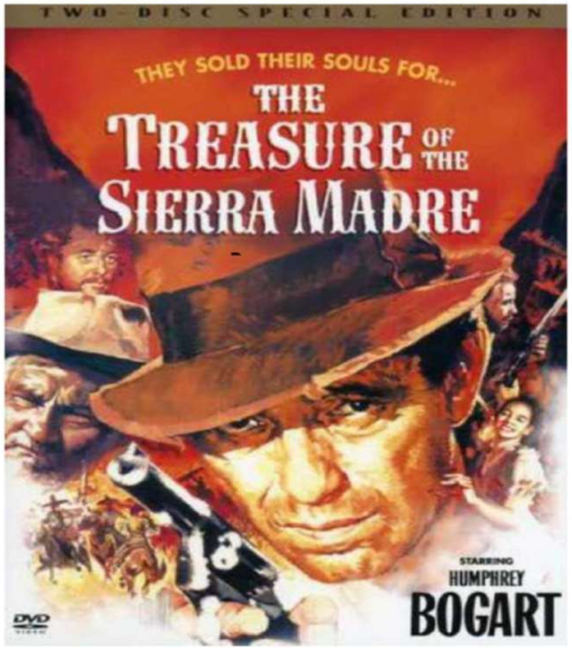
[More] Treasures from Kaons



Amarjit Soni
HET@BNL

ICHEP 2020

07/28/20

Outline

I

- **Unforgetable Memories!**
- **Exciting history .....**
- **Bagged 2 Nobels for BNL!**
- **Magics of –QM-mixings-K0**
- **A very important consequence of the QM mixing: K_LONG….**
- **Delta Mk constraints =>stirs up flavor and CP puzzles of BSMs**
- **Another important consequence of Δmk => K-mixing, BSM vs SM**
- **Lattice BK=> Epsilon_K => precision constraints on the modern day UT fit**

An extremely powerful tool!

outline

- Basics of Direct CP in K=> $\pi\pi$ i.e. ε'
- Early attempt(s), hurdles & resolution
- I. Breakthrough: Domain wall & chiral symmetric formulation
- II. Another key development: Lellouch-Luscher method
- 1^{st} completion ~2015 & indication of difficulty
- Improved stats & systematic=> new result
- some implications
- Summary + Outlook

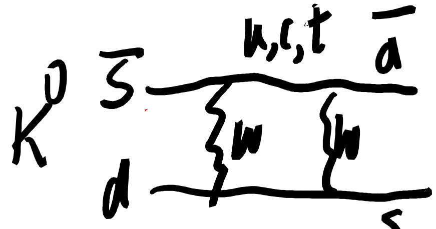
ΔmK^{expt} vs ΔmK^{theory}

- **ΔmK^{expt} extremely precise** → $[3.484 \pm 0.006] \times 10^{-12}$ MeV

N+H; B+G → pert theory NLO

- **ΔmK^{theory} O(50%) errors…….LD, non-local, 4-q OP as OPE is NOT valid ….intermediate pipi states make significant contribution.**
- **Historically, therefore, the very well measured experimental # cannot be used as a precision tool for constraining SM or BSM**
- **RBC-UKQCD past ~5 years with new lattice methodology is working to change this situation…CU PhD students 1. JiangLei Yu 2.Ziyuan Bai , 3. Bigeng Wang [NOW] ….$\delta(\Delta mK^{theory})$ ~O(20%)……checks underway NOW**

II $K^0 - \overline{K^0}$ Mixing, Decay, Indirect CP violation



QM mixing

$\Delta S = 2$

$O(G_F^2)$

2 States K_L, K_S If CP is exact

$$K_L \equiv \frac{K^0 - \overline{K^0}}{\sqrt{2}} \; ; \quad K_S \equiv \frac{K^0 + \overline{K^0}}{\sqrt{2}}$$

CP−   CP+

$\to 3\pi$ $\to 2\pi$

$\not\to 2\pi$ $\not\to 3\pi$

The long lifetime of K_L a very important blessing; lead to one of the most important discoveries in Particle Physics ie CP violation

$\frac{\Delta m_K}{m_K} \sim 7 \times 10^{-15}$ But $\tau_{K_L} / \tau_{K_S} \sim O(500) \gg 1$

$c\tau$ ~ 15 m!

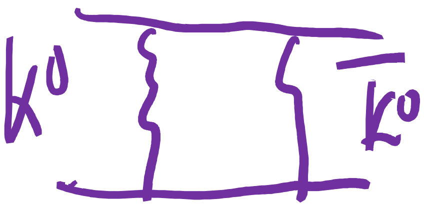
III Indirect CP violation

BNL 1964 Fitch, Cronin, Christensen + Turlay

NOBLE
CRONIN
+
FITCH

$$\frac{A(K_L \to \pi\pi)}{A(K_S \to \pi\pi)} \neq 0 \ !$$

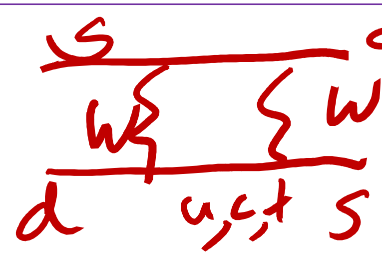
$$\epsilon_K \sim 2.23 \times 10^{-3}$$



CPV in state mixing, $\Delta S=2$ H_{eff}

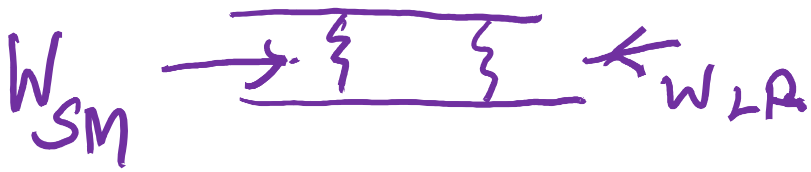
ΔmK : a powerful constraint on BSM

In SM $\Delta S=2$



$+\ \cdots\cdots$

an explicit illustration: LRSM; Beall + Bander + AS PRL 1982



$$\left(\frac{\Delta m_K}{m_K}\right)^{expt} \approx 10^{-14}!!$$

$\Rightarrow m_R \gtrsim 1.6\ \text{TeV}$

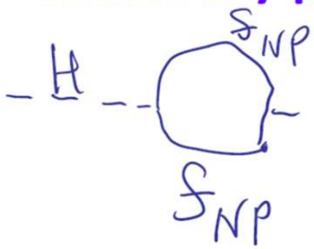
$\Rightarrow$ Flavor + CP puzzles of BSMs

MR > ~20 MW

- **An interesting tale of 4 factor of a few all going in one direction to render MR larger than MW.**
  - **Starting point is the simple observation that LXR is NOT Fierz invariant unlike SM LXL**
- **Indeed, LXL =-2 [S+P]X[S-P] whereas for SM, LXL = LXL**
- **Thus M_LR => Const as mq=>0 whereas M_LL i.e. SM =>0 as m_q => 0 with naïve factorization MLR ~ (O5) X SM**
- **Soon one realizes there are another 3 factors of O(2) all causing enhancement including NLO QCD.**
- **Very soon these factors pile up to O(20)**

Outstanding Th.puzzles of our times

- **Hierarchy puzzle**



$\sim \frac{g_{NP}^2}{16\pi^2}\Lambda_{NP}^2 \Rightarrow \Lambda_{NP} \lesssim$ TeV

to avoid fine tuning m_H

- **Flavor puzzle** Δ flava = 2 e.g.

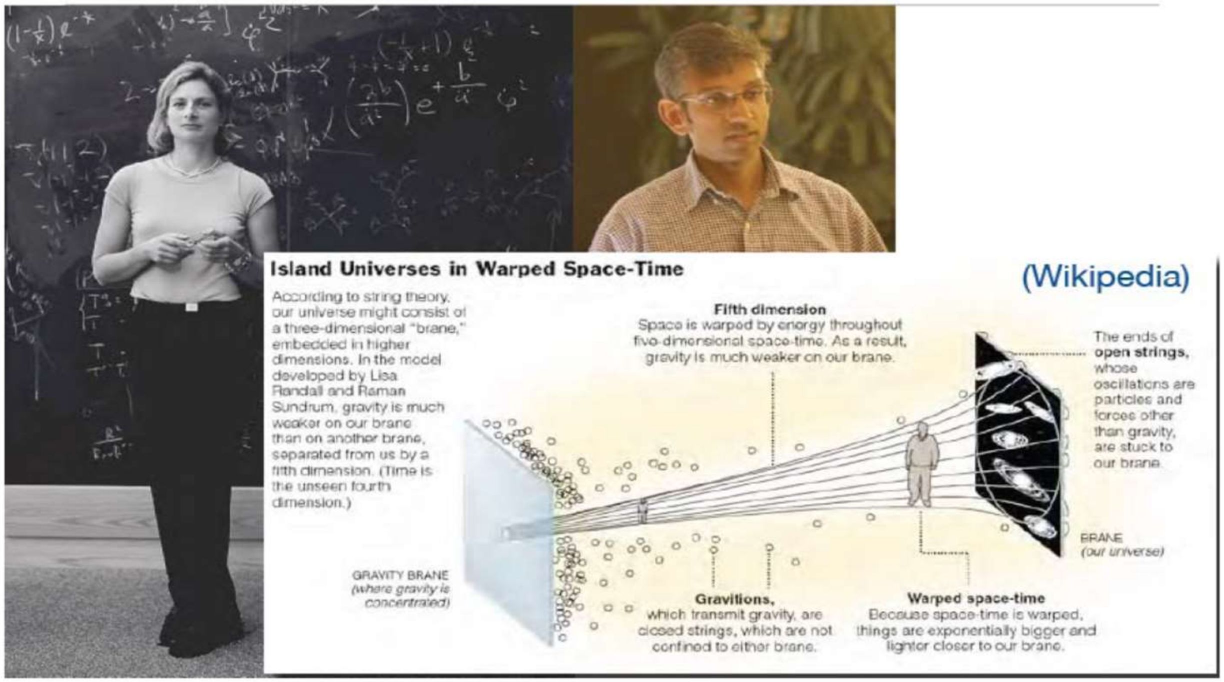
$\sim \frac{g^2_{NP}}{\Lambda^2_{NP}} \Rightarrow \Lambda_{NP} \gtrsim 10^3$ TeV

to avoid Constraint from Δm_K, ϵ_K . . .

$\Rightarrow$ "NO LOSE" Thm

By IGNORING FLAVOR BUT Flavors exist

The Randall-Sundrum (RS) idea



[Stolen from Neubert]

9/13/13 A. Soni HET-BNL 13

PHYSICAL REVIEW D **71,** 016002 (2005)

Flavor structure of warped extra dimension models

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(Received 14 September 2004; published 6 January 2005)

We recently showed that warped extra-dimensional models with bulk custodial symmetry and few TeV Kaluza-Klein (KK) masses lead to striking signals at B factories. In this paper, using a spurion analysis, we systematically study the flavor structure of models that belong to the above class. In particular we find that the profiles of the zero modes, which are similar in all these models, essentially control the underlying flavor structure. This implies that our results are robust and model independent in this class of models. We discuss in detail the origin of the signals in B physics. We also briefly study other new physics signatures that arise in rare K decays ($K \to \pi\nu\nu$), in rare top decays [$t \to c\gamma(Z,\text{gluon})$], and the possibility of CP asymmetries in D^0 decays to CP eigenstates such as $K_S\pi^0$ and others. Finally we demonstrate that with light KK masses, ~3 TeV, the above class of models with anarchic 5D Yukawas has a "CP problem" since contributions to the neutron electric dipole moment are roughly 20 times larger than the current experimental bound. Using AdS/CFT correspondence, these extra-dimensional models are dual to a purely 4D strongly coupled conformal Higgs sector thus enhancing their appeal.

DOI: 10.1103/PhysRevD.71.016002 PACS numbers: 11.25.Wx, 11.30.Hv

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www.elsevier.com/locate/physletb

The little Randall–Sundrum model at the large hadron collider

"LRS"

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ABSTRACT

We present a predictive warped model of flavor that is cut off at an ultraviolet scale $\mathcal{O}(10^3)$ TeV. This "Little Randall–Sundrum (LRS)" model is a volume-truncation, by a factor $y \approx 6$, of the RS scenario and is holographically dual to dynamics with number of colors larger by y. The LRS couplings between Kaluza–Klein states and the Standard Model fields, including the proton constituents, are explicitly calculable without *ad hoc* assumptions. Assuming separate gauge and flavor dynamics, a number of unwanted contributions to precision electroweak, $Zb\bar{b}$ and flavor observables are suppressed in the LRS framework, compared with the corresponding RS case. An important consequence of the LRS truncation, independent of precise details, is a significant enhancement of the clean (golden) di-lepton LHC signals, by $\mathcal{O}(y^3)$, due to a larger "ρ-photon" mixing and a smaller inter-composite coupling.

Little Randall-Sundrum models: ϵ_K strikes again

M. Bauer, S. Casagrande, L. Gründer, U. Haisch and M. Neubert
Institut für Physik (THEP), Johannes Gutenberg-Universität, D-55099 Mainz, Germany

(Dated: November 22, 2008)

A detailed phenomenological analysis of neutral kaon mixing in "little Randall-Sundrum" models is presented. It is shown that the constraints arising from the CP-violating quantity ϵ_K can, depending on the value of the ultra-violet cutoff, be even stronger than in the original Randall-Sundrum scenario addressing the hierarchy problem up to the Planck scale. The origin of the enhancement is explained, and a bound Λ_{UV} > several 10^3 TeV is derived, below which vast corrections to ϵ_K are generically unavoidable. Implications for non-standard $Z^0 \to b\bar{b}$ couplings are briefly discussed.

PACS numbers: 11.10.Kk,12.60.-i,12.90.+b,13.20.Eb,13.38.Dg

In both cases as in SUSY-like & most BSM, the LXL of SM $\Delta S=2$ becomes LXR-like as in LRSM leading to enhanced MEs for K-$\bar{K}$

II. K => ππ, ΔI = ½ Rule & ε’

Delta I=1/2 rule (puzzle): a challenge for generations

$I=0,2$

$\pi^+\pi^-$

MAIN MODES $\qquad \tau$

- K_s $\qquad \pi^+\pi^-$ $\qquad \sim 0.9\times10^{-10}\,s$ $\qquad K^0$ $\quad \bar{s}$, d

$\Delta I = 1/2, 3/2$

$\pi^+\pi^0$

- K^+ $\qquad \pi^+\pi^0$ $\quad \cancel{\Delta I=3/2}$ $\sim 1.2\times10^{-8}\,s$ $\qquad K^+$ $\quad \bar{s}$, u

$I=2$

K_L $\qquad \pi^+\pi^-\pi^0$ $\qquad \sim 5\times10^{-8}\,s$

phase space suppressed

IV: ε' / ε: Direct CPV

EXPERIMENTAL ROUTE

$$\eta_{+-} = |\eta_{+-}| e^{i\phi_{+-}} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}$$

$$\eta_{00} = |\eta_{00}| e^{i\phi_{00}} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}$$

$$\eta_{+-} = \epsilon + \epsilon', \qquad \eta_{00} = \epsilon - 2\epsilon'$$

$$\epsilon' \simeq \frac{1}{3}\left[\eta_{+-} - \eta_{00}\right] \Rightarrow O(10^{-3}) - O(10^{-3}) \Rightarrow 10^{-6}$$

$$\epsilon = \frac{1}{3}\left[2\eta_{+-} + \eta_{00}\right]$$

Conf on NP@LHC 02/29/16; A. Soni 10

BSM-CP: Theoretical motivation

- **Since CP violation was experimentally seen in 1964, this means CP is NOT a symmetry of nature.**
- **Therefore, we cannot set the CP-odd phase(s) naturally to zero.**
- **BSMs are naturally endowed with CP-odd phases.**
- **Since epsilon' is a lot smaller than even eps, it should be extremely sensitive to the new phase(s).**
- ***This naturalness based argument is a compelling argument for us to try understand esp' quantitatively as precisely as possible.***
- **Moreover, SM cannot account for baryogenesis…..CKM CP not enough**
- **Due to all of the above (and some more) reasons searching for BSM CP-phase(s) is just about the most powerful way to look for NP…..*an early realization & a driving force for pursuing eps' for past few decades***

K→2π

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

I=2

I=0

Use lattice to calculate 6 quantities: ReA0, ReA2 known from expt; δ0,δ2 via ChPT etc..So very good checks; ImA. ImA2 unknown

$\omega \equiv \mathrm{Re}A_2/\mathrm{Re}A_0 \sim 0.045$

Indirect CP

$$|\epsilon| = 2.228(11) \times 10^{-3},$$

$$\mathrm{Re}(\epsilon'/\epsilon) = 1.65(26) \times 10^{-3}.$$

DIRECT CP

$\epsilon' << \epsilon$

$\epsilon' \sim 10^{-6}$!

A.S. in Proceedings of Lattice '85 (FSU)..1st Lattice meeting ever attended

The matrix elements of some penguin operators control in the standard model another CP violation parameter, namely ϵ'/ϵ.[6,8] Indeed efforts are now underway for an improved measurement of this important parameter.[10] In the absence of a reliable calculation for these parameters, the experimental measurements, often achieved at tremendous effort, cannot be used effectively for constraining the theory. It is therefore clearly important to see how far one can go with MC techniques in alleviating this old but very difficult

With C. Bernard [UCLA]

Serves as a template for the need of Lattice calculations for more economical use of almost all experimental data From IF

MOTHER of all (lattice) calculations to date:
A Personal Perspective

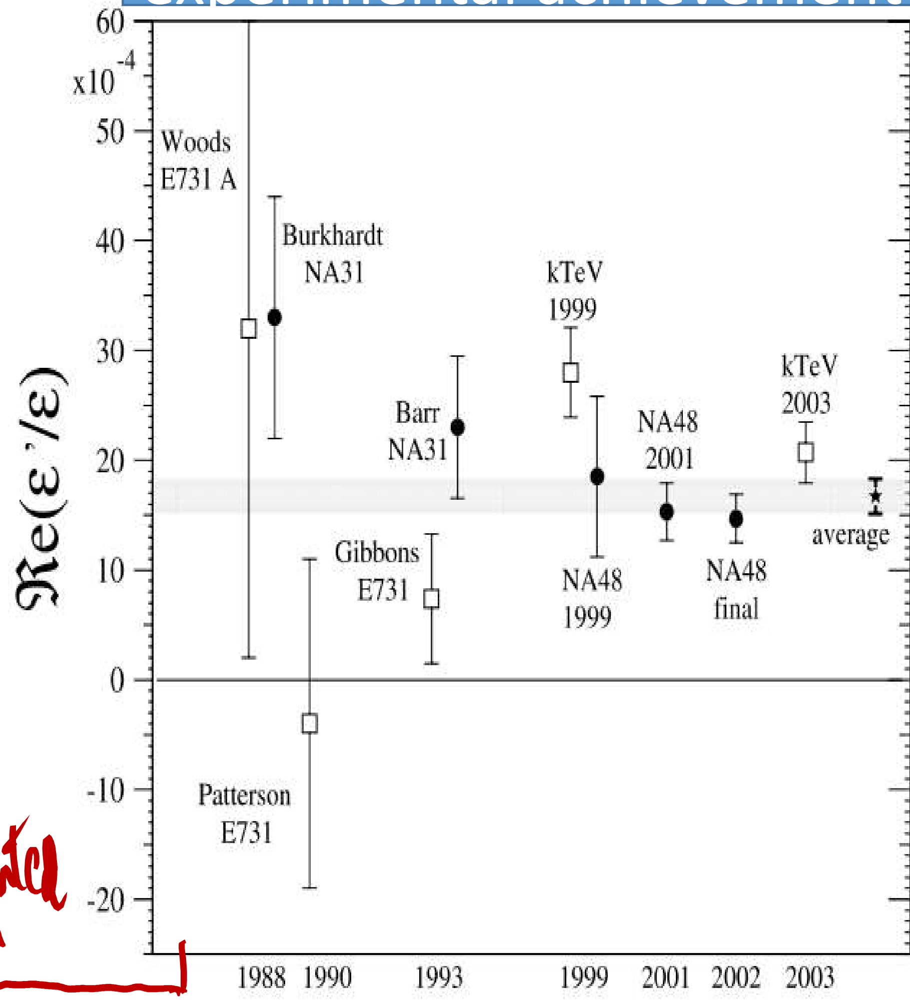
Tianle WANG, Dan Hoying

- Calculation K=> $\pi\pi$ & ε' were the reasons I went into lattice over 1/3 of a century ago!
- *9 + (3 new) PhD thesis*: Terry Draper (UCLA'84), George Hockney(UCLA'86), Cristian Calin (Columbia=CU'01), Jack Laiho(Princeton'04), Sam Li(CU'06), Matthew Lightman(CU'09), Elaine Goode(Southampton'10), Qi Liu(CU'12), Daiqian Zhang(CU'15)+ [new ones starting from CU, U Conn and Southampton] + many PD's & junior facs.. obstacles & challenges (and of course "mistakes"!) ad infinitum.....

EXTREMELY valuable inputs from countless:

- **Fred Gilman and Mark Wise**
- **Andrzej Buras et al**
- **Guido Martinelli et al**
- **Yigal Shamir**
- **Laurent Lellouch + Martin Luscher**
- **……**
- **……..**
- **……..**

A monumental experimental achievement!



Konrad Kleinknecht "Uncovering CPV"

$16.6(2.3) \times 10^{-4}$ PDG 2014

LATTICE work started

Basic calculational framework

Δ S=1 H_W

WC to NLO
Buchalla, Buras, Lautenbacher
RMP '96; Ciuchini et al 95

$$H_W = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \sum_{i=1}^{10} [z_i(\mu) + \tau y_i(\mu)] Q_i(\mu).$$

$$\tau = -V_{ts}^* V_{td} / V_{us}^* V_{ud}.$$

$M_i = \langle K | Q_i | \pi\pi \rangle$ Needed to all orders in α_s

For simplicity: 1st strategy via ChPT

PHYSICAL REVIEW D VOLUME 32, NUMBER 9 1 NOVEMBER 1985

Application of chiral perturbation theory to $K \to 2\pi$ decays

⇒ LEEFT

BDSPW-85

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Department of Physics, University of California, Los Angeles, California 90024

H. David Politzer and Mark B. Wise
Department of Physics, California Institute of Technology, Pasadena, California 91125

(Received 3 December 1984)

Chiral perturbation theory is applied to the decay $K \to 2\pi$. It is shown that, to quadratic order in meson masses, the amplitude for $K \to 2\pi$ can be written in terms of the unphysical amplitudes $K \to \pi$ and $K \to 0$, where 0 is the vacuum. One may then hope to calculate these two simpler amplitudes with lattice Monte Carlo techniques, and thereby gain understanding of the $\Delta I = \frac{1}{2}$ rule in K decay. The reason for the presence of the $K \to 0$ amplitude is explained: it serves to cancel off unwanted renormalization contributions to $K \to \pi$. We make a rough test of the practicability of these ideas in Monte Carlo studies. We also describe a method for evaluating meson decay constants which does not require a determination of the quark masses.

12/20/2017 USED extensively on lattice for ~20 years ⇒ NLO J. LAIHO PhD Thesis ~'03

A key point to emphasize is that overcoming each major obstacle led to significant application to phenomenology and/or lattice

[necessity is the parent of…….]

Lattice computation of the decay constants of B and D mesons

Claude W. Bernard
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Department of Physics FM-15, University of Washington, Seattle, Washington 98195

Amarjit Soni
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

(Received 1 July 1993)

Semileptonic decays on the lattice: The exclusive 0^- to 0^- case

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Aida X. El-Khadra
Theory Group, Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, Illinois 60510

Amarjit Soni
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(Received 21 December 1990)

PHYSICAL REVIEW D VOLUME 45, NUMBER 3 1 FEBRUARY 1992

Lattice study of semileptonic decays of charm mesons into vector mesons

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Amarjit Soni
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

(Received 30 September 1991)

We present our lattice calculation of the semileptonic form factors for the decays $D \to K^*$, $D_s \to \phi$, and $D \to \rho$ using Wilson fermions on a $24^3 \times 39$ lattice at $\beta = 6.0$ with 8 quenched configurations. For $D \to K^*$, we find for the ratio of axial form factors $A_2(0)/A_1(0) = 0.70 \pm 0.16$ [illegible]. Results for other form factors and ratios are also given

PIONEERING WORKS Leading to Modern Day UT ?

PHYSICAL REVIEW D, VOLUME 58, 014501

SU(3) flavor breaking in hadronic matrix elements for B-$\bar{B}$ oscillations

Later ΔM_s CDF, DØ

C. Bernard
Department of Physics, Washington University, St. Louis, Missouri 63130

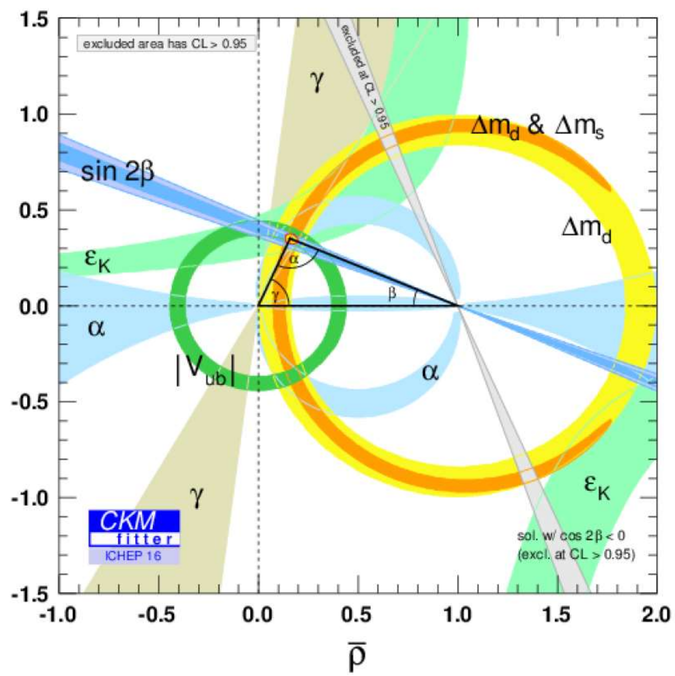
T. Blum and A. Soni
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

(Received 28 January 1998; published 5 May 1998)

Use exptal data + lattice WME to test SM & search for new physics

http://ckmfitter.in2p3.fr
see also http://www.utfit.org

Looks great; but looks can be deceiving…
In fact at level of O(2σ) tension(s) exist



O(10-15%) new physics is possible and is HUGE!

NOW
B_K, f_B, f_{B_s}, f_{B_s}/f_B
ξ all with errors $\lesssim 2\%$
But $f_+(q^2)$, $f_0(q^2)$
errors ~ 5 – 10%

Two key papers

PHYSICAL REVIEW D VOLUME 56, NUMBER 1 1 JULY 1997

QCD with domain wall quarks

1st Simulation with DWQ

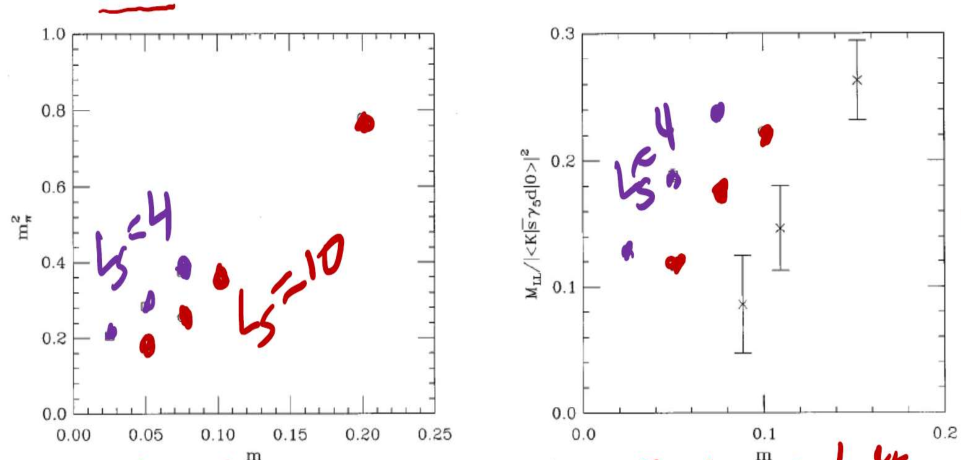
96-'97

T. Blum* and A. Soni†
Department of Physics, Brookhaven National Laboratory, Upton, New York 11973

(Received 27 November 1996)

We present lattice calculations in QCD using Shamir's variant of Kaplan fermions which retain the continuum $SU(N)_L \times SU(N)_R$ chiral symmetry on the lattice in the limit of an infinite extra dimension. In particular, we show that the pion mass and the four quark matrix element related to K_0-$\overline{K_0}$ mixing have the expected behavior in the chiral limit, even on lattices with modest extent in the extra dimension, e.g., $N_s = 10$. [S0556-2821(97)00113-6]

excellent Chiral symmetry with ~10 sites in 5th dim.



12/20/2017

MAJOR BREAKTHROUGH FOR K→ππ Lattice Calculations i.e. No ChPT

cmp/01.

a new method

another major development for K→ππ on lattice

<u>Direct</u> K-> $\pi\pi$ (a la Lellouch-Luscher), using finite volume correlation* functions, [i.e. w/o ChPT] RBC initiates around 2006

CONTINUED BY RBC-UKQCD {(mostly) Edinburgh – Southampton}

* Allows to bypass Maini-Testa theorem

COMMON interest: use of DWQ for simulations

Relating lattice ME to physical amplitudes

$$A_{2/0} = F\frac{G_F}{\sqrt{2}}V_{ud}V_{us}\sum_{i=1}^{10}\sum_{j=1}^{7}\left[\Big(z_i(\mu)+\tau y_i(\mu)\Big)Z_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}}M_j^{\frac{3}{2}/\frac{1}{2},\mathrm{lat}}\right]$$

F

F is the Lellouch-Luscher factor which relates finite volume ME to the infinite volume

$$A = \frac{1}{\pi q}\sqrt{\frac{\partial\phi}{\partial q}+\frac{\partial\delta}{\partial q}}\sqrt{m_K}E_{\pi\pi}L^{2/3}M$$

Phase shift

A/M is LL factor F

$q = \frac{pL}{2\pi}$;

$\sim \frac{\delta}{q}$ for small p

φ is a somewhat complicated function of q and boundary Conditions [See Daiqian Zhang thesis]

12/20/2017 IMSC; HET-BNL;soni 98

Results for ε'

RBC-UKQCD PRL2015

EWP

QCDP ~80% {Central Value}

- Using Re(A_0) and Re(A_2) from experiment and our lattice value for Im(A_0) and Im(A_2) and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$= 1.38(5.15)(4.43) \times 10^{-4},$$

$$16.6(2.3) \times 10^{-4}$$

RBC-UKQCD PRL'15
EDITOR'S CHOICE

LARGE CANCELLATION!!

Bearing in mind the largish errors in this first calculation, we interpret that our result are consistent with experiment at ~2σ level

$$w = \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \sim 0.045$$

with expt
Computed ReA0 good agreement with expt
Offered an "explanation" of the Delta I=1/2 enhancement

A UNIQUE ASPECT OF OUR CALCULATION

- **REAL A0, the strong phase (δ0) and Im A0 are being calculated simultaneously from 1st principles in the same calculation**

- **Re A0 is also known from EXPERIMENT…& strong phase deduced via ChPT + expt; therefore, these provide a powerful check [amongst many others] of what we are doing**

- **If a non-perturbative calculation of ImA0 and of eps' is done w/o also calculating ReA0 & δ0 in the same framework, then its repercussions for eps' (in the very least) raises questions.**

A possible difficulty: strong phases

- **The continuum and our lattice determinations of strong phase**

differ

Colangelo et al
ChPT etc

- $$\phi_{\epsilon'} = \delta_2 - \delta_0 + \frac{\pi}{2} = \begin{cases} (42.3 \pm 1.5)^{o} & \text{[illegible] [2]} \\ (54.6 \pm 5.8)^{o} & \text{RBC [47, 48]} \end{cases}$$

RBC-UKQCD

OK

off by $\simeq 2\sigma \Rightarrow$ a Concern

Statistics increase

CKelly (LAT/18

- Original goal was a 4x increase in statistics over 216 configurations used in 2015 analysis.
- 4x reduction in configuration generation time obtained via algorithmic developments (exact one-flavor implementation) → ① Murphy
- Large-scale programme performed involving many machines:

SCs over 3 continents

Source	Determinant computation	Independent configs.
Blue Waters	RHMC	34+18+4+3
KEKSC	RHMC	106
BNL	RHMC	208
DiRAC	RHMC	151
KEKSC	EOFA	275+215
BNL	EOFA	245
		1259 total

- Measurements performed using IBM BG/Q machines at BNL and the Cori computer (Intel KNL) at NERSC largely complete.
- Including original data, now have 6.7x increase in statistics!

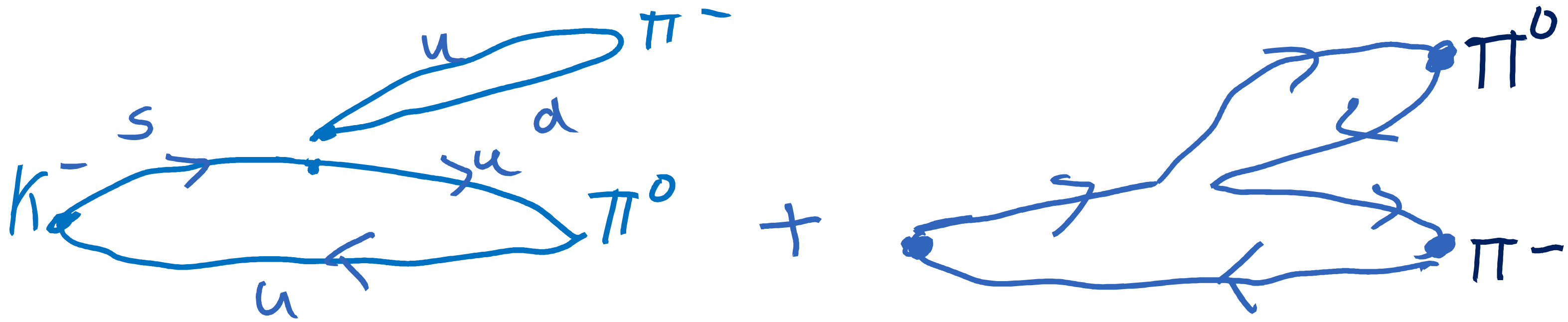
NOW ~1440 g.c

Implications for $K \to \pi\pi$ and resolution

- Despite vast increase in statistics, *this second state cannot be resolved from the time dependence using only a single $\pi\pi$ operator.*
- Possibly a significant underestimate of excited state systematic error in $K \to \pi\pi$ calculation that can only be resolved by adding additional operators.
- In response we have **expanded the scope of the calculation**:
  - Added $K \to \sigma$ matrix elements
  - Added $K \to \pi\pi$ matrix element of new $\pi\pi$ operator with larger relative pion momenta (still $p_{CM}=0$)
- Result is **3x increase in the number of I=0 $\pi\pi$ operators in $K \to \pi\pi$ calc.**
- Also added $\pi\pi$ 2pt functions with non-zero total $\pi\pi$ momenta.
  Calculate phase shift at several (smaller) additional center-of-mass energies.
  - Additional points that can be compared to dispersive result / experiment
  - Improve ~11% systematic on Lellouch-Luscher factor associated with slope of phase shift.
- Currently have 152 measurements with new operators! Adding ~ 100/month

Unravelling the $\Delta I=1/2$ rule

Dissecting (the much easier) ΔI=3/2 [I=2 ππ] Amp on the lattice: 2 contributing topologies only

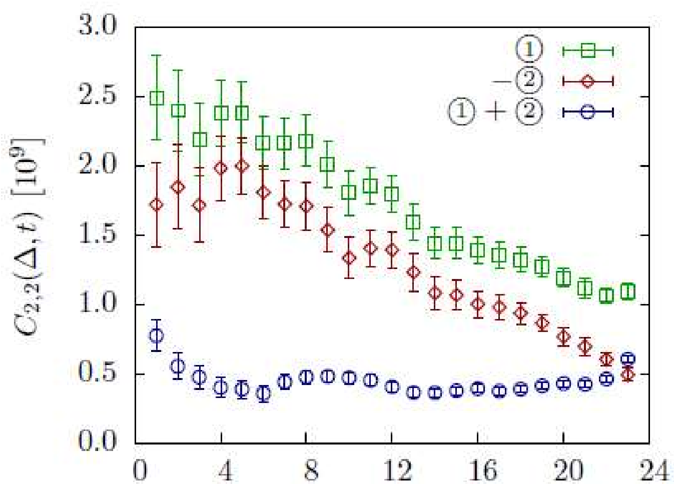


Tr X Tr

(N^2)

Tr (N)

Simplest basic step is Significantly different from phenomenological Expectations!



DRAMATIC CANCELLATION!
$(m_\pi \sim 140\ \text{MeV})$

Im A0 & ε'

arXiv: 2004.09440

RBC-UKQCD 2020

14

Parameter	Value	
	2-state fit	3-state fit
Fit range	6-15	4-15
$A^0_{\pi\pi(111)}$	0.3682(31)	0.3718(22)
$A^0_{\pi\pi(311)}$	0.00380(32)	0.00333(27)
A^0_{σ}	−0.0004309(41)	−0.0004318(42)
E_0	0.3479(11)	0.35030(70)
$A^1_{\pi\pi(111)}$	0.1712(91)	0.1748(67)
$A^1_{\pi\pi(311)}$	−0.0513(27)	−0.0528(30)
A^1_{σ}	0.000314(17)	0.000358(13)
E_1	0.568(13)	0.5879(65)
$A^2_{\pi\pi(111)}$	—	0.116(29)
$A^2_{\pi\pi(311)}$	—	0.063(10)
A^2_{σ}	—	0.000377(94)
E_2	—	0.94(10)
p-value	0.314	0.092

TABLE III: Fit parameters in lattice units and the p-values for multi-operator fits to the $I = 0$ $\pi\pi$ two-point functions. Here E_i are the energies of the states and A^i_α represents the matrix element of the operator α between the state i and the vacuum, given in units of $\sqrt{1\times 10^{13}}$. The second column gives the parameters for our primary fit which uses two-states and three operators. The third column shows a fit with the same three operators and one additional state that is used to probe the systematic effects of this third state on the $K \to \pi\pi$ matrix element fits.

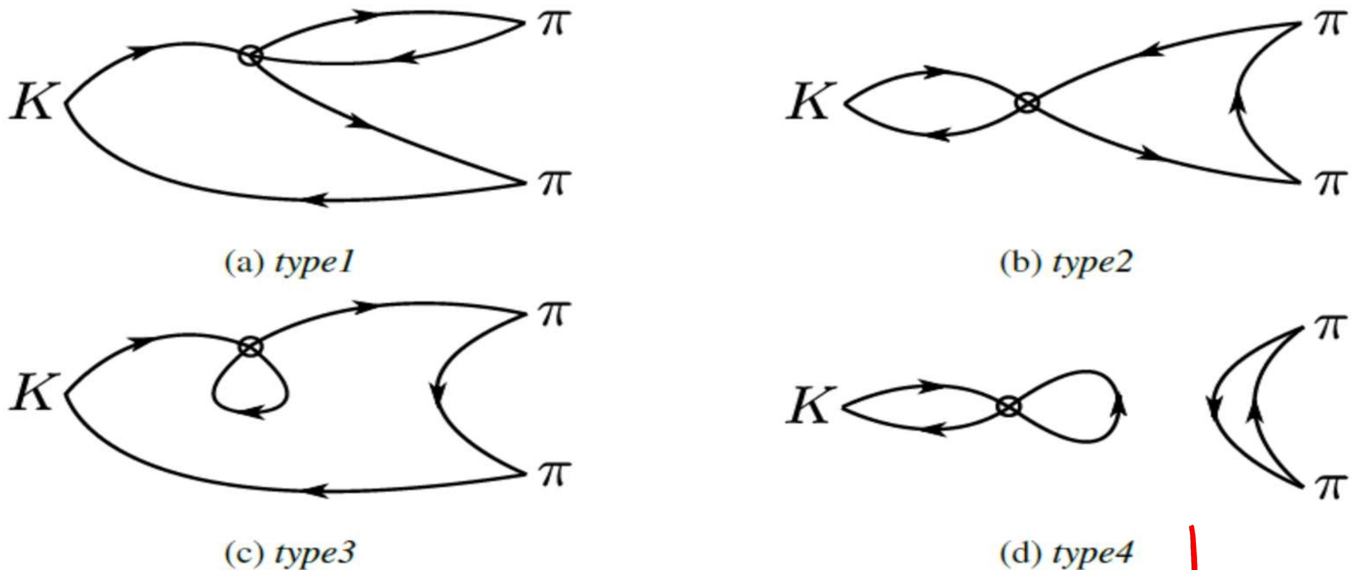


FIG. 2: The four classes of $K \to \pi\pi$ Wick contractions.

ALL ARE INCLUDED!

"DISCONNECTED" extremely difficult

Tree

Re A0

Im A0

Q_i^*

59

	Re(A_0)		Im(A_0)	
i	($\not{q}$, $\not{q}$) ($\times 10^{-7}$ GeV)	(γ^μ, γ^μ) ($\times 10^{-7}$ GeV)	($\not{q}$, $\not{q}$) ($\times 10^{-11}$ GeV)	(γ^μ, γ^μ) ($\times 10^{-11}$ GeV)
1	0.383(77)	0.335(64)	0	0
2	2.89(30)	2.81(28)	0	0
3	0.0081(58)	0.0050(42)	0.20(14)	0.12(10)
4	0.081(23)	0.088(17)	1.24(35)	1.34(27)
5	0.0380(68)	0.0339(53)	0.552(99)	0.492(77)
6	−0.410(28)	−0.398(27)	−8.78(60)	−8.54(57)
7	0.001863(56)	0.001900(56)	0.02491(75)	0.02540(75)
8	−0.00726(14)	−0.00708(13)	−0.2111(40)	−0.2060(39)
9	$-8.7(1.5) \times 10^{-5}$	$-8.5(1.4) \times 10^{-5}$	−0.133(22)	−0.128(21)
10	$2.37(38) \times 10^{-4}$	$2.13(32) \times 10^{-4}$	−0.0304(49)	−0.0273(41)
Total	2.99(32)	2.86(31)	−7.15(66)	−6.93(64)

Dominant

Very small

Dominant

TABLE XVIII: The contributions of each of the ten four-quark operators to Re(A_0) and Im(A_0) for the two different RI-SMOM intermediate schemes. The scheme and units are listed in the column headers. The errors are statistical, only.

Systematic errors

Error source	Value
Excited state	-
Unphysical kinematics	5%
Finite lattice spacing	12%
Lellouch-Lüscher factor	1.5%
Finite-volume corrections	7%
Missing G_1 operator	3%
Renormalization	4%
Total	15.7%

TABLE XXV: Relative systematic errors on the infinite-volume matrix elements of $\overline{\mathrm{MS}}$-renormalized four-quark operators Q'_j.

Re A_0 → ~20%

Error source	Value	
	$\mathrm{Re}(A_0)$	$\mathrm{Im}(A_0)$
Matrix elements	15.7%	15.7%
Parametric errors	0.3%	6%
Wilson coefficients	12%	12%
Total	19.8%	20.7%

TABLE XXVI: Relative systematic errors on $\mathrm{Re}(A_0)$ and $\mathrm{Im}(A_0)$.

Im A_0 → ~21%

	Quantity	Our Value	EXPT
	$\mathrm{Re}(A_0)$	$2.99(0.32)(0.59)\times 10^{-7}$ GeV	$\sim 3.32\times 10^{-7}$ GeV
	$\mathrm{Im}(A_0)$	$-6.98(0.62)(1.44)\times 10^{-11}$ GeV	
$\Delta I = \frac{1}{2}$ Rule	$\mathrm{Re}(A_0)/\mathrm{Re}(A_2)$	19.9(2.3)(4.4)	~ 22.45
	$\mathrm{Re}(\varepsilon'/\varepsilon)$	0.00217(26)(62)(50)	due IB see foll pages

TABLE I: A summary of the primary results of this work. The values in parentheses give the statistical and systematic errors, respectively. For the last entry the systematic error associated with electromagnetism and isospin breaking is listed separately as a third error contribution.

IB+EM effects…..not yet from lattice

We use

$$\frac{\varepsilon'}{\varepsilon} = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right]$$

→ isospin sym formula.

$\sim (17 \pm 9.1) \times 10^{-2}$

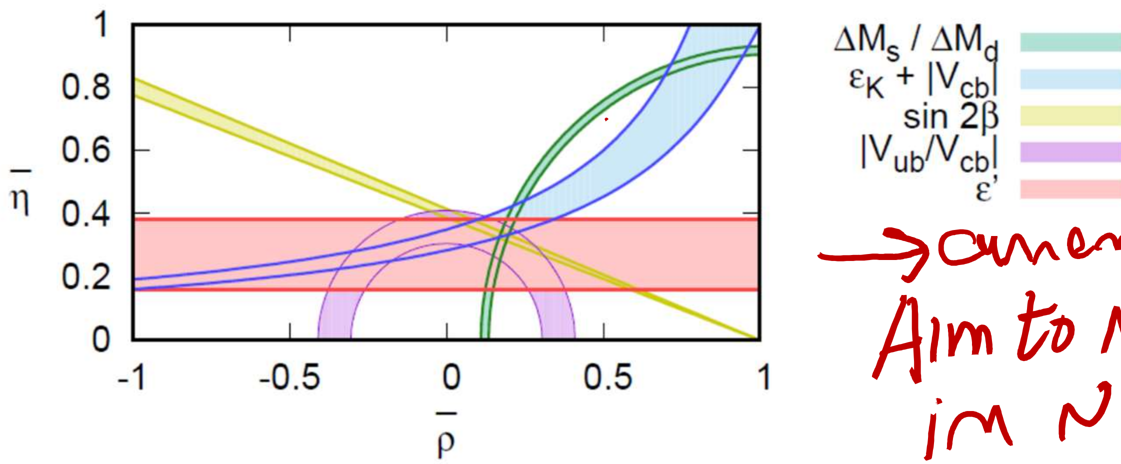
IB+EM eff

$$\Rightarrow \quad \frac{\varepsilon'}{\varepsilon} = \frac{i\omega_+ e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}(A_2^{\mathrm{emp}})}{\mathrm{Re}(A_2^{(0)})} - \frac{\mathrm{Im}(A_0^{(0)})}{\mathrm{Re}(A_0^{(0)})}\left(1-\hat{\Omega}_{\mathrm{eff}}\right)\right].$$

See Cirigliano et al 1911.01359

THIS is NOT our work

WE CHOOSE To include THIS in our syst error



→ current systematic ~35%
Aim to reduce this
in ~3 yrs to ≈15%

FIG. 12: The horizontal-band constraint on the CKM matrix unitarity triangle in the $\bar{\rho} - \bar{\eta}$ plane obtained from our calculation of ε', along with constraints obtained from other inputs [6, 70, 71]. The error bands represent the statistical and systematic errors combined in quadrature. Note that the band labeled ε' is historically (e.g. in Ref. [72]) labeled as ε'/ε, where ε is taken from experiment.

Naturalness: an important consideration

A firm believer in naturalness

My Car's Licence plate in CA

- **Used to be OSCILL8 (through the 80's while @UCLA)**

Implications for NP & Naturalness

DRAWING STRONG CONCLUSIONS BASED on 20% tests is TOO RISKY!

& Too Premature

OR 35%

⇒ Its imperative that we continue to improve our precision

scalars'13; 9/13/13 A. Soni HET-BNL 9

A lesson from history (I)

"A special search at Dubna was carried out by E. Okonov and his group. They did not find a single $K_L \rightarrow \pi^+ \pi^-$ event among 600 decays into charged particles [12] (Anikira et al., JETP 1962). At that stage the search was terminated by the administration of the Lab. The group was unlucky."

-Lev Okun, "The Vacuum as Seen from Moscow"

1964: BF= 2×10^{-3}

A failure of imagination ? Lack of patience ?

CHristenson, Cronin, FITCH + TURLAY BNL '64

Summary + Outlook 1 of 2 pages

- **After decades of effort, overcoming major hurdles, using DWQ with essentially continuum-like fermions along with improved renormalization methodology, cutting edge statistical analysis and algorithmic advances, RBC-UKQCD is presenting an updated result on SM-eps' ~ 21.7(26)(62)(50)X10^{-4} which is compatible [within errors] with the measured value 16.6(2.3)X10^{-4}**
- **Bearing in mind that this is an extremely treacherous calculation loaded with numerous avenues of errors and oversights, an independent calculation has been in process for about ~3 years within RBC-UKQCD. This effort is led by Tom Blum with (then g.s.) Dan Hoying/Masaaki Tomii, U Conn-BNL, Taku Izubuchi et al. This path uses PBC unlike the currently finished result which used GPBC…we hope to have 1st results from PBC in ~ 2 years.**
- **Also GPBC effort will be continued at other lattice spacing(s)**

Summary + Outlook

- **Lattice efforts to incorporate IB + EM effects are being studied but have some ways to go before they can tackle K=> pi pi and eps'**
- **With physical pions, kaons and such first glance at lattice ChPT is quite encouraging, see RBC-UKQCD, David Murphy et al 2015 and DM, PhD thesis, Columbia Univ**
- **This begs the question that much simpler path could now be used via BDSPW [LO ChPT] and/or L+S [NLOChPT] to address eps'…This could be tens of times simpler though at some cost in accuracy……….all this needs to be studied…Mattia Bruno, Christoph Lehner + AS et al**
- **Hope to have an improved result on eps' with O(15%) errors in ~3 years**

EXTRAS

Lattice used

RBC

RIKEN-BNL-Research Center

B=BNL-HET

C=Columbia

The RBC & UKQCD collaborations

UKQCD subgroup of UKQCD that uses DWQ

Edinburgh

Southampton

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni
Bigeng Wang
Tianle Wang
Yidi Zhao

UC Boulder

Oliver Witzel

CERN

Mattia Bruno

Columbia University

Ryan Abbot
Norman Christ
Duo Guo
Christopher Kelly
Bob Mawhinney
Masaaki Tomii
Jiqun Tu

University of Connecticut

Tom Blum
Dan Hoying (BNL)
Luchang Jin (RBRC)
Cheng Tu

Edinburgh University

Peter Boyle
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
Andrew Yong
Azusa Yamaguchi

UAM Madrid

Julien Frison

University of Liverpool

Nicolas Garron

MIT

David Murphy

Peking University

Xu Feng

University of Regensburg

Christoph Lehner (BNL)

University of Southampton

Nils Asmussen
Jonathan Flynn
Ryan Hill
Andreas Jüttner
James Richings
Chris Sachrajda

Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

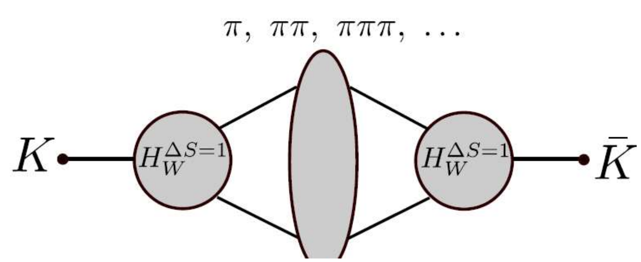
CKeCKM'18

- Neutral kaon mixing induced by 2nd order weak processes gives rise to mass difference between K_L and K_S

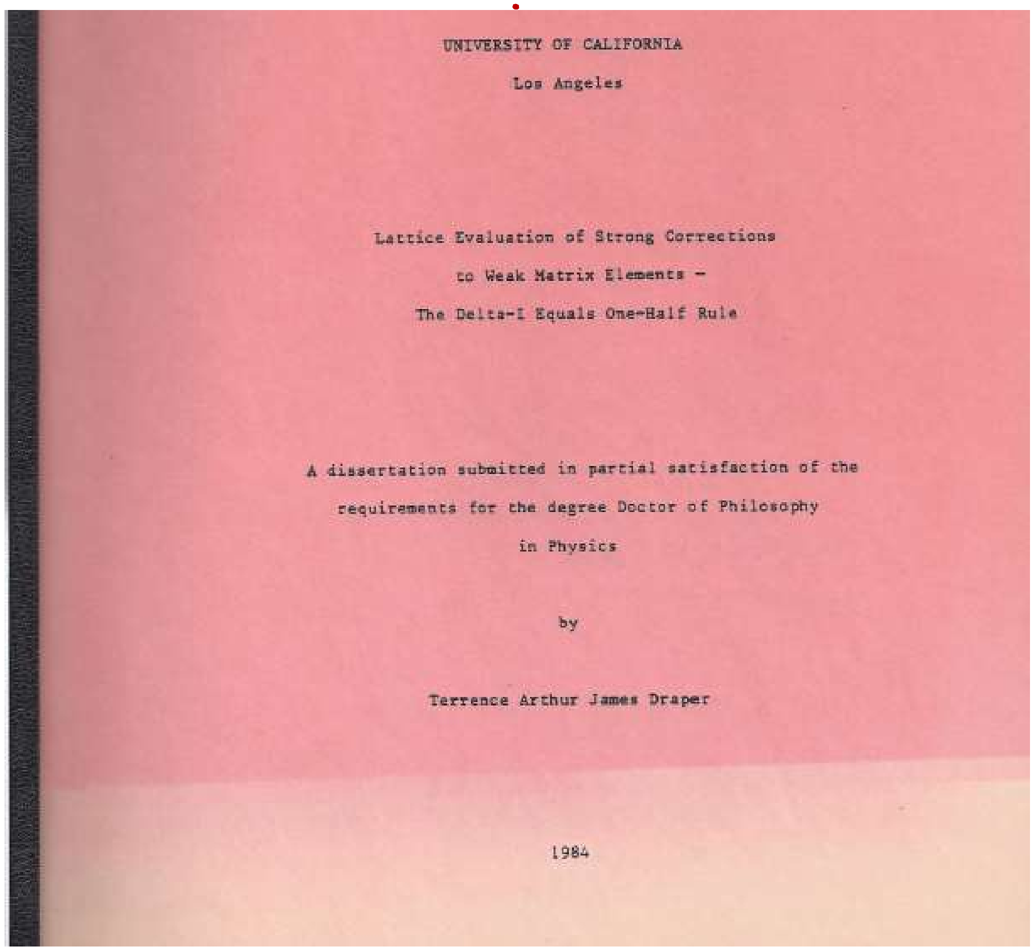
$$\Delta M_K = 2\sum_n \frac{\langle \overline{K}^0 | H_W | n\rangle\langle n | H_W | K^0\rangle}{M_K - E_n}$$

B. Wang e
LAT'18

- FCNC → highly suppressed in SM due to GIM mechanism: $\Delta m_K = 3.483(6)\times 10^{-12}$ MeV small and highly sensitive to new BSM FCNC.
- PT calc using weak EFT with ΔS=2 eff. Hamiltonian (charm integrated out) dominated by $p \sim m_c$: poor PT convergence at charm scale → ~36% PT sys error.
- PT calc neglects long-distance effects arising when 2 weak operators separated by distance $\sim 1/\Lambda_{QCD}$.
- Use lattice to evaluate matrix element of product of $H_W^{\Delta S=1,\,\mathrm{eff}}$ directly:



The 1st PhD Thesis



UNIVERSITY OF CALIFORNIA

Los Angeles

Lattice Evaluation of Strong Corrections

to Weak Matrix Elements –

The Delta-I Equals One-Half Rule

A dissertation submitted in partial satisfaction of the

requirements for the degree Doctor of Philosophy

in Physics

by

Terrence Arthur James Draper

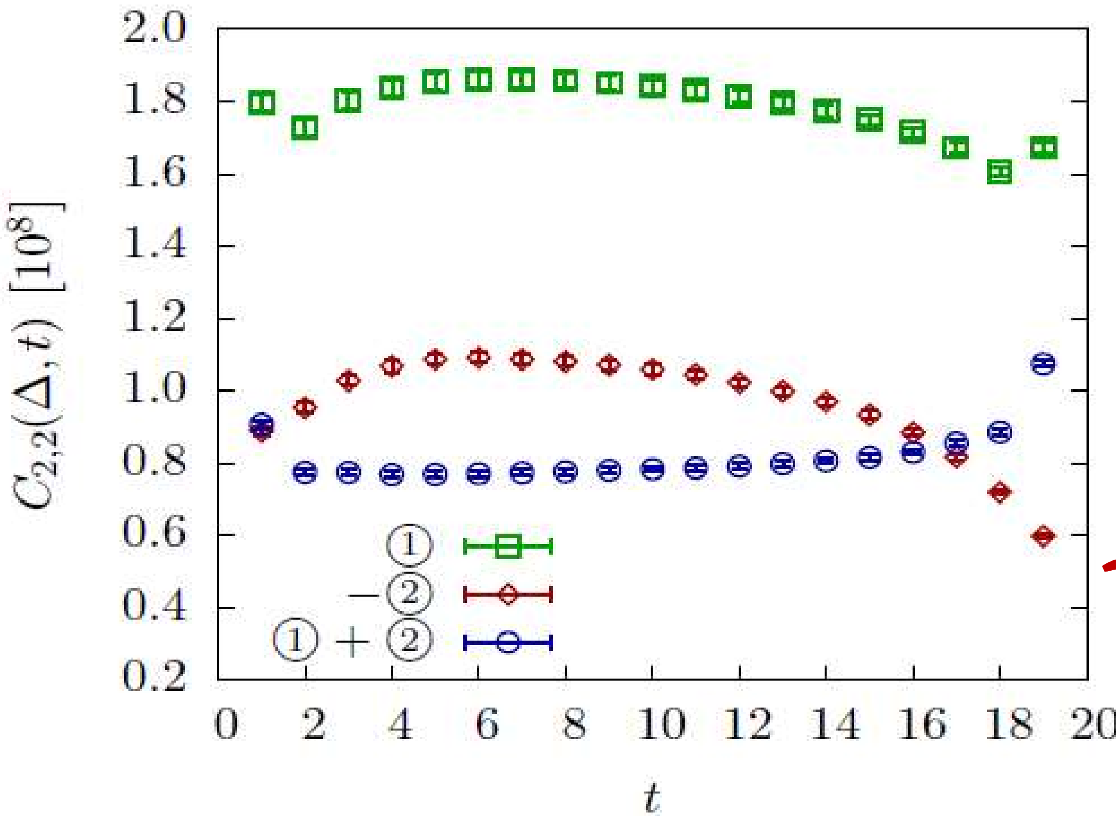
1984

Grew from end of year Beer Party ~June 20, 1982!!

[UCLA]

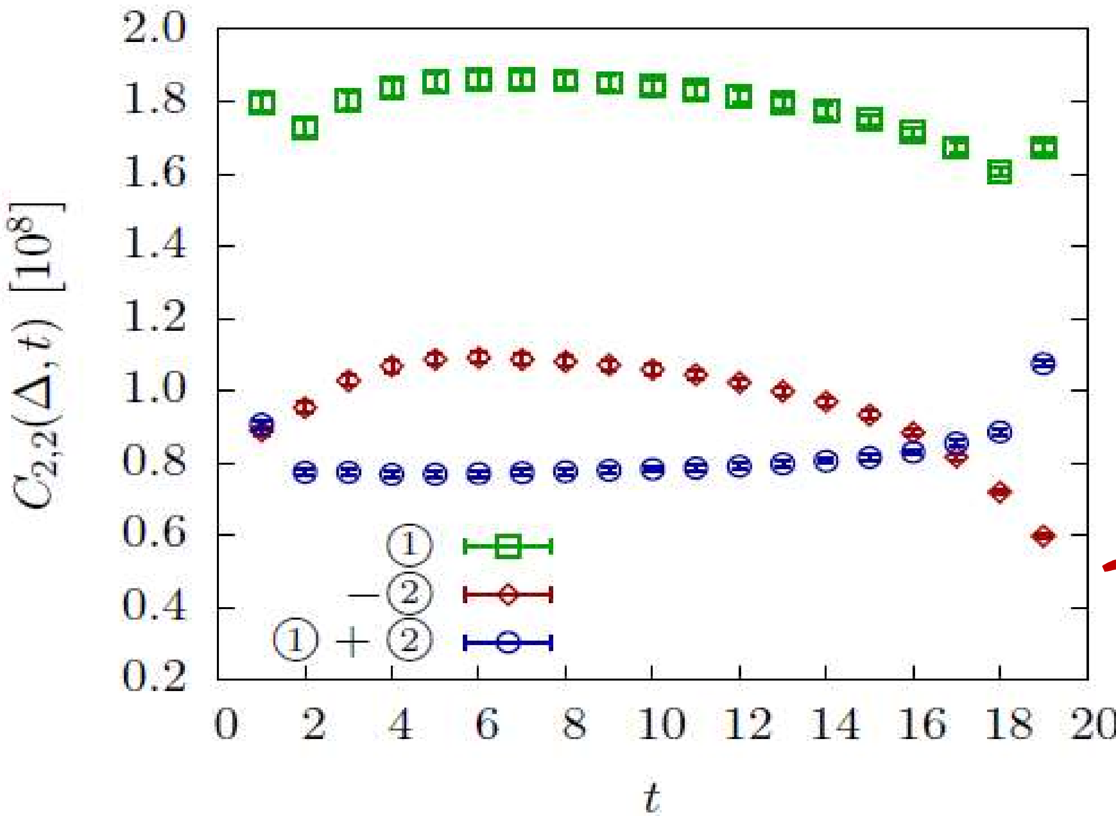
Beware of End of year Beer Parties!

ODE to YESTERYEARS!



For heavier π, $m_\pi \sim 330$ MeV less cancellation bet. N^2 & N Large N begins to improve!

FIG. 3: Contractions ①, -② and ① + ② as functions of t from the simulation at threshold with $m_\pi \simeq 330\,\mathrm{MeV}$ and $\Delta = 20$.



For heavier π, $m_\pi \sim 330$ MeV less cancellation bet. N^2 & N Large N begins to improve!

FIG. 3: Contractions ①, -② and ① + ② as functions of t from the simulation at threshold with $m_\pi \simeq 330\,\mathrm{MeV}$ and $\Delta = 20$.

Net effect

- **This large cancellation between N^2 and N [N=3,for QCD] leads to a reduction in ReA2 compared to "naïve expectations" by a factor of about 4 to 5 in the original effect of around 22.5**
- **Then there is a factor of 2 to 3 from renorm…=> bringing the total to [8 to 15] of the needed 22.5**
- **The remaining factor of ~ [1.5 to 2.8] … comes from ReA0 over "naïve expectations"**

More on A0

$Q_2 \equiv$

- **Another important fact about Re A0 is that at a scale of ~1.3 GeV or more,**

the contribution from penguin operators, Q3,Q4,Q5,Q6,is negligibly small.

- **Indeed, ~85% of ReA0 originates at these scales from Q2 which is just the original Weak interaction 4-q operator: [s-bar gamma_muL u]X[d-bar gamma_uL u], which originates from integrating out the W-boson.**
- **The essential moral is that if you take the original weak interaction 4q operraor**

and non-pertubatively compute its matrix element between K to pi pi in the I=0 channel then it accounts for most (~85%) of Re A0…..

- **Lastly, but equally importantly, it should be stressed that the SVZ-penguin operator Q6 is in fact the dominant contributor to Im A0**.

s u

Tree

$$Q_1 = (\bar{s}_\alpha d_\alpha)_L (\bar{u}_\beta u_\beta)_L,$$

$$Q_2 = (\bar{s}_\alpha d_\beta)_L (\bar{u}_\beta u_\alpha)_L,$$

QCDP

$$Q_3 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_L,$$

$$Q_4 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_L,$$

$$Q_5 = (\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_R,$$

I=0 $\Rightarrow$

$$Q_6 = (\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_R,$$

$\rightarrow 0$

$m_q \rightarrow 0$

EWP

$$Q_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_R,$$

$$Q_8 = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_R,$$

I=2

$$Q_9 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_L,$$

$$Q_{10} = \frac{3}{2}(\bar{s}_\alpha d_\beta)_L \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_L,$$

$\longrightarrow$ const

$m \rightarrow 0$

s w d

g

QCDP

s d

γ, Z

EWP

Why EWK cannot be neglected : 3 Reasons

- **Despite $\alpha_{QED,EWK}$ << α_{QCD} , EWK contributions are extremely important and CANNOT be neglected:**
- **EWK are (8,8) and QCD are (8,1), and (8,8) go to constant whereas (8,1) vanish in the chiral limit**
- **EWK, i.e. those due Z exch have Wilson coeff that go as mt^2 /mW^2**
- **In ε' they enter as**

$$\left[\frac{\overbrace{\mathrm{Im}A_2}^{\text{EWP}}}{\underbrace{\mathrm{ReA}_2}_{\text{small}}} - \frac{\overbrace{\mathrm{Im}A_0}^{\text{QCDP}}}{\underbrace{\mathrm{ReA}_0}_{\text{large}}}\right]$$

$$\frac{ReA_0}{ReA_2} \sim 22$$

Why EWK cannot be neglected : 3 Reasons

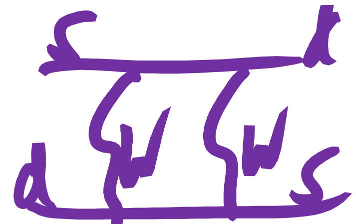
- Despite $\alpha_{QED,EWK}$ << α_{QCD} , EWK contributions are extremely important and CANNOT be neglected:
- EWK are (8,8) and QCD are (8,1), and (8,8) go to constant whereas (8,1) vanish in the chiral limit
- EWK, i.e. those due Z exch have Wilson coeff that go as mt^2/mW^2
- In ε' they enter as

EWP QCDP

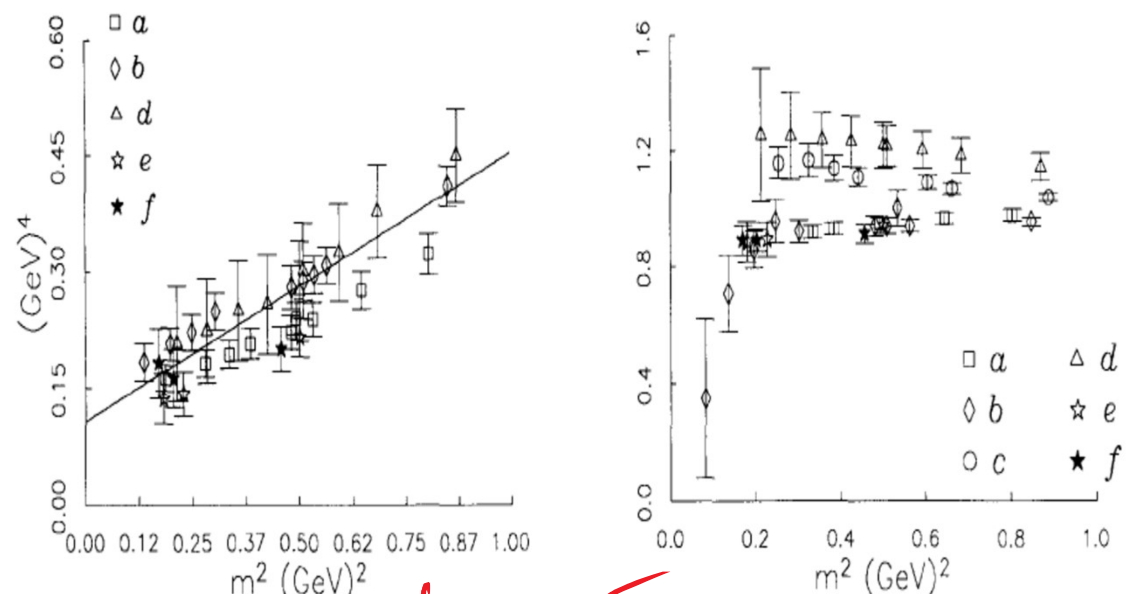
$$\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]$$

$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \sim 22$ small large

$\langle K | (\bar{s}\gamma_{\mu L} d)^2 | \bar{K} \rangle$



162 C. Bernard, A. Soni / Weak matrix elements on the lattice



χS Violation by K-$\bar{K}$ ⇒ FINE TUNING PROBLEM

RBC-UKQCD
2004.09440

7-ops. 10-ops.

i	SMOM($\not{q},\not{q}$) (GeV^3)	SMOM(γ^μ,γ^μ) (GeV^3)	$\overline{\text{MS}}$ via SMOM($\not{q},\not{q}$) (GeV^3)	$\overline{\text{MS}}$ via SMOM(γ^μ,γ^μ) (GeV^3)
1	0.060(39)	0.059(38)	−0.107(22)	−0.093(18)
2	−0.125(19)	−0.106(16)	0.147(15)	0.143(14)
3	0.142(17)	0.128(14)	−0.086(61)	−0.053(44)
4	-	-	0.185(53)	0.200(40)
5	−0.351(62)	−0.313(48)	−0.348(62)	−0.311(48)
6	−1.306(90)	−1.214(82)	−1.308(90)	−1.272(86)
7	0.775(23)	0.790(23)	0.769(23)	0.784(23)
8	3.312(63)	3.092(58)	3.389(64)	3.308(63)
9	-	-	−0.117(20)	−0.114(19)
10	-	-	0.137(22)	0.123(19)

TABLE XIV: Physical, infinite-volume matrix elements in the SMOM($\not{q},\not{q}$) and SMOM(γ^μ,γ^μ) schemes at $\mu = 4.006$ GeV given in the 7-operator chiral basis, as well as those converted perturbatively into the $\overline{\text{MS}}$ scheme at the same scale in the 10-operator basis. The errors are statistical only.

2 Schemes

VOLUME 48, NUMBER 13 PHYSICAL REVIEW LETTERS 29 MARCH 1982

Constraint on the Mass Scale of a Left-Right-Symmetric Electroweak Theory from the K_L-K_S Mass Difference

G. Beall and Myron Bander
Department of Physics, University of California, Irvine, California 92717

and

A. Soni
Department of Physics, University of California, Los Angeles, California 90024
(Received 21 December 1981)

The K_L-K_S mass difference provides a stringent constraint on the mass (M_R) of the charged right-handed gauge field occurring in a "manifest" left-right-symmetric electroweak theory, yielding $M_R \gtrsim 1.6$ TeV. Taken in the context of a grand-unifying gauge theory, e.g., O(10), such a large bound on M_R, along with the measured value of $\sin^2\theta_W$, implies that $M_R \gtrsim 10^9$ GeV.

PACS numbers: 12.10.Ck, 11.30.Ly, 14.80.Er

$$O_{LL} \equiv [\bar\psi_s{}^\alpha \gamma_\mu \tfrac{1}{2}(1-\gamma_5)\psi_d{}^\alpha][\bar\psi_s{}^\beta \gamma^\mu \tfrac{1}{2}(1-\gamma_5)\psi_d{}^\beta].$$

$$\mathfrak{M}_{LL} \simeq \mathfrak{M}_{LL}{}^{\mathrm{vac}} = \tfrac{2}{3} f_K{}^2 m_K{}^2/2m_K. \quad (5)$$

⇐ SM

To evaluate the LR contribution we also use the divergence equation $\bar\psi_1\gamma_5\psi_2 = -i\partial_\mu(\bar\psi_1\gamma^\mu\gamma_5\psi_2)/(m_1+m_2)$ to obtain

$$A_{LR}(d\bar s \to \bar d s) = \left(\frac{g}{\sqrt2}\right)^4 \left(\frac{O_{LR}}{8\pi^2 M_R{}^4}\right) \sum_{i,j=u,c,t}\sum m_i U_{is}{}^{R*} U_{id}{}^L m_j U_{js}{}^{L*} U_{jd}{}^R$$

$$\times\left[\frac{\beta\ln\beta}{(1-\beta)(\epsilon_i-\beta)(\epsilon_j-\beta)} + \frac{\epsilon_i\ln\epsilon_i}{(1-\epsilon_i)(\beta-\epsilon_i)(\epsilon_j-\epsilon_i)} + \frac{\epsilon_j\ln\epsilon_j}{(1-\epsilon_j)(\beta-\epsilon_j)(\epsilon_i-\epsilon_j)}\right], \quad (2)$$

⇒

$$\mathfrak{M}_{LR} \simeq \mathfrak{M}_{LR}{}^{\mathrm{vac}} = \tfrac{1}{2}[m_K{}^2/(m_s+m_d)^2 + \tfrac{1}{6}] f_K{}^2 m_K{}^2/2m_K \simeq 7.7\mathfrak{M}_{LL}{}^{\mathrm{vac}}, \quad (6)$$

7.7

$$O_{LR} \equiv [\bar\psi_s{}^\alpha \tfrac{1}{2}(1-\gamma_5)\psi_d{}^\alpha][\bar\psi_s{}^\beta \tfrac{1}{2}(1+\gamma_5)\psi_d{}^\beta]$$

⇐ LRSM

used $m_s = 150$ MeV

$\frac{M_{LRM}}{M_{SM}} \sim 7.7$

PRL 1981-82

P. Boyle et al 1812.04981

the five operators O_i. In our framework we are interested only in the parity-even operators. In the so-called SUSY basis introduced in [2], the parity-even operators are,

$$
\begin{aligned}
O_1 &= (\bar{s}_a\gamma_\mu(1-\gamma_5)d_a)(\bar{s}_b\gamma_\mu(1-\gamma_5)d_b) \quad \Leftarrow SM\\
O_2 &= (\bar{s}_a(1-\gamma_5)d_a)(\bar{s}_b(1-\gamma_5)d_b)\\
O_3 &= (\bar{s}_a(1-\gamma_5)d_b)(\bar{s}_b(1-\gamma_5)d_a)\\
O_4 &= (\bar{s}_a(1-\gamma_5)d_a)(\bar{s}_b(1+\gamma_5)d_b) \quad \Rightarrow LRM\\
O_5 &= (\bar{s}_a(1-\gamma_5)d_b)(\bar{s}_b(1+\gamma_5)d_a).
\end{aligned}
\tag{1.2}
$$

$$\frac{M_i^{BSM}}{M^{SM}}$$

$i \neq 1$, 2

Table 3: Comparison of the results of this work in $\overline{\text{MS}}(\mu = 3\text{GeV})$ alongside our collaboration's previous results presented in [5].

	RBC-UKQCD16[5]	This Work
N_f =	2+1	2+1
scheme	RI-SMOM	RI-SMOM
R_2	-19.48(44)(52)	-18.83(17)(55)
R_3	6.08(15)(23)	5.815(63)(125)
R_4	43.11(89)(230)	41.58(37)(119)
R_5	10.99(20)(88)	10.81(9)(37)

LRM is an extremely interesting extension of SM
$\Rightarrow m_\nu \neq 0$