

# *Going off-shell: matrix element, parton densities and shower*



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Institute of Nuclear Physics  
Polish Academy of Sciences

*Krzysztof Kutak*



NCN

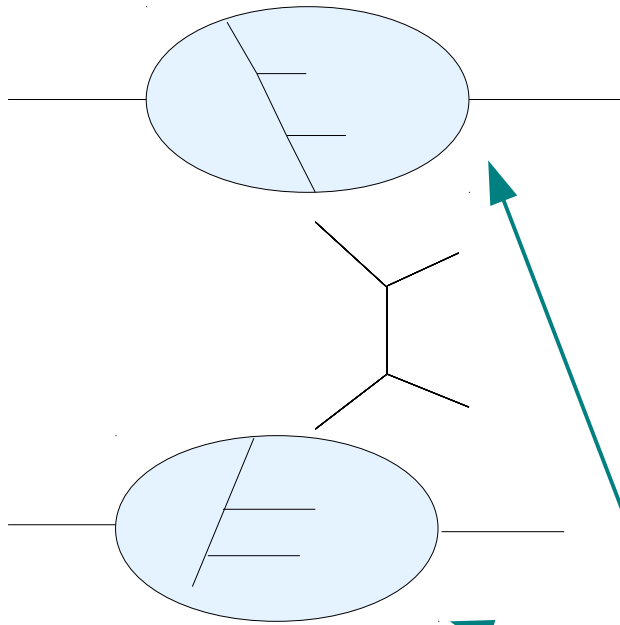


*Based on projects with:*

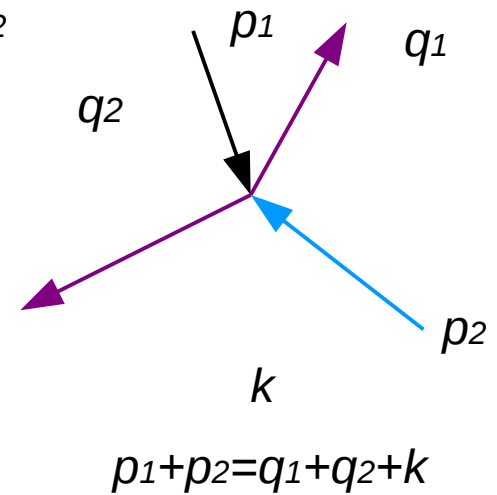
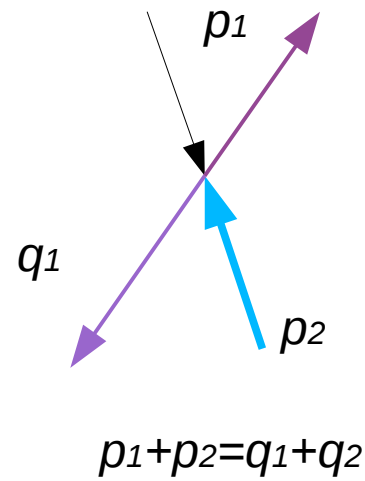
*E. Blanco, M. Deak, F. Hautmann, M. Hentschinski, A. Kusina, L. Keersmaekers, O. Lelek, P. Van Mechelen, A. Van Hameren, H. Van Haeuvermaet, H. Jung, M. Serino, P. Kotko*

*$k_T$  - factorization*

# QCD at high energies – $k_T$ factorization



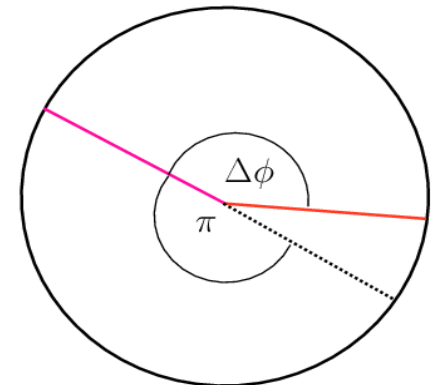
Strongly decreasing  
Longitudinal momentum  
fractions of off-shell partons



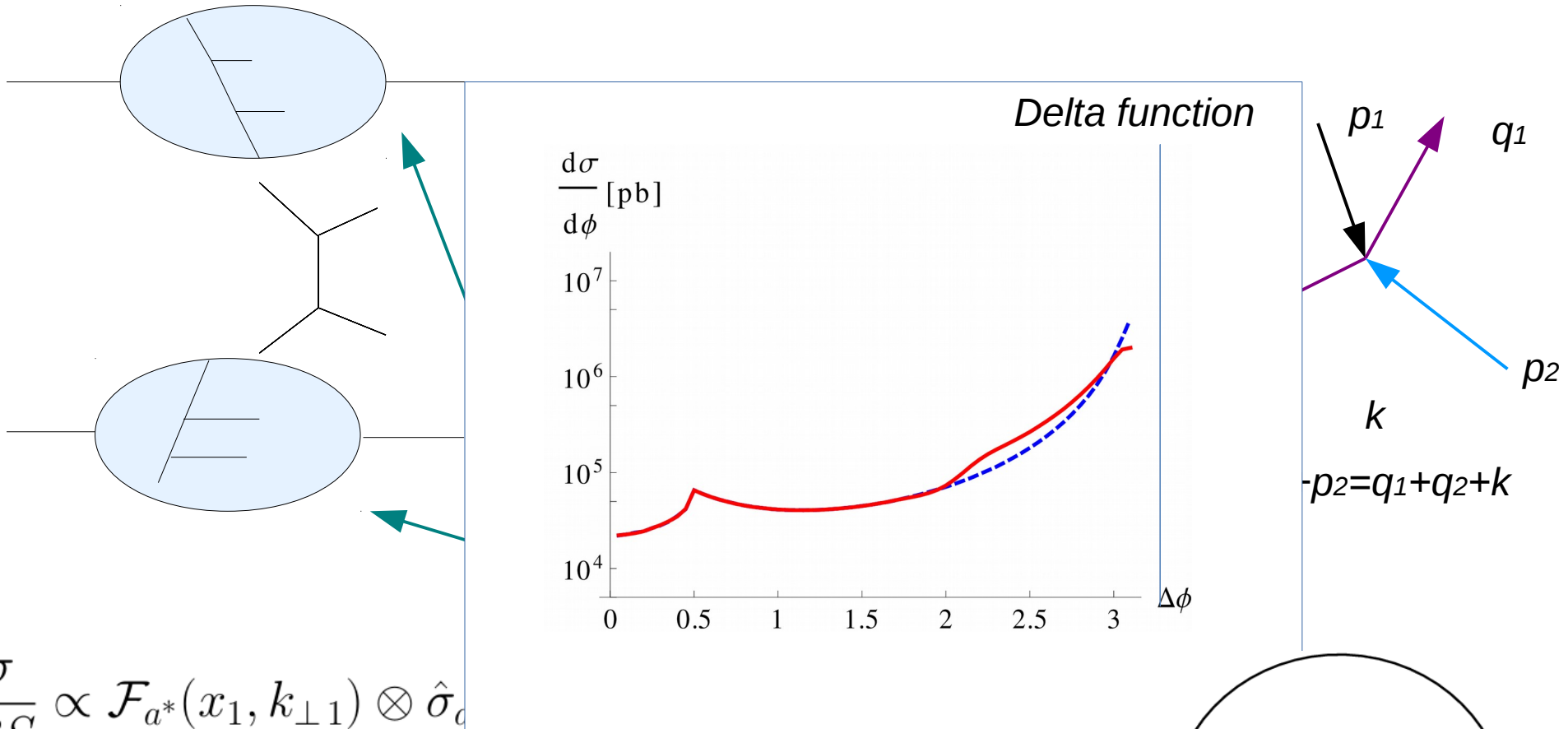
$$\frac{d\sigma}{dPS} \propto \mathcal{F}_{a^*}(x_1, k_{\perp 1}) \otimes \hat{\sigma}_{ab \rightarrow cd}(x_1, x_2) \otimes \mathcal{F}_{b^*}(x_2, k_{\perp 2})$$

Ciafaloni, Catani, Hautman '93  
Collins, Ellis '93

New helicity based methods for ME  
Kotko, K.K, van Hameren, '12



# QCD at high energies – $k_T$ factorization

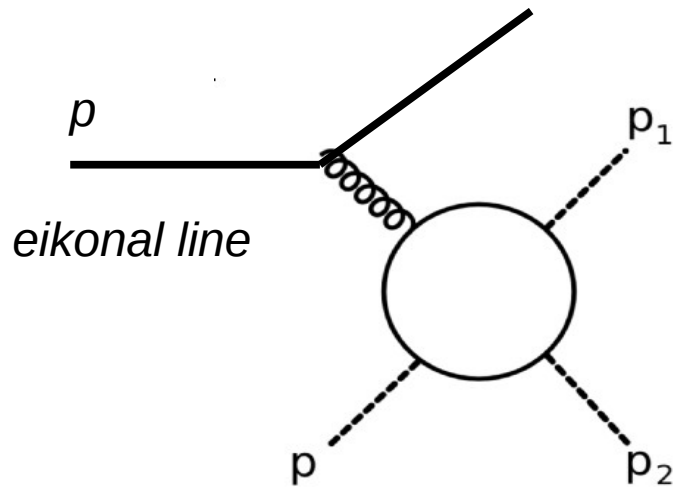


$$\frac{d\sigma}{dPS} \propto \mathcal{F}_{a^*}(x_1, k_{\perp 1}) \otimes \hat{\sigma}_d$$

Ciafaloni, Catani, Hautman '93  
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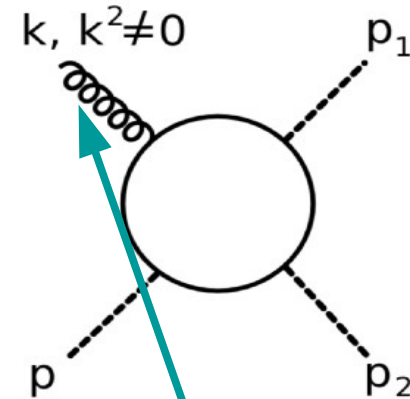
# Example of matrix element for hybrid factorization



Decomposition  
of polarization sum

$$g_{\mu\nu} = g_{\mu\nu\perp} + \frac{p_{A\mu}p_\nu + p_{A\nu}p_\mu}{s}$$

$$k = x p_A + k_T$$



$$\epsilon_\mu^0 = \frac{i\sqrt{2}x}{|k_t|} p_{A\mu}$$

Polarization sum  
for onshell gluons

$$\sum_{\lambda=\pm} \epsilon_\mu^\lambda \epsilon_\nu^{\lambda*} = g_{\mu\nu} - \frac{p_{A\mu}q_\nu + q_\mu p_{A\nu}}{q^\rho p_{A\rho}}$$

Polarization of off-shell gluon  
One applies usual methods  
but polarization summ deiffers

Effective action based approach  
*Lipatov 95, Lipatov, Vyazovsky 2000*

Gauge link based derivation  
*Kotko'14*

# Hard coefficient functions in kt factorization:

One consider embedding off-shell amplitude in on-shell and introduces eikonal lines

Kotko, KK, van Hameren 2013,  
KK, Salwa, van Hameren 2013

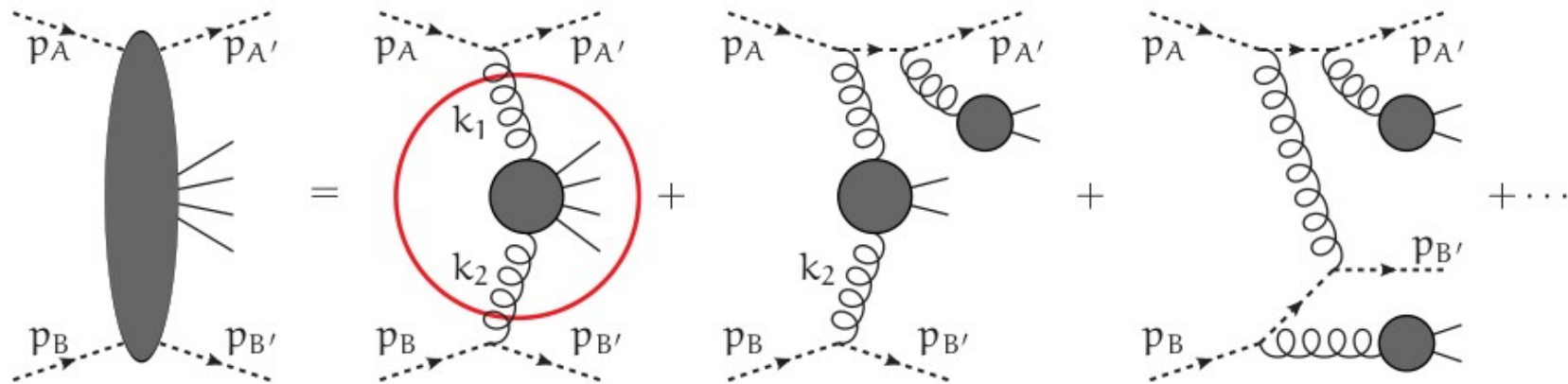
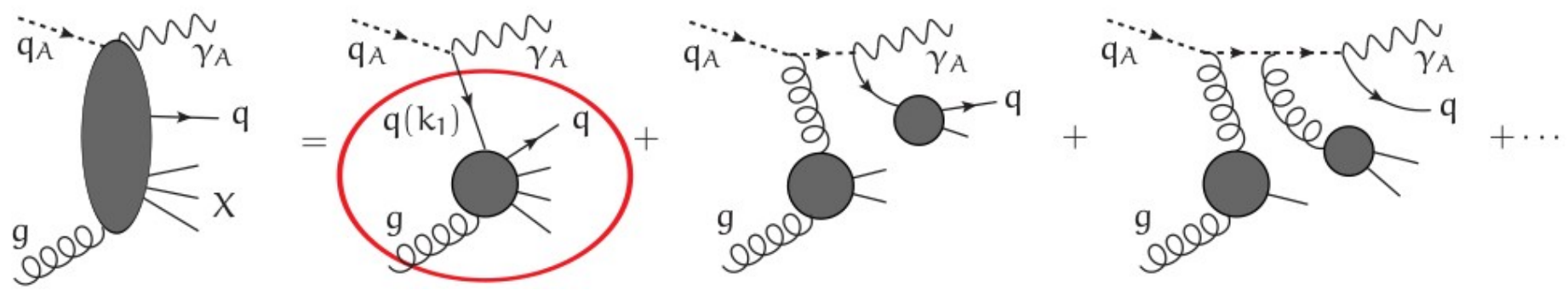


Diagram showing a fermion eikonal line (wavy line) connecting vertex  $j$  to vertex  $i$ . The equation is:
 
$$= -i \delta_{i,j} u(p_1)$$

Diagram showing a gluon eikonal line (coiled line) connecting vertex  $j$  to vertex  $i$ . The equation is:
 
$$= -i T_{i,j}^a p_1^\mu$$

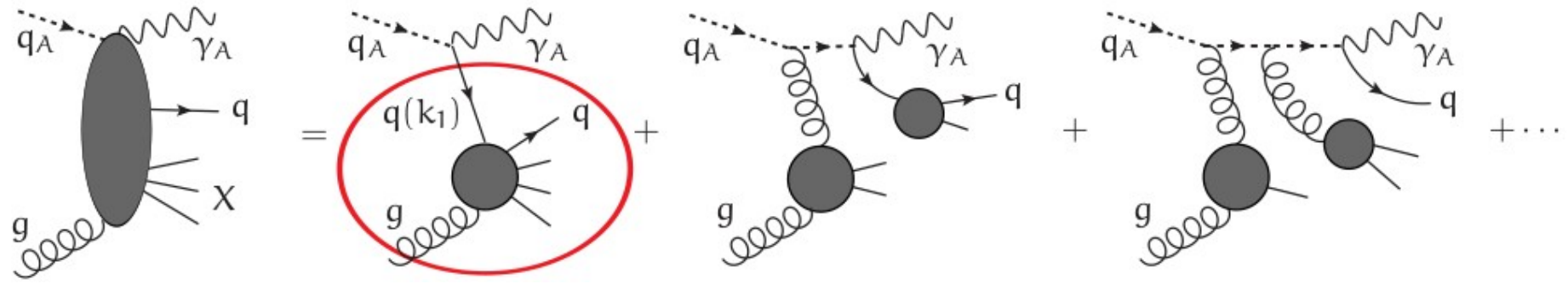
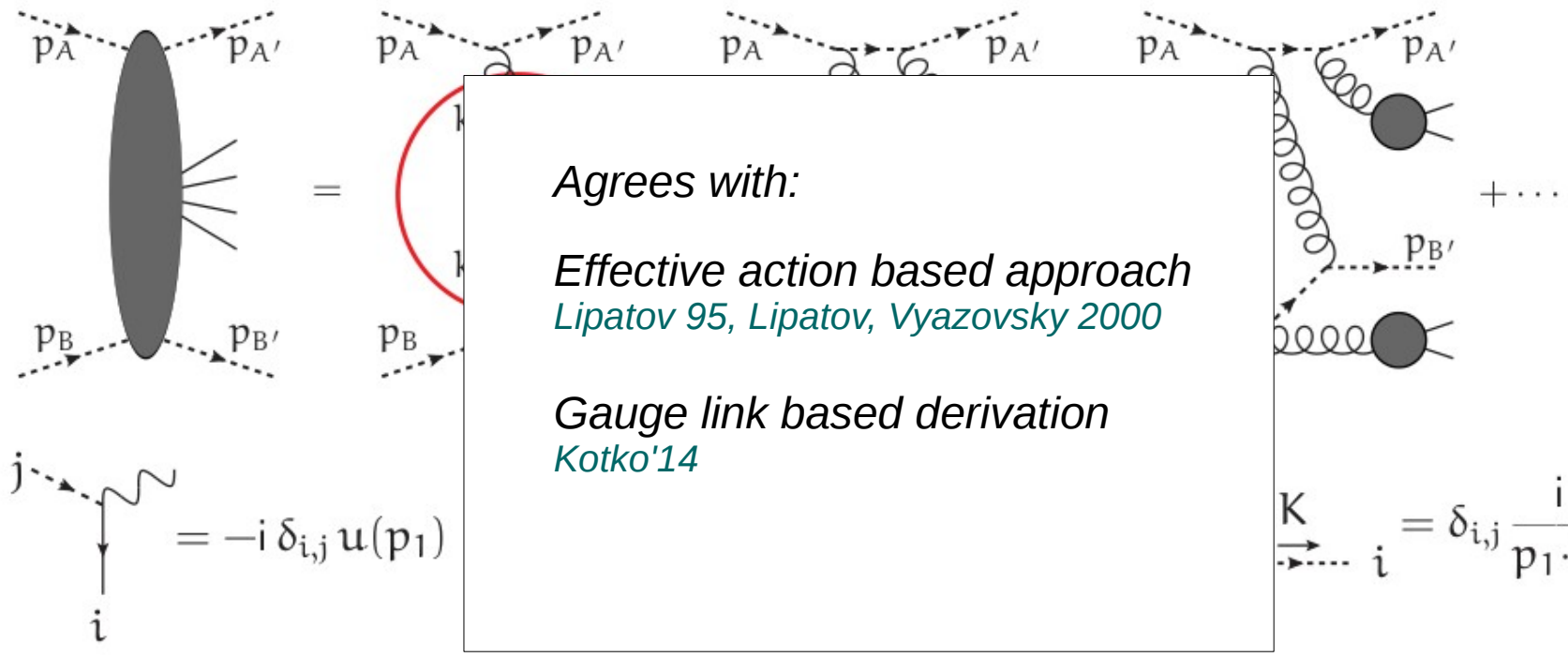
Diagram showing a scalar eikonal line (dashed line) connecting vertex  $j$  to vertex  $i$ . The equation is:
 
$$= \delta_{i,j} \frac{i}{p_1 \cdot K}$$



# Hard coefficient functions in HEF:

Kotko, KK, van Hameren 2013,  
KK, Salwa, van Hameren 2013

Implemented in KaTie Monte Carlo

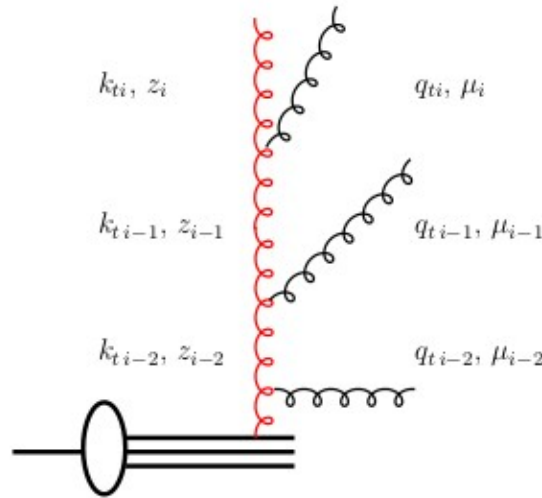


# Monte Carlo – parton branching method

The idea: construct such parton shower that gives also TMD dependent parton density.  
On integrated level the pdf obeys DGLAP equation.

A. Bermudez Martinez, P. Connor, H. Jung, A. Lelek, R. Žlebčík, F. Hautmann, V. Radescu  
Phys.Rev. D99 (2019) no.7, 074008

See talks by  
Bermudez Martinez  
S. Taheri



$$\mathcal{A}_a(x, \mathbf{k}, \mu^2) = \Delta_a(\mu^2) \mathcal{A}_a(x, \mathbf{k}, \mu_0^2) + \sum_b \int \frac{d^z \mathbf{q}'}{\pi \mathbf{q}'^2} \frac{\Delta_a(\mu^z)}{\Delta_a(\mathbf{q}'^2)} \Theta(\mu^2 - \mathbf{q}'^2) \Theta(\mathbf{q}'^2 - \mu_0^2) \\ \times \int_x^{z_M} \frac{dz}{z} P_{ab}^{(R)}(\alpha_s, z) \mathcal{A}_b\left(\frac{x}{z}, \mathbf{k} + (1-z)\mathbf{q}', \mathbf{q}'^2\right)$$

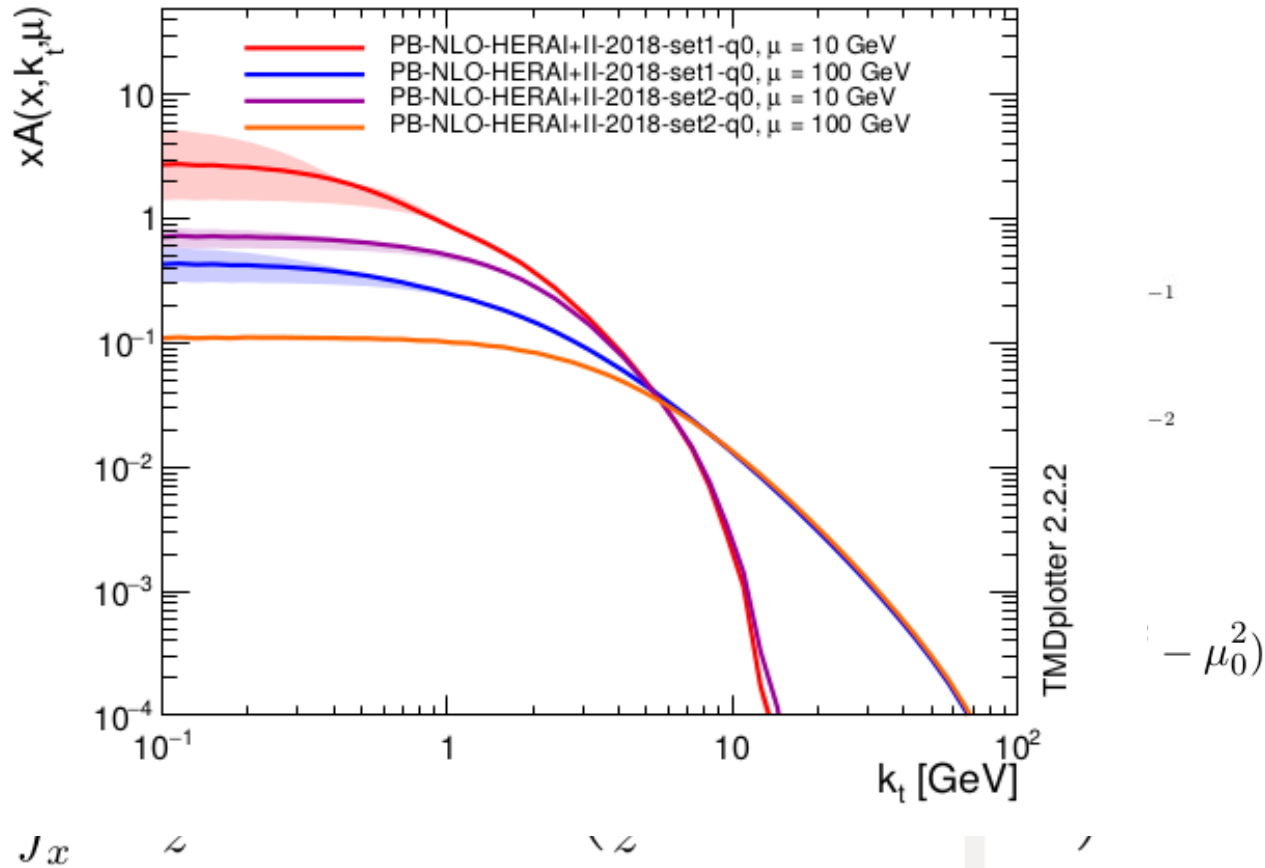
The method provides first consistent and complete set of TMD's which are applicable in Monte Carlo simulations and can be generalized to account for small x effects at least in linear regime. Applies in the regime above saturation scale.



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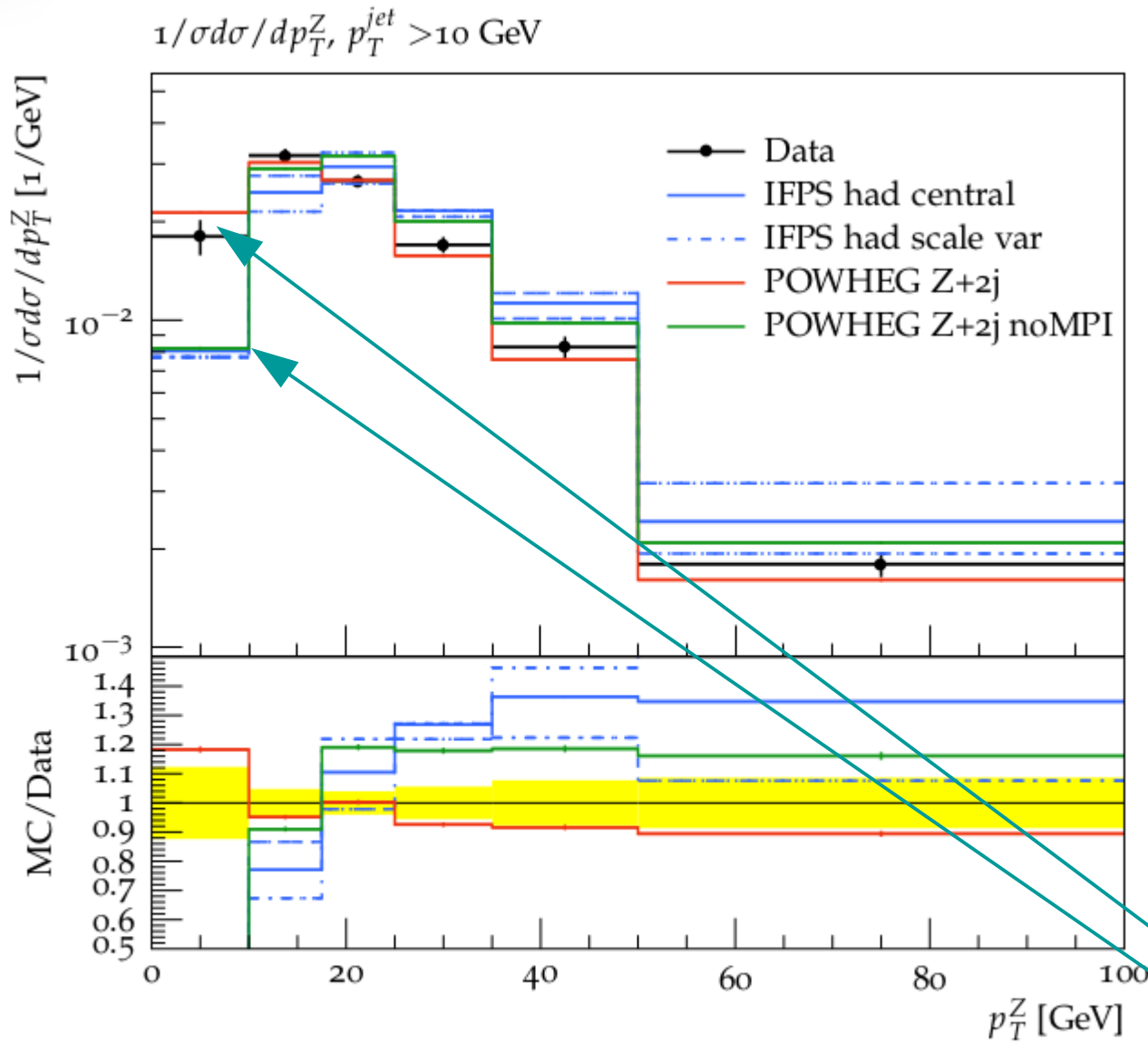
gluon,  $x = 0.01$



The method provides first consistent and complete set of TMD's which are applicable in Monte Carlo simulations and can be generalized to account for small  $x$  effects at least in linear regime. Applies in the regime above saturation scale.

# Example: Z+jet: $p_T$ of Z

van Hameren, Deak, Jung, Kusina, Kutak, M. Serino '18



*Simulation in  
KaTie+ CASCADE*

*Parton Branching  
pdfs used*

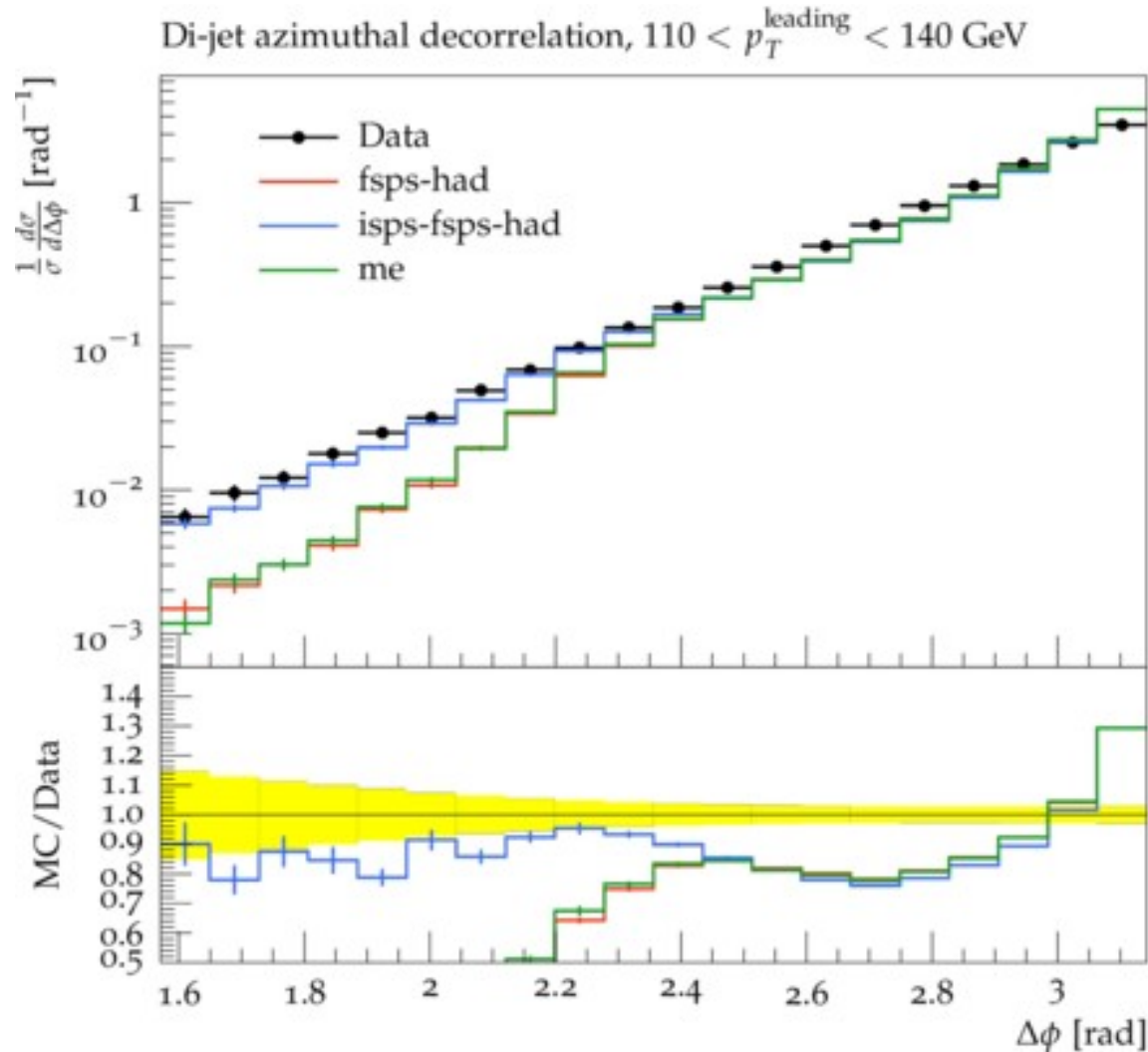
*Together with  
off-shell ME  
as obtained  
from*

*KaTie  
ME generator*

*Large contribution  
from MPI in  
POWHEG*

# Example- dijets- azimuthal angle correlations – central region

M. Bury, A. van Hameren, H. Jung, KK, S. Sapeta, M. Serino '17



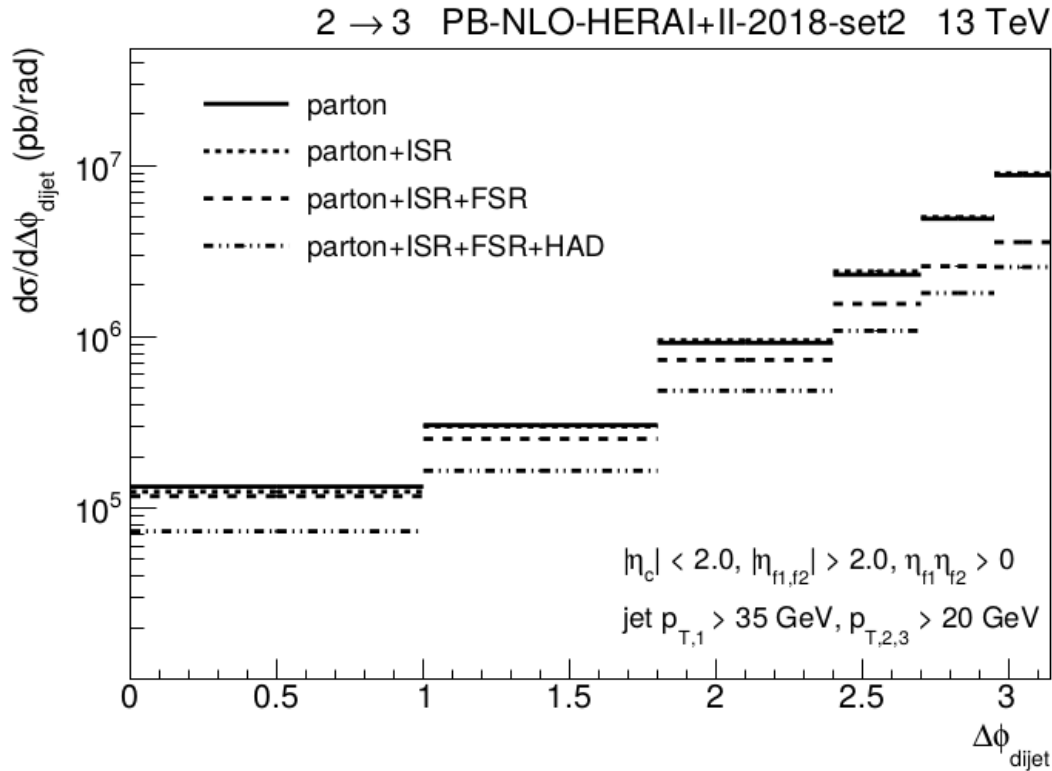
Simulation in  
KaTie+ CASCADE

KMR  
pdfs used  
unintegrated  
DGLAP based

Together with  
off-shell ME

as obtained  
from KaTie  
ME generator

# Decorelations 3 jets forward-central

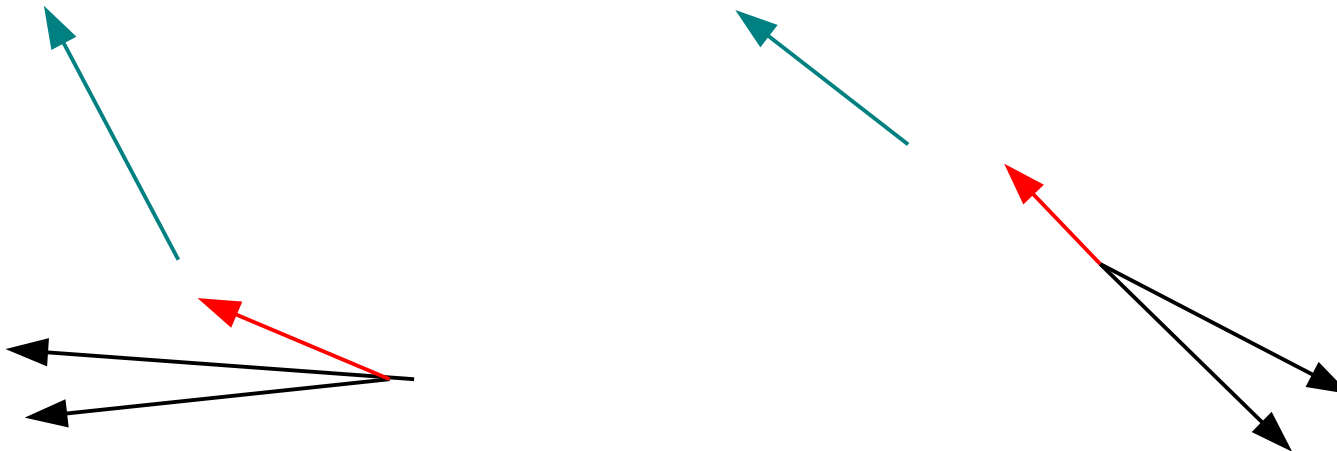


H. Van Haeve, Van Hameren, Kotko, K. Kutak, Van Mechelen '20

Results obtained within *kt* factorization calculations + ISR are on top on *kt* factorization without ISR used Parton Branching TMDs

Hautmann, H. Jung, A. Lelek, V. Radescu and R. Zlebcik '18

Bermudez Martinez, P. Connor, H. Jung, A. Lelek, R. Zlebcik, F. Hautmann '19



## *Parton branching method for lead unintegrated pdfs*

$$x \mathcal{A}_a^{\text{Pb}}(x, k_t^2, \mu^2) = \int dx' \mathcal{A}_{0,b}^{\text{Pb}}(x', k_{t,0}^2, \mu_0^2) \frac{x}{x'} \mathcal{K}_{ba} \left( \frac{x}{x'}, k_{t,0}^2, k_t^2, \mu_0^2, \mu^2 \right)$$

$$\mathcal{A}_{0,b}^{\text{Pb}}(x, k_{t,0}^2, \mu_0^2) = f_{0,b}^{\text{Pb}}(x, \mu_0^2) \cdot \exp(-|k_{t,0}^2|/\sigma^2)$$

$$\sigma^2 = q_0^2/2$$

$$q_0 = 0.5 \text{ GeV}$$

*Momentum conservation → exact kinematics*

*The splitting function → NLO DGLAP*

*The shower is consistent with the transversal momentum dependent distribution it uses*

# Available collinear nuclear PDFs

## Multiplicative correction factor

$$f_i^{p/A}(x_N, \mu_0) = R_i(x_N, \mu_0, A) f_i^{\text{free proton}}(x_N, \mu_0)$$

**HKN:** Hirai, Kumano, Nagai [[PRC 76, 065207 \(2007\)](#)]

**DSSZ:** de Florian, Sassot, Stratmann, Zurita [[PRD 85, 074028 \(2012\)](#)]

**EPS09:** Eskola, Paukkunen, Salgado [[JHEP 04 \(2009\) 065](#)]

**EPPS16:** Eskola, Paakkinen, Paukkunen, Salgado [[EPJC 77 \(2017\) 163](#)]

**KT16:** Khanpour, Tehrani [[PRD 93, 014026 \(2016\)](#)]

## Native nuclear PDFs

$$f_i^{p/A}(x_N, \mu_0) = f_i(x_N, A, \mu_0)$$
$$f_i(x_N, A = 1, \mu_0) \equiv f_i^{\text{free proton}}(x_N, \mu_0)$$

**nCTEQ15:** Kovarik, Kusina, Jezo, Clark, Keppel, Lyonnet, Morfin, Olness, Owens, Schienbein, Yu [[PRD 93, 085037 \(2016\)](#), [arXiv:1509.00792](#)]

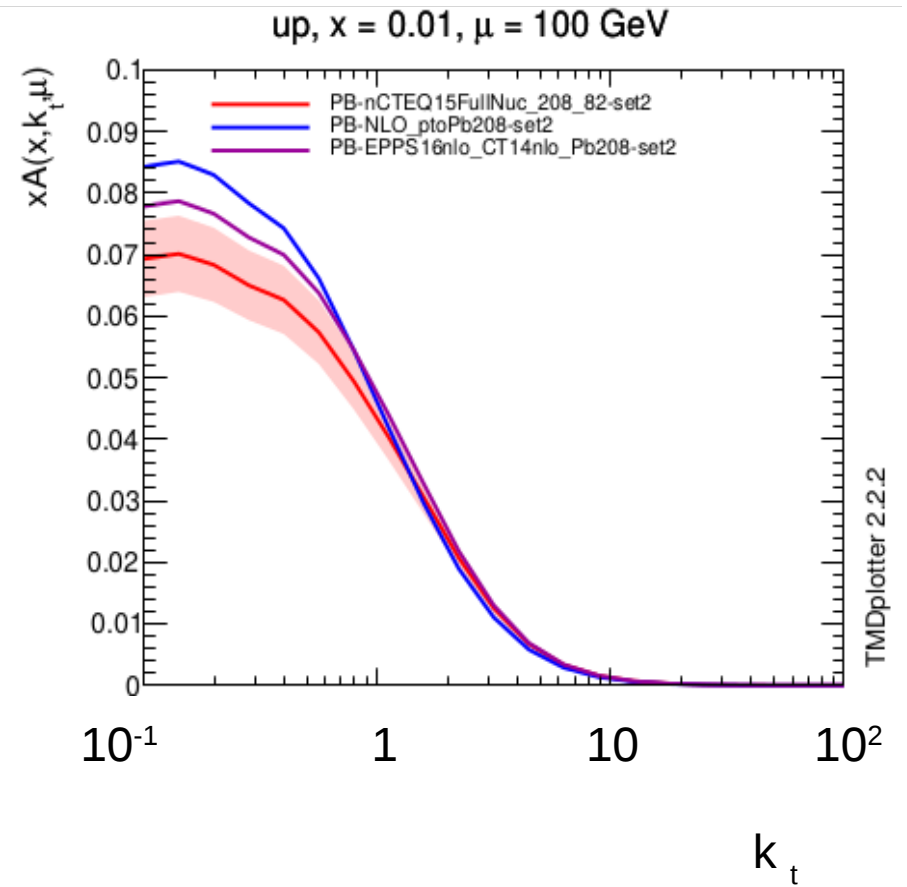
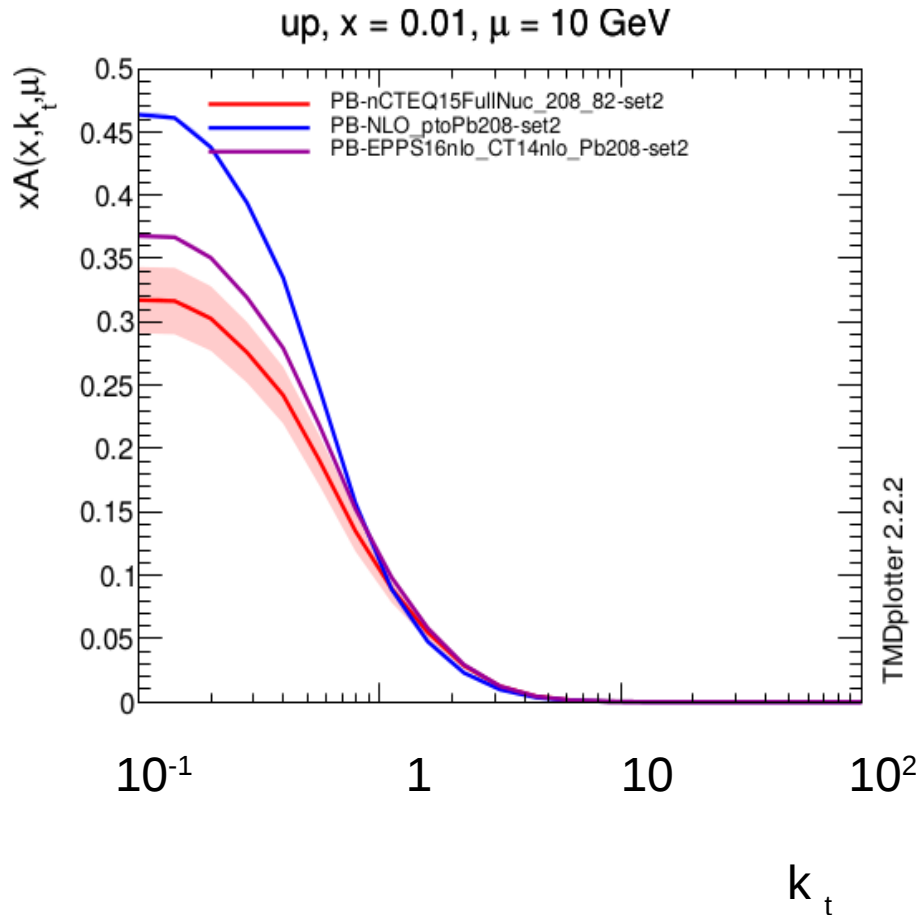
**NNPDF** Khalek, Ethier, Rojo [arXiv:1904.00018](#)

**WHV** Marina Walt, Ilkka Helenius, Werner Vogelsang [arXiv:1908.03355](#)

*See also talks by*  
*V. Guzey*  
*M. Zurita*

# Transverse momentum dependence

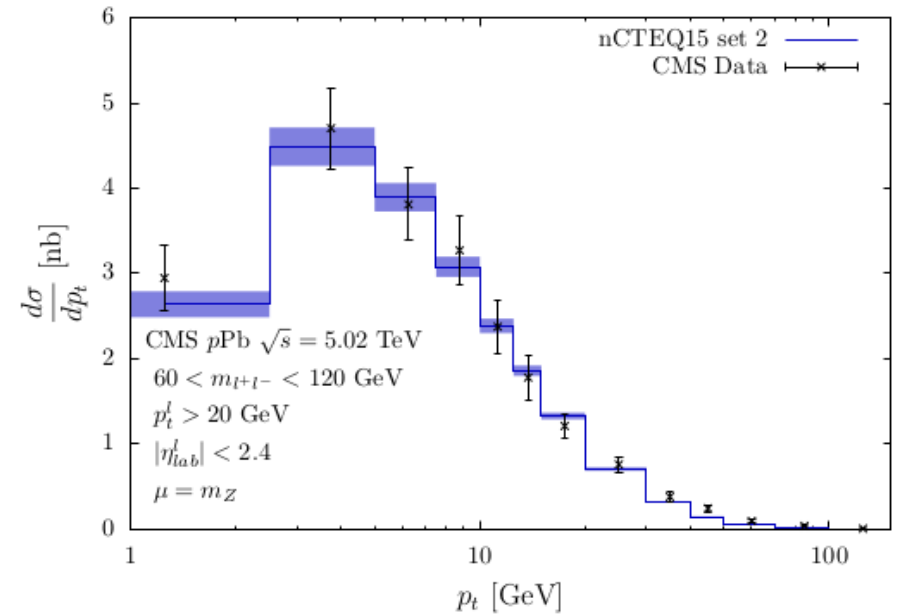
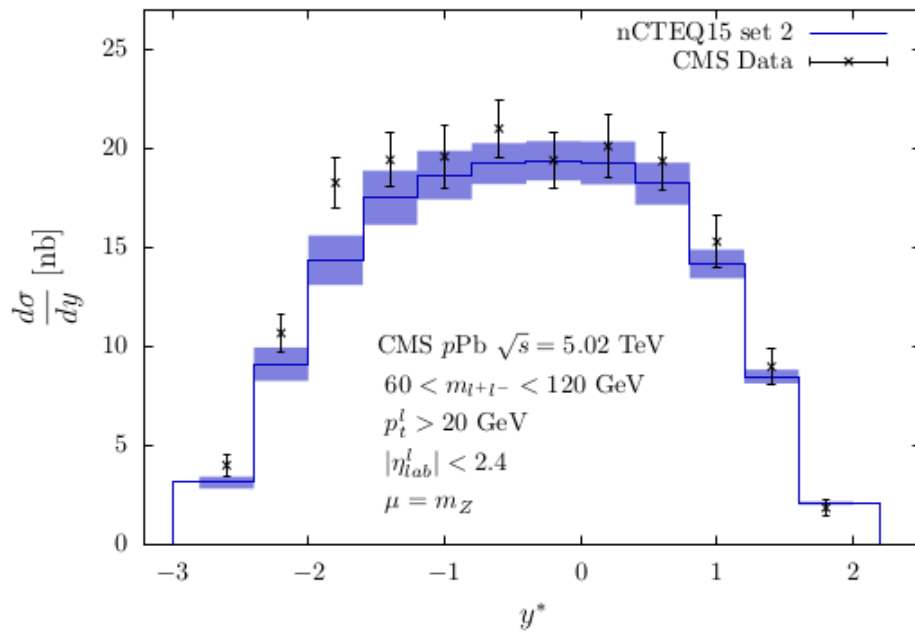
Blanco, Jung, Van Hameren, Kusina, Kutak '19



# Drell-Yan – with PDF uncertainty

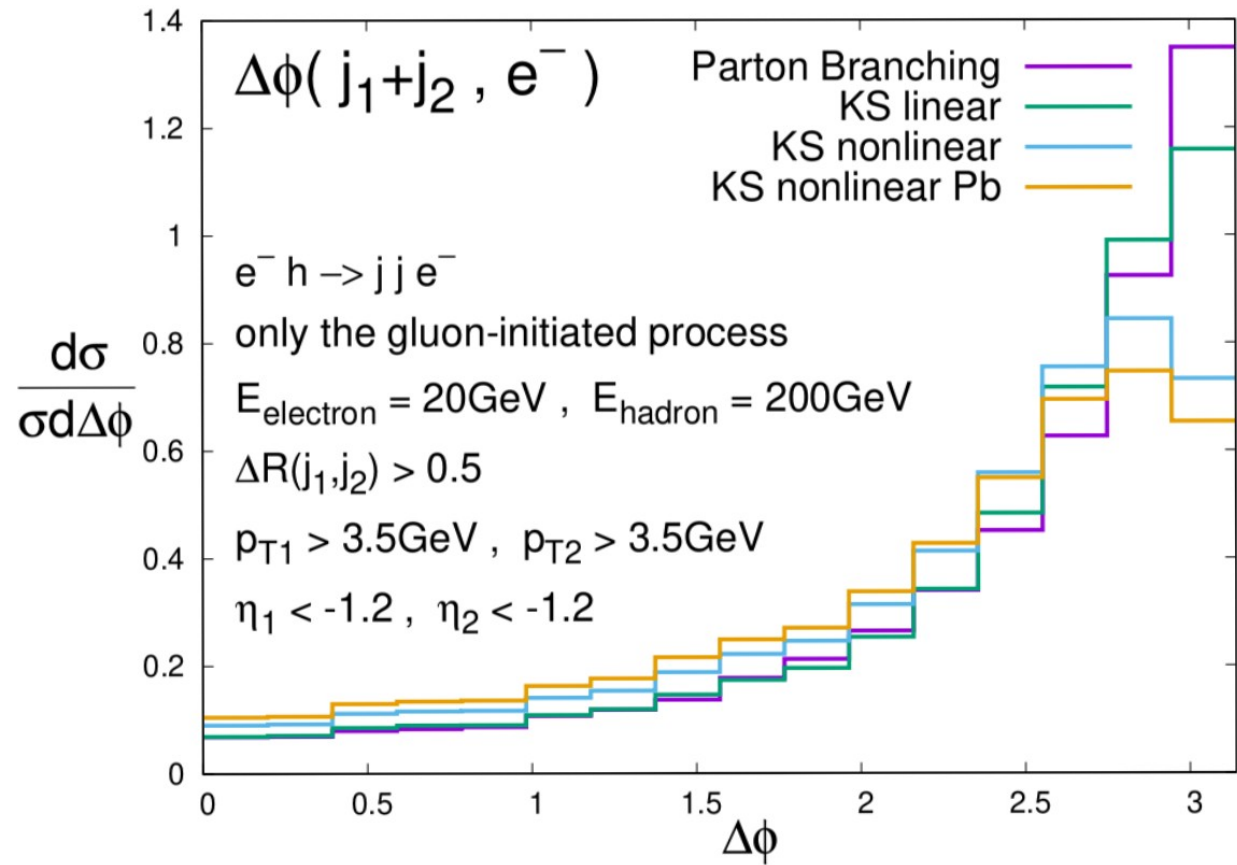
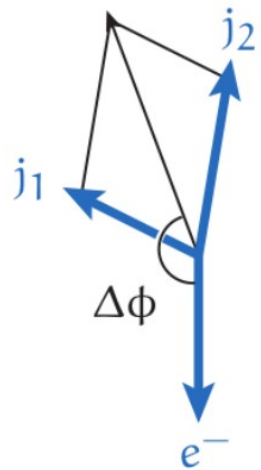
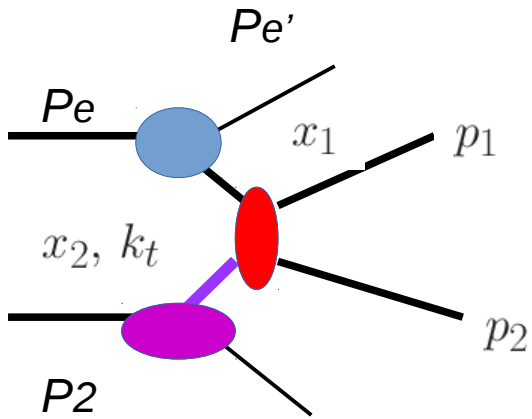
Blanco, Jung, Van Hameren, Kusina, Kutak '19

$$p + Pb \rightarrow Z^* \rightarrow \mu^+ + \mu^-$$





# Preliminary: towards DIS in Electron Ion Collider



from A. Van Hameren

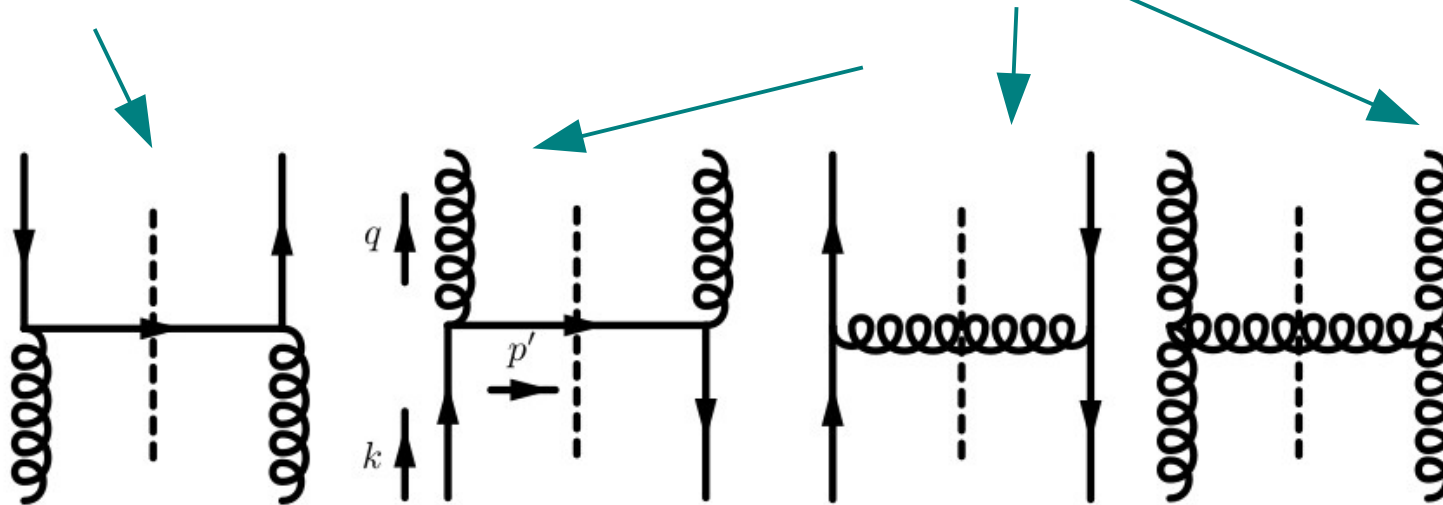
# Kernels with $kt$ dependent splittings

In the collinear physics based Monte Carlo's the  $k_T$  dependence is generated by kinematics. However, by using low  $x$  methods one can obtain kernels that depend on transverse momentum.

Real emission kernels obtained in

Catani and Hautmann '94

O. Gituliar, M. Hentschinski, K.K; JHEP 1601 (2016) 181  
Hentschinski, Kusina, KK, Serino '17



At present we have a conjecture for complete kernels.  
However, the results distributions are still to be obtained.

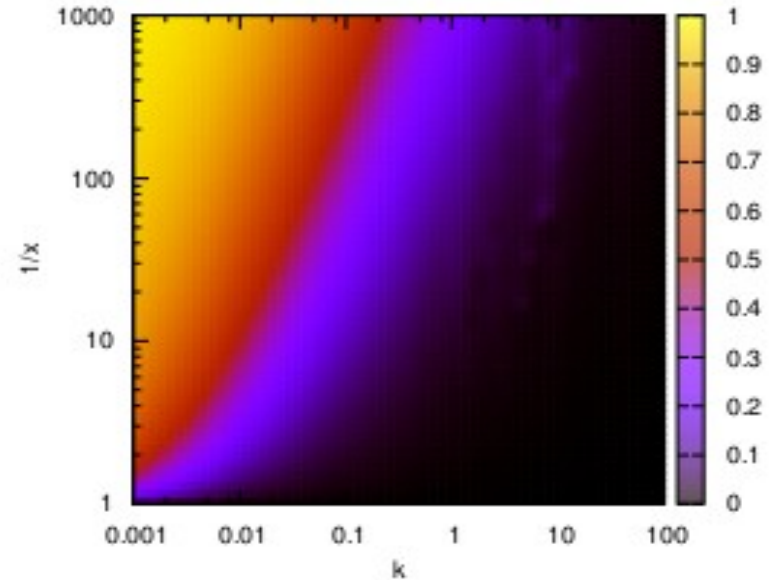
*Saturation*

# Saturation

**Saturation** – state where number of gluons stops growing due to high occupation number. Way to fulfill unitarity requirements in high energy limit of QCD.

L.V. Gribov, E.M. Levin, M.G. Ryskin  
*Phys.Rept.* 100 (1983) 1-150

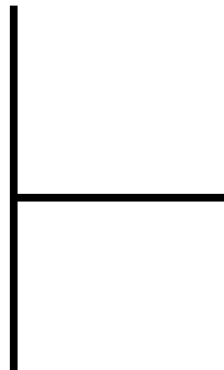
Larry D. McLerran, Raju Venugopalan  
*Phys.Rev. D*49 (1994) 3352-3355



On microscopic level it means that gluon apart splitting recombine

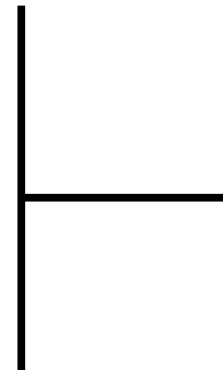
*splitting*

Linear evolution equation  
 BFKL

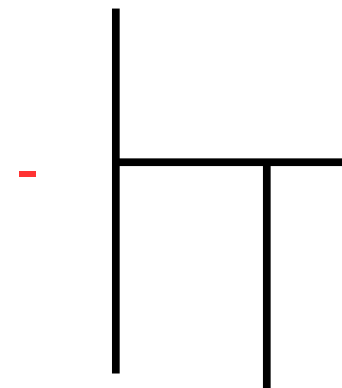


*Nonlinear evolution equations*

*splitting*

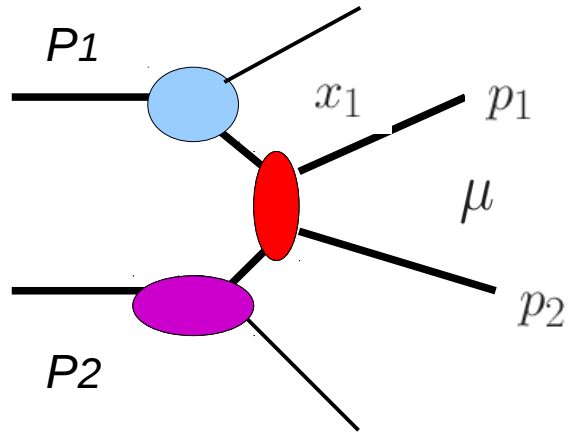


*recombination*

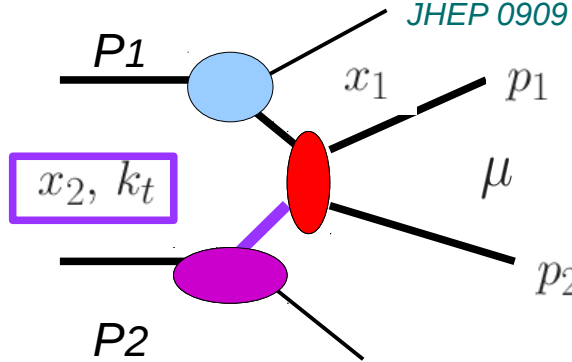


# Improved Transversal Momentum Dependent Factorization

$$\frac{d\sigma_{\text{SPS}}^{P_1 P_2 \rightarrow \text{dijets} + X}}{dy_1 dy_2 dp_{1t} dp_{2t} d\Delta\phi} = \frac{p_{1t} p_{2t}}{8\pi^2 (x_1 x_2 s)^2} \sum_{a,c,d} x_1 f_{a/P_1}(x_1, \mu^2) |\overline{\mathcal{M}}_{ag^* \rightarrow cd}|^2 \mathcal{F}_{g/P_2}(x_2, k_t^2) \frac{1}{1 + \delta_{cd}}$$



A, Dumitru, A. Hayashigaki J. Jalilian-Marian  
Nucl.Phys. A765 (2006) 464-482  
M. Deak, F. Hautmann, H. Jung, K. Kutak  
JHEP 0909 (2009) 121



Using HEF motivated sum over polarization  
for low x gluons we included  $k_t$  in ME

Conjecture P. Kotko K. Kutak, C. Marquet, E. Petreska, S. Sapeta,  
A. van Hameren, JHEP 1509 (2015) 106  
Appropriate in any configuration

Can be obtained from CGC

T. Altinoluk, R. Boussarie, Piotr Kotko JHEP 1905 (2019) 156

Generalization of hybrid formula but no  $k_t$  in ME

Fabio Dominguez, Bo-Wen Xiao, Feng Yuan  
Phys.Rev.Lett. 106 (2011) 022301

F. Dominguez, C. Marquet, Bo-Wen Xiao, F. Yuan  
Phys.Rev. D83 (2011) 105005

Appropriate in back-to-back configuration

gauge invariant amplitudes with  $k_t$  and TMDs

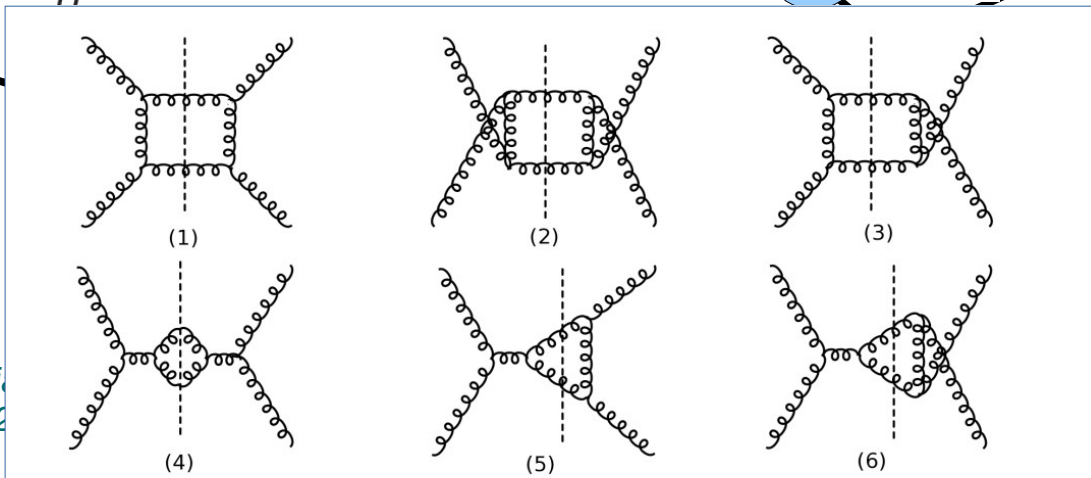
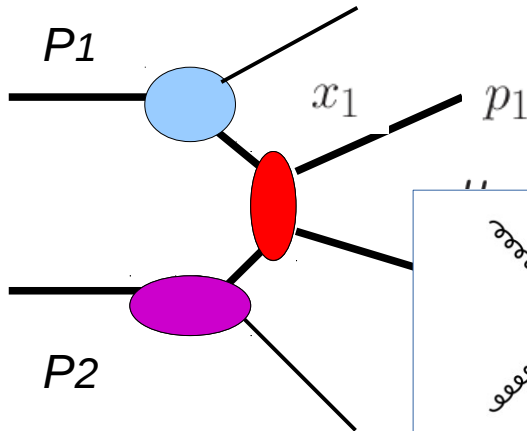
Example for  $g^* g \rightarrow g g$

$$\frac{d\sigma^{pA \rightarrow ggX}}{d^2 P_t d^2 k_t dy_1 dy_2} = \frac{\alpha_s^2}{(x_1 x_2 s)^2} x_1 f_{g/p}(x_1, \mu^2) \sum_{i=1}^6 \mathcal{F}_{gg}^{(i)} H_{gg \rightarrow gg}^{(i)}$$

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A, Dumitru, A. Hayashigaki J. Jalilian-Marian  
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 F. Hautmann, H. Jung, K. Kutak  
 JHEP 0909 (2009) 121



$p_2$   
 or polarization  
 ct in ME

Generalization of hybrid  
 ME

Fabio Dominguez, Bo-Wen Xi  
 Phys.Rev.Lett. 106 (2011) 022

F. Dominguez, C. Marquet, Bo  
 Phys.Rev. D83 (2011) 105005

Appropriate in back-to-back configuration

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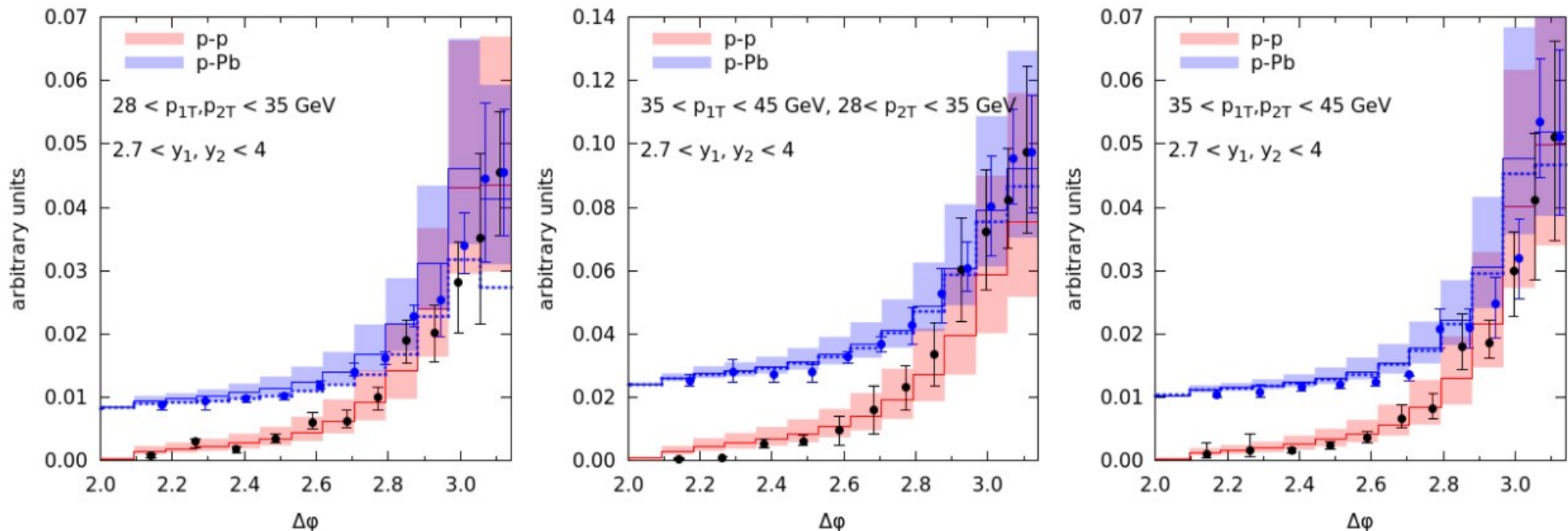
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# Signature of broadening in forward-forward dijets

ATLAS Phys.Rev. C100 (2019) no.3, 034903

A. Hameren, P. Kotko, K. Kutak, S. Sapeta  
Phys.Lett. B795 (2019) 511-515



Data: number of dijets normalized to number of single inclusive jets. We can not calculate that. We can compare shapes.

Procedure: fit normalization to p-p data.

Use that both for p-p and p-Pb. Shift p-Pb data

The procedure allows for visualization of broadening

# Summary

- *New TMD introduced: i.e. nTMD's using PB method*
- *Good description of data within kt factorization*
- *Parton branching shower works well*
- *Some steps towards generalization of splittings done*
- *Preliminary results for DIS*
-