Dense QCD(-like) matter in strong magnetic fields

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Chiral soliton lattice phase in QCD

- Dense QCD matter in strong magnetic field studied in [1, 2] using chiral perturbation theory with two light flavors
- Chiral anomaly captured by Wess-Zumino-Witten term [3, 4]

$$\mathcal{L} = \frac{f_\pi^2}{4}\left[\mathrm{Tr}\left(D_\mu \Sigma^{-1} D^\mu \Sigma\right) + m_\pi^2 \mathrm{Tr}\left(\Sigma + \Sigma^{-1}\right)\right] + \mathcal{L}_{\mathrm{WZW}} \qquad (1)$$

- Σ: matrix Goldstone field. If $\Sigma = \exp\left(\frac{i\pi^0\sigma_3}{f_\pi}\right)$ and constant magnetic field $\mathbf{B} = (0, 0, B)$ assumed:

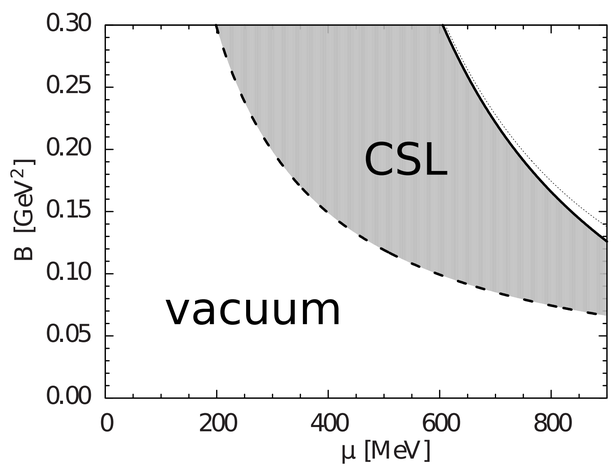
$$\mathcal{L}_{\mathrm{WZW}} = \frac{1}{4\pi^2 f_\pi}\mu B\, \partial_z \pi^0$$

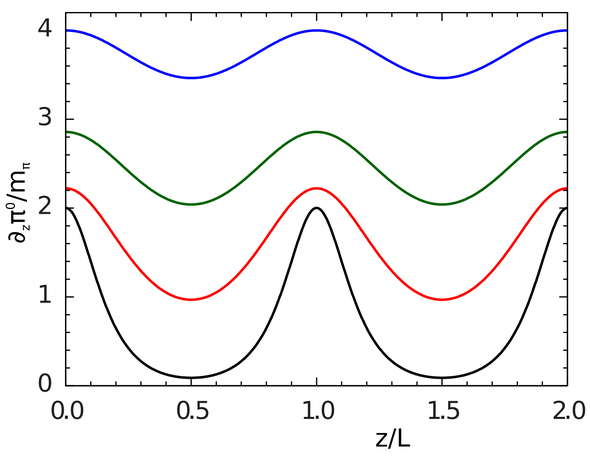
where μ is the baryon chemical potential. For

$$\mu B \geq 16\pi m_\pi f_\pi^2$$

a z-dependent periodic configuration of neutral pions energetically preferable = Chiral Soliton Lattice phase [2]

- Phase diagram in the μ-B plane found in [2] (left plot)
- Dependence of the gradient of the pion field on the position divided by the lattice spacing L (right plot, blue and black curves: highest and lowest value of the product $\mu \cdot B$, respectively)



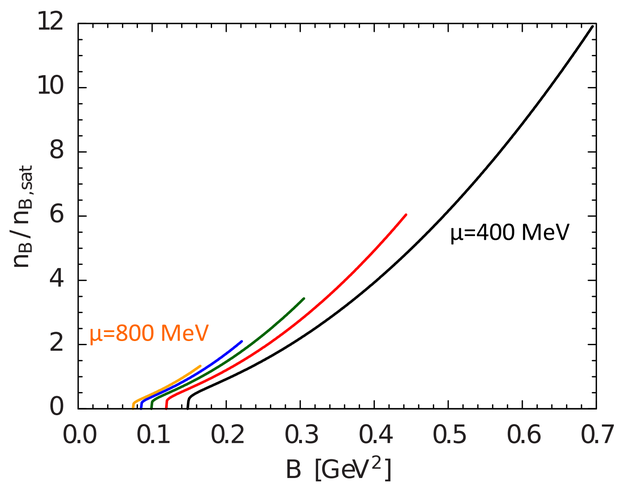


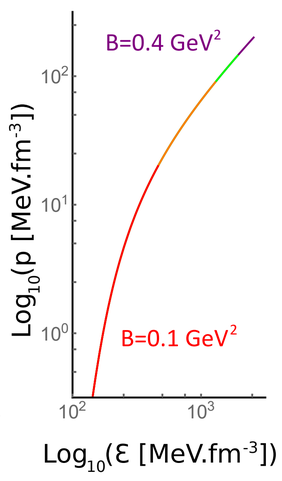
Relevance for the neutron stars?

- Neutral pion condensate carries non-zero baryon number density:

$$n_B(z) = \frac{B}{4\pi^2}\partial_z \pi^0(z)$$

- Already for chemical potentials accessible by chiral perturbation theory, baryon density relevant for neutron stars reached [2] (left plot)
- Dependence of the pressure p on the energy density ϵ calculated for different values of the magnetic field (right plot)





- Can magnetic fields $\sim 10^{19}$ G appear in cores of the neutron stars?
- Could the CSL phase (and/or the corresponding phase transition) play any role for the gravitational-wave observations?

Tell me your opinion!

Similar phase accessible to lattice simulations?

- QCD with finite baryon chemical potential suffers from "sign problem" $\Rightarrow$ standard lattice Monte-Carlo techniques do not work
- QCD-like theories with quarks in real or pseudoreal representation of the gauge group: free of sign problem, their behavior in strong magnetic field studied [5]
- Two light flavors: effective field theory based on $SU(4)/SO(4)$ and $SU(4)/Sp(4)$ coset spaces in real and pseudoreal case, respectively
- Strong magnetic field: in both cases effectively coset space $SU(2) \times SU(2) \times U(1)_Q / SU(2) \times U(1)_Q$
- Σ: $SU(2)$ matrix field including electrically neutral Goldstone bosons: π^0 and a diquark-antidiquark pair: $d, \bar{d}$
- Effective field theory: (1) with [6]

$$\mathcal{L}_{\mathrm{WZW}} = -\frac{C}{6}\epsilon^{\mu\nu\alpha\beta} A_\mu \mathrm{Tr}(\partial_\nu \Sigma \partial_\alpha \Sigma^{-1} \partial_\beta \Sigma \Sigma^{-1}) + \frac{iC}{8}\epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} A^B_\alpha \mathrm{Tr}[\sigma_3(\partial_\beta \Sigma \Sigma^{-1} - \partial_\beta \Sigma^{-1}\Sigma)]$$

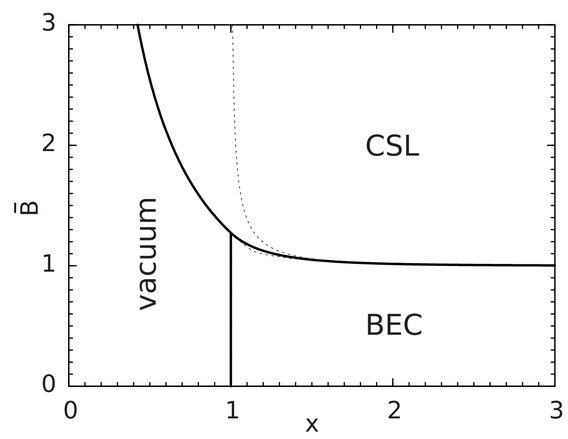
where A_μ is the electromagnetic field and $A^B_\mu = (\mu, \mathbf{0})$ is the auxiliary field including the baryon chemical potential. The constant C is determined by the dimension d of the color gauge group representation that the quarks transform in

$$C = \frac{d}{4\pi^2}$$

- Constant magnetic field B assumed, dimensionless variables introduced (the above mentioned class of theories can be then described by a single phase diagram):

$$x = \frac{\mu}{m_\pi}, \quad \bar{B} = \frac{CB}{f_\pi^2}$$

- CSL-like condensate of neutral pions can appear for magnetic fields satisfying $\bar{B} \geq 1$
- for low $\bar{B}$ and $x \geq 1$, Bose-Einstein condensation of diquarks preferable



- Set of counterexamples to conjecture of [7] claiming that in vector-like gauge theories, absence of the sign problem implies absence of inhomogeneous phases in the phase diagram
- Confirmation of a CSL-like phase, e.g., for the 2-color QCD on lattice?

References

[1] D. T. Son and M. A. Stephanov, "Axial anomaly and magnetism of nuclear and quark matter", Phys. Rev. **D77** (2008) 014021, arXiv:0710.1084 [hep-ph].

[2] T. Brauner and N. Yamamoto, "Chiral Soliton Lattice and Charged Pion Condensation in Strong Magnetic Fields", JHEP **04** (2017) 132, arXiv:1609.05213 [hep-ph].

[3] J. Wess and B. Zumino, "Consequences of anomalous Ward identities", Phys. Lett. **B37** (1971) 95–97.

[4] E. Witten, "Global aspects of current algebra", Nucl. Phys. **B223** (1983) 422–432.

[5] T. Brauner, G. Filios, and H. Kolešová, "Anomaly-Induced Inhomogeneous Phase in Quark Matter without Sign Problem", arXiv:1902.07522 [hep-ph].

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