Towards establishing the second b-flavored CKM unitarity triangle

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Work in collaboration with my Ph.D student Di Zhang:
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The CKM matrix

★ In the SM, the quark fields interact with both the gauge fields and the Higgs field —— the very origin of flavor mixing and CP violation.

• If the quark Yukawa interactions were absent, those weak charged-current interactions would always be flavor-diagonal;

• If the quark interactions with gauge fields were absent, the Yukawa interactions of quarks would always be flavor-diagonal.

★ So a nontrivial mismatch between the flavor and mass eigenstates of quarks results in flavor mixing ($N_q \geq 2$) and CP violation ($N_q \geq 3$).

★ In the mass-eigenstate basis, it's the Cabibbo-Kobayashi-Maskawa matrix that describes quark flavor mixing and CP violation. The only constraint on the CKM matrix, imposed by the SM itself, is unitarity.

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}} \, \overline{(u, \quad c, \quad t)_{\rm L}} \, \gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{\rm L} W^+_\mu + {\rm h.c.}$$

• Way (1): measuring the moduli to test the normalization conditions
• Way (2): measuring the triangles to test the orthogonality relations

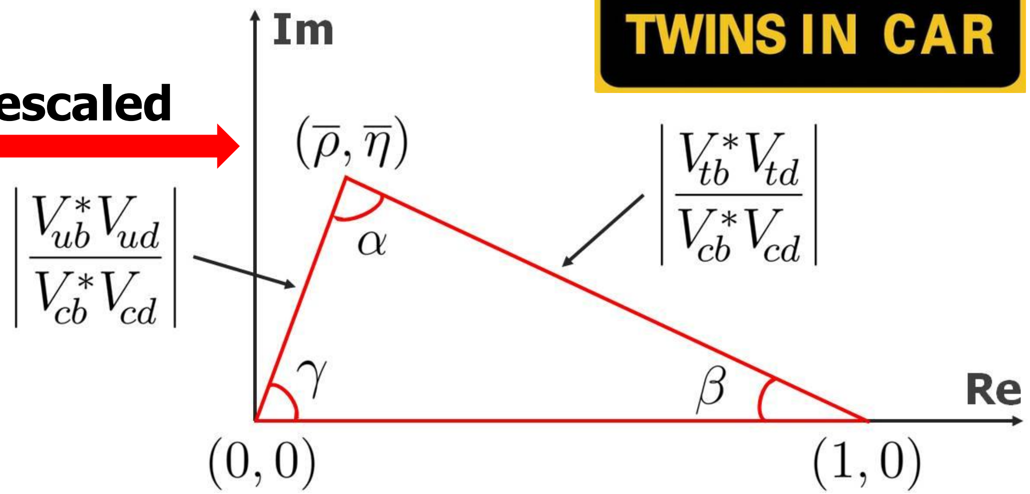
The b-flavored twins

★ **Higher-precision measurements of b-flavored twin CKM unitarity triangles will be available, at both super-B factory and High-luminosity LHCb.**

- **The most studied triangle:**

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

rescaled



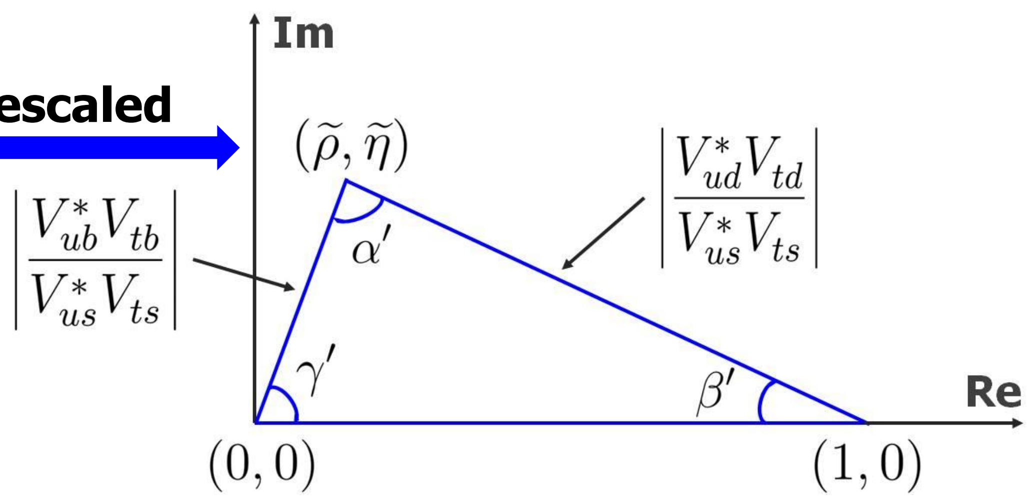
Its apex: $\overline{\rho} + \mathrm{i}\overline{\eta} = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}$

$\alpha = \left(84.5^{+5.9}_{-5.2}\right)^\circ$ **(from data)**

- **The b-flavored twin sister:**

$$V_{ud}^* V_{td} + V_{us}^* V_{ts} + V_{ub}^* V_{tb} = 0$$

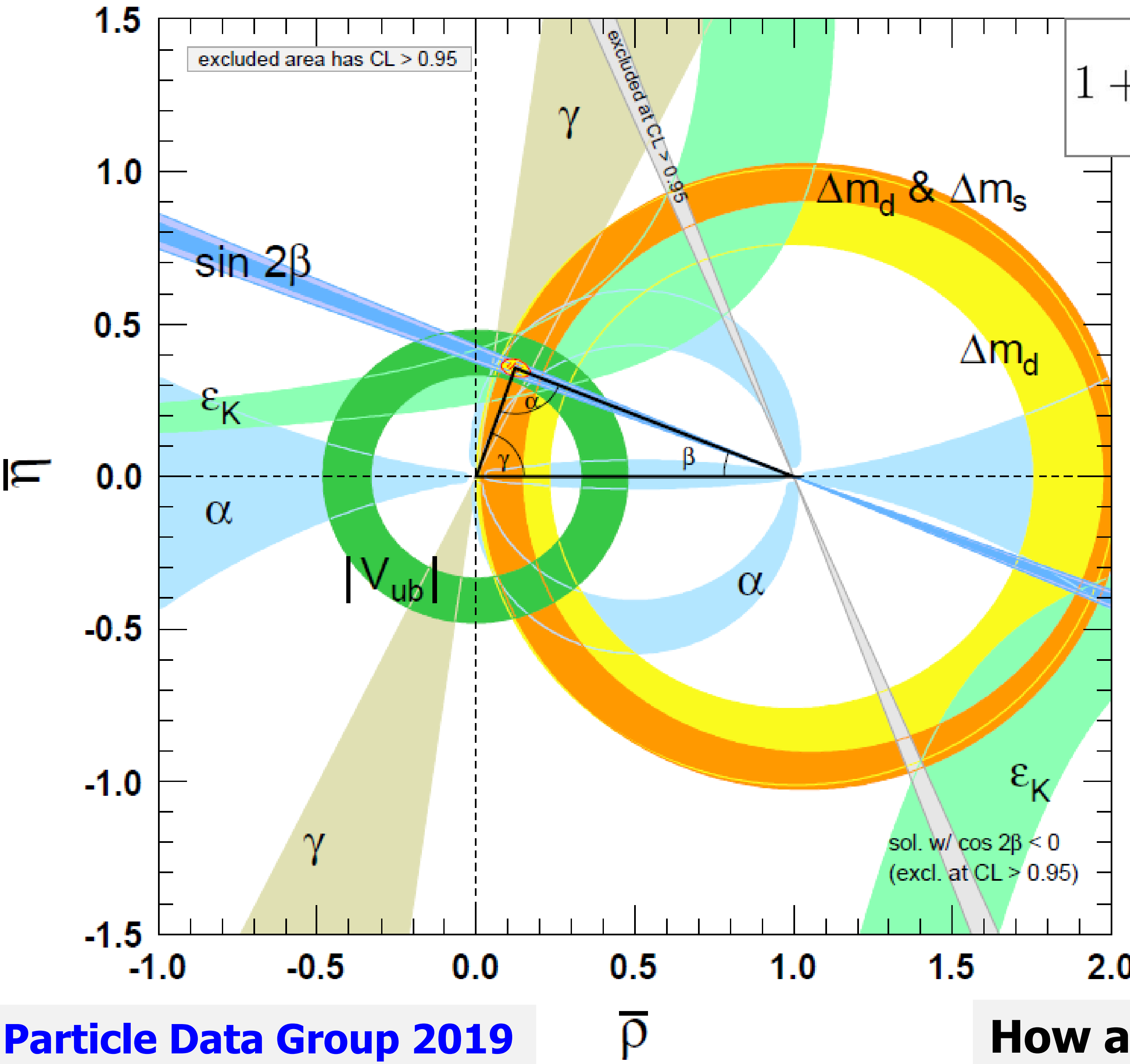
rescaled



Its apex: $\widetilde{\rho} + \mathrm{i}\widetilde{\eta} = -\frac{V_{ub}^* V_{tb}}{V_{us}^* V_{ts}}$

$\alpha' = \alpha$ **(by definition)**

Constraints on the "red" sister



$$1 + \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} + \frac{V_{tb}^* V_{td}}{V_{cb}^* V_{cd}} = 0$$

Its vertex:

$$\bar{\rho} + i\bar{\eta} = -\frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}}$$

$$\alpha = (84.5^{+5.9}_{-5.2})^\circ$$

Constraints from:

- **CPV in K mixing**
- **Bd mixing**
- **Bs mixing**
- **B decays**
- **CPV in B decays**

Particle Data Group 2019

How about its twin sister?

Wolfenstein's expansion

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$+\frac{1}{2}\lambda^4 \begin{pmatrix} -\frac{1}{4} & 0 & 0 \\ A^2\lambda[1-2(\rho+i\eta)] & -\frac{1}{4}\left(1+4A^2\right) & 0 \\ A\lambda(\rho+i\eta) & A[1-2(\rho+i\eta)] & -A^2 \end{pmatrix} + \mathcal{O}(\lambda^6)$$

L. Wolfenstein 83

A. Buras et al 94,
M. Kobayashi 94,
Z. Xing 94,
J. Charles et al 04

$$\overline{\rho} = \rho\left\{1-\frac{1}{2}\lambda^2 - \left[\frac{1}{8} - \left(\frac{1}{2} - \rho + \frac{\eta^2}{\rho}\right)A^2\right]\lambda^4\right\} + \mathcal{O}(\lambda^6) ,$$

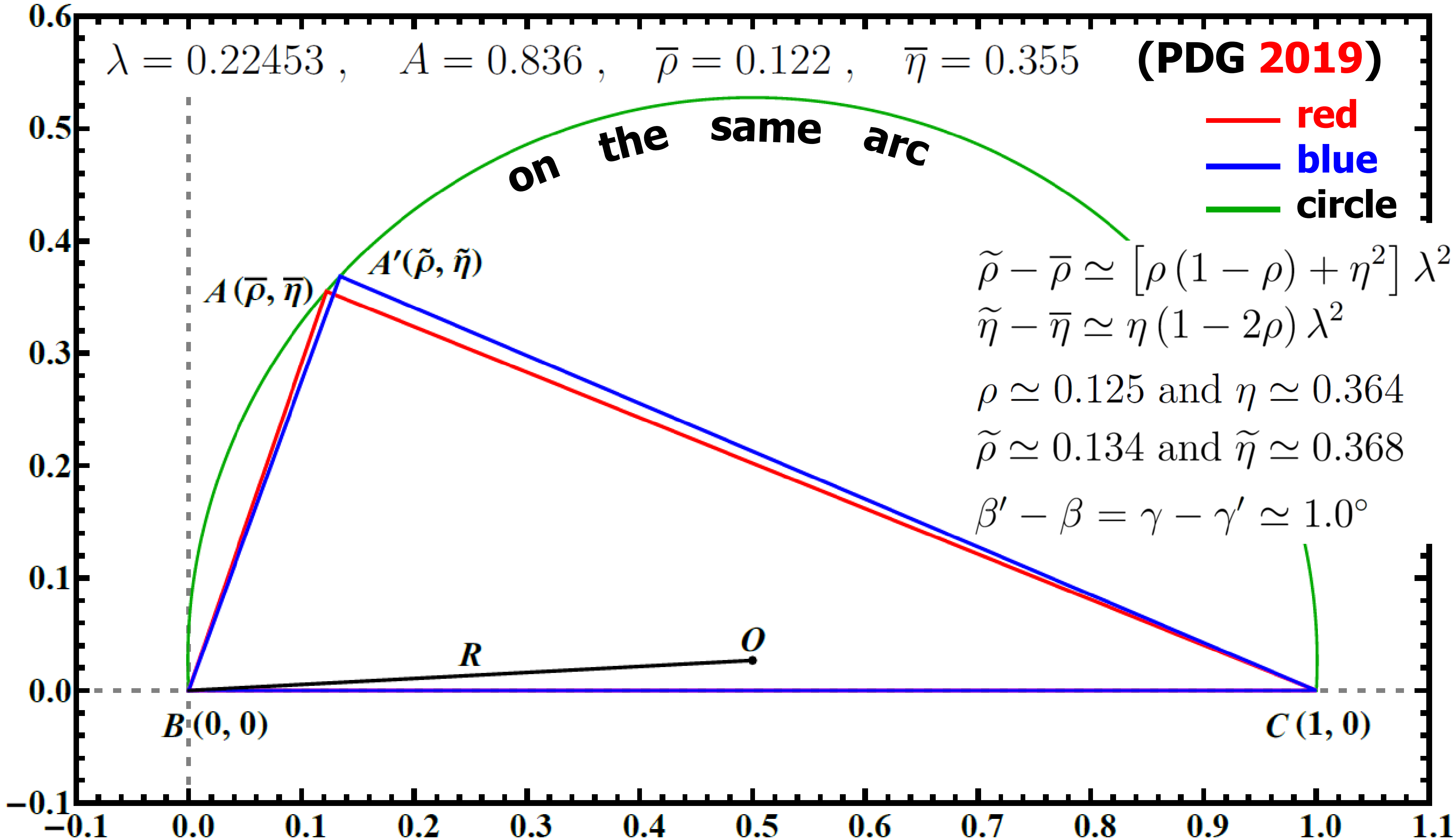
$$\overline{\eta} = \eta\left\{1-\frac{1}{2}\lambda^2 - \left[\frac{1}{8} - \left(\frac{1}{2} - 2\rho\right)A^2\right]\lambda^4\right\} + \mathcal{O}(\lambda^6) ;$$

Z. Xing, D. Zhang 2019

$$\widetilde{\rho} = \rho\left\{1+\left(\frac{1}{2} - \rho + \frac{\eta^2}{\rho}\right)\lambda^2 + \left[\frac{3}{8} - \frac{1}{2}A^2 - \rho(1-\rho) - 3\eta^2 + \frac{\eta^2}{\rho}\right]\lambda^4\right\} + \mathcal{O}(\lambda^6) ,$$

$$\widetilde{\eta} = \eta\left\{1+\left(\frac{1}{2} - 2\rho\right)\lambda^2 + \left[\frac{3}{8} - \frac{1}{2}A^2 - \eta^2 - (2-3\rho)\rho\right]\lambda^4\right\} + \mathcal{O}(\lambda^6) .$$

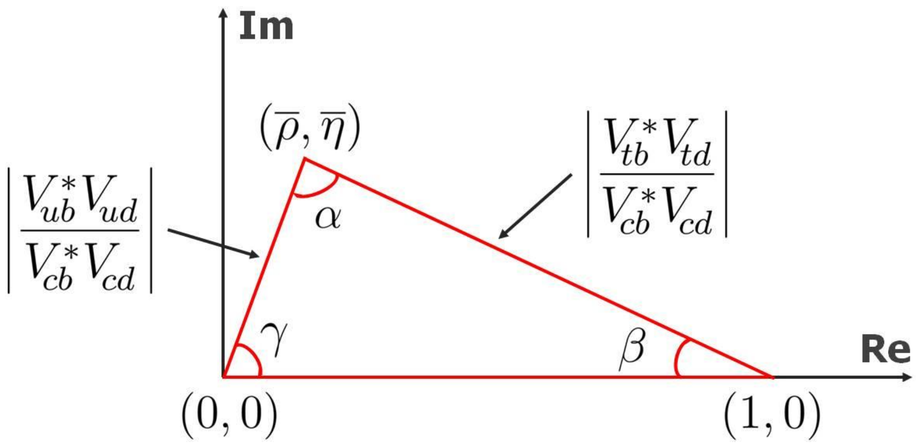
The twin: how similar?



Orbit: $$\left(\widetilde{\rho} - \frac{1}{2}\right)^2 + \left(\widetilde{\eta} - \frac{1}{2}\cot\alpha\right)^2 = \left(\overline{\rho} - \frac{1}{2}\right)^2 + \left(\overline{\eta} - \frac{1}{2}\cot\alpha\right)^2 = \left(\frac{1}{2}\csc\alpha\right)^2$$

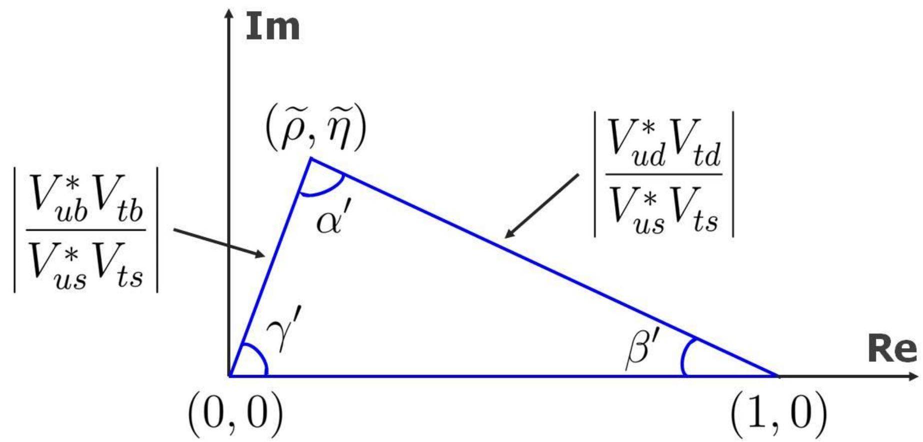
How to test experimentally?

★ **To measure apexes of the red and blue triangles, so as to examine the consistency of the CKM unitarity, if the latter can be established.**



$$\rho + \mathrm{i}\eta = \frac{\sqrt{1 - A^2\lambda^4}\,(\overline{\rho} + \mathrm{i}\overline{\eta})}{\sqrt{1-\lambda^2}\,[1 - A^2\lambda^4\,(\overline{\rho} + \mathrm{i}\overline{\eta})]}$$

Available to extract ρ and η now.



$$\rho + \mathrm{i}\eta = \frac{\sqrt{1 - \lambda^2}\,(\widetilde{\rho} + \mathrm{i}\widetilde{\eta})}{\sqrt{1-A^2\lambda^4}\,[1 - \lambda^2\,(\widetilde{\rho} + \mathrm{i}\widetilde{\eta})]}$$

Is it possible to extract ρ and η?

Constraints from:

- **CPV in K mixing**
- **Bd mixing**
- **Bu / Bd decays**
- **CPV in Bu / Bd decays**

indistinguishable?

a tiny difference?

Constraints from:

- **CPV in K mixing**
- **Bs mixing**
- **Bu / Bs decays**
- **CPV in Bu / Bs decays**

A useful reference

On the Other Five Unitarity Triangles

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Abstract

A comprehensive program of **CP** studies in heavy flavour decays has to go beyond observing large **CP** asymmetries in nonleptonic B decays and finding that the sum of the three angles of the unitarity triangle is consistent with 180°. There are many more correlations between observables encoded in the KM matrix; those can be expressed through five unitarity triangles in addition to the one usually considered. To test the completeness of the KM description one has to obtain a highly over-constrained data set sensitive to $\mathcal{O}(\lambda^2)$ effects with $\lambda = \sin\theta_C$. Those fall into two categories: (i) Certain large angles agree to leading order only, yet differ in order λ^2 in a characteristic way. (ii) Two observable angles are – for reasons specific to the KM ansatz – $\mathcal{O}(\lambda^2)$ and $\mathcal{O}(\lambda^4)$ thus generating an asymmetry of a few percent and of about 0.1 %, respectively. The former can be measured in $B_s \to \psi\eta$, $\psi\phi$ *without* hadronic uncertainty, the latter in Cabibbo suppressed D decays. The intervention of New Physics could boost these effects by an order of magnitude. A special case is provided by $D^+ \to K_{S,L}\pi^+$ vs. $D^- \to K_{S,L}\pi^-$. Finally, **CP** asymmetries involving $D^0 - \bar{D}^0$ oscillations could reach observable levels only due to New Physics.

Two-loop RGEs

Two-loop RGEs for quark Yukawa couplings and CKM matrix elements (M. Machacek, M. Vaughn 84; V. Barger et al 93; M. Luo, Y. Xiao 03):

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}|\end{pmatrix} \simeq (S_1 + S_2)\begin{pmatrix}0 & 0 & |V_{ub}| \\ 0 & 0 & |V_{cb}| \\ |V_{td}| & |V_{ts}| & 0\end{pmatrix}$$

V. Barger et al 93

$$\frac{\mathrm{d}\mathcal{J}}{\mathrm{d}t} \simeq 2\,(S_1 + S_2)\,\mathcal{J} \ , \qquad \boxed{\mathcal{J} \simeq A^2\lambda^6\eta}$$

We find them valid up to $\mathcal{O}(\lambda^4)$.

1-loop

$$S_1 = -\frac{1}{16\pi^2}\left(C^{\mathrm{u}}_{\mathrm{d}} y_t^2 + C^{\mathrm{d}}_{\mathrm{u}} y_b^2\right) \ ,$$

2-loop

$$S_2 = -\frac{1}{(16\pi^2)^2}\left[D^{\mathrm{u}}_{\mathrm{d}} y_t^2 + D^{\mathrm{d}}_{\mathrm{u}} y_b^2 + \left(D^{\mathrm{ud}}_{\mathrm{d}} + D^{\mathrm{du}}_{\mathrm{u}}\right) y_t^2 y_b^2 + D^{\mathrm{uu}}_{\mathrm{d}} y_t^4 + D^{\mathrm{dd}}_{\mathrm{u}} y_b^4\right]$$

SM:

$$C^{\mathrm{u}}_{\mathrm{d}} = C^{\mathrm{d}}_{\mathrm{u}} = -\frac{3}{2} \ , \quad D^{\mathrm{u}}_{\mathrm{d}} \simeq -\frac{79}{80}g_1^2 + \frac{9}{16}g_2^2 - 16g_3^2 + \frac{15}{4}\left(y_t^2 + y_b^2\right) \ ,$$

$$D^{\mathrm{d}}_{\mathrm{u}} \simeq -\frac{43}{80}g_1^2 + \frac{9}{16}g_2^2 - 16g_3^2 + \frac{15}{4}\left(y_t^2 + y_b^2\right) \ , \quad D^{\mathrm{uu}}_{\mathrm{d}} = D^{\mathrm{dd}}_{\mathrm{u}} = \frac{11}{4} \ , \quad D^{\mathrm{ud}}_{\mathrm{d}} = D^{\mathrm{du}}_{\mathrm{u}} = -1$$

MSSM:

$$C^{\mathrm{u}}_{\mathrm{d}} = C^{\mathrm{d}}_{\mathrm{u}} = 1 \ , \quad D^{\mathrm{u}}_{\mathrm{d}} \simeq \frac{4}{5}g_1^2 - 3y_t^2 \ , \quad D^{\mathrm{d}}_{\mathrm{u}} \simeq \frac{2}{5}g_1^2 - 3y_b^2$$

$$D^{\mathrm{uu}}_{\mathrm{d}} = D^{\mathrm{dd}}_{\mathrm{u}} = -2 \ , \quad D^{\mathrm{ud}}_{\mathrm{d}} = D^{\mathrm{du}}_{\mathrm{u}} = 0 \ ,$$

Some observations

- **Wolfenstein parameters run in this way, up to an accuracy of $\mathcal{O}(\lambda^4)$:**

$$\frac{\mathrm{d}\lambda}{\mathrm{d}t} \simeq \frac{\mathrm{d}\rho}{\mathrm{d}t} \simeq \frac{\mathrm{d}\eta}{\mathrm{d}t} \simeq 0 \;, \qquad \frac{\mathrm{d}A}{\mathrm{d}t} \simeq (S_1 + S_2)A \;, \qquad \frac{\mathrm{d}\overline{\rho}}{\mathrm{d}t} \simeq \frac{\mathrm{d}\overline{\eta}}{\mathrm{d}t} \simeq \frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}t} \simeq \frac{\mathrm{d}\widetilde{\eta}}{\mathrm{d}t} \simeq 0$$

The twin rescaled UTs are stable against changes of the energy scale;
The original UTs enlarge or shrink, but their shapes keep unchanged.

- **In the SM:** $\dfrac{S_2}{S_1} \sim \dfrac{1}{16\pi^2} \cdot \dfrac{D^{\mathrm{u}}_{\mathrm{d}} + D^{\mathrm{uu}}_{\mathrm{d}} y_t^2}{C^{\mathrm{u}}_{\mathrm{d}}} \sim \mathcal{O}(\lambda^2)$ **,**

in the MSSM this ratio is roughly $\mathcal{O}(\lambda^3)$ to $\mathcal{O}(\lambda^2)$ **.**

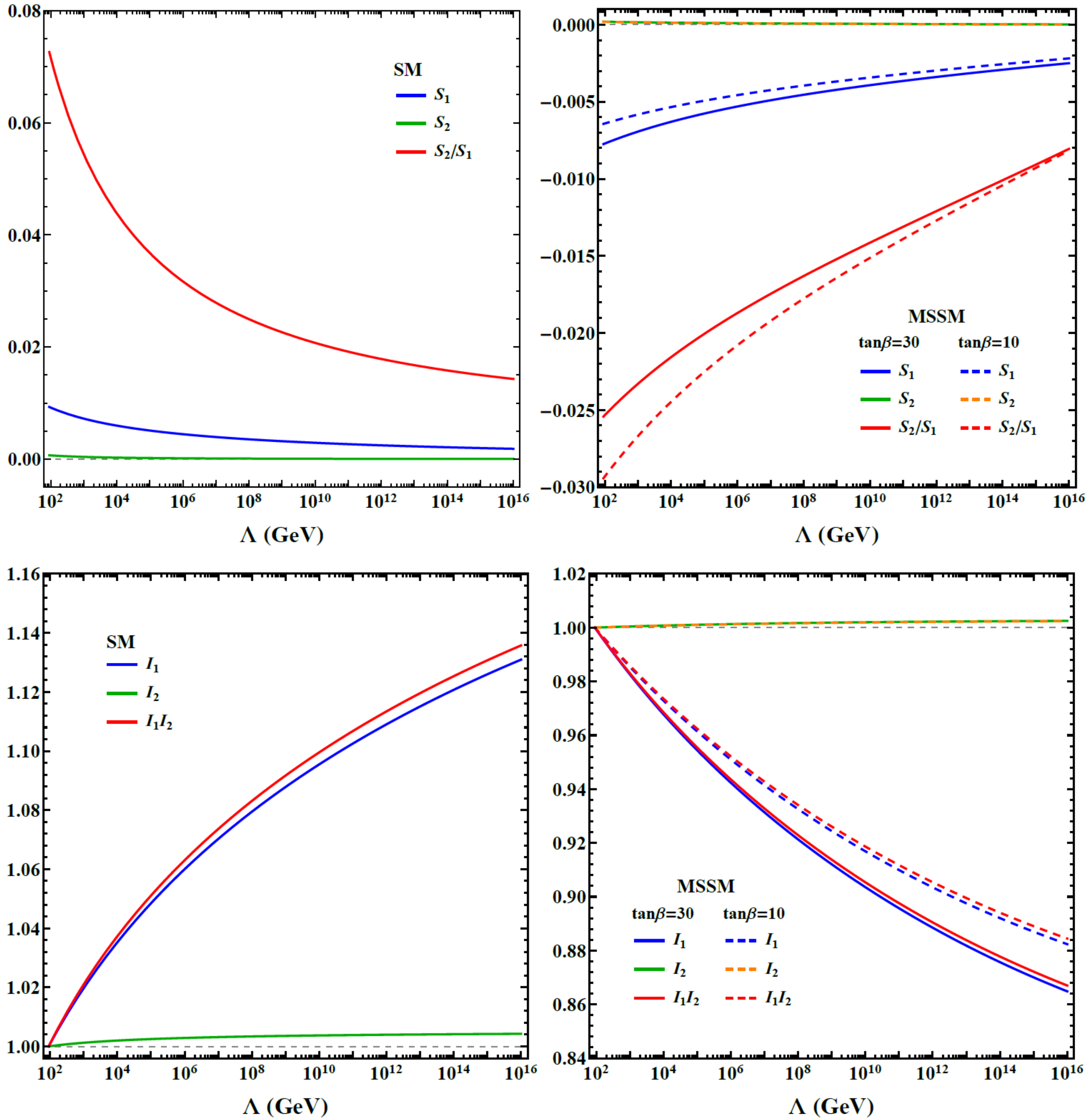
- **Integral form of the evolution of Wolfenstein parameters:**

$$\lambda(\Lambda) \simeq \lambda(\Lambda_{\mathrm{EW}}) \;, \quad \rho(\Lambda) \simeq \rho(\Lambda_{\mathrm{EW}}) \;, \quad \eta(\Lambda) \simeq \eta(\Lambda_{\mathrm{EW}}) \;, \quad A(\Lambda) \simeq I_1 I_2 A(\Lambda_{\mathrm{EW}})$$

$$\overline{\rho}(\Lambda) \simeq \overline{\rho}(\Lambda_{\mathrm{EW}}) \;, \quad \overline{\eta}(\Lambda) \simeq \overline{\eta}(\Lambda_{\mathrm{EW}}) \;, \quad \widetilde{\rho}(\Lambda) \simeq \widetilde{\rho}(\Lambda_{\mathrm{EW}}) \;, \quad \widetilde{\eta}(\Lambda) \simeq \widetilde{\eta}(\Lambda_{\mathrm{EW}}) \;,$$

One- and two-loop functions: $I_i = \exp\left(\displaystyle\int_0^{\ln(\Lambda/\Lambda_{\mathrm{EW}})} S_i \mathrm{d}t\right)$

A numerical illustration is given below.



SM
S1
S2
S2/S1
Λ (GeV)
MSSM
tanβ=30
tanβ=10
S1
S2
S2/S1
S1
S2
S2/S1
Λ (GeV)
SM
I1
I2
I1I2
Λ (GeV)
MSSM
tanβ=30
tanβ=10
I1
I2
I1I2
I1
I2
I1I2
Λ (GeV)

Conclusions

★ **In the high precision measurement era signified by super-B factory and High-luminosity LHCb, it makes sense to probe and establish the other CKM unitarity triangles.**

★ **It is especially interesting to separately establish and compare the b-flavored twin triangles, to test consistency of the SM prediction.**

★ **Our calculations show tiny differences between the twins, and their stability against two-loop RGE running with energy scales. The same study can be extended to the other four CKM triangles.**

★ **A similar study can be extended to the PMNS unitarity triangles in the lepton sector, in particular after CP violation is observed.**

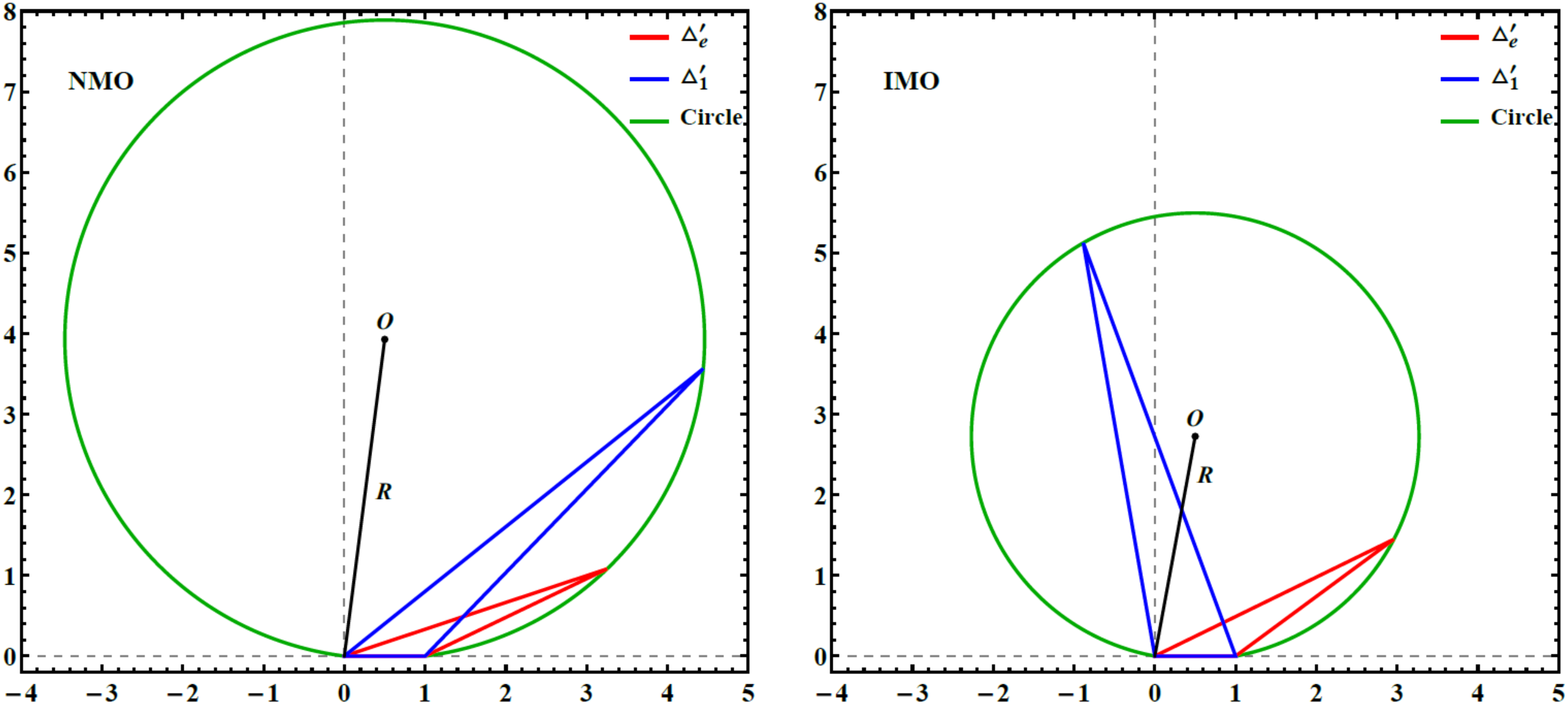
THANK YOU FOR YOUR ATTENTION

Backup: on leptons

The lepton unitarity triangles in the complex plane can be studied in a similar way. Here let us take a pair of them for the sake of illustration.

$$\triangle'_e : \quad 1 + \frac{U_{\mu 2}U^*_{\tau 2}}{U_{\mu 1}U^*_{\tau 1}} + \frac{U_{\mu 3}U^*_{\tau 3}}{U_{\mu 1}U^*_{\tau 1}} = 0$$

$$\triangle'_1 : \quad 1 + \frac{U_{\mu 3}U^*_{\mu 2}}{U_{e3}U^*_{e2}} + \frac{U_{\tau 3}U^*_{\tau 2}}{U_{e3}U^*_{e2}} = 0$$



$$\left(\rho_e - \frac{1}{2}\right)^2 + \left(\eta_e - \frac{1}{2}\cot\alpha_e\right)^2 = \left(\rho_1 - \frac{1}{2}\right)^2 + \left(\eta_1 - \frac{1}{2}\cot\alpha_e\right)^2 = \left(\frac{1}{2}\csc\alpha_e\right)^2$$