## TMD densities at leading and higher order from the Parton Branching method

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## On be half of

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## Outline

(1) Recap of Parton Branching method
(2) Determination of PDFs at 5FL-LO, 5FL-NLO \& 4FL-NLO
(3) What is the gain with exclusive evolution?

## Recap of Parton Branching method

- Including the $\Delta_{s}$ in to the differential form of the DGLAP eq.

$$
\mu^{2} \frac{\partial}{\partial \mu^{2}} \frac{f\left(x, \mu^{2}\right)}{\Delta_{s}\left(\mu^{2}\right)}=\int \frac{d z}{z} \frac{\alpha_{s}}{2 \pi} \frac{\mathcal{P}(z)}{\Delta_{s}\left(\mu^{2}\right)} f\left(\frac{x}{z}, \mu^{2}\right)
$$

- Integral form with a very simple physical interpretation:

$$
f\left(x, \mu^{2}\right)=f\left(x, \mu_{0}^{2}\right) \Delta_{s}\left(\mu^{2}\right)+\int \frac{d z}{z} \frac{d \mu^{\prime 2}}{\mu^{\prime 2}} \cdot \frac{\Delta_{s}\left(\mu^{2}\right)}{\Delta_{s}\left(\mu^{\prime 2}\right)} P^{R}(z) f\left(\frac{x}{z}, \mu^{\prime 2}\right)
$$

- Solve integral equation via iteration:

$$
\begin{aligned}
& f_{0}\left(x, \mu^{2}\right)=f\left(x, \mu_{0}^{2}\right) \Delta_{s}\left(\mu^{2}\right) \\
& f_{1}\left(x, \mu^{2}\right)=f\left(x, \mu_{0}^{2}\right) \Delta_{s}\left(\mu^{2}\right) \\
& +\int_{\mu_{0}^{2}}^{\mu^{2}} \frac{d \mu^{\prime 2}}{\mu^{\prime 2}} \frac{\Delta_{s}\left(\mu^{2}\right)}{\Delta_{s}\left(\mu^{\prime 2}\right)} \int \frac{d z}{z} P^{R}(z) f\left(x / z, \mu_{0}^{2}\right) \Delta\left(\mu^{\prime 2}\right)
\end{aligned}
$$



- iterating with second branching and so on to get the full solution


## Transverse Momentum Dependence

- Parton Branching evolution generates every single branching:
- kinematics can be calculated at every step
- give physics interpretation of evolution scale:
- in high energy limit: $p_{T}$-ordering:
$\mu=q_{T}$
- angular ordering: $\mu=q_{T} /(1-z)$



## Determination of PDFs

## PDFs from PB method: fit to HERA data

- A kernel obtained from the MC solution of the evolution equation for any initial parton
- Kernel is folded with the non-perturbative starting distribution

$$
\begin{aligned}
x f_{a}\left(x, \mu^{2}\right) & =x \int d x^{\prime} \int d x^{\prime \prime} \mathcal{A}_{0, b}\left(x^{\prime}\right) \tilde{\mathcal{A}}_{a}^{b}\left(x^{\prime \prime}, \mu^{2}\right) \delta\left(x^{\prime} x^{\prime \prime}-x\right) \\
& =\int d x^{\prime} \mathcal{A}_{0, b}\left(x^{\prime}\right) \cdot \frac{x}{x^{\prime}} \tilde{\mathcal{A}}_{a}^{b}\left(\frac{x}{x^{\prime}}, \mu^{2}\right)
\end{aligned}
$$

- Fit performed using $\times$ Fitter frame (with collinear Coefficient functions at both LO \& NLO)
- LO PDFs are of especial interest for MC event generators, based on LO ME + PS.
- full coupled-evolution with all flavors
- using full HERA I+II inclusive DIS (neutral current, charged current) data
- $3.5<Q^{2}<50000 \mathrm{GeV}^{2} \& 4.10^{-5}<x<0.65$
- Can be easily extended to include any other measurement for fit.


## Standard 5FL-NLO full fit with different scale in $\alpha_{s}$




- Set1- $\alpha_{s}\left(\mu^{2}\right) \rightarrow \chi^{2} /$ dof $=1.21$
- Set2- $\alpha_{s}\left(p_{T}^{2}\right) \rightarrow \chi^{2} /$ dof $=1.21$

$$
\begin{aligned}
& x g(x)=A_{g} x^{B_{g}}(1-x)^{C_{g}}-A_{g}^{\prime} x^{B_{g}}(1-x)^{C_{g}}, \\
& x u_{v}(x)=A_{u_{v}} x^{B_{u_{v}}}(1-x)^{C_{U_{v}}}\left(1+E_{u_{v}} x^{2}\right), \\
& x d_{v}(x)=A_{d_{v}} x^{B_{d_{v}}}(1-x)^{C_{d_{v}}}, \\
& x \bar{U}(x)=A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\left(1+D_{\bar{U}} x\right), \\
& x \bar{D}(x)=A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}} .
\end{aligned}
$$

- fits are as good as HERAPDF2.0.
- very different gluon distribution obtained at small $Q^{2}$
- the differences are washed out at higher $Q^{2}$
A. Martinez, P. Connor, H. Jung, A. Lelek, R. Žlebčík, F. Hautmann and V. Radescu, Phys. Rev. D 99, no. 7, 074008 (2019).


## Standard 5FL-LO full fit with different scale in $\alpha_{s}$

## LO TMDs are important for LO multi-jet merging

$\rightarrow$ see talk by Armando Bermudes: 28th July; Strong Interactions and Hadron Physics



- Set1- $\alpha_{s}\left(\mu^{2}\right) \rightarrow \chi^{2} /$ dof $=1.24$
- Set2- $\alpha_{s}\left(p_{T}^{2}\right) \rightarrow \chi^{2} /$ dof $=1.37$

$$
\begin{aligned}
& x g(x)=A_{g} x^{B_{g}}(1-x)^{C_{g}}, \\
& x u_{v}(x)=A_{U_{v}} x^{B_{u_{v}}}(1-x)^{C_{U_{v}}}\left(1+E_{U_{v}} x^{2}\right), \\
& x d_{v}(x)=A_{d_{v}} x^{B_{d_{v}}}(1-x)^{C_{d_{v}}}, \\
& x \bar{U}(x)=A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\left(1+D_{\bar{U}} x\right), \\
& x \bar{D}(x)=A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}} .
\end{aligned}
$$




- very different gluon distribution obtained at small and large $Q^{2}$
- the uncertainty is smaller at LO compared to NLO


## Standard 4FL-NLO full fit with different scale in $\alpha_{s}$

## NEW



- Set1- $\alpha_{s}\left(\mu^{2}\right) \rightarrow \chi^{2} /$ dof $=1.20$
- Set2- $\alpha_{s}\left(p_{T}^{2}\right) \rightarrow \chi^{2} /$ dof $=1.23$

$$
\begin{aligned}
& x g(x)=A_{g} x^{B_{g}}(1-x)^{C_{g}}-A_{g}^{\prime} x^{B_{g}^{\prime}}(1-x)^{C_{g}^{\prime}} \\
& x u_{v}(x)=A_{u_{v}} x^{B_{u_{v}}}(1-x)^{C_{u_{v}}}\left(1+E_{u_{v}} x^{2}\right), \\
& x d_{v}(x)=A_{d_{v}} x^{B_{d_{v}}(1-x)^{C_{d_{v}}}} \\
& x \bar{U}(x)=A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\left(1+D_{\bar{U}} x\right), \\
& x \bar{D}(x)=A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}
\end{aligned}
$$

- very different gluon distribution obtained at small $Q^{2}$
- the differences are washed out at higher $Q^{2}$


## Fit to DIS x-section at 5FL-NLO, 5FL-LO \& 4FL-NLO: $F_{2}$

How well can we describe inclusive DIS cross section with the two sets at NLO \& LO?


## What is the gain with exclusive evolution?

## $k_{t}$ behavior at LO and NLO








- difference coming from different starting distribution and also the evolution
- with the same starting distribution we still get differences at small $k_{t}$
- at larger $k_{t}$, more splitting $\rightarrow$ The differences between LO and NLO are washed out


## $k_{t}$ behavior at 4FL-NLO and 5FL-NLO



- 4-FL gluon is larger than 5-FL gluon at small $k t$ region.
- at small $k_{t} \rightarrow$ starting distribution
- at large $k_{t} \rightarrow$ the differences are washed out due to having more splittings.


## The basic contribution to Bottom Flavor Production

Fred Olness's talk-U Manchester-22 April 2016
( $\alpha_{S}{ }^{1}$

- data in the $\mathrm{Z}+\geq 1$ b-jet and $\mathrm{Z}+\geq 2$ b-jets cases are better described by 5 FL prediction and 4 FL prediction, respectively.


## PB-TMD, PB-TMD shower \& MC@NLO : Z+b jets

CMS Measurements of the associated production of $a \mathbf{Z}$ boson and $b$ jets in pp collisions at 8 TeV , Eur. Phys. J., C77(11), 751, CMS-SMP-14-010, arxiv:1611.06507

- cuts:
- leptons: $|\eta|<2.4, p_{T}>20 \mathrm{GeV}, 71 \mathrm{GeV}<m_{\| /}<111 \mathrm{GeV}$
- jets: anti- $k_{T}, \mathrm{R}=0.5,|\eta|<2.4, p_{T}>30 \mathrm{GeV}$, b-Hadron

CMS, $8 \mathrm{TeV}, \mathrm{Z}$ boson pt, at least two b jets


- $p_{t}$ spectrum of $Z$ boson is nicely described with both $4 F L$ and $5 F L$ schemes


## $\mathrm{Z}+2 \mathrm{~b}$ jets: sensitivity to initial state shower for 5 FL



- TMD has little impact
- IPS has significant large effect
- FPS has contribution at small $\Delta \phi: g \rightarrow b b$


## Z +2 b jets: comparison between 5FL \& 4FL



- ME calculation at 5 FL is $\mathrm{Z}+1$ jet NLO
- PS is important in the 5-FL scheme

CMS, 8 TeV, DeltaPhi bb, at least two $b$ jets


- ME calculation at 4 FL is $\mathrm{Z}+2$ jet NLO
- PS and TMD has very little impact in 4-FL scheme


## Z +2 b -jets: $\Delta \phi(b b)$ - comparison to measurement

- $\Delta \phi$ between the b-b system is well described with the 4FL \& 5FL scheme

- In 4FL, ME plays a rule (little contribution from TMD and PS)
- the 5 FL , quite a lot contribution from the TMD + the initial + final state PS
- both calculations do very nicely agree
- the calculation which involves TMD + PS is consistent with the full ME calculations even at NLO


## Conclusion

- PB method to solve DGLAP equation at LO, NLO, NNLO.
- advantages of PB method (angular ordering)
- method directly applicable to determine $k_{t}$ distribution (as would be done in PS)
- TMD distributions for all flavors determined at LO \& NLO
- Application to pp processes:
- NEW: application to Z+b-jets
- Z+b-jets interesting tool for studying initial state parton radiation in very detail: TMD and TMD showers
- 4-FL and 5-FL results including:TMD+IPS+FPS do agree


## Thank you

Feel invited to connect to the following link for more discussions on these topics after today's sessions ( $\sim 21: 00$ ):
https://cern.zoom.us/j/97149635135?pwd=NnJDRGpaOU9KenBVa3Zsck1LM05RQT09 Same ICHEP Zoom password

## Backup

## Evolution equation and parton branching method

- use momentum weighted PDFs with real emission probability

$$
\begin{aligned}
x f_{a}\left(x, \mu^{2}\right) & =\Delta_{a}\left(\mu^{2}\right) x f_{a}\left(x, \mu_{0}^{2}\right) \\
& +\sum_{b} \int_{\mu_{0}^{2}}^{\mu^{2}} \frac{d \mu^{\prime 2}}{\mu^{\prime 2}} \frac{\Delta_{s}\left(\mu^{2}\right)}{\Delta_{a}\left(\mu^{\prime 2}\right)} \int_{x}^{z_{M}} d z P_{a b}^{R}\left(\alpha_{a}, z\right) \frac{x}{z} f_{b}\left(x / z, \mu^{2}\right)
\end{aligned}
$$

- due to step by step individual branchings, all kinematics can be calculated exactly.
- $z_{M}$ introduced to separate real from virtual and non-resolvable branching
- reproduces DGLAP up to $\mathcal{O}\left(1-z_{M}\right)$
- make use of momentum sum rule to treat virtual corrections
- use Sudakov form factor for non-resolvable and virtual corrections

$$
\Delta_{a}\left(z_{M}, \mu^{2}, \mu_{0}^{2}\right)=\exp \left(-\sum_{b} \int_{\mu_{0}^{2}}^{\mu^{2}} \frac{d \mu^{\prime 2}}{\mu^{\prime 2}} \int_{0}^{z_{M}} d z z P_{b a}^{R}\left(\alpha_{s}, z\right)\right)
$$

## PDFs from PB method: fit to HERA data

- two angular ordered sets with different argument in $\alpha_{s}$ (either $\mu$ or $q_{t}$ )
- $q_{c u t}$ in, $\alpha_{s}\left(\max \left(q_{c u t}^{2},\left|q_{t, i}^{2}\right|\right)\right)$, to avoid the non-perturbative region, $\left|q_{t, i}^{2}\right|=\left(1-z_{i}\right)^{2} \mu_{i}^{2}$
- for both LO \& NLO:
- $\mu_{0}^{2}=1.9 \mathrm{GeV}^{2}$ for set1 (as in HERAPDF)
- $\mu_{0}^{2}=1.4 \mathrm{GeV}^{2}$ for set2 (the best $\chi^{2} /$ dof)
- fits to HERA measurements performed using $\chi^{2} /$ dof minimization
- the experimental uncertainties defined with the Hessian method with $\Delta \chi^{2}=1$.
- the model dependence obtained by varying charm and bottom masses and $\mu_{0}^{2}$.
- the uncertainty coming from the $q_{c u t}$ in set2

|  | Central <br> value | Lower <br> value | Upper <br> value |
| :---: | :---: | :---: | :---: |
| PB Set1 $\mu_{0}^{2}\left(\mathrm{GeV}^{2}\right)$ | 1.9 | 1.6 | 2.2 |
| PB Set 2 $\mu_{0}^{2}\left(\mathrm{GeV}^{2}\right)$ | 1.4 | 1.1 | 1.7 |
| PB Set $2 q_{\text {cut }}(\mathrm{GeV})$ | 1.0 | 0.9 | 1.1 |
| $m_{c}(\mathrm{GeV})$ | 1.47 | 1.41 | 1.53 |
| $m_{b}(\mathrm{GeV})$ | 4.5 | 4.25 | 4.75 |

A. Martinez, P. Connor, H. Jung, A. Lelek, R. Žlebčík, F. Hautmann and V. Radescu, Phys. Rev. D 99, no. 7, $\underline{0} 74008$ (2019),

## TMD distributions from fit to HERA data



- Different shape and dependence of the uncertainty as a function of $k_{t}$.
- Model dependence larger than experimental uncertainties.
- Difference essentially in low $k_{t}$ region.
- Introducing $p_{T}$ instead of $\mu$ suppresses further soft gluons at low $k_{t}$.
A. Martinez, P. Connor, H. Jung, A. Lelek, R. Žlebčík, F. Hautmann and V. Radescu, Phys. Rev. D 99, no. 7, $\underline{\underline{\underline{0}}}$ 年4008_(2019),


## Kinematic coverage of the HERA data in the $\left(x, Q^{2}\right)$ plane

- This is mainly small $Q^{2}$ effects rather than small $\times$ one.

Kinematic coverage


- Going from $Q_{\text {min }}=3.5$ to $5 \mathrm{GeV}^{2}$, no obvious change on $x$ while $\chi^{2}$ change significantly.
- No $x$ dependence $\rightarrow$ No direct need for any small-x modification


## PB-TMD, PB-TMD shower \& MC@NLO : Z+b jets

- MC@NLO for Z+b : (5-FL scheme \& 4-FL scheme)
- using herweg6 subtraction terms
- PB-TMD to generate initial state $k_{T}$
- initial state parton shower following PB TMD
- uncertainties:
- MC@NLO for Z+b : (5-FL scheme \& 4-FL scheme)
- using herweg6 subtraction terms
- PB-TMD to generate initial state $k_{T}$
- initial state parton shower following PB TMD


## z+b-jets: $\Delta \phi(Z b)$-comparison to measurements II

comparison between 4FL-PB-TMD and 5FL-PB-TMD
CMS, 8 TeV , DeltaPhi_Zb, at least one $b$ jet


## Z-jets: sensitivity to initial state $k_{T}$

8 TeV , DeltaPhi_Zb, at least one b jet


- TMD important at large $\Delta \phi$
- Initial state PS only small effect
- FSR only small effect at large $\Delta \phi$

