

## 1. Motivation: net-proton number fluctuations

- baryon number susceptibilities  $\chi^B$  can be calculated on the lattice
- enhancement of susceptibilities near the critical point
- susceptibilities are (in principle) measurable as cumulants of baryon number distribution
- Problems with the measurement:
  - susceptibilities are calculated in grand-canonical ensemble, however cumulants are measured in real collisions which conserve  $B$
  - baryon number involved in the collision fluctuates due to centrality fluctuations
  - baryon number is not measurable, net-proton number is used as proxy (e.g. since no neutrons are measured)

## 2. Features of this model: Monte Carlo simulation

- conservation of the baryon number based on the number of participants (wounded nucleons), which fluctuates e-by-e
- rapidity distribution of wounded vs. produced (anti)baryons
- wounded nucleons (may) remember their isospin
- only protons and neutrons (and their antiparticles) in the simulations
- Two sorts of nucleons (and antinucleons) in the final state:
  - originally wounded nucleons
  - produced as nucleon-antinucleon pairs
- Glauber Monte Carlo
  - we use GLISSANDO 2 [1]
  - centrality is determined based on deposited energy measure

## 3. Rapidity distribution of wounded nucleons

$$\frac{dN_w}{dy}(y) = \frac{N_w}{2\sqrt{2\pi\sigma_y^2}} \left\{ \exp\left(-\frac{(y-y_m)^2}{2\sigma_y^2}\right) + \exp\left(-\frac{(y+y_m)^2}{2\sigma_y^2}\right) \right\}$$

- $\sigma_y = 0.8$
- $y_m$  set to reproduce 
$$N_{p-\bar{p}} = \frac{Z}{A} \int_{-y_b}^{y_b} \frac{dN_w}{dy} dy$$
- $y_b = 0.25$
- data on net-proton number taken from [2,3]

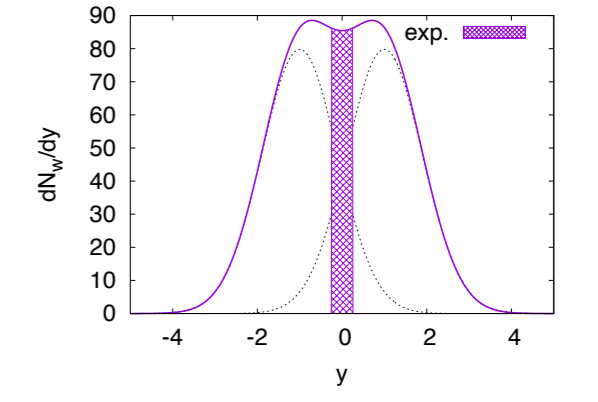


Illustration for:  $y_m = 1, \sigma_y = 0.8$

## 4. Rapidity distribution of produced nucleon pairs

$$\frac{dN_{B\bar{B}}}{dy} = N_{B\bar{B}} \frac{C}{1 + \exp\left(\frac{|y| - y_m}{a}\right)}$$

- $C$  is normalization to 1
- $N_{B\bar{B}}$  set to reproduce the observed number of antiprotons

$$N_{\bar{p}} = \frac{1}{2} \int_{-y_b}^{y_b} \frac{dN_{B\bar{B}}}{dy} dy$$

- $y_b = 0.25$
- data on antiproton numbers taken from [2,3]

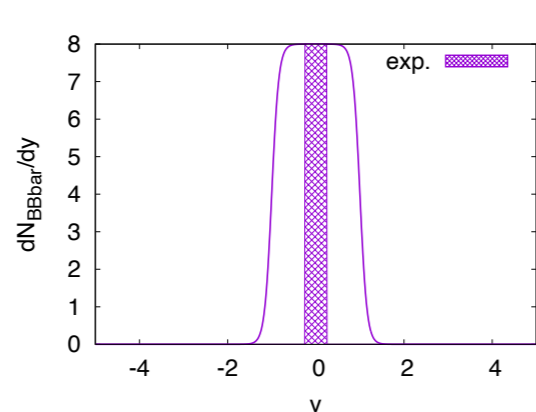


Illustration for:  $y_m = 1, a = 0.08$

## 5. Details of Glauber MC simulation

- Centrality is determined according to the multiplicity

$$M \propto \frac{1-\alpha}{2} N_w + \alpha N_{bin}$$

$$\alpha(\sqrt{s_{NN}}) = \alpha_0 + \alpha_1 \ln \sqrt{s_{NN}}$$

- The number of NNbar pairs fluctuates according to Poissonian with the mean proportional to  $N_w$

$$\mu_{N\bar{N}} = \frac{dN_{p\bar{p}}}{dy} y_m \frac{N_w}{\langle N_w \rangle}$$

where  $dN_{p\bar{p}}/dy$  is measured for given energy and centrality, and  $\langle N_w \rangle$  is mean number of wounded nucleons at given centrality

## 6. Model parameters for different energies

$\sqrt{s_{NN}}$ [GeV]	$\alpha$	$y_m$	$N_{B\bar{B}}$
7.7	0.110	0.519	0.8265
11.5	0.114	0.770	4.4790
19.6	0.120	1.019	16.946
27	0.123	1.128	27.1070
39	0.127	1.308	44.4262
62.4	0.132	1.384	75.2842
200	0.145	1.665	177.794

## 7. Definitions

Central moments

$$\mu_1 = \langle n \rangle = \bar{n}$$

$$\mu_2 = \langle (n - \bar{n})^2 \rangle = \sigma^2$$

$$\mu_3 = \langle (n - \bar{n})^3 \rangle$$

$$\mu_4 = \langle (n - \bar{n})^4 \rangle$$

Scaled skewness and kurtosis

$$S\sigma = \frac{\mu_3}{\mu_2} = \frac{\chi_3}{\chi_2}$$

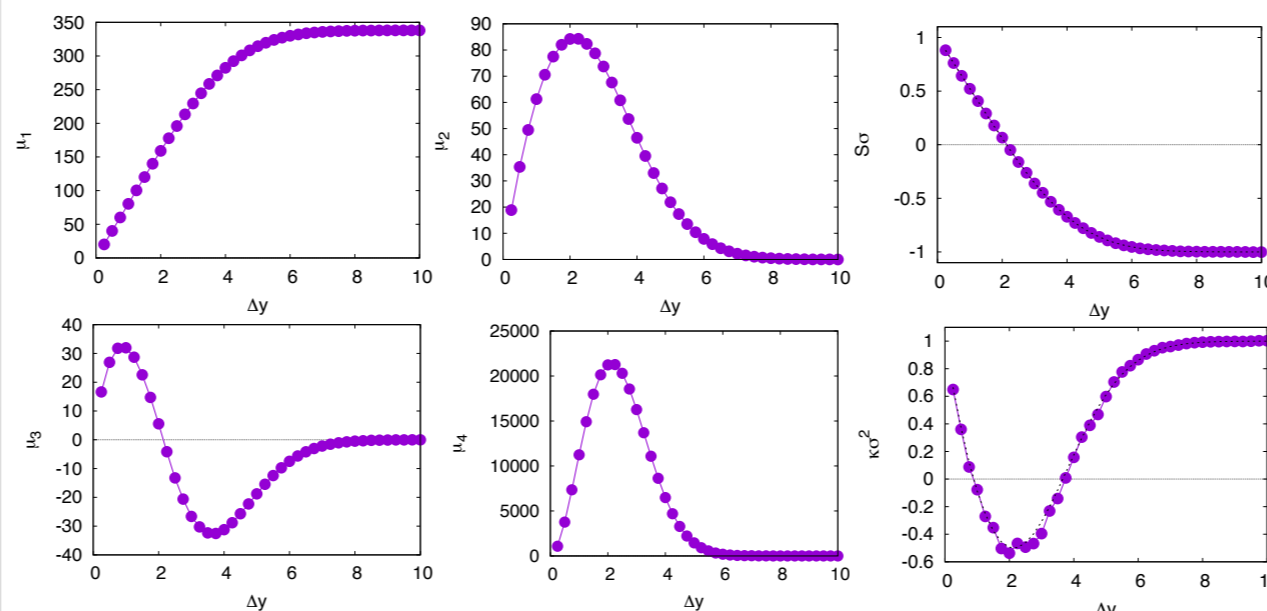
$$\kappa\sigma^2 = \frac{\mu_4}{\mu_2} - 3\mu_2^2 = \frac{\chi_4}{\chi_2}$$

$$\frac{\kappa\sigma^4}{\bar{n}} = \frac{\mu_4 - 3\mu_2^2}{\mu_1} = \frac{\chi_4}{\chi_1}$$

## 8. Basic exercise: baryon number conservation

Moments of  $B$  number distribution depending on rapidity bin width around central rapidity – reproduced by binomial distribution.

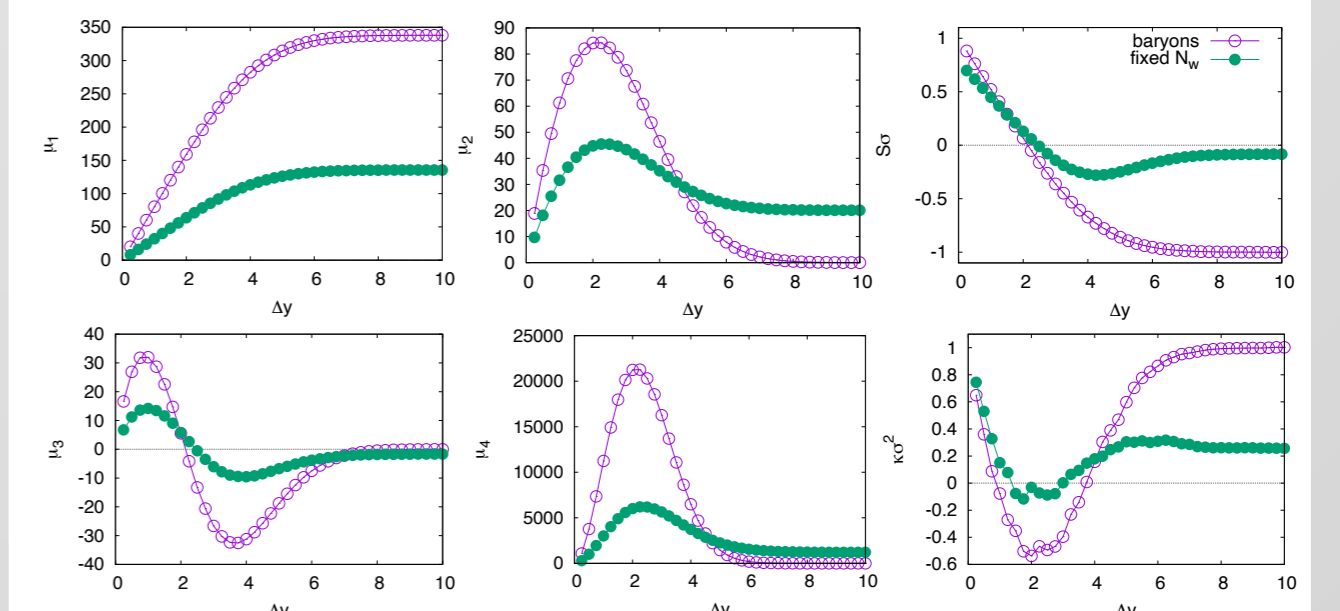
$\sqrt{s_{NN}} = 19.6$  GeV,  $N_{B\bar{B}} = 16.94$ ,  $y_m = 1.019$ ,  $N_w = 338$   
 $5 \times 10^7$  events



## 9. Basic exercise: net-protons vs. net baryons

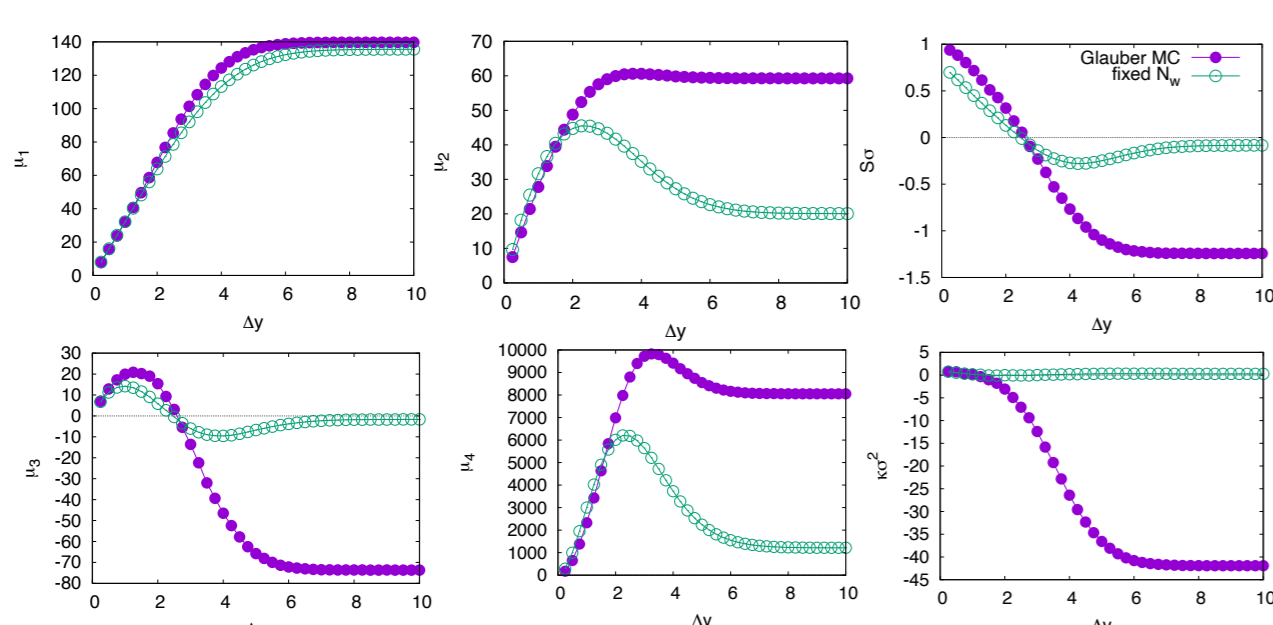
Fluctuations of net baryon number compared to net-proton number

$\sqrt{s_{NN}} = 19.6$  GeV,  $N_{B\bar{B}} = 16.94$ ,  $y_m = 1.019$ ,  $N_w = 338$   
 $5 \times 10^7$  events



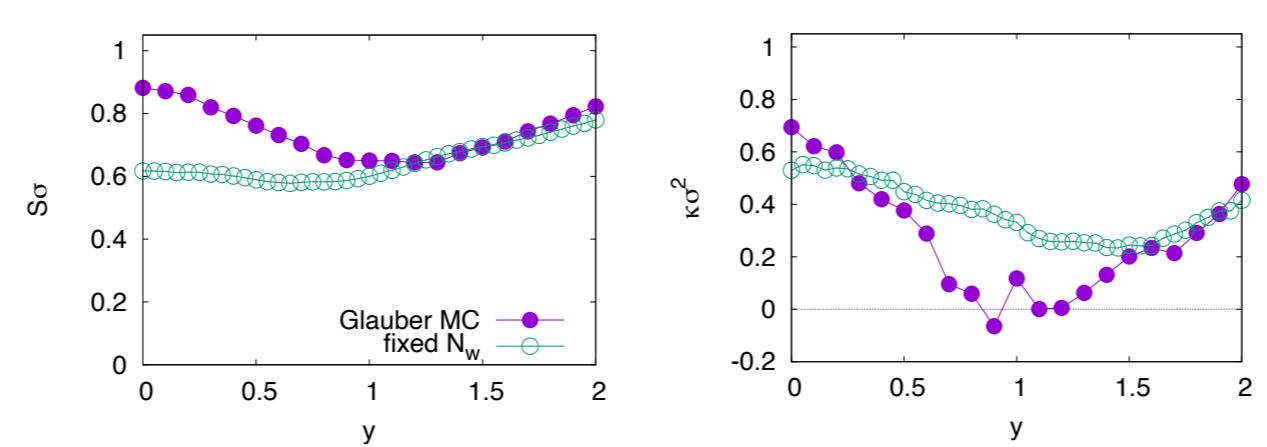
## 10. Basic exercise: net protons + fluctuating $N_w$

Comparison: fixed number of wounded nucleons vs. fluctuating  $N_w$   
 $\sqrt{s_{NN}} = 19.6$  GeV,  $N_{B\bar{B}} = 16.94$ ,  $y_m = 1.019$ ,  $N_w = 338$   
 $5 \times 10^7$  events, Glauber MC  $1.2 \times 10^6$  events



## 11. Results: net-proton number as function of $y$

Brewer et al [4]: search for critical point by looking at the rapidity dependence of cumulants, because  $\mu_B$  depends on rapidity. Here: non-critical dependence of the moments on rapidity.

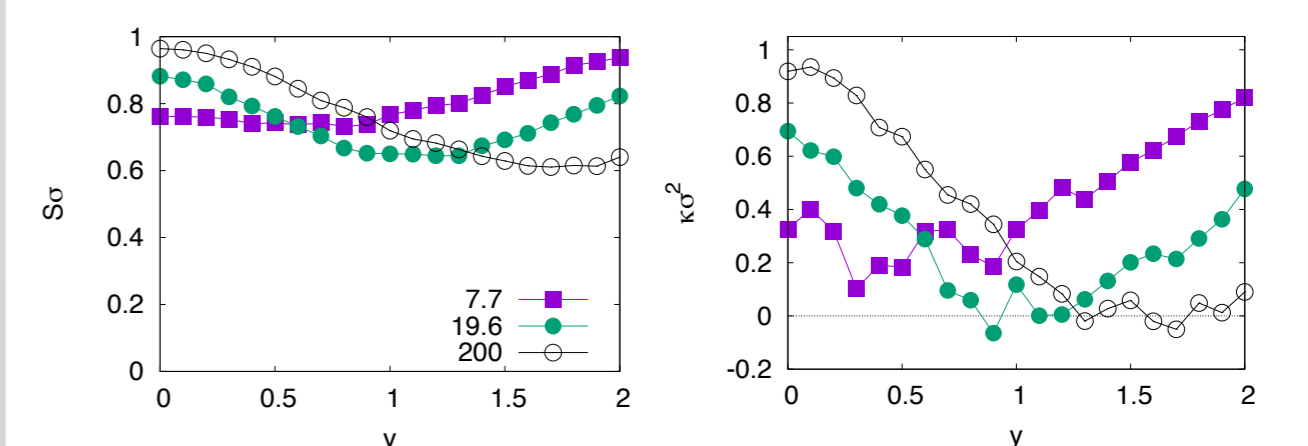


$\sqrt{s_{NN}} = 19.6$  GeV,  $N_{B\bar{B}} = 16.94$ ,  $y_m = 1.019$ ,  $N_w = 338$   
 Glauber MC  $1.2 \times 10^6$  events,  $\Delta y = 0.5$

## 12. Rapidity dependence for different energies

As panel 11, but for different energies.

Glauber MC,  $1.2 \times 10^6$  events

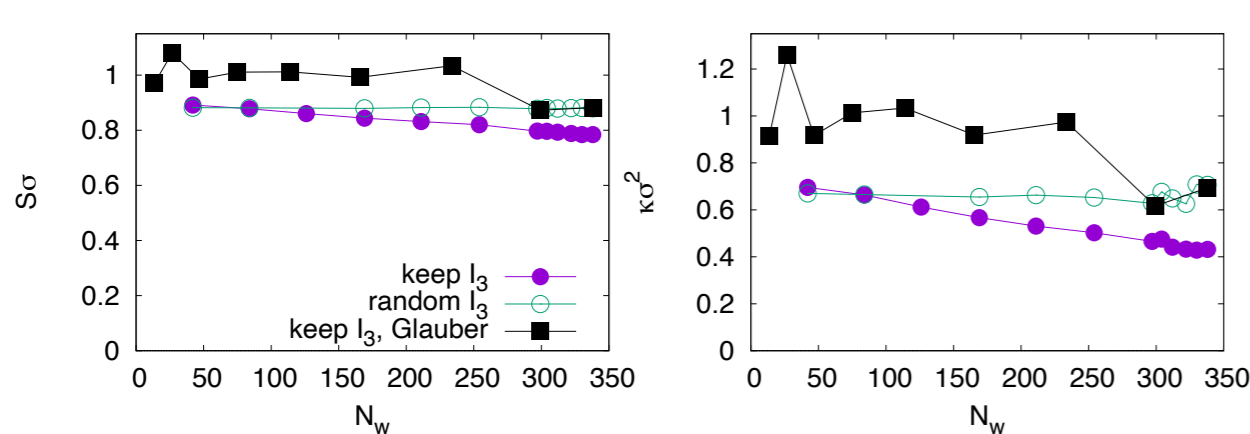


The minimum corresponds with the width of the rapidity distribution.

## 13. Centrality dependence

$\sqrt{s_{NN}} = 19.6$  GeV,  $y_m = 1.019$ ,  $N_{B\bar{B}}/N_w = 0.050$

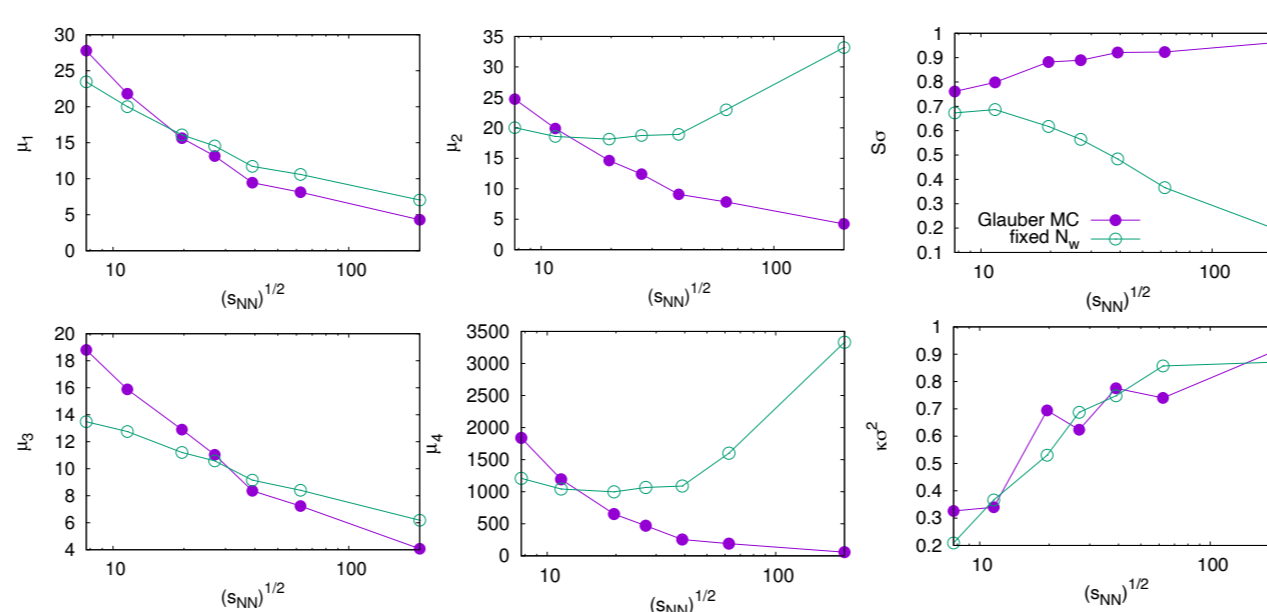
Statistics:  $2 \times 10^7$  for fixed  $N_w$ ,  $\sim 5 \times 10^5$  for Glauber MC



$S\sigma$  and  $\kappa\sigma^2$  are lowered towards more central events of wounded protons nucleons remember their isospin.

## 14. Collision energy dependence of net protons

rapidity bin  $\Delta y = 0.5$  around  $y = 0$   
 $2 \times 10^7$  events for fixed  $N_w$ ,  $1.2 \times 10^6$  events for Glauber MC



The importance of produced BBbar pairs grows with energy.

## 15. Conclusions

A “minimal” model for proton number fluctuations:

- rapidity dependent composition through two components: wounded nucleons and produced BBbar pairs
- Glauber MC (GLISSANDO 2)
- General formalism recently published by Braun-Munzinger *et al.* [5]

Findings:

- rapidity dependence of  $\kappa\sigma^2$  with  $\sqrt{s_{NN}}$ -dependent minimum
- baryon number conservation: decrease of  $S\sigma$  and  $\kappa\sigma^2$  with lower energies

## References

- [1] M. Rybczyński *et al.*, *Comp. Phys. Commun.* 185 (2014) 1759
- [2] STAR collab., *Phys. Rev. C* 79 (2009) 034909
- [3] STAR collab., *Phys. Rev. C* 96 (2017) 044904
- [4] J. Brewer *et al.*, *Phys. Rev. C* 98 (2018) 061901
- [5] P. Braun-Munzinger *et al.*, arXiv:2007.02463