Nonperturbative calculations of form factors for exclusive semileptonic $B_{(s)}$ decays

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Nonperturbative calculations of form factors for exclusive semileptonic $B_{(s)}$ decays

in collaboration with
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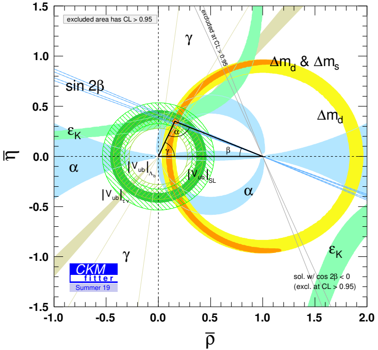
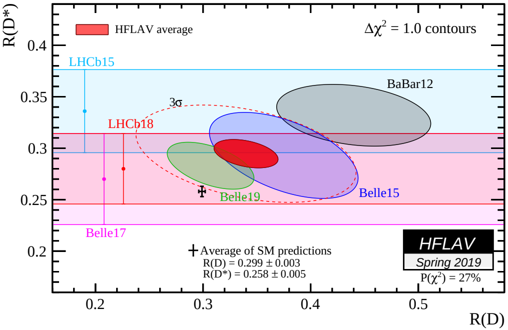
introduction

Motivation

[CKMfitter]



[HFLAV spring 2019]



- Determine fundamental parameters of the Standard Model e.g. $|V_{ub}|$, $|V_{cb}|$, $|V_{td}|$, $|V_{ts}|$
- May address interesting observations or challenge the Standard Model
  $\to$ e.g. test lepton flavor universality via $R(D^{(*)})$

Why B_s meson decays?

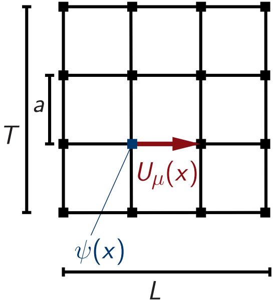
- Experimentally measured by LHCb e.g. [LHCb PRD 101 (2020) 072004]
- Alternative, tree-level determination of $|V_{cb}|$ and $|V_{ub}|$ from $B_s \to D_s\ell\nu$ and $B_s \to K\ell\nu$
  - → Cross-checking commonly used $B \to \pi\ell\nu$ and $B \to D^{(*)}\ell\nu$
  - → Only spectator quark changes; flavor symmetry should hold very well
  - → Longstanding $2-3\sigma$ discrepancy between exclusive ($B \to \pi\ell\nu$) and inclusive ($B \to X_u\ell\nu$)
- Likely more precise than $B \to \tau\nu$
- Alternative, exclusive ($\Lambda_b \to p\ell\nu$) determination [Detmold, Lehner, Meinel, PRD92 (2015) 034503]
- Strange-quarks are easier for nonperturbative lattice calculations

Lattice calculation

- Wick-rotate to Euclidean time $t \to i\tau$
- Discretize space-time and set up a hypercube of finite extent $L^3 \times T$ and spacing a
- Use path integral formalism

$$\langle\mathcal{O}\rangle_E = \frac{1}{Z}\int \mathcal{D}[\psi,\overline{\psi}]\,\mathcal{D}[U]\,\mathcal{O}[\psi,\overline{\psi},U]\,e^{-S_E[\psi,\overline{\psi},U]}$$

$\Rightarrow$ Large but finite dimensional path integral



- Finite volume of length L $\to$ IR regulator
  - $\to$ Study physics in a finite box of volume $(aL)^3$
  - $\to$ Strongly prefer decays with 1 (QCD-stable) hadronic final state (narrow width approximation)

- Finite lattice spacing a $\to$ UV regulator
  - $\to$ Quark masses need to obey $am < 1$

Simulating charm and bottom (schematic)

$a^{-1} > 1.5$ GeV

charm: RHQ; extrapolations of fully relativistic actions (?)

bottom: HQET, NRQCD, RHQ

$a^{-1} > 2.2$ GeV

charm: fully relativistic action

bottom: (guided) extrapolation of fully relativistic action

$a^{-1} > 4.6$ GeV

bottom: fully relativistic action

HQET: static limit, relatively noisy

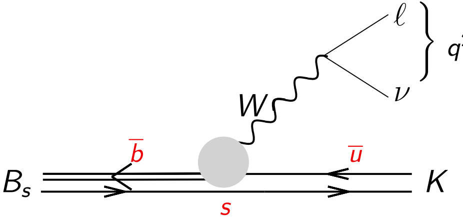
NRQCD: non-relativistic QCD, no continuum limit

RHQ or Fermilab: relativistic heavy quark action, complicated discretization errors

(heavy) HISQ or (heavy) MDWF: fully relativistic, clean nonperturbative renormalization

$$B_s \to K\ell\nu$$

$|V_{ub}|$ from exclusive semileptonic $B_s \to K\ell\nu$ decay



$$q^2 = M_{B_s}^2 + M_K^2 - 2M_{B_s}E_K$$

- Conventionally parametrized by (B_s meson at rest)

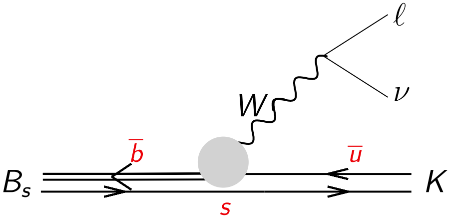
$$\frac{d\Gamma(B_s \to K\ell\nu)}{dq^2} = \frac{\eta_{EW} G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_K^2 - M_K^2}}{q^4 M_{B_s}^2}$$

experiment CKM known

$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2 (E_K^2 - M_K^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_{B_s}^2 - M_K^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

$|V_{ub}|$ from exclusive semileptonic $B_s \to K\ell\nu$ decay



- $f_+(q^2)$ and $f_0(q^2)$
  - → Parametrizes interactions due to the (nonperturbative) strong force
  - → Use operator product expansion (OPE) to identify short distance contributions
  - → Calculate matrix element of the flavor changing currents as point-like operators using lattice QCD

- Conventionally parametrized by (B_s meson at rest)

$$\underbrace{\frac{d\Gamma(B_s \to K\ell\nu)}{dq^2}}_{\text{experiment}} = \underbrace{\frac{\eta_{EW} G_F^2 |V_{ub}|^2}{24\pi^3}}_{\text{CKM}} \underbrace{\frac{(q^2 - m_\ell^2)^2 \sqrt{E_K^2 - M_K^2}}{q^4 M_{B_s}^2}}_{\text{known}}$$

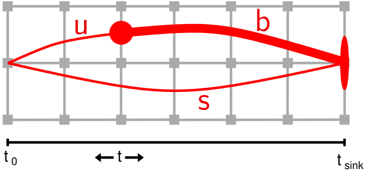
$$\times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2 (E_K^2 - M_K^2) |f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2} (M_{B_s}^2 - M_K^2)^2 |f_0(q^2)|^2 \right]$$

nonperturbative input

$B_s \to K\ell\nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K|V^\mu|B_s\rangle = f_+(q^2)\left(p^\mu_{B_s} + p^\mu_K - \frac{M^2_{B_s} - M^2_K}{q^2} q^\mu\right) + f_0(q^2)\frac{M^2_{B_s} - M^2_K}{q^2} q^\mu$$



- Calculate 3-point function by
  - → Inserting a quark source for a strange quark propagator at t_0
  - → Allow it to propagate to t_{sink}, turn it into a sequential source for a b quark
  - → Propagate a light quark from t_0 and contract with b quark at t with $t_0 \leq t \leq t_{sink}$

$B_s \to K\ell\nu$ form factors

- Parametrize the hadronic matrix element for the flavor changing vector current V^μ in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$

$$\langle K|V^\mu|B_s\rangle = f_+(q^2)\left(p^\mu_{B_s} + p^\mu_K - \frac{M^2_{B_s} - M^2_K}{q^2} q^\mu\right) + f_0(q^2)\frac{M^2_{B_s} - M^2_K}{q^2} q^\mu$$

- Prefer to compute

$$f_\parallel(E_K) = \langle K|V^0|B_s\rangle/\sqrt{2M_{B_s}} \quad \text{and} \quad f_\perp(E_K)p^i_K = \langle K|V^i|B_s\rangle/\sqrt{2M_{B_s}}$$

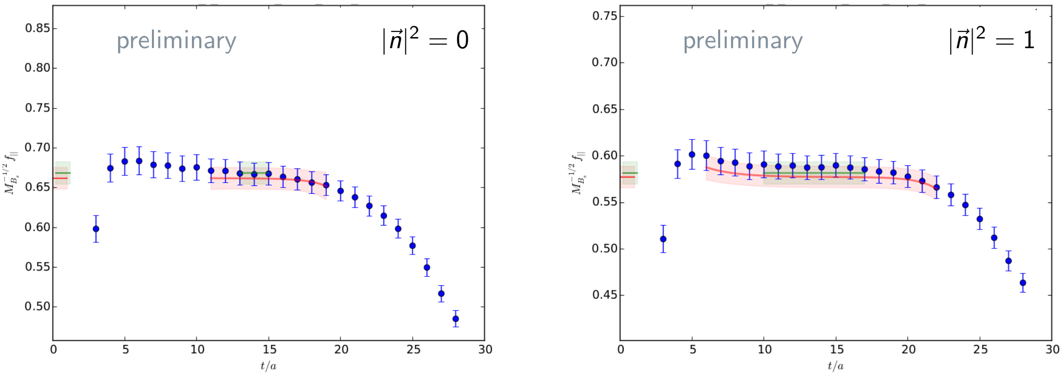
which are directly proportional to 3-point functions

- Both are related by

$$f_0(q^2) = \frac{\sqrt{2M_{B_s}}}{M^2_{B_s} - M^2_K}\left[(M_{B_s} - E_K)f_\parallel(E_K) + (E^2_K - M^2_K)f_\perp(E_K)\right]$$

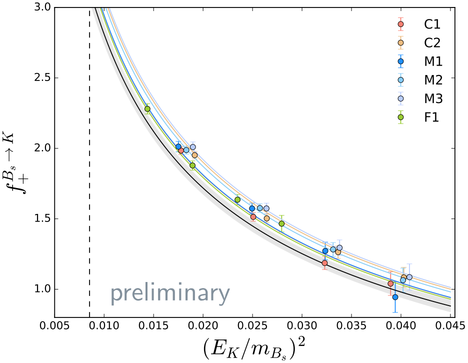
$$f_+(q^2) = \frac{1}{\sqrt{2M_{B_s}}}\left[f_\parallel(E_K) + (M_{B_s} - E_K)f_\perp(E_K)\right]$$

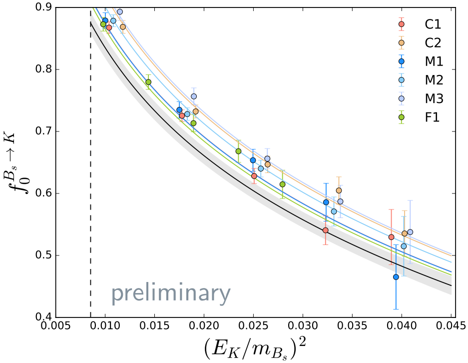
$B_s \to K\ell\nu$ form factors: F1S ensemble



- Comparison of fit to the ground state only with fit including one excited state term for K and B_s

Chiral-continuum extrapolation using SU(2) hard-kaon χPT





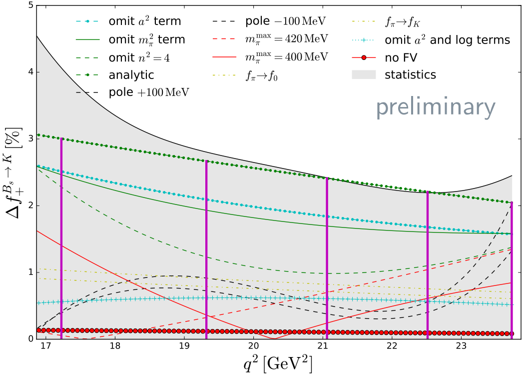
- Updating [Flynn et al. PRD 91 (2015) 074510] adding third, finer lattice spacing, improved values for a^{-1} and RHQ parameters
- $f_{pole}(M_K, E_K, a^2) = \frac{c_0\Lambda}{E_K+\Delta} \times \left[1 + \frac{\delta f}{(4\pi f)^2} + c_1\frac{M_\pi^2}{\Lambda^2} + c_2\frac{E_K}{\Lambda} + c_3\frac{E_K^2}{\Lambda^2} + c_4(a\Lambda)^2\right]$
- δf non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \to 0$

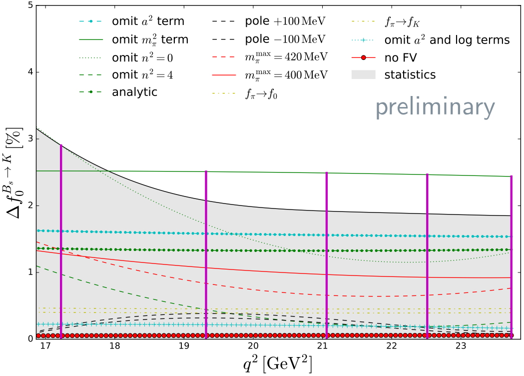
Estimate systematic errors due to

- Chiral-continuum extrapolation
  - → Use alternative fit functions, vary pole mass, etc.
  - → Impose different cuts on the data
- Discretization errors of light and heavy quarks
  - → Estimate via power-counting
- Uncertainty of the renormalization factors
  - → Estimate effect of higher loop corrections
- Finite volume, isospin breaking, ...
- Uncertainty due to RHQ parameters and lattice spacing (a^{-1})
  - → Carry out additional simulations to test effects on form factors
- Uncertainty of strange quark mass
  - → Repeat simulation with different valence quark mass

$\Longrightarrow$ full error budget

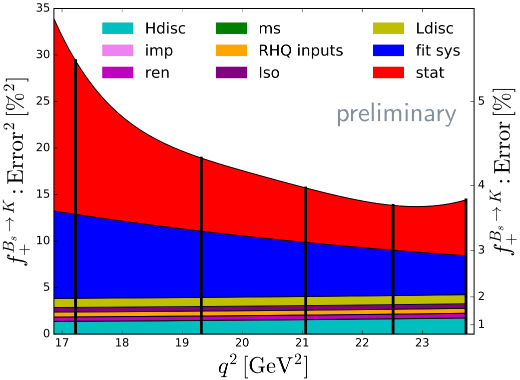
Error budget $B_s \to K\ell\nu$

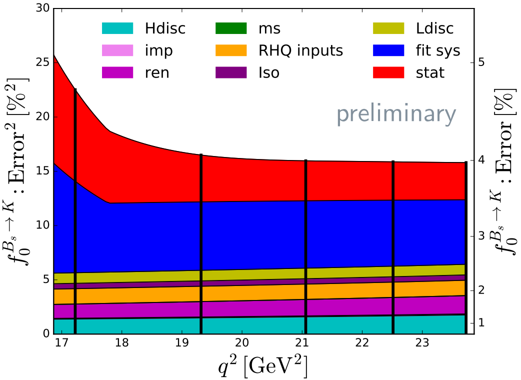




- $\Delta f = \left| f^{\text{variation}} - f^{\text{central}} \right| / f^{\text{central}}$

Error budget $B_s \to K\ell\nu$





Kinematical extrapolation (z-expansion)

[Boyd, Grinstein, Lebed, PRL 74 (1995) 4603] [Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Map complex q^2 plane with cut $q^2 > t_*$ onto the unit disk in z

$$z(q^2, t_*, t_0) = \frac{\sqrt{t_* - q^2} - \sqrt{t_* - t_0}}{\sqrt{t_* - q^2} + \sqrt{t_* - t_0}}$$

with

$$t_* = (M_B + M_\pi)^2 \qquad \text{(two-particle production threshold)}$$
$$t_\pm = (M_{B_s} \pm M_K)^2 \qquad \text{(with } t_- = q^2_{max}\text{)}$$
$$t_0 \equiv t_{\rm opt} = t_* - \sqrt{t_*(t_* - t_-)} \qquad \text{(symmetrize range of } z\text{)}$$

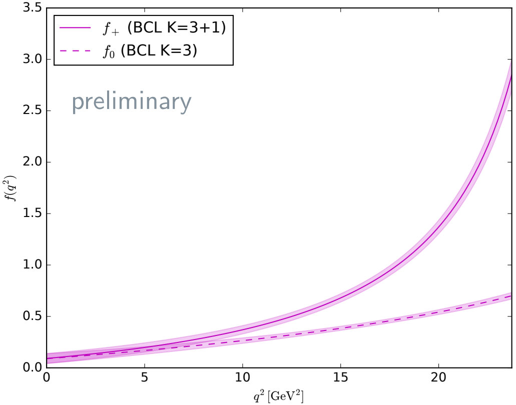
- BCL express form factor f_+ for $B \to \pi\ell\nu$

$$f_+(q^2) = \frac{1}{1 - q^2/M^2_{pole}} \sum_{k=0}^{K-1} b_k^+(t_0) z^k$$

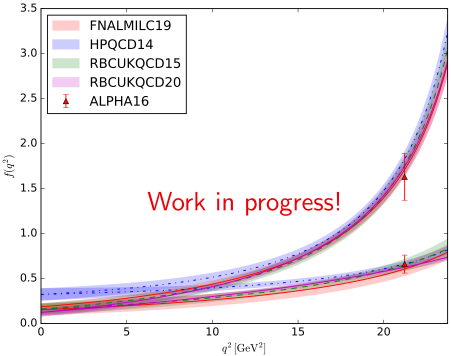
- For other decays use product of factors for subthreshold poles for both f_+ and f_0 paralleling the Blaschke factors for a BGL fit to the same decay

Kinematical extrapolation (z-expansion)

- Perform fit in z-space with K parameters
- Then convert back to physical q^2
- BCL with pole $M_+ = B^* = 5.33$ GeV for f_+
- Exploit kinematic constraint $f_+ = f_0\Big|_{q^2=0}$
- Include HQ power counting to constrain size of f_+ coefficients (work in progress)



Kinematical extrapolation (z-expansion)



- Compare form factors to other determinations
  - → FNAL MILC19 [Bazavov et al. arXiv:1901.02561]
  - → HPQCD14 [Bouchard et al. PRD 90 (2014) 054506]
  - → HPQCD18 [Monahan et al. PRD 98 (2014) 114509]
  - → RBC UKQCD15 [Flynn et al. PRD 91 (2015) 074510]
  - → ALPHA16 [Bahr et al. PLB757(2016)473]
  - → Analytic predictions at $q^2 = 0$

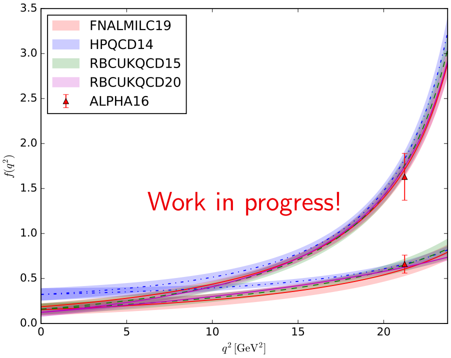
    [Duplancic et al. PRD78 (2008) 054015]

    [Faustov et al. PRD87 (2013) 094028]

    [Wang et al. PRD86 (2012) 114025]

    [Khodjamirian et al. JHEP08 (2017) 112]

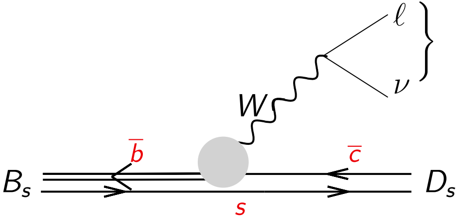
Kinematical extrapolation (z-expansion)



- Combination with experimental results gives $|V_{ub}|$
- Determine ratios to test lepton flavor universality
- Predict forward-backward asymmetries
- ...

$$B_s \to D_s \ell \nu$$

$|V_{cb}|$ from exclusive semileptonic $B_s \to D_s\ell\nu$ decay



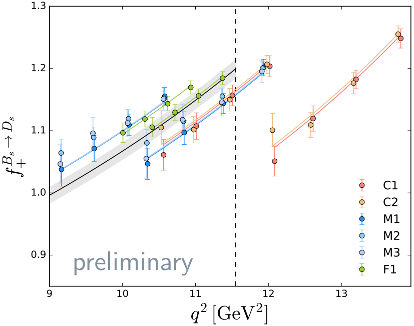
$$q^2 = M_{B_s}^2 + M_{D_s}^2 - 2M_{B_s}E_{D_s}$$

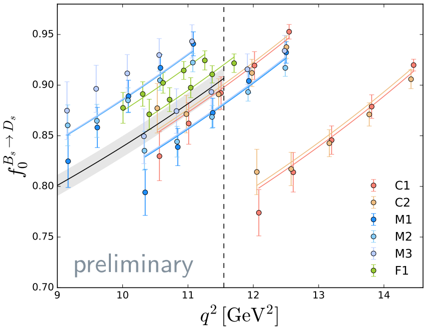
- Conventionally parametrized by (B_s meson at rest)
- Accommodate charm quarks

$$\underbrace{\frac{d\Gamma(B_s \to D_s\ell\nu)}{dq^2}}_{\text{experiment}} = \underbrace{\frac{\eta_{EW} G_F^2 |V_{ub}|^2}{24\pi^3}}_{\text{CKM}} \underbrace{\frac{(q^2 - m_\ell^2)^2 \sqrt{E_{D_s}^2 - M_{D_s}^2}}{q^4 M_{B_s}^2}}_{\text{known}}$$

$$\times \underbrace{\left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2 (E_{D_s}^2 - M_{D_s}^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(M_{B_s}^2 - M_{D_s}^2)^2 |f_0(q^2)|^2\right]}_{\text{nonperturbative input}}$$

Global fit $B_s \to D_s\ell\nu$

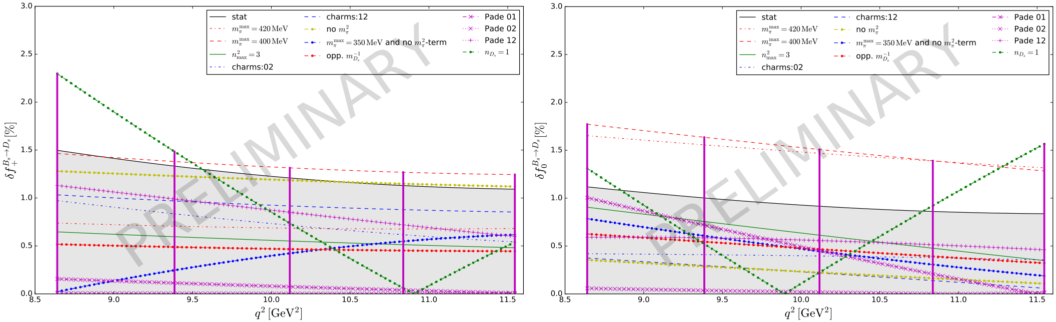




- $$f(q^2, a, M_\pi, M_{D_s}) = \left[c_0 + c_1 \frac{M_\pi^2}{\Lambda^2} + \sum_{j=1}^{n_{D_s}} c_{2,j} \left[\Lambda \Delta M_{D_s}^{-1}\right]^j + c_3 (a\Lambda)^2 + c_4 (am_q)^2 \right] P_{a,b}\left(\frac{q^2}{M_{B_s}^2}\right)$$

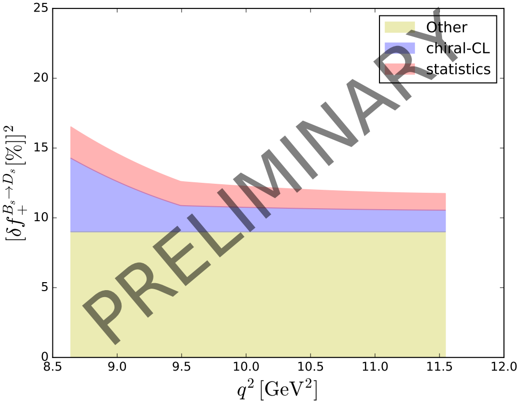
with $$\Delta M_{D_s}^{-1} \equiv \left(\frac{1}{M_{D_s}} - \frac{1}{M_{D_s}^{\text{phys}}}\right), \quad P_{a,b}(x) = \frac{1 + \sum_{i=1}^a a_i x^i}{1 + \sum_{i=1}^b b_i x^i}$$

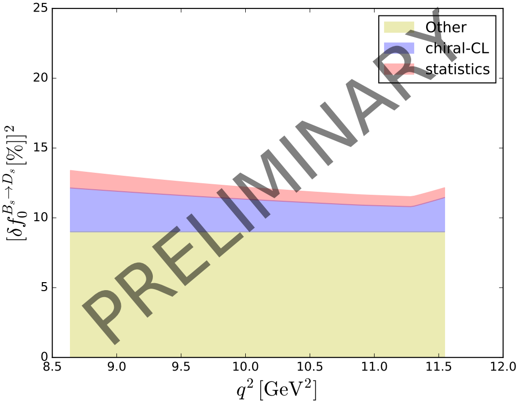
PRELIMINARY error budget $B_s \to D_s\ell\nu$



- $\delta f = \left| f^{\text{variation}} - f^{\text{central}} \right| / f^{\text{central}}$

PRELIMINARY error budget $B_s \to D_s\ell\nu$





- "Other": 3% placeholder to cover higher order corrections, lattice spacing, finite volume, ...

summary

Summary

- Calculation for $B_s \to K\ell\nu$ form factors essentially completed
  - → Working on comparison to other determinations
  - → Extracting phenomenologically interesting quantities

- Finalizing systematic error budget for $B_s \to D_s\ell\nu$ decays