Production of $\eta_c(1S, 2S)$ in e^+e^- and pp collisions

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References:

[1] I. Babiarz, V. P. Goncalves, R. Pasechnik, W. Schäfer and A. Szczurek, Phys. Rev. D **100**, no.5, 054018 (2019),[arXiv:1908.07802 [hep-ph]].

[2] I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek, JHEP **02**, 037 (2020),[arXiv:1911.03403 [hep-ph]].

Contest

- Introduction
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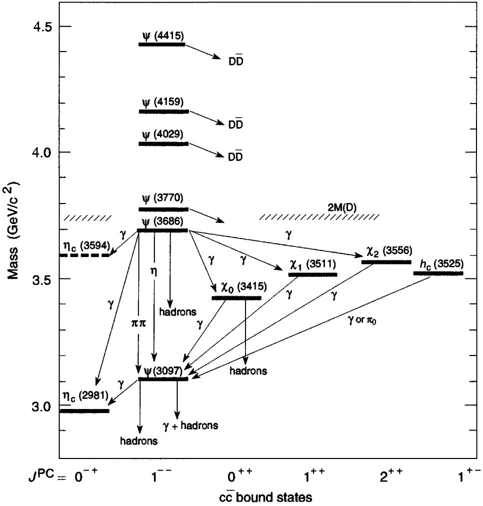
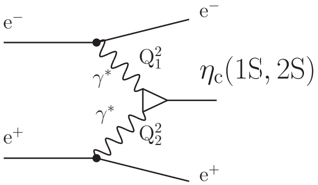
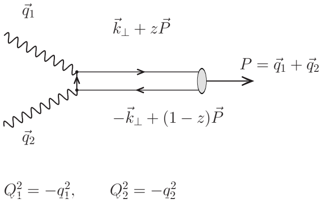
Introduction



Description of the mechanism $\gamma^*\gamma^* \to \eta_c(1S, 2S)$



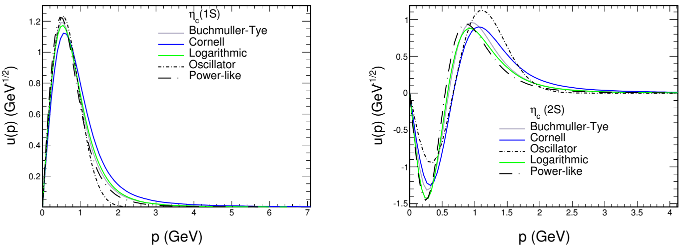


$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)$$

Light-front representation of the transition form factor:

$$F(Q_1^2, Q_2^2) = e_c^2\sqrt{N_c}4m_c \cdot \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3}\psi(z, \mathbf{k})$$
$$\left\{\frac{1-z}{(\mathbf{k}-(1-z)\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2} + \frac{z}{(\mathbf{k}+z\mathbf{q}_2)^2 + z(1-z)\mathbf{q}_1^2 + m_c^2}\right\}.$$

Nonrelativistic quarkonium wave functions



Radial momentum space wave function for different potentials.
Radial space wave function are obtained from the Schrödinger equation

J. Cepila, J. Nemchik, M. Krelina and R. Pasechnik, Eur. Phys. J. C **79**, no.6, 495 (2019),[arXiv:1901.02664 [hep-ph]].

$$\frac{\partial^2 u(r)}{\partial r^2} = (V_{\rm eff}(r) - \epsilon)u(r)\,, \qquad u(r) = \sqrt{4\pi}\, r\psi(r)\,,$$

$$\int_0^\infty |u(r)|^2 dr = 1 \quad \Rightarrow \quad \int_0^\infty |u(p)|^2 dp = 1$$

Light-front wave functions

Fock-state expansion:

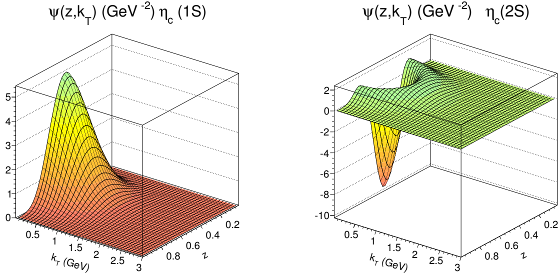
$$|\eta_c; P_+, \mathbf{P}\rangle = \sum_{i,j,\lambda,\bar\lambda} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \Psi_{\lambda\bar\lambda}(z,\mathbf{k}) |c_{i\lambda}(zP_+, \mathbf{p}_c) \bar{c}^j_{\bar\lambda}((1-z)P_+, \mathbf{p}_{\bar c})\rangle + \dots$$

$$\Psi_{\lambda\bar\lambda}(z,\mathbf{k}) = \bar{\mathrm{U}}_\lambda(zP_+,\mathbf{k})\gamma_5 \mathrm{V}_{\bar\lambda}((1-z)P_+, -\mathbf{k})\psi(z,\mathbf{k})$$

$$\psi(z,\mathbf{k}) = \frac{\pi}{\sqrt{2M_{c\bar c}}} \frac{u(p)}{p} \,.$$

Terentev prescription valid for weakly bound system

$\mathbf{p} = \mathbf{k}, \quad p_z = (z - \frac{1}{2})M_{c\bar c},$



Radial light-front wave function for Buchmüller-Tye potential.

$F(0,0)$ transition for both on-shell photons

$$F(0,0) = e_c^2 \sqrt{N_c}\, 4m_c \cdot \int \frac{dz d^2\mathbf{k}}{z(1-z)16\pi^3} \frac{\psi(z,\mathbf{k})}{\mathbf{k}^2 + m_c^2} \,,$$

$F(0,0)$ is related to the two-photon decay width by the formula:

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{\pi}{4} \alpha_{\rm em}^2 M_{\eta_c}^3 \left| F(0,0) \right|^2 .$$

$F(0,0)$ can be rewrite in the terms of radial momentum space wave function $u(p)$:

$$F(0,0) = e_c^2 \sqrt{2N_c}\, \frac{2m_c}{\pi} \int_0^\infty \frac{dp\, p\, u(p)}{\sqrt{M_{c\bar{c}}^3}(p^2 + m_c^2)} \frac{1}{2\beta} \log\left(\frac{1+\beta}{1-\beta} \right) ,$$

In the non-relativistic (NR) limit, where $p^2/m_c^2 \ll 1, \beta \ll 1$, and $2m_c = M_{c\bar{c}} = M_{\eta_c}$, we obtain

$$F(0,0) = e_c^2 \sqrt{N_c} \sqrt{2} \frac{4}{\pi \sqrt{M_{\eta_c}^5}} \int_0^\infty dp\, p\, u(p) = e_c^2 \sqrt{N_c} \frac{4\, R(0)}{\sqrt{\pi M_{\eta_c}^5}} \,,$$

where $\beta = \frac{p}{\sqrt{p^2 + m_c^2}}$, the velocity v/c of the quark in the $c\bar{c}$ cms-frame and R(0) radial wave function at the origin.

$F(0,0)$ for both on-shell photons

Transition form factor $|F(0,0)|$ for $\eta_c(1S)$ at $Q_1^2 = Q_2^2 = 0$.

potential type	m_c [GeV]	$\lvert F(0,0)\rvert$ [GeV^{-1}]	$\Gamma_{\gamma\gamma}$ [keV]	f_{η_c} [GeV]
harmonic oscillator	1.4	0.051	2.89	0.2757
logarithmic	1.5	0.052	2.95	0.3373
power-like	1.334	0.059	3.87	0.3074
Cornell	1.84	0.039	1.69	0.3726
Buchmüller-Tye	1.48	0.052	2.95	0.3276
experiment	-	0.067 ± 0.003 [1]	5.1 ± 0.4 [1]	0.335 ± 0.075 [2]

[1] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no.3, 030001 (2018).

[2] K. W. Edwards *et al.* [CLEO Collaboration], Phys. Rev. Lett. **86**, 30 (2001) [hep-ex/0007012].

R(0) and $\gamma\gamma$-width for $\eta_c(1S)$ derived in **the non-relativistic limit**.

potential type	R(0)[GeV$^{3/2}$]	$\Gamma_{\gamma\gamma}$ [keV] $\mathrm{M} = \mathrm{M}_{\eta_c}$	$\Gamma_{\gamma\gamma}$ [keV] $\mathrm{M} = 2\mathrm{m}_c$
harmonic oscillator	0.6044	5.1848	5.8815
logarithmic	0.8919	11.290	11.157
power-like	0.7620	8.2412	10.297
Cornell	1.2065	20.660	13.568
Buchmüller-Tye	0.8899	11.240	11.409

$$f_{\eta_c}\varphi(z,\mu_0^2) = \frac{1}{z(1-z)}\frac{\sqrt{N_c}\,4m_c}{16\pi^3}\int d^2\mathbf{k}\,\theta(\mu_0^2 - \mathbf{k}^2)\,\psi(z,\mathbf{k}) \text{ and } \int_0^1 \mathrm{d}z\,\varphi(z,\mu_0^2) = 1$$

$F(0,0)$ for both on-shell photons

Transition form factor $|F(0,0)|$ for $\eta_c(2S)$ at $Q_1^2 = Q_2^2 = 0$.

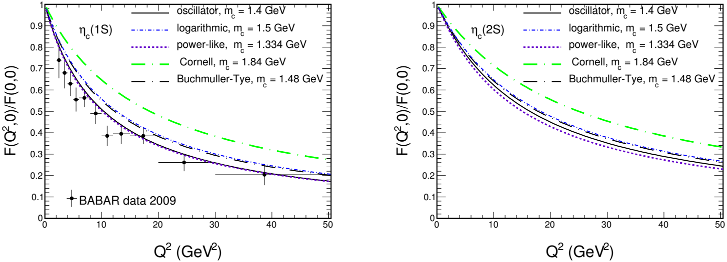
potential type	m_c [GeV]	$\|F(0,0)\|$ [GeV^{-1}]	$\Gamma_{\gamma\gamma}$ [keV]	f_{η_c} [GeV]
harmonic oscillator	1.4	0.03492	2.454	0.2530
logarithmic	1.5	0.02403	1.162	0.1970
power-like	1.334	0.02775	1.549	0.1851
Cornell	1.84	0.02159	0.938	0.2490
Buchmüller-Tye	1.48	0.02687	1.453	0.2149
experiment [1]	-	0.03266 ± 0.01209	2.147 ± 1.589	

[1] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, no.3, 030001 (2018).

R(0) and $\gamma\gamma$-width for $\eta_c(2S)$ derived in the **non-relativistic limit.**

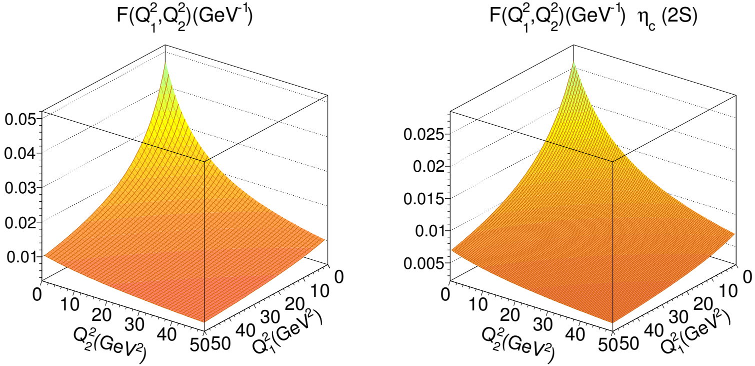
potential type	R(0) [GeV$^{3/2}$]	$\Gamma_{\gamma\gamma}$ [keV] $\mathrm{M} = \mathrm{M}_{\eta_c}$	$\Gamma_{\gamma\gamma}$ [keV] $\mathrm{M} = 2\mathrm{m_c}$
harmonic oscillator	0.7402	5.2284	8.8214
logarithmic	0.6372	3.8745	5.6946
power-like	0.5699	3.0993	5.7594
Cornell	0.9633	8.8550	8.6493
Buchmüller-Tye	0.7185	4.9263	7.4374

Normalized transition form factor $\tilde{F}(Q^2, 0)$

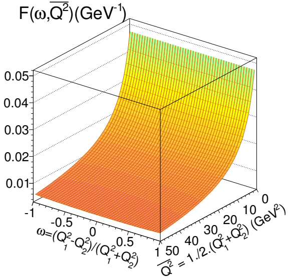


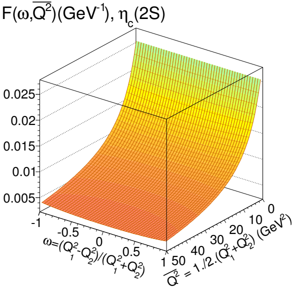
Normalized transition form factor $\tilde{F}(Q^2, 0)$ as a function of photon virtuality Q^2. The BaBar data are shown for comparison. J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **81**, 052010 (2010) [arXiv:1002.3000 [hep-ex]].

Transition form factor $F(Q_1^2, Q_2^2)\, \gamma^*\gamma^* \to \eta_c(1S, 2S)$



Transition form factor for $\eta_c(1S)$ and $\eta_c(2S)$ for Buchmüller -Tye potential. The sign of Bose symmetry Q_1^2, Q_2^2.

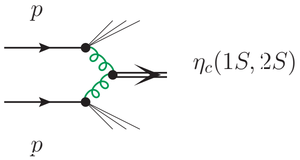




The $\gamma^*\gamma^* \to \eta_c$ (1S) and $\gamma^*\gamma^* \to \eta_c$ (2S) form factor as a function of (Q_1^2, Q_2^2) and $(\omega, \bar{Q}^2)$ for the Buchmüller-Tye potential for illustration.

$$\omega = \frac{Q_1^2 - Q_2^2}{Q_1^2 + Q_2^2} \quad \text{and} \quad \bar{Q}^2 = \frac{Q_1^2 + Q_2^2}{2} \, .$$

Production of η_c(1S,2S) in pp collisions



In the k_T-factorization approach, gluons are off-shell, $q_i^2 = -\boldsymbol{q}_i^2$ and their four momenta:

$$q_1 = (q_{1+}, 0, \boldsymbol{q}_1)\,,\; q_2 = (0, q_{2-}, \boldsymbol{q}_2)\,,$$
$$q_{1+} = x_1\sqrt{\frac{s}{2}}\,,\; q_{2-} = x_2\sqrt{\frac{s}{2}}\,.$$

The phase space element:

$$d\Phi(2 \to 1) = (2\pi)^4 \delta^{(4)}(q_1 + q_2 - p)\, \frac{d^4 p}{(2\pi)^3} \delta(p^2 - M_{\eta_c}^2)\,.$$

I. Babiarz, R. Pasechnik, W. Schäfer and A. Szczurek, JHEP **02**, 037 (2020), [arXiv:1911.03403 [hep-ph]]

$$d\sigma = \int \frac{dx_1}{x_1} \int \frac{d^2\boldsymbol{q}_1}{\pi \boldsymbol{q}_1^2} \mathcal{F}(x_1, \boldsymbol{q}_1^2, \mu_F^2) \int \frac{dx_2}{x_2} \int \frac{d^2\boldsymbol{q}_2}{\pi \boldsymbol{q}_2^2} \mathcal{F}(x_2, \boldsymbol{q}_2^2, \mu_F^2) \frac{1}{2x_1x_2s} \overline{|\mathcal{M}|^2}\, d\Phi(2 \to 1)\,,$$

$$\mathcal{M}^{ab} = \frac{q_{1\perp}^{\mu} q_{2\perp}^{\nu}}{|\boldsymbol{q}_1||\boldsymbol{q}_2|} \mathcal{M}_{\mu\nu}^{ab} = \frac{q_{1+}q_{2-}}{|\boldsymbol{q}_1||\boldsymbol{q}_2|} n^{+\mu} n^{-\nu} \mathcal{M}_{\mu\nu}^{ab} = \frac{x_1x_2s}{2|\boldsymbol{q}_1||\boldsymbol{q}_2|} n^{+\mu} n^{-\nu} \mathcal{M}_{\mu\nu}^{ab}\,.$$

In covariant form, the matrix element reads:

$$\mathcal{M}_{\mu\nu}^{ab} = (-i)4\pi\alpha_s\, \varepsilon_{\mu\nu\alpha\beta} q_1^{\alpha} q_2^{\beta} \frac{\mathrm{Tr}[t^a t^b]}{\sqrt{N_c}}\, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)\,.$$

Matrix element and form factor

The explicit form of matrix element reads:

$$
\begin{aligned}
n^{+\mu} n^{-\mu} \mathcal{M}^{ab}_{\mu\nu} &= 4\pi\alpha_s(-i)[\boldsymbol{q}_1, \boldsymbol{q}_2] \frac{\mathrm{Tr}[t^a t^b]}{\sqrt{N_c}} \, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \\
&= 4\pi\alpha_s(-i) \frac{1}{2} \delta^{ab} \frac{1}{\sqrt{N_c}} |\boldsymbol{q}_1||\boldsymbol{q}_2| \sin(\phi_1 - \phi_2) \, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \,,
\end{aligned}
$$

and averaging over colors, we obtain our final result:

$$
\begin{aligned}
\frac{d\sigma}{dy d^2\boldsymbol{p}} = \int \frac{d^2\boldsymbol{q}_1}{\pi \boldsymbol{q}_1^4} \mathcal{F}(x_1, \boldsymbol{q}_1^2) \int \frac{d^2\boldsymbol{q}_2}{\pi \boldsymbol{q}_2^4} \mathcal{F}(x_2, \boldsymbol{q}_2^2) \, \delta^{(2)}(\boldsymbol{q}_1 + \boldsymbol{q}_2 - \boldsymbol{p}) \\
\times \frac{\pi^3 \alpha_s^2}{N_c(N_c^2 - 1)} ||\boldsymbol{q}_1||\boldsymbol{q}_2| \sin(\phi_1 - \phi_2) \, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)|^2.
\end{aligned}
$$

The relation $I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2)$ with the form factor $\gamma^*\gamma^* \to \eta_c$:

$$
F(Q_1^2, Q_2^2) = e_c^2 \sqrt{N_c} \, I(\boldsymbol{q}_1^2, \boldsymbol{q}_2^2) \,,
$$

The transition form factor in NRQCD limit:

$$
F_{\mathrm{NRQCD}}(Q_1^2, Q_2^2) = \frac{4 e_c^2 \sqrt{N_c}}{\sqrt{\pi M_{\eta_c}}} \frac{1}{M_{\eta_c}^2 + Q_1^2 + Q_2^2} \, R(0) \,,
$$

Normalization of the form factor

$$\Gamma_{\rm LO}(\eta_c \to \gamma\gamma) = \frac{\pi}{4}\alpha_{\rm em}^2 M_{\eta_c}^3 |F(0,0)|^2 \,.$$

At NLO, the expressions for the widths read (J. P. Lansberg and T. N. Pham, Phys. Rev. D **74**, 034001 (2006))

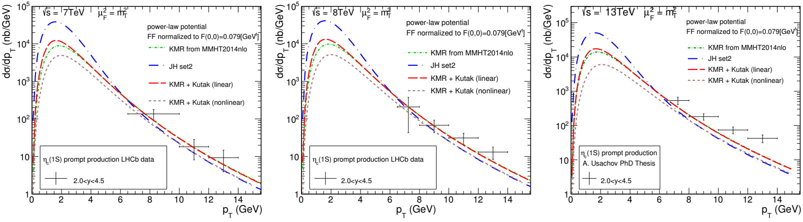
$$\Gamma_{\rm NLO}(\eta_c \to \gamma\gamma) \quad = \quad \Gamma_{\rm LO}(\eta_c \to \gamma\gamma)\left(1 - \frac{20-\pi^2}{3}\frac{\alpha_s}{\pi}\right),$$

Radiative decay widths as well as $|F(0,0)|$ obtained from $\Gamma_{\gamma\gamma}$ using leading order and next-to-leading order approximation. [1] M. Tanabashi *et al.* [Particle Data Group] Phys. Rev. D **98**, no. 3, 030001 (2018)

	Experimental values $\Gamma_{\gamma\gamma}$(keV)[1]	Derived from LO $\|F(0,0)\|[GeV^{-1}]$	Derived from NLO $\|F(0,0)\|_{\gamma\gamma}[GeV^{-1}]$
$\eta_c(1S)$	5.0 $\pm$0.4	0.067$\pm$0.003	0.079$\pm$0.003
$\eta_c(2S)$	1.9 $\pm$1.3 $\cdot 10^{-4} \cdot \Gamma_{\eta_c(2S)}$	0.033$\pm$0.012	0.038$\pm$0.014

Results for $\eta_c(1S)$ production

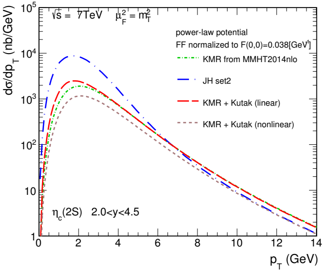
Differential cross section as a function of transverse momentum for prompt $\eta_c(1S)$ production compared with the LHCb data [1]for $\sqrt{s} = 7, 8\,\mathrm{TeV}$ and preliminary experimental data [2] for $\sqrt{s}$ = 13 TeV. Different UGDs were used. Here we used the $g^*g^* \to \eta_c(1S)$ form factor calculated from the power-law potential.

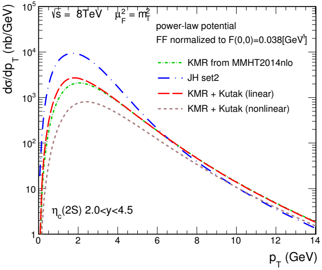


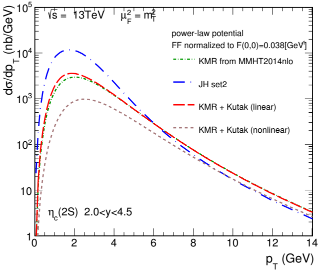
[1] R. Aaij *et al.* [LHCb Collaboration], Eur. Phys. J. C **75**, no. 7, 311 (2015), [2] A. Usachov, arXiv:1910.08796 [hep-ex].

Results for $\eta_c(2S)$ production

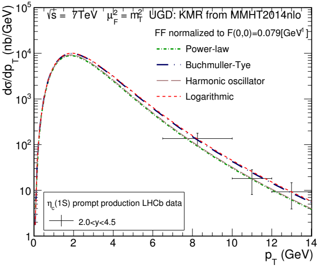
Differential cross section as a function of transverse momentum for prompt $\eta_c(2S)$ production for $\sqrt{s} = 7, 8, 13\,\mathrm{TeV}$.

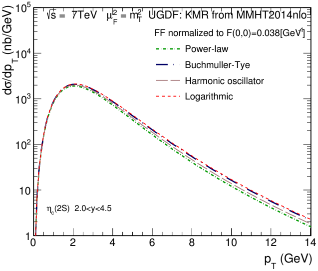


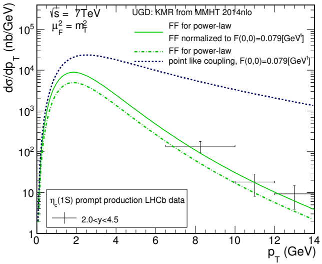




Several different form factors with one common normalization







Conclusions

- The transition form factor for different wave functions obtained as a solution of the Schrödinger equation for the $c\bar{c}$ system for different phenomenological $c\bar{c}$ potentials from the literature, was calculated.
- We have studied the transition form factors for $\gamma^* \gamma^* \to \eta_c$ (1S,2S) for two space-like virtual photons, which can be accessed experimentally in future measurements of the cross section for the $e^+ e^- \to e^+ e^- \eta_c$ process in the double - tag mode.
- The transition form factor for only one off-shell photon as a function of its virtuality,was studied and compared to the BaBar data for the $\eta_c(1S)$ case.
- There is practically no dependence on the asymmetry parameter ω, which could be verified experimentally at Belle 2.
- The application of the form factor into k_T-factorization cross section was presented.
- Comparison to LHCb data was shown.
- The uncertainty of the choice of the UGDs is higher than the choice of the potential for form factor.