

Overview of the Path to 0.01% Theoretical Luminosity Precision for the FCCee and its Possible Synergistic Effects for Other FCC Precision Theory Requirements

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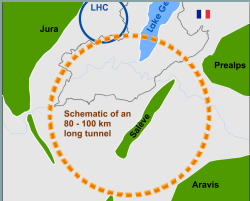
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in collaboration with

S. Jadach, W. Placzek, M. Skrzypek, S. A. Yost

—see Phys.Lett.B **790**(2019)314





General context: QED uncertainties in EW observables



Observable	From	Present {QED}	FCC stat.	FCC syst.	$\frac{\text{Now}\{\text{QED}\}}{\text{FCC}(\text{exp.})}$
M_Z [MeV]	Z linesh. [2]	$91187.5 \pm 2.1\{0.3\}$	0.005	0.1	3
Γ_Z [MeV]	Z linesh. [2]	$2495.2 \pm 2.1\{0.2\}$	0.008	0.1	2
Γ_h/Γ_l	$\sigma(M_Z)$ [3]	$20.767 \pm 0.025\{0.012\}$	$1 \cdot 10^{-4}$	$\sim 10^{-3}$	12
N_ν	$\sigma(M_Z)$ [2]	$2.984 \pm 0.008\{0.006\}$	$0.8 \cdot 10^{-4}$	$4 \cdot 10^{-4}$	15
N_ν	Z+ γ [4]	$2.69 \pm 0.15\{0.06\}$	$1 \cdot 10^{-3}$	$< 10^{-3}$	60
$\sin^2 \theta_W^{eff}$	$A_{FB}^{lept.}$ [3]	$0.23099 \pm 0.00053\{06\}$	$0.6 \cdot 10^{-5}$	$< 10^{-5}$	10
$\sin^2 \theta_W^{eff}$	$A_{pol.}^\tau$ [2,3]	$0.23159 \pm 0.00041\{12\}$	$0.6 \cdot 10^{-5}?$	$< 10^{-5}?$	20?
M_W [MeV]	ADLO [5]	$80376 \pm 33\{7\}$	0.3	0.5	14
$A_{FB,\mu}^{M_Z \pm 3.5\text{GeV}}$	$\frac{d\sigma}{d\cos\theta}$ [2]	$\pm 0.020\{0.001\}$	$1.0 \cdot 10^{-5}$	$0.3 \cdot 10^{-5}$	100
$\alpha_{QED}^{-1}(M_Z)$	$\leq 10\text{GeV}$ [6]	128.952 ± 0.014	0.004	0.001	-

Table 1: Experimental precision of electroweak observables, which are most sensitive to QED effects. In the braces {...} in 3-rd column are estimates of the systematic error due to QED calculation uncertainty. The necessary improvement factors of QED calculations for FCCee experiments are shown in the last column. FCCee systematic is without QED component. Uncertain numbers are marked with the question mark.

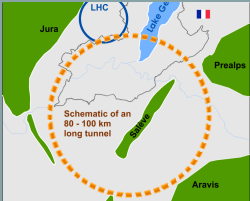
To be discussed in the following 

[2] DELPHI, ALEPH, OPAL, L3, SLD Collaboration, S. Schael *et al.*, *Phys. Rept.* **427** (2006) 257–454, [hep-ex/0509008](#).

[3] DELPHI, ALEPH, OPAL, L3, SLD Collaboration, D. Abbaneo *et al.*, [hep-ex/0112021](#).

[4] OPAL Collaboration, G. Abbiendi *et al.*, *Eur. Phys. J.* **C18** (2000) 253–272, [hep-ex/0005002](#).

[5] DELPHI, OPAL, ALEPH, L3 Collaboration, S. Schael *et al.*, *Phys. Rept.* **532** (2013) 119–244, [1302.3415](#).



QED challenges at FCCee

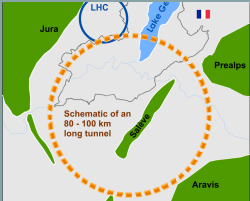


QED challenges at FCCee are of 2-fold type:

- A. More higher (fixed) orders, better resummation, more sophisticated Monte Carlo programs
- B. Possibly completely new methodology of the QED “deconvolution” and related new definition of the EW pseudo-observables (EWPO’s)
--S. Jadach, arXiv:1903.09895; J. Gluza, this meeting

An illustrative example:

Low angle Bhabha for [luminosity](#) measurement which enters into many observables, notably neutrino counting.



Example of low angle Bhabha (luminosity) at FCCee
in IFJPAN-IV-2018-07, BU-EPP-18-03, MPP-2018-91 by S. Jadach,
W. Płaczek, M. Skrzypek, B.F.L.W, S.A.Yost (PLB 790 (2019) 314)



- Motivation: better measurement of invisible Z width from Z peak x-section
- LEP legacy:

$$R_{inv}^0 = \frac{\Gamma_{inv}}{\Gamma_{\ell\ell}} = \sqrt{\frac{12\pi R_{\ell}^0}{\sigma_{had}^0 m_Z^2}} - R_{\ell}^0 - (3 + \delta_{\tau})$$

- assuming lepton universality

$$(R_{inv}^0)_{exp} = N_v \left(\frac{\Gamma_{v\bar{v}}}{\Gamma_{\ell\ell}} \right)_{SM}$$

- from LEP Z-peak measurements

$$N_v = 2.9840 \pm 0.0082$$

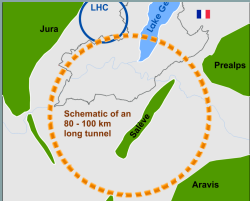
$$\delta N_v \simeq 10.5 \frac{\delta n_{had}}{n_{had}} \oplus 3.0 \frac{\delta n_{lept}}{n_{lept}} \oplus 7.5 \frac{\delta \mathcal{L}}{\mathcal{L}}$$

$$\frac{\delta \mathcal{L}}{\mathcal{L}} = 0.061\% \Rightarrow \delta N_v = 0.0046$$

ADLO, SLD and LEPEWWG, Phys. Rept. 427 (2008) 257, hep-ex/0509008

- Recently, Janot and Jadach, arXiv:1912.02067, current status for LEP lumi theory error
 $\Leftrightarrow 0.037\% \cup$ improved measurement error analysis
 $\Rightarrow N_v = 2.9975 \pm 0.0074$ with $\delta N_v = 0.0028$ from $\frac{\delta \mathcal{L}}{\mathcal{L}}$

- $7.5 \times 0.061\% = 0.0046$. Shall we do better at FCCee?? YES!
- In 1999 lumi TH error 0.061% was dominated by VP \Rightarrow No motivation to improve QED components
 Now, 0.037% (JJ) dominated by photonic correction \Rightarrow motivation already to improve QED error. At FCCee VP error will be reduced by another factor 2 compared to today! New reality!
- Low angle Bhabha luminometer already defined, Mogens Dam, FCC Week 2018, 2019 wkshp



Example of low angle Bhabha (luminosity) at FCCee

Overview of IFJPAN-IV-2018-07, BU-EPP-18-03, MPP-2018-91 by S.Jadach, W. Płaczek, M. Skrzypek, B.F.L.W., S.A.Yost (PLB 790 (2019) 314)



LEP legacy, lumi TH error budget

LEP update 2018(2019)

Type of correction/error	LEP1		LEP2	
	1996	1999	1996	1999
(a) Missing photonic $O(\alpha^2)$ [4,5]	0.10%	0.027%	0.20%	0.04%
(b) Missing photonic $O(\alpha^3 L^3)$ [6]	0.015%	0.015%	0.03%	0.03%
(c) Vacuum polarization [7,8]	0.04%	0.04%	0.10%	0.10%
(d) Light pairs [9,10]	0.03%	0.03%	0.05%	0.05%
(e) Z-exchange [11,12]	0.015%	0.015%	0.0%	0.0%
Total	0.11% [12]	0.061% [13]	0.25% [12]	0.12% [13]

Type of correction / Error	1999	Update 2018
(a) Photonic $O(L_e \alpha^2)$	0.027% [5]	0.027%
(b) Photonic $O(L_e^3 \alpha^3)$	0.015% [6]	0.015%
(c) Vacuum polariz.	0.040% [7,8]	0.013% (0.011%(JJ))
(d) Light pairs	0.030% [10]	0.010% [18,19]
(e) s-channel Z-exchange	0.015% [11,12]	0.015%
(f) Up-down interference	0.0014% [27]	0.0014%
(f) Technical Precision	–	(0.027)%
Total	0.061% [13]	0.038% (0.037%(JJ))

Table 1: Summary of the total (physical+technical) theoretical uncertainty for a typical calorimetric detector. For LEP1, the above estimate is valid for a generic angular range within 1° - 3° (18-52 mrad), and for LEP2 energies up to 176 GeV and an angular range within 3° - 6° . Total uncertainty is taken in quadrature. Technical precision included in (a).

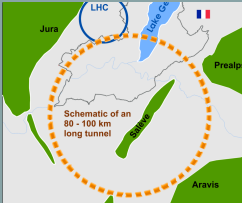
- By the time of FCCee VP contribution will be merely 0.006%(F. Jegerlehner)
- QED corrections and Z contrib. come back to front!
- Z contr. easy to master, even if rises at FCCee, because (28-58)->(64-86) mrad.

Our FCCee forecast is 0.001%, provided QED

is improved.

Bibliography in last slides

Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4 \alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2 \alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
(d) Light pairs	0.010% [18,19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}



arXiv:1812.01004v1 [hep-ph] 3 Dec 2018

The Path to 0.01% Theoretical Luminosity Precision for the FCC-ee*



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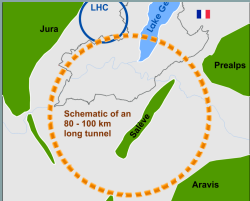
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Abstract

The current status of the theoretical precision for the Bhabha luminometry is critically reviewed and pathways are outlined to the requirement targeted by the FCC-ee precision studies. Various components of the pertinent error budget are discussed in detail – starting from the context of the LEP experiments, through their current updates, up to prospects of their improvements for the sake of the FCC-ee. It is argued that with an appropriate upgrade of the Monte Carlo event generator `BHLUMI` and/or other similar MC programs calculating QED effects in the low angle Bhabha process, the total theoretical error of 0.01% for the FCC-ee luminometry can be reached. A new study of the Z and s -channel γ exchanges within the angular range of the FCC-ee luminometer using the `BHWIDE` Monte Carlo was instrumental in obtaining the above result. Possible ways of `BHLUMI` upgrade are also discussed.

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Details follow...



LEP legacy and update 2018



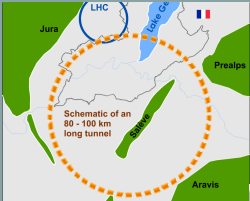
[25] F. Jegerlehner, “qed(mz) and future prospects with low energy e+e collider data”, FCC-ee Mini-Workshop, Physics Behind Precision <https://indico.cern.ch/event/469561/>.

[18] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, “Light pair correction to Bhabha scattering at small angle”, *Nucl. Phys.* **B547** (1999) 39–59, [hep-ph/9811436](https://arxiv.org/abs/hep-ph/9811436).

[19] G. Montagna, M. Moretti, O. Nicrosini, A. Pallavicini, and F. Piccinini, “Light pair corrections to small angle Bhabha scattering in a realistic set up at LEP”, *Phys. Lett.* **B459** (1999) 649–652, [hep-ph/9905235](https://arxiv.org/abs/hep-ph/9905235).

Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4 \alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2 \alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
(d) Light pairs	0.010% [18, 19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

- All of LEP/SLD luminosity QED error estimates represent corrections **missing in BHLUMI v.4.04** Monte Carlo, used by all LEP and SLD collaborations.
- BHLUMI features $O(\alpha^1)$ and $O(L_e^2 \alpha^2)$ corrections with YFS resummation, neglecting photonics interferences between e^+ and e^- lines, where $L_e = \ln(|t|/m_e^2)$.
- Vacuum polarisation and pairs not dominant any more —
QED photonic corrections and Z-exchange come back to front line!



QED photonics corrs.



Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4 \alpha^4)$	0.027%	0.6×10^{-5}
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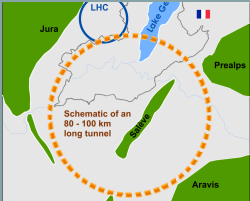
1. Photonic corrections are large, but higher orders contrib. known, hence soft/collinear **re-summation is mandatory!**
2. M.E. in BHLUMI includes $O(\alpha^1)$ and $O(L_e^2 \alpha^2)$ corrections within YFS soft photon re-summation, neglecting photonics interferences between e^+ and e^- lines (suppressed by $|t|/s$ factor).
3. Photonics 2nd order NLO $O(L_e \alpha^2)$ and 3rd order LO $O(\alpha^3 L_e^3)$ corrections were calculated long ago [4], [6]. Presently they are not in BHLUMI v4.02 and accounted for in the error budget. Once included, error estimate is done for $O(L_e^0 \alpha^2)$, $O(\alpha^4 L_e^4)$ and $O(\alpha^3 L_e^2)$ corrections.
4. Corrections $O(L_e^0 \alpha^2) \sim 10^{-5}$ are not quoted in FCC error budget because are known.
5. Using scaling rules of thumb we estimate $O(\alpha^4 L_e^4)$ as $0.015\% \times \gamma = 0.6 \times 10^{-5}$ and $O(\alpha^3 L_e^2) \sim \gamma^2 \alpha / \pi \simeq 10^{-5}$.
6. N.B. BHLUMI with $O(L_e \alpha^2)$ has been already realised but not published because VP was dominant in 1998.

$$\gamma = \frac{\alpha}{\pi} \ln \frac{|\vec{t}|}{m_e^2} = 0.042$$

$$|\vec{t}|^{1/2} = \langle |t| \rangle^{1/2} \simeq 3.25 \text{ GeV}$$

[4] S. Jadach, M. Melles, B. F. L. Ward, and S. A. Yost, "Exact results on $O(\alpha)$ corrections to the single hard bremsstrahlung process in low angle Bhabha scattering in the SLC / LEP energy regime", *Phys. Lett.* **B377** (1996) 168–176, [hep-ph/9603248](https://arxiv.org/abs/hep-ph/9603248).

[6] S. Jadach and B. F. L. Ward, "Missing third order leading log corrections in the small angle Bhabha calculation", *Phys. Lett.* **B389** (1996) 129–136.



Z and s-channel gamma exchange for FCCee angular range 64-86mrad



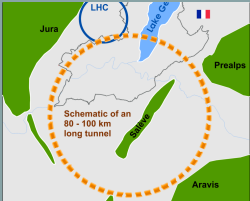
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(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

1. With respect to **dominant** t-channel gamma exchange $|\gamma_t|^2 = \gamma_t \otimes \gamma_t$, all other contributions are suppressed (near Z) by factor $\langle |t| \rangle / s = 1.3 \cdot 10^{-3}$ (instead $0.4 \cdot 10^{-3}$ for LEP!)
2. However, **resonant** Z_s exchange gets enhanced by M_Z / Γ_Z and $\gamma_t \otimes Z_s$ term will be up to 1%. It is included in BHLUMI at the complete 1-st order level (with QED running couplings). Using results of ref. [11] its uncertainty due to QED corrections is **presently** estimate above as 0.090%
3. **Non-resonant** $\gamma_t \otimes \gamma_s \sim 0.1\%$ is included in BHLUMI, gets small QED cor. with uncertainty 0.01%
4. Other contribution not in BHLUMI are: $|Z_s|^2 \sim 0.01\%$, $\gamma_t \otimes Z_t \sim 3 \cdot 10^{-5}$, $|\gamma_s|^2 \sim 10^{-6}$ and $|Z_t|^2 \sim 10^{-6}$
5. It will be straightforward to reduce the above uncertainties to $\sim 10^{-4}$ level by means of upgrade of the BHLUMI matrix element to the level of BHWIDE (EEX type).
6. With the implementation of the mat.el. of the CEEX type, as in KKMC, one could get for this group of contributions precision level of $\sim 10^{-5}$.

Study of Z and s-channel γ exchanges using BHWIDE

E_{CM} [GeV]	Δ_{tot} [%]	$\delta_{O(\alpha)}^{QED}$ [%]	$\delta_{h.o.}^{QED}$ [%]	δ_{tot}^{weak} [%]
90.1876	+0.642 (12)	-0.152 (59)	+0.034 (38)	-0.005 (12)
91.1876	+0.041 (11)	+0.148 (59)	-0.035 (38)	+0.009 (12)
92.1876	-0.719 (13)	+0.348 (59)	-0.081 (38)	+0.039 (13)

[11] S. Jadach, W. Placzek, and B. F. L. Ward, "Precision calculation of the gamma - Z interference effect in the SLC / LEP luminosity process", *Phys. Lett.* **B353** (1995) 349-361.



Vacuum polarization



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(a) Photonic $O(L_e^4 \alpha^4)$	0.027%	0.6×10^{-5}
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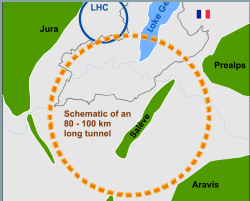
1. The error due to imprecise knowledge of the QED coupling constant for the t-channel exchange is $\frac{\delta_{VP}\sigma}{\sigma} = 2 \frac{\delta\alpha_{eff}(\bar{t})}{\alpha_{eff}(\bar{t})}$
2. With $\Delta\alpha^{(5)}(-s_0) = (64.09 \pm 0.63) \times 10^{-4}$, of ref. [26], at $s_0=2\text{GeV}$ we get $(\delta_{VP}\sigma)/\sigma = 1.3 \times 10^{-4}$
3. Anticipating improvement of hadronic e^+e^- cross section we expect by the FCCee time factor 2 improvement down to $\delta_{VP}\sigma/\sigma = 0.65 \times 10^{-4}$
4. N.B. The above is part of strategy of obtaining $\alpha_{eff}(M_Z^2)$ in two steps:
 - (a) obtaining $\Delta\alpha^{(5)}(-s_0)$ from $\sigma_{had}(s)$, $s^{1/2} \leq 2.5 \text{ GeV}$, using dispersion relations,
 - (b) calculating $\Delta\alpha^{(5)}(M_Z^2) - \Delta\alpha^{(5)}(-s_0)$ using perturbative QCD.
 Getting $\Delta\alpha^{(5)}(-s_0)$ for Bhabha luminometry from $\alpha_{eff}(M_Z^2)$ could be an interesting crosscheck:)

[25] F. Jegerlehner, “qed(mz) and future prospects with low energy e+e collider data”, FCC-ee Mini-Workshop, Physics Behind Precision <https://indico.cern.ch/event/469561/>,

[26] F. Jegerlehner, “Variations on Photon Vacuum Polarization”, 1711.06089.

[48] F. Jegerlehner, “Precision measurements of $\sigma(\text{hadronic})$ for $\alpha_{eff}(E)$ at ILC energies and $(g-2)(\mu)$ ”, *Nucl. Phys. Proc. Suppl.* **162** (2006) 22–32, [,22(2006)], [hep-ph/0608329](https://arxiv.org/abs/hep-ph/0608329).

[49] F. Jegerlehner, “The Anomalous Magnetic Moment of the Muon”, *Springer Tracts Mod. Phys.* **274** (2017) pp.1–693.

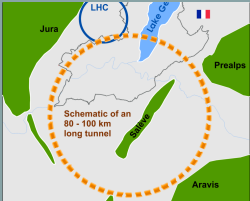


Light fermion pairs



Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4 \alpha^4)$	0.027%	0.6×10^{-5}
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(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

1. Additional light fermion pair production in Bhabha process $e^-e^+ \rightarrow e^-e^+ f\bar{f}$, $f = e, \mu, \tau, u, d, s$ together with the corresponding virtual correction (fermion loop on photon line) is a valid 2nd order correction.
2. Numerically most sizeable is electron pair production subprocess $e^-e^+ \rightarrow e^-e^+ \gamma^*$, $\gamma^* \rightarrow e^-e^+$ which very well known [9,10,18,19,53-60] and its precision is usually quoted to be $\sim 0.5 \cdot 10^{-4}$.
3. Second pair production $e^-e^+ \rightarrow e^-e^+ 2(e^-e^+)$ and addition photon production $e^-e^+ \rightarrow e^-e^+ e^-e^+ \gamma$ are calculable [10,18,54] and quoted to be negligible.
4. Contributions from heavier leptons and light quarks $f = \mu, \tau, u, d, s$ are typically $\sim 0.8 \cdot 10^{-4}$ and in LEP context were entirely accounted as part of an error. They can be however calculated with the precision $\ll 0.5 \cdot 10^{-4}$.
5. These corrections can be incorporated only partly in BHLUMI (electron pair exponentiation in [10]), most likely auxiliary MC programs will be needed to calculate them.

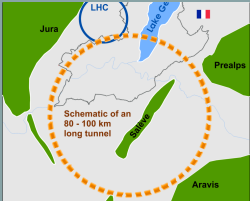


Up-down interference



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(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

1. From ref. [27] this photonics (1st order correction) is known to be $\delta\sigma/\sigma \simeq 0.07 |t|/s$ and for the luminometry it was negligible.
2. For FCCee it will come in a natural way in the upgrade M.E. of BHLUMI, to be done either as in BHWIDE or in KKMC.
3. We use conservatively factor $2\gamma \simeq 0.1$ in its precision estimate.



Technical precision

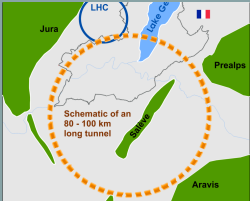


Type of correction / Error	Update 2018	FCCee forecast
(a) Photonic $O(L_e^4 \alpha^4)$	0.027%	0.6×10^{-5}
(b) Photonic $O(L_e^2 \alpha^3)$	0.015%	0.1×10^{-4}
(c) Vacuum polariz.	0.014% [25]	0.6×10^{-4}
(d) Light pairs	0.010% [18, 19]	0.5×10^{-4}
(e) Z and s-channel γ exchange	0.090% [11]	0.1×10^{-4}
(f) Up-down interference	0.009% [27]	0.1×10^{-4}
(f) Technical Precision	(0.027)%	0.1×10^{-4}
Total	0.097%	1.0×10^{-4}

1. Technical precision is the hardest problem!
2. In LEP workshop ref. [29] (1998) it was based on two pillars: comparison with semi-analytical calculation in ref. [45] and on comparison of BHLUMI with two hybrid MCs, LUMLOG+OLBBIS and SABSPV.
3. It was established to be 0.27%, together with missing photonics corrections.
4. Later on another BabaYaga MC was developed [20-24] based on the parton shower algorithm, and in principle could be used to evaluate technical precision independently.
5. However, once BHLUMI will be upgraded to include complete $O(L_e \alpha^2)$ and $O(\alpha^3 L_e^3)$ the problem will come back, because it will be much harder to upgrade BabaYaga to the same NNLO level due to known peculiarities of the parton shower methodology.
6. Alternative solution could/should be worked out. See S. Frixione, 1909.03886, V. Bertone *et al.*, 1911.12040 .

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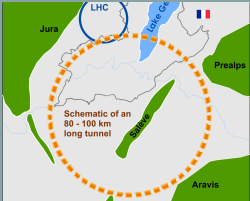
[45] S. Jadach and B. F. L. Ward, “Semianalytical third order calculations of the small angle Bhabha cross-sections”, *Acta Phys. Polon.* **B28** (1997) 1907–1979.



SYNERGIES



- Historically, our exact $O(\alpha^2 L_e)$ corrections were done for BHLUMI 4 precision => Combined via crossing with CEEEX => KKMC for state-of-art 2f production => KKMC-hh for Z production in pp
- KKMC-hh => MG5_aMC@NLO/KKMC-hh (to appear) => exact QCD NLO \otimes exact $O(\alpha^2 L_e)$ EW
- **When we add to BHLUMI QED matrix element corrections of $O(L_e \alpha^2)$ and $O(\alpha^3 L_e^3)$**
- => **Already reduce δN_γ from $\delta \mathcal{L}/\mathcal{L}$ to 0.0015.**
- We now need to take CEEEX to BHLUMI (a technical precision solution)
- => For FCCee, take CEEEX to all the EEX YFS realizations for LEP:
- YFSWW3 & KORALW (see M. Skrzypek, in 2020 FCC Wkshp)
- YFSZZ
- BHWIDE
- **We do need sufficient theory resources.**



SYNERGIES



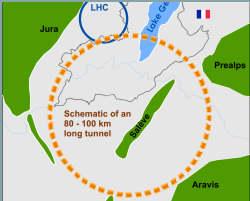
- For example, A_{FB} : Jadach & Yost, arXiv: 1801.08611 => use CEEX
=> KKMC for state-of-art 2f production => already have $\Delta [A_{FB}]_{|F|} \sim 10^{-4}$
- ΔMW : (Skrzypek(FCCee Workshp,2020)):
- **Threshold & Reconstruction: Need ~0.3 MeV for FCC-ee**
- CEEX extension of the LEP2 MC YFSWW3 & KORALW needed in both cases:
- In progress: Jadach et al., arXiv:1906.09071 -- CEEX formalism applied
- to $e^+e^- \rightarrow WW + n\gamma \rightarrow 4f + n'\gamma$
- Note: Contact with the usual Kleiss-Stirling spinor product-based photon helicity infrared factors in CEEX via

$$e j_X^\mu(k_i) = e Q_X \theta_X \frac{2 p_X^\mu}{2 p_X k_i} \rightarrow s_i(k_i) = e Q_X \theta_X \frac{b_{\sigma_i}(k_i, p_X)}{2 p_X k_i},$$

with

$$b_\sigma(k, p) = \sqrt{2} \frac{\bar{u}_\sigma(k) \not{p} u_\sigma(\zeta)}{\bar{u}_{-\sigma}(k) u_\sigma(\zeta)}$$

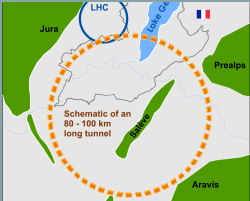
- The way forward is open.



SUMMARY



- All of LEP/SLD luminosity QED error estimates represent corrections **missing in BHLUMI v.4.04** Monte Carlo, used by all LEP and SLD collaborations.
- BHLUMI features $O(\alpha^1)$ and $O(L_e^2 \alpha^2)$ corrections with YFS resummation, neglecting photonics interferences between e^+ and e^- lines, where $L_e = \ln(|t|/m_e^2)$.
- **One has to add to BHLUMI QED matrix element corrections of $O(L_e \alpha^2)$ and $O(\alpha^3 L_e^3)$**
- They were calculated by Cracow-Knoxville collaboration long time ago (1996-99), but there was no strong motivation to publish them in the MC form, because of large VP uncertainty.
- Interferences between e^+ and e^- lines should be added at 1-st order, with resummation.
- This class of corrections are implemented in the KKMC and BHWIDE since 1999.
- Corrections due to Z exchange and s-channel gamma are big but easy to master (ME upgrade).
- There is (almost) enough auxiliary programs and calculations to control light pair corrections.
- **Summarising there is no hard obstacles on the way to 0.01% QED precision on the theory side.**
- The sticky issue is that of “technical precision”.
If BabaYaga Monte Carlo team makes sufficient progress this problem is solved (**Piccinini**).
- Alternative solutions are available: **comparing CEEX and EEX upgrades of BHLUMI, Frixione *et al.*, Sherpa,**
- **We do need sufficient theory resources.**

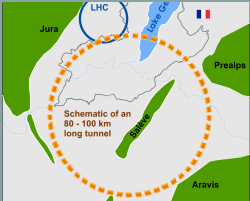


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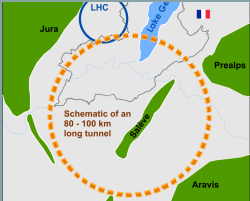
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