Soft-gluon effective coupling

Daniel de Florian
ICAS - UNSAM
Argentina

S.Catani, D.deF., M.Grazzini

+S.Devoto and J.Mazzitelli (in preparation)

ICHEP 2020

Outline

- Soft-gluon effective coupling introduction
- NLO
- NNLO and beyond
- Conformal relation (all orders)
- Conclusions

- Hard scattering observables sensitive to soft-gluon effects

  - originate from boundaries of phase space
  - real-radiation strongly suppressed : unbalance virtual-radiation
  - generate large logarithmic corrections $\alpha_s^n L^{2n}$

low transverse momentum $\log \frac{q_T}{Q}$

threshold $\log \left(1 - \frac{Q^2}{\hat{s}}\right)$

- Need to be resumed to some logarithmic accuracy to improve convergence of perturbative expansion

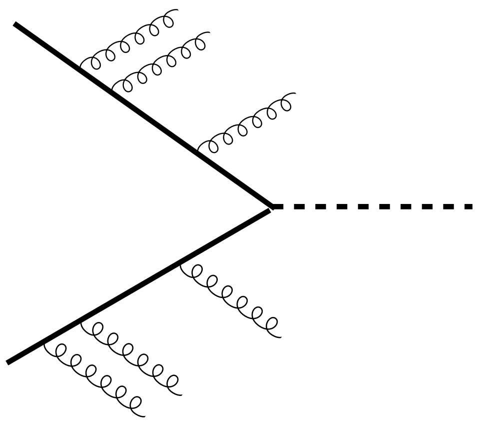
Threshold resummation (invariant mass M)

$c\bar{c} \rightarrow F$

DY $q\bar{q}$

Higgs gg

- resummed partonic cross-section in Mellin space (N)



$$\hat{\sigma}^{F(\mathrm{res})}_{i\bar{i},N}\left(M^2;\alpha_S\left(M^2\right)\right) = \sigma^{(0)}_{i\bar{i}\to F}\left(M^2;\alpha_S\left(M^2\right)\right) C^{\mathrm{th}}_{i\bar{i}\to F}\left(\alpha_S\left(M^2\right)\right)\Delta_{i,N}\left(M^2\right)$$

Sudakov form factor

$$\Delta_{i,N}\left(M^2\right) = \exp\left\{\int_0^1 dz\,\frac{z^{N-1}-1}{1-z}\left[2\int_{\mu_F^2}^{(1-z)^2M^2}\frac{dq^2}{q^2}A_i\left(\alpha_S\left(q^2\right)\right) + D_i\left(\alpha_S\left((1-z)^2M^2\right)\right)\right]\right\}$$

- process independent (and free of logs)

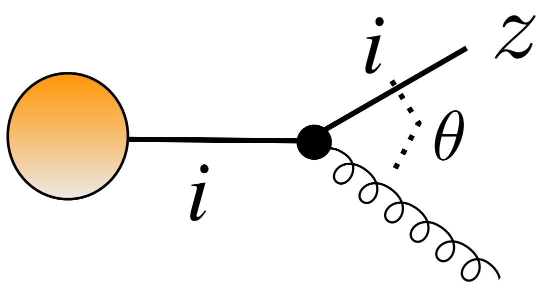
$$P_{ii}\left(\alpha_S;z\right) = \frac{1}{1-z}A_i\left(\alpha_S\right) + \dots$$

$$A_i\left(\alpha_S\right) = \sum_{n=1}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n A_i^{(n)}$$ soft-collinear emission

$$D_i\left(\alpha_S\right) = \sum_{n=2}^{\infty}\left(\frac{\alpha_S}{\pi}\right)^n D_i^{(n)}$$ soft non-collinear radiation

soft limit of splitting function
cusp anomalous dimension

- Another way to resum large logs (and more) → Monte Carlo



collinear branching driven by splitting function

$$dw_i^{DL} = \frac{\alpha_S}{2\pi} P_{ii}(z)\, dz \frac{d\theta^2}{\theta^2} \simeq C_i \frac{\alpha_S}{\pi} \frac{dz}{1-z} \frac{dq_T^2}{q_T^2}$$

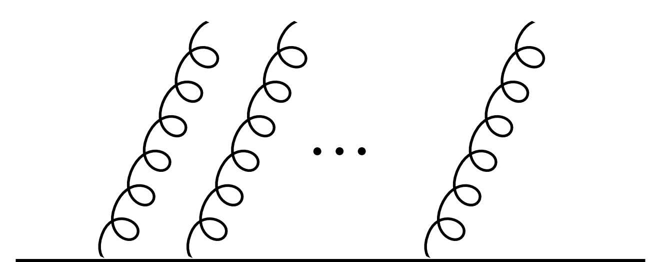
soft and collinear emission (DL accuracy)

- Intensity of soft-gluon radiation at LO given by $C_i \dfrac{\alpha_S}{\pi}$ $\quad C_i = C_F(q), C_A(g)$

- The resummation of soft-collinear terms at LL achieved by

$$C_i \frac{\alpha_S}{\pi} \to C_i \frac{\alpha_S\left(q_T^2\right)}{\pi}$$

coupling evaluated at q_T (resum)



- The resummation of soft-collinear terms at NLL achieved by (MC@NLL)

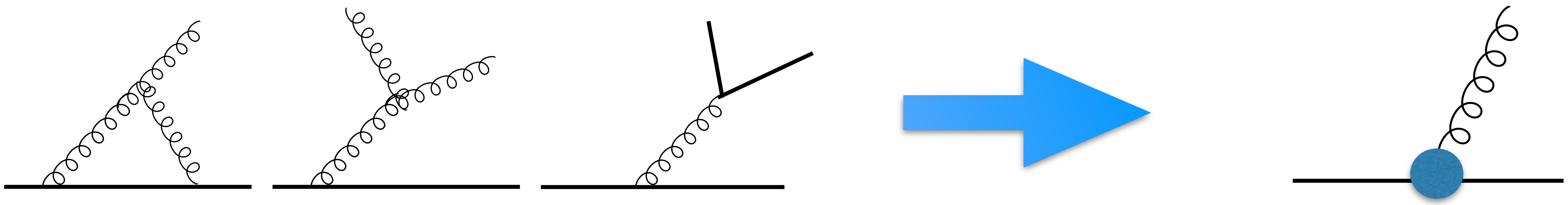
$$C_i \frac{\alpha_S}{\pi} \to \mathcal{A}_i^{CMW}\left(\alpha_S\left(q_T^2\right)\right) = C_i \frac{\alpha_S^{CMW}\left(q_T^2\right)}{\pi} = C_i \frac{\alpha_S\left(q_T^2\right)}{\pi}\left(1 + \frac{\alpha_S\left(q_T^2\right)}{2\pi} K\right)$$

Catani, Marchesini, Webber (1991)

soft effective coupling at NLL

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right) C_A - \frac{5}{9} n_F$$

NLL



- Up to 2-loops, the soft-gluon effective coupling is still given by the cusp anomalous dimension

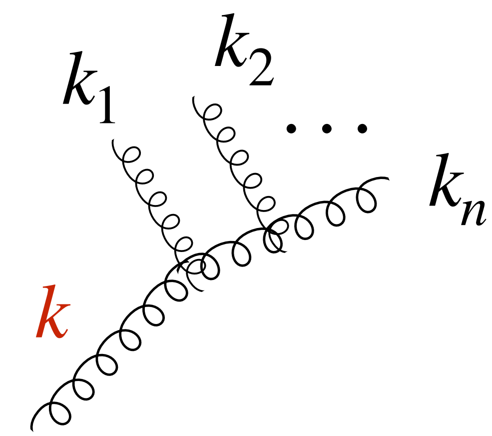
$$A_i^{(1)} = C_i, \quad A_i^{(2)} = \frac{1}{2} C_i \left[C_A \left(\frac{67}{18} - \frac{1}{6}\pi^2 \right) - \frac{5}{9} N_f \right] \equiv \frac{1}{2} C_i K$$

  - Higher orders of cusp known

- But, cusp=soft coupling beyond 2-loops?
  - All order definition for soft effective coupling

- all-order definition provided in terms of a *web* Banfi, El-Menoufi, Monni (2018)

$$w\left(k;\epsilon\right) = \sum_{n=1}^{\infty} \int \left(\prod_{i=1}^{n} \left[dk_i\right] \right) \tilde{M}_{\rm s}^2\left(k_1,\ldots,k_n\right)(2\pi)^d \delta^{(d)}\left(k - \sum_i k_i\right)$$



depends only on k_T and $m_T^2 = k_T^2 + k^2$ and finite

- given the two variables, propose two definitions for soft-coupling

$$\widetilde{\mathscr{A}}_{T,i}\left(\alpha_{\rm S}\left(\mu^2\right);\epsilon\right) = \frac{1}{2}\mu^2 \int_0^\infty dm_T^2\, dk_T^2\, \delta\left(\mu^2 - k_T^2\right) w_i(k;\epsilon)$$

defined at fixed value of k_T

Banfi, El-Menoufi, Monni (2018) suitable for q_T-related observables

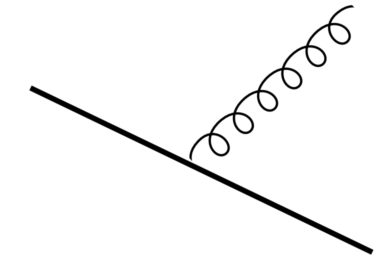
$$\widetilde{\mathscr{A}}_{0,i}\left(\alpha_{\rm S}\left(\mu^2\right);\epsilon\right) = \frac{1}{2}\mu^2 \int_0^\infty dm_T^2\, dk_T^2\, \delta\left(\mu^2 - m_T^2\right) w_i(k;\epsilon)$$

defined at fixed value of m_T

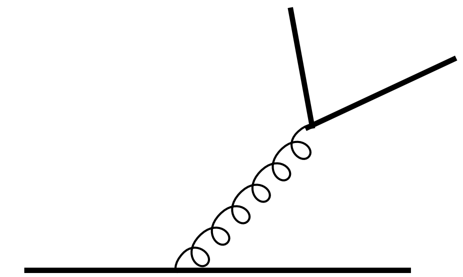
suitable for threshold-related observables

- can take limit $\epsilon \to 0$ to obtain the physical couplings: keep *D*-dimensional

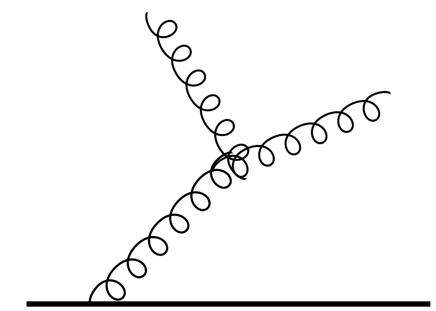
- at lowest order both couplings agree to all orders in ϵ



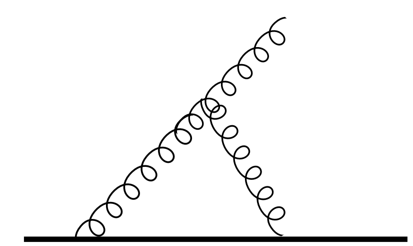
- we computed both couplings at α_s^2 (all orders in ϵ)



$$\widetilde{\mathscr{A}}^{(2)}_{T,i}(\epsilon) = A_i^{(2)} + \epsilon C_i \left[C_A \left(\frac{101}{27} - \frac{11\pi^2}{144} - \frac{7\zeta_3}{2} \right) + n_F \left(\frac{\pi^2}{72} - \frac{14}{27} \right) \right]$$
$$+\epsilon^2 C_i \left[C_A \left(\frac{607}{81} - \frac{67\pi^2}{216} - \frac{77\zeta_3}{36} - \frac{7\pi^4}{120} \right) + n_F \left(\frac{5\pi^2}{108} - \frac{82}{81} + \frac{7\zeta_3}{18} \right) \right] + \mathcal{O}\left(\epsilon^3\right)$$



$$\widetilde{\mathscr{A}}^{(2)}_{0,i}(\epsilon) = A_i^{(2)} + \epsilon C_i \left[C_A \left(\frac{101}{27} - \frac{55\pi^2}{144} - \frac{7\zeta_3}{2} \right) + n_F \left(\frac{5\pi^2}{72} - \frac{14}{27} \right) \right]$$
$$+\epsilon^2 C_i \left[C_A \left(\frac{607}{81} - \frac{67\pi^2}{72} - \frac{143\zeta_3}{36} - \frac{\pi^4}{36} \right) + n_F \left(\frac{5\pi^2}{36} - \frac{82}{81} + \frac{13\zeta_3}{18} \right) \right] + \mathcal{O}\left(\epsilon^3\right)$$



- agree with cusp anomalous dimension in 4 dimensions
  - ϵ terms different (phase space integration)

$$\Delta = \frac{\pi^2}{3}\pi\beta_0\epsilon + \ldots$$

Why interested in *D*-dimensional expression?

$$\widetilde{\mathscr{A}}_{T,i}\Big(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\Big)=\widetilde{\mathscr{A}}_{0,i}\Big(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\Big)=A_i\left(\alpha_S\right)$$

'D-dimensional' $\beta = 0$ conformal point $\epsilon = \beta(\alpha_s)$

in QCD $$\frac{d\ln\alpha_s(\mu^2)}{d\ln\mu^2}=-\epsilon+\beta(\alpha_s(\mu^2)) \qquad \beta=-(\beta_0\alpha_s+\beta_1\alpha_s^2+\ldots)$$

- Obtained from Sudakov (threshold resummation) using soft-coupling

Expanding conformal equation to third order one directly obtains

$$\mathscr{A}_i^{(3)}=A_i^{(3)}-\left(\beta_0\pi\right)^2\widetilde{\mathscr{A}}_i^{(1;2)}+\left(\beta_0\pi\right)\widetilde{\mathscr{A}}_i^{(2;1)}$$

3-loop soft effective coupling (4-*D*) is simply given by the cusp anomalous + ϵ^n terms to (*D*-dimensional) second order (different for each definition)

q_T resummation

$$\mathscr{A}_{T,i}^{(3)} = A_i^{(3)} + C_i\left(\beta_0\pi\right)^2\frac{\pi^2}{12} + C_i\left(\beta_0\pi\right)\left[C_A\left(\frac{101}{27}-\frac{11\pi^2}{144}-\frac{7\zeta_3}{2}\right)+n_F\left(\frac{\pi^2}{72}-\frac{14}{27}\right)\right]$$

- Agrees with previous result and 3rd order coefficient for q_T resummation

Banfi, El-Menoufi, Monni (2018) — Becher, Neubert (2011)

Threshold resummation (new!)

$$\mathscr{A}_{0,i}^{(3)} = A_i^{(3)} + C_i\left(\beta_0\pi\right)^2\frac{\pi^2}{12} + C_i\left(\beta_0\pi\right)\left[C_A\left(\frac{101}{27}-\frac{55\pi^2}{144}-\frac{7\zeta_3}{2}\right)+n_F\left(\frac{5\pi^2}{72}-\frac{14}{27}\right)\right]$$

- Provides the full Sudakov factor for threshold resummation at 3rd order
- Both soft-collinear and soft non-collinear contributions (DY-like)

$$\Delta = C_i\frac{\pi^2}{3}(\pi\beta_0)^2$$

- Comparing to Sudakov form factor at 4th order (threshold resummation)
- 4th order soft effective coupling (threshold related)

$$\mathscr{A}_{0,i}(\alpha_S) = C_i \frac{\alpha_S}{\pi} \left[1 + 0.54973\alpha_S - 1.7157\alpha_S^2 - \left(5.980\,\delta_{iq} + 6.236\,\delta_{ig} \right) \alpha_S^3 + \mathcal{O}\left(\alpha_S^4\right) \right]$$

- Full Soft radiation from two hard partons in color singlet
- Processes involving several hard partons more complicated (MC)
  - need to take into account soft wide-angle emission and collinear radiation
- But soft coupling $\mathscr{A}_i$ not affected
  - intensity of soft-collinear radiation from parton i
  - exactly known to 4th order (for threshold related)

Conclusions

- Definition(s) of Soft-gluon effective coupling at higher orders
- Explicit results for NNLO
- N^3LO soft-coupling for threshold related observables
- Conformal relation (all orders)

$$\widetilde{\mathcal{A}}_{T,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=\widetilde{\mathcal{A}}_{0,i}\left(\alpha_S;\epsilon=\beta\left(\alpha_S\right)\right)=A_i\left(\alpha_S\right)$$


thank you
danke
謝謝
ngiyabonga
teşekkür ederim
gracias
merci
obrigado
go raibh maith agat
mochchakkeram
dziękuje
bedankt
sukriya
kop khun krap
terima kasih
спасибо
grazie
arigatô
dakujem
tapadh leat
dank je
мерси
ありがとう
감사합니다
hvala
mauruuru
takk
sagolun
taiku
ευχαριστώ
Баярлалаа
рахмат
mersi
barka
welalin
tack
spas
faafetai lava
vinaka
blagodaram
misaotra
matondo
paldies
grazzi
mahalo
kiitos
dankie
dhanyavad
хвала
asante
manana
obrigada
tenki
murakoze
chokrane
mamnun
djiere dieuf
tau
дякую
enkosi
bayarlalaa
gracie
koszonom
nanni
nandri
dankon
akun
aciu
chnorakaloutioun
gratias ago
gracies
sulpay
sobodi
dekuji
mesi
didi madloba
najis tuke
kam sah hamnida
rahmat
tanemirt
rahmet
xiexie
diolch
dhanyavadagalu
shukriya
merce
trugarez