Three flavour order parameters of chiral symmetry in low energy QCD

Marián Kolesár

(in collaboration with Jiří Novotný)

A) Quantum chromodynamics at low energies

B) Overview of chiral perturbation theory

C) Quark masses

D) Order parameters - two and three flavours

E) Results: $\eta \to 3\pi$ and $\pi\pi$ scattering

F) Phase structure of QCD with varying number of light quarks

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Institute of particle and nuclear physics, Charles University, Prague

A) Quantum chromodynamics

Theory of strong interactions

Quarks

- six flavor u,d,s,c,b,t
- three colors

Principle of local gauge invariance of $SU(3)_c \rightarrow$ gluons

Lagrangian:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}(i\gamma_\mu D^\mu - \mathcal{M})q$$

$$G^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu + ig\,[\,G^\mu, G^\nu\,], \qquad D^\mu = \partial^\mu + igG^\mu$$

At low energies

- gluon self-interactions $\rightarrow$ the coupling constant g grows
- failure of the usual QFT perturbative expansion
- quarks and gluons are not the appropriate degrees of freedom
- quark confinement and hadron spectrum

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A) Approximate symmetries of QCD

The quark flavors in $\mathcal{L}_{QCD}$ differ only in their masses

Isospin symmetry $m_u = m_d$ $SU(2)$ symmetry group

- nucleon doublet (p,n)
- pion triplet (π^+, π^-, π^0)
- kaon quadruplet $(K^+, K^-, K^0, \bar{K}^0)$

Eightfold way $m_u = m_d = m_s$ $SU(3)$ symmetry group

- baryon octet (p,n,Σ,Ξ) and decuplet (Δ,Σ*,Ξ*,Ω)
- pseudoscalar meson octet (π,K,η)
- vector meson octet (ρ, K^*,ω)
- scalar multiplet? (σ,κ, a_0,f_0's) $\rightarrow$ large N_c and Zweig rule problems

Chiral symmetry $0 = m_u = m_d (= m_s)$ $SU(N_f)_L \times SU(N_f)_R$

- parity doublets $\rightarrow$ not observed in the spectrum

Strong evidence for chiral symmetry $\rightarrow$ spontaneously broken

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B) Chiral perturbation theory

Effective theory of QCD at the lowest energies

- relevant degrees of freedom: pseudoscalar mesons
- Goldstone bosons of spontaneously broken $SU(N_f)_L \times SU(N_f)_R$
- $\mathcal{L}$ as the most general form respecting the approximate symmetry
- organized as expansion in derivatives and quark masses
- $SU(2)$: $\Lambda < M_K = 496\text{MeV}$, $SU(3)$: $\Lambda < M_\rho = 770\text{MeV}$

Building blocks: $\pi^a \sim \pi, K, \eta$

$$U(x) = \text{Exp}\frac{i}{F_0}\pi^a(x)\lambda^a, \quad \mathcal{M} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & [m_s] \end{pmatrix}$$

Transformational properties: $U_L \in SU(N_f)_L \quad U_R \in SU(N_f)_R$

$$U'(x) = U_R U(x) U_L^+, \quad \mathcal{M}' = U_R \mathcal{M} U_L^+$$

Power counting: $D = 2 + 2L + \sum_n V_n(n-2)$

- renormalizable order by order

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B) Chiral perturbation theory $SU(2)\times SU(2)$

Standard power counting: $m_q \sim O(p^2)$ (Gasser,Leutwyler 1984)

Effective Lagrangian: $$\mathcal{L}_{eff} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \mathcal{L}^{(6)} + \ldots$$

$$\mathcal{L}^{(2(k+l))} \sim p^{2k}\chi^l, \quad \chi = 2B\mathcal{M}$$

$$\mathcal{L}^{(2)} = \frac{F^2}{4}\mathrm{Tr}[D_\mu U D^\mu U^+ + (U^+\chi + \chi^+ U)]$$

$$\mathcal{L}^{(4)} = \mathcal{L}^{(4)}(l_1 \ldots l_7) + \mathcal{L}^{(4)}_{WZ}$$

(Gasser,Leutwyler 1984)
(Wess,Zumino 1971)

$$\mathcal{L}^{(6)} = \mathcal{L}^{(6)}(c_1 \ldots c_{52}) + \mathcal{L}^{(6)}_{WZ}(c_1^W \ldots c_{13}^W)$$

(Bijnens et al. 1999)
(Bijnens et al. 2002)

Free parameters (in addition to m_u,m_d):

$\mathcal{O}(p^2)$: F, B

$\mathcal{O}(p^4)$: $l_1 \ldots l_7$

$\mathcal{O}(p^6)$: $c_1 \ldots c_{52}$ and $c_1^W \ldots c_{13}^W$

Predictive power - information needed to fix the LEC's

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Free parameters (in addition to m_u,m_d,m_s):

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$\mathcal{O}(p^6)$: $C_1 \ldots C_{90}$ and $C_1^W \ldots C_{23}^W$

C) Quark masses

Mass parameters (3 flavour):

$\hat{m}$ - light quark mass average

r - strange to light quark mass ratio

R - isospin violation (light quark mass difference)

$$\hat{m} = \frac{m_u + m_d}{2}, \qquad r = \frac{m_s}{\hat{m}}, \qquad R = \frac{(m_s - \hat{m})}{(m_d - m_u)}$$

lattice		$\hat{m}$	r	R
FLAG'19 N_f=2+1	average	3.364±0.041	27.42±0.12	38.1±1.5
FLAG'19 N_f=2+1+1	average	3.410±0.043	27.23±0.10	40.7±2.7
phenomenology				
Kampf et al.'12	$\eta \to 3\pi$			37.8±3.3
Colangelo et al.'17	$\eta \to 3\pi$			34.2±2.2

FLAG - Flavour Lattice Averaging Group

D) Leading order: $SU(2)\times SU(2)$

Convenient normalization: $(\hat{m}=(m_u+m_d)/2)$

$$Z(2)=\frac{F^2}{F_\pi^2},\qquad X(2)=\frac{2\,\hat{m}\Sigma}{F_\pi^2 M_\pi^2}=\frac{2\,\hat{m}BF^2}{F_\pi^2 M_\pi^2}$$

Two flavour values:

phenomenology		$Z(2)$	$X(2)$
Descotes et al.'01	$\pi\pi$ scattering	0.89±0.03	0.81±0.07
lattice			
Bernard et al.'12	RBC/UKQCD	0.86±0.01	0.89±0.01
FLAG'19 N_f=2+1	average	0.89±0.01	0.84±0.05
FLAG'19 N_f=2+1+1	average	0.862±0.005	0.98±0.24

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Three flavour values:

phenomenology		$Z(3)$	$X(3)$
Bijnens, Ecker'14	NNLO χPT (main fit)	0.59	0.63
Bijnens, Ecker'14	NNLO χPT (free fit)	0.48	0.45
Amoros et al.'01	NNLO χPT ("fit 10")	0.89	0.66
lattice			
Bernard et al.'12	RBC/UKQCD+ReχPT	0.54±0.06	0.38±0.05
Ecker et al.'13	RBC/UKQCD+large N_c	0.91±0.08	
FLAG'19 $N_f=2+1$	MILC 09A	0.72±0.06	0.61±0.06

E) Bayesian statistical analysis: $\eta \to 3\pi$

$\eta \to 3\pi$ observables calculated in resummed CHPT

(M.K.,J.Novotný 2018)

Decay widths (PDG): $\Gamma_+ = 300 \pm 12$ eV, $\Gamma_0 = 428 \pm 17$ eV

Dalitz plot parameters (KLOE): $a = -1.095 \pm 0.004$

Higher Dalitz slopes not predicted reliably from theory

(M.K.,J.Novotný 2017)

Bayes' theorem

$$P(X_i|\text{data}) = \frac{P(\text{data}|X_i)P(X_i)}{\int \mathrm{d}X_i\, P(\text{data}|X_i)P(X_i)}$$

$\boldsymbol{P(X_i|\text{data})}$ - probability density of X_i being true given data
$\boldsymbol{P(\text{data}|X_i)}$ - experimental distributions of independent observables O_k
$\boldsymbol{P(X_i)}$ - probability distribution of X_i (prior)

num.integration $\to$ Monte Carlo sampling

$\sim 10^7$ total samples $\equiv$ ensemble of theoretical predictions

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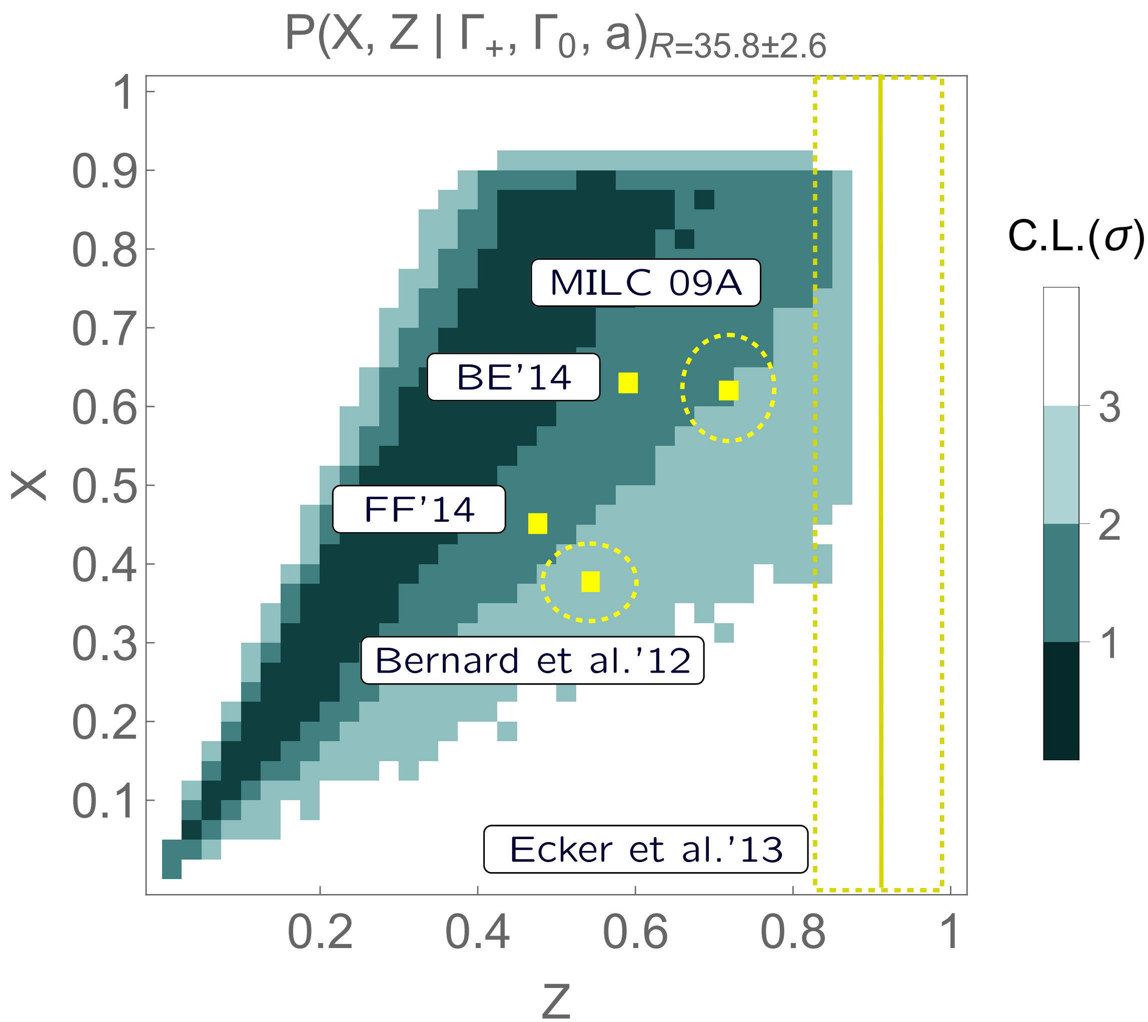
Extraction from $\eta \to 3\pi$ data: *(M.K.,J.Novotný 2018)*



$$Y = \frac{X}{Z} = 1.44 \pm 0.32 \quad Z < 0.78 \ (2\sigma \ \text{CL})$$

$\pi\pi$ subthreshold parameters calculated in resummed CHPT

(Stern et al. 2003; M.K.,J.Novotný 2018)

$$A_{\pi\pi}(s,t,u) = \frac{\alpha_{\pi\pi}}{F_\pi^2}\frac{M_\pi^2}{3} + \frac{\beta_{\pi\pi}}{F_\pi^2}\left(s - \frac{4M_\pi^2}{3}\right) + \ldots$$

BNL-E865: $\alpha_{\pi\pi} = 1.381 \pm 0.242, \quad \beta_{\pi\pi} = 1.081 \pm 0.023$

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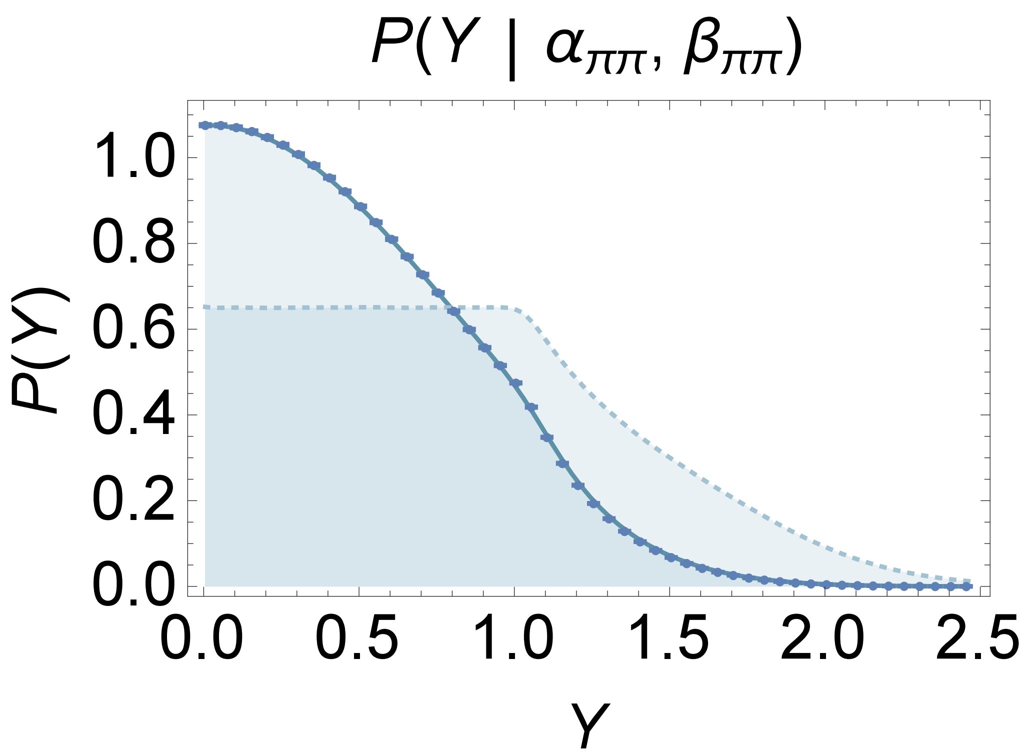

More precise data available (NA48/2)

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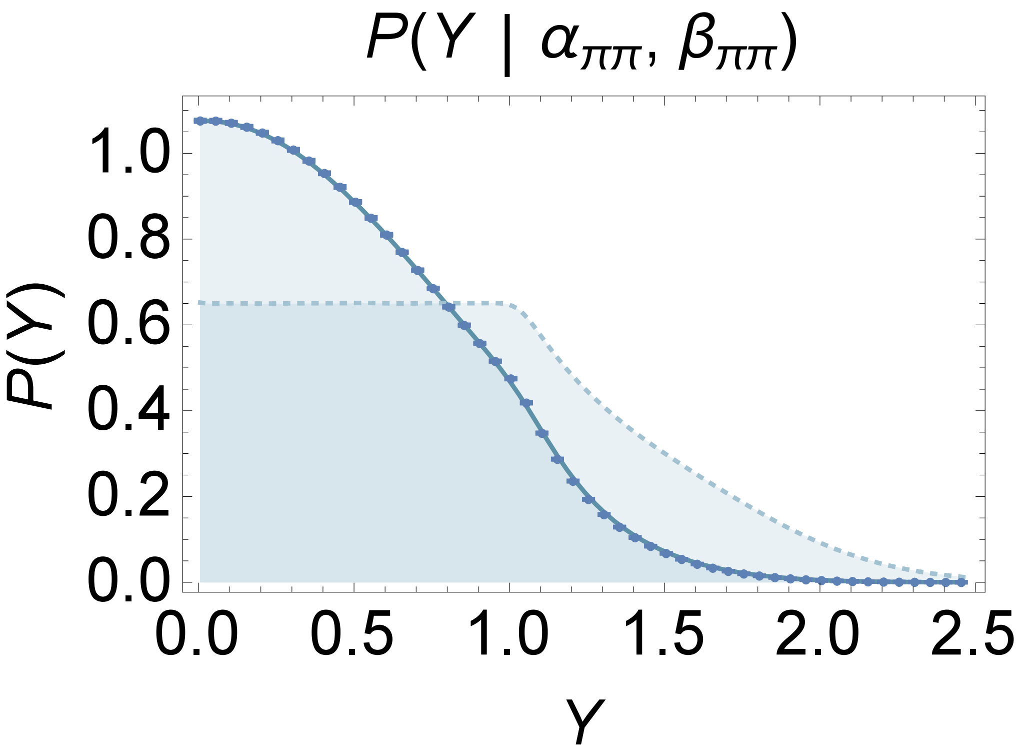

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F) Phase structure of QCD with varying N_f

$N_f < N_f^c$	quark confinement, SBχS, hadron spectrum
$N_f = N_f^c$	chiral phase transition
$N_f^c < N_f < N_f^A$	conformal window
$N_f \geq N_f^A = 11/2N_c$	asymptotic freedom lost

(Appelquist et al.1998)

Interpretation - counter-play:

- condensating effect of gluon self-interactions
- screening of light-quark-loop vacuum fluctuations

"Paramagnetic" inequality: dependence on N_f

$$F_0(N_f+1) < F_0(N_f), \; \Sigma(N_f+1) < \Sigma(N_f)$$

$$F_0(2)^2 \; = \; F_0(3)^2 + 16 m_s B_0 L_4^r - 2\bar{\mu}_K + \mathcal{O}(m_s^2)$$

$$\Sigma(2) \; = \; \Sigma(3)(1 + \frac{32 m_s B_0}{F_0^2} L_6^r - 2\bar{\mu}_K - \frac{1}{3}\bar{\mu}_\eta) + \mathcal{O}(m_s^2)$$

- effect of s-quark vacuum fluctuations
- L_4, L_6 - $1/N_c$ suppressed LECs, connected to the scalar sector
- possible $1/N_c$ and Zweig rule violation?

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"Paramagnetic" inequality: dependence on N_f *(Stern et al.2000)*

$$F_0(N_f+1) < F_0(N_f), \ \Sigma(N_f+1) < \Sigma(N_f)$$

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$$\Sigma(2) = \Sigma(3)(1 + \frac{32 m_s B_0}{F_0^2} L_6^r - 2\bar{\mu}_K - \frac{1}{3}\bar{\mu}_\eta) + \mathcal{O}(m_s^2)$$

- effect of s-quark vacuum fluctuations
- L_4, L_6 - $1/N_c$ suppressed LECs, connected to the scalar sector
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→ **difference between $SU(2)$ and $SU(3)$ theory?**

F) Phase structure of QCD with varying N_f

$N_f < N_f^c$	quark confinement, SBχS, hadron spectrum
$N_f = N_f^c$	chiral phase transition
$N_f^c < N_f < N_f^A$	conformal window
$N_f \geq N_f^A = 11/2N_c$	asymptotic freedom lost

(Appelquist et al.1998)

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Thank you for your attention!