Green Functions of Chiral Currents within OPE

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Green Functions of Chiral Currents

- The amplitudes of physical processes can be computed using the LSZ reduction formula from the Green functions.
- The interpolating fields for the external sources in QCD are:
  - the vector and axial-vector currents:

$$V_\mu^a(x) = \overline{q}(x)\gamma_\mu T^a q(x)\,, \qquad A_\mu^a(x) = \overline{q}(x)\gamma_\mu\gamma_5 T^a q(x)\,,$$

  - or the scalar and pseudoscalar densities:

$$S^a(x) = \overline{q}(x)T^a q(x)\,, \qquad P^a(x) = i\overline{q}(x)\gamma_5 T^a q(x)\,.$$

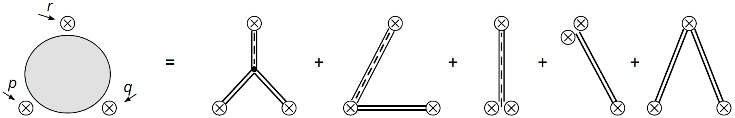
- Nontrivial three-point Green functions in QCD:
  - Set I: $\langle SSS\rangle$, $\langle SPP\rangle$, $\langle VVP\rangle$, $\langle AAP\rangle$, $\langle VAS\rangle$, $\langle VVS\rangle$, $\langle AAS\rangle$, $\langle VAP\rangle$.
    - Order parameters of χSB.
  - Set II: $\langle ASP\rangle$, $\langle VSS\rangle$, $\langle VPP\rangle$, $\langle VVA\rangle$, $\langle AAA\rangle$, $\langle AAV\rangle$, $\langle VVV\rangle$.

Odd-intrinsic Parity Sector of QCD

- Special interest of ours: odd-intrinsic parity sector of QCD.
  - $\langle VVP \rangle$, $\langle AAP \rangle$, $\langle VAS \rangle$, $\langle VVA \rangle$, $\langle AAA \rangle$.
- We assume the saturation of dynamics with the lightest resonances.
- At the NLO, the relevant Lagrangian in the odd-intrinsic parity sector was formulated for the first time in [K. Kampf and J. Novotný '11]:

$$\mathcal{L}_R^{(6)} = \sum_X \sum_i \kappa_i^X \widehat{\mathcal{O}}^X_{i\,\mu\nu\alpha\beta} \varepsilon^{\mu\nu\alpha\beta}$$

  - 67 operators and 67 corresponding unknown couplings κ_i^X in total.
  - X: the single-resonance fields V, A, S, P, double-resonance fields VV, AA, SA, SV, VA, PA, PV and triple-resonance fields VVP, VAS, AAP.
- Topology of the Feynman diagrams (the crossing is implicitly assumed):



- How to obtain the unknown coupling constants?
  - Phenomenological constraints or high-energy behavior.

Example: $\langle VVA \rangle$ Green Function

- Calculation is straightforward [TK, K. Kampf and J. Novotný (in prep.)].
  - Only 6 types of tree-level diagrams in RχT at $\mathcal{O}(p^6)$.
  - The result is given in terms of scalar formfactors $\mathcal{F}_{VVA}^{\mathrm{R}\chi\mathrm{T}}$, $\mathcal{G}_{VVA}^{\mathrm{R}\chi\mathrm{T}}$ and $\mathcal{H}_{VVA}^{\mathrm{R}\chi\mathrm{T}}$.
- Phenomenologically important formfactor $w_T(Q^2)$:

$$w_T(Q^2) = -16\pi^2 \left[\mathcal{F}_{VVA}^{\mathrm{R}\chi\mathrm{T}}(-Q^2, 0, -Q^2) + \mathcal{H}_{VVA}^{\mathrm{R}\chi\mathrm{T}}(-Q^2, 0, -Q^2)\right].$$

- OPE for $\langle V^* VA \rangle$ is known [P. Colangelo *et al.* '12]:

$$w_T(Q^2) = \frac{N_c}{Q^2} + \frac{128\pi^3 \alpha_s \chi \langle \bar{q}q \rangle^2}{9Q^6} + \mathcal{O}\left(\frac{1}{Q^8}\right).$$

- Matching gives the following predictions:
  - $\kappa_5^{VA} = -0.086$.
    - The decay $f_1(1285) \to \rho\gamma$ suggests $\kappa_5^{VA} = -0.062 \pm 0.030$.
  - Deviation from the Brodsky-Lepage behavior as $\delta_{\mathrm{BL}} = -1.342$.
    - Contradiction with phenomenologically compatible value $\delta_{\mathrm{BL}} = -0.055 \pm 0.025$ obtained from the $\langle VVP \rangle$.

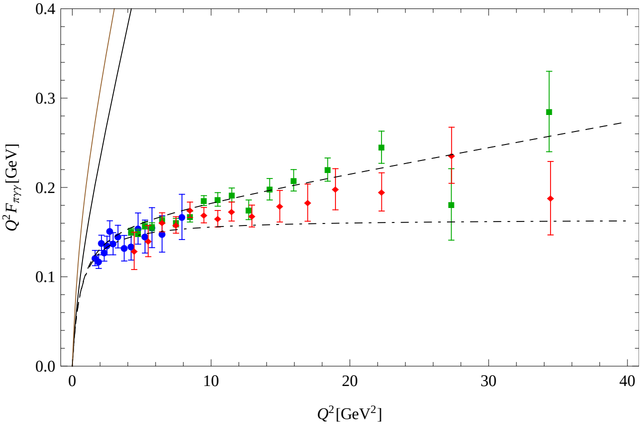


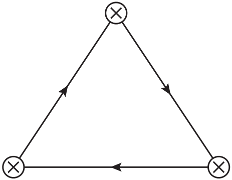
Figure: A plot of BABAR (green), BELLE (red) and CLEO (blue) data fitted with the formfactor $\mathcal{F}^{\mathrm{R}\chi\mathrm{T}}_{\pi^0\gamma\gamma}(0,-Q^2;0)$ using the modified B-L condition. The full black line is our fit with $\delta_{\mathrm{BL}} = -1.342$, the full brown line is a fit using the LMD formfactor. The dashed line stands for $\delta_{\mathrm{BL}} = -0.055$ and the dot-dashed line for $\delta_{\mathrm{BL}} = 0$.

Operator Product Expansion

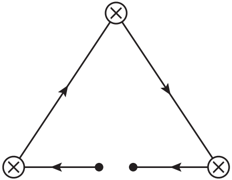
- Framework to study behaviour at high energies.
- At short distances (i.e. for large external momenta), the Green function can be expanded into a series of nonperturbative parameters with c-number coefficients:

$$
\begin{aligned}
\Pi_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(\lambda p, \lambda q; \lambda r) \xrightarrow{\lambda\to\infty} \quad & \lambda\, C^{\mathbb{1}}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r) \\
&+ \frac{1}{\lambda^2} C^{\langle\overline{q}q\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r)\langle\overline{q}q\rangle \\
&+ \frac{1}{\lambda^3} C^{\langle G^2\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r)\langle G^a_{\mu\nu}G^{\mu\nu,a}\rangle \\
&+ \frac{1}{\lambda^4} C^{\langle\overline{q}\sigma\cdot Gq\rangle}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r)\langle\overline{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle \\
&+ \frac{1}{\lambda^5} C^{\langle\overline{q}q\rangle^2}_{\mathcal{O}_1^a\mathcal{O}_2^b\mathcal{O}_3^c}(p,q;r)\langle(\overline{q}Xq)(\overline{q}Xq)\rangle \\
&+ \dots
\end{aligned}
$$

OPE: Perturbative Contribution and Quark Condensate



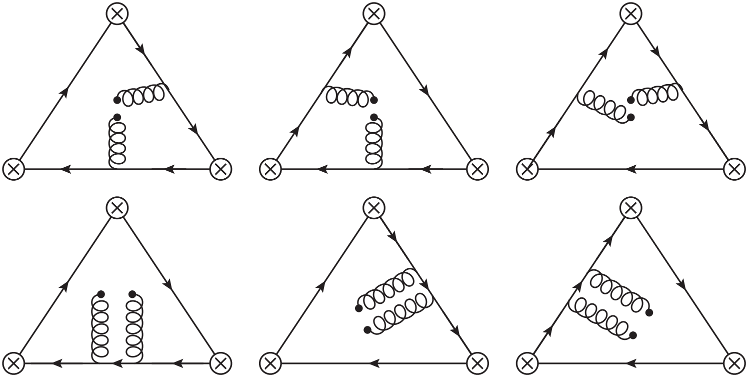
- Known to several loops.
- Canonical example: $\langle VVA \rangle$
  - Anomaly [S. L. Adler '69, J. S. Bell and R. Jackiw '69].
  - Triangle diagram is not modified by QCD radiative corrections at two loops.
  - Three-loop corrections do not vanish and are proportional to the QCD β-function [J. Mondejar and K. Melnikov '12].



- Implicates the chiral symmetry breaking.
- Contributes to the order parameters of the χSB.
- LO was studied a long time ago [B. Moussallam '94].
- NLO: gluonic corrections at $\mathcal{O}(\alpha_s)$ [M. Jamin and V. Mateu '08].
  - An opportunity to explore the renormalisation dependence of such condensate in full QCD.

OPE: Gluon Condensate $\langle G_{\mu\nu} G^{\mu\nu} \rangle$

- Straightforward, but lengthy evaluation, easiest in the Fock-Schwinger gauge.
- Fourier transform needed to convert the result into momentum representation.
- Although it is a one-loop calculation, cancelation of logarithmic terms occurs.



OPE: Effective Propagation of QCD Condensates

- Higher QCD condensates are obtained through the effective propagation of nonlocal condensates [TK, K. Kampf and J. Novotný '20]:

$$\langle \bar{q}^A_{i,\alpha}(x) q^B_{k,\beta}(y)\rangle = \Bigg(\frac{1}{2^2\cdot 3^2}\langle\bar{q}q\rangle\delta_{ik} - \frac{g_s\langle\bar{q}\sigma\cdot Gq\rangle}{2^5\cdot 3^2}\left[F^{\langle\bar{q}q\rangle}(x,y)\right]_{ki} + \\ + \frac{i\pi\alpha_s\langle\bar{q}q\rangle^2}{2^3\cdot 3^7}\left[G^{\langle\bar{q}q\rangle}(x,y)\right]_{ki} + \ldots\Bigg)\delta_{\alpha\beta}\delta^{AB}\,,$$

$$g_s\langle \bar{q}^A_{i,\alpha}(x)\mathcal{A}^a_\mu(u) q^B_{k,\beta}(y)\rangle = \Bigg(\frac{g_s\langle\bar{q}\sigma\cdot Gq\rangle}{2^7\cdot 3^2}\left[F_\mu^{\langle\bar{q}\mathcal{A}q\rangle}(x,u,y)\right]_{ki} + \\ + \frac{\pi\alpha_s\langle\bar{q}q\rangle^2}{2^3\cdot 3^5}\left[G_\mu^{\langle\bar{q}\mathcal{A}q\rangle}(x,u,y)\right]_{ki} + \ldots\Bigg)(T^a)_{\beta\alpha}\delta^{AB}\,,$$

where

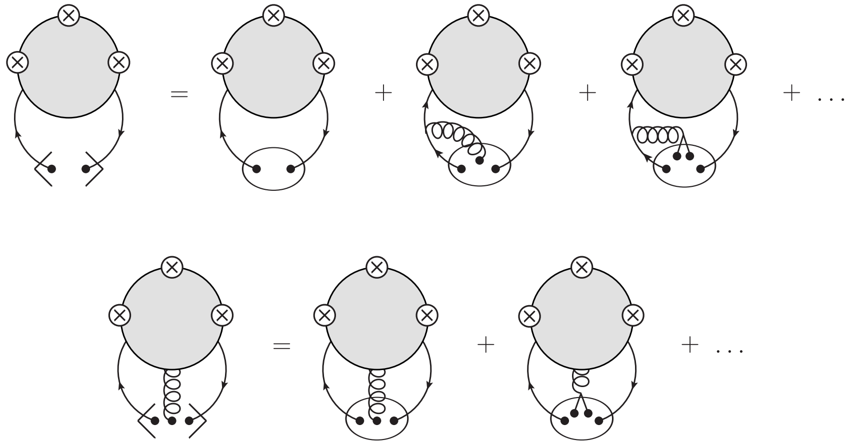
$$F^{\langle\bar{q}q\rangle}(x,y) = \frac{1}{2}(x-y)^2 + \frac{i}{3}\sigma^{(x)(y)}\,,$$

$$G^{\langle\bar{q}q\rangle}(x,y) = 4(x\cdot y)(\not{x}-\not{y}) - (x^2-y^2)(\not{x}+\not{y})\,,$$

$$F_\mu^{\langle\bar{q}\mathcal{A}q\rangle}(x,u,y) = \sigma^{(u)\mu}\,,$$

$$G_\mu^{\langle\bar{q}\mathcal{A}q\rangle}(x,u,y) = \frac{1}{6}\gamma^\mu\left[3u\cdot(x+y)-4u^2\right] + \frac{1}{6}\not{u}\left[4u^\mu - 3(x+y)^\mu\right] - \frac{i}{2}\varepsilon^{\mu(x-y)(u)\alpha}\gamma_\alpha\gamma_5\,.$$

OPE: Effective Propagation of QCD Condensates

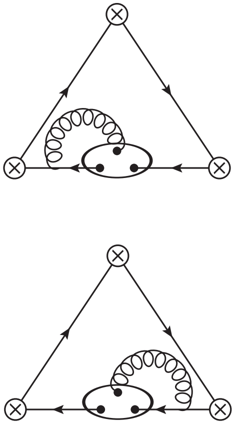


- Omitting the effective propagation of the QCD condensates *"has been one of the main source of errors in the existing QSSR literature"* [S. Narison '04]

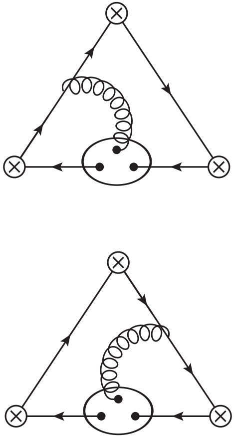
OPE: Quark-gluon Condensate $\langle \overline{q} \sigma_{\mu\nu} G^{\mu\nu} q \rangle$

- Two types of contributions due to the effective propagation of nonlocal...

...quark condensate:

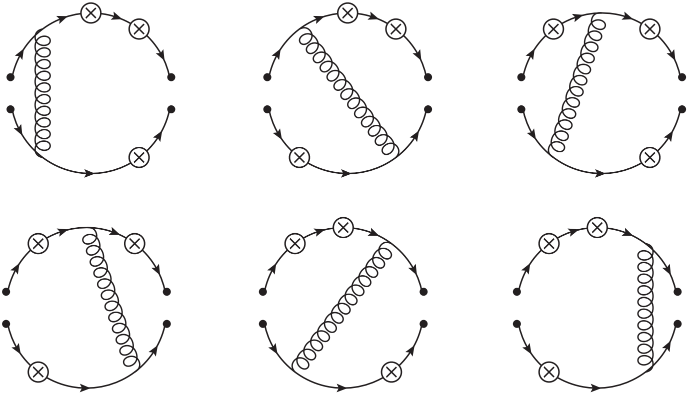


...quark-gluon condensate:



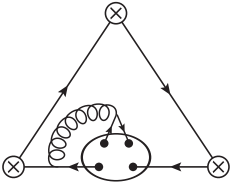
OPE: Four-quark Condensate $\langle \overline{q} q \rangle^2$

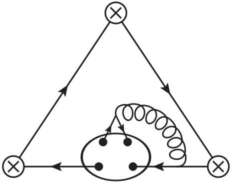
- Factorization hypothesis assumed: $\langle (\overline{q}\gamma_\mu T^a q)(\overline{q}\gamma^\mu T^a q) \rangle = -\frac{4}{27}\langle \overline{q} q \rangle^2$.
- Direct contribution:



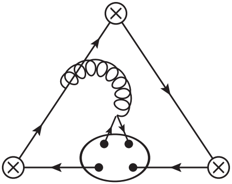
- Two other types of contributions due to the effective propagation of nonlocal...

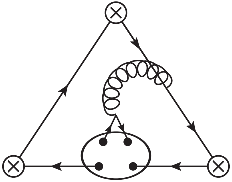
...quark condensate:





...quark-gluon condensate:





Matching OPE-RχT

- We have calculated:
  - the perturbative contribution and contributions of the gluon and four-quark condensates to the $\langle ASP \rangle$, $\langle VSS \rangle$, $\langle VPP \rangle$, $\langle VVA \rangle$, $\langle AAA \rangle$, $\langle AAV \rangle$ and $\langle VVV \rangle$ Green functions,
  - the contributions of the quark and quark-gluon condensates to the $\langle SSS \rangle$, $\langle SPP \rangle$, $\langle VVP \rangle$, $\langle AAP \rangle$, $\langle VAS \rangle$, $\langle VVS \rangle$, $\langle AAS \rangle$ and $\langle VAP \rangle$ Green functions.
- Having the OPE for all momenta large ready at hand, will be the matching OPE-RχT successful?

Work in progress!

- The matching is complicated!
  - How to deal with the logarithmic terms at $\mathcal{O}(\alpha_s)$?
  - Need for infinite tower of resonances?!

Conclusion

- OPE with two large momenta is not phenomenologically consistent.
- Will the OPE with three large momenta help?
- Contributions of the QCD condensates up to (and including) dimension 6 have been evaluated in the chiral limit for all the Green functions of chiral currents.
- Special emphasis on the odd-intrinsic parity sector of QCD.
- Should be available soon on arXiv.
- However, matching RχT with OPE is complicated.

Thank you for your attention!

See arXiv:2006.13006 [hep-ph] for details