) \sim \tan^2 \theta_c$

  Important to confirm this ratio for all three P_c-states