Explaining the SM flavor structure with grand unified theories

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Based on work done in collaboration with

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Grand Unification

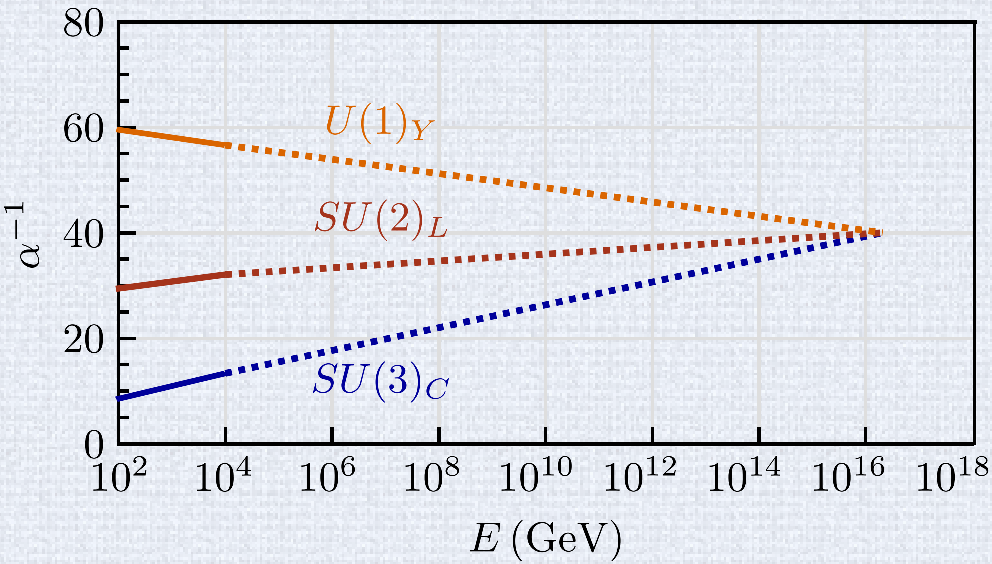
SM

The Standard Model (SM) is based on the group

$$G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$$

Thus ... there are 3 gauge couplings to be explained

$$D_\mu = \partial_\mu - i\underline{g_3}\lambda^a G^a - i\underline{g_2}\sigma^a W^a - i\underline{g_1} Y B$$



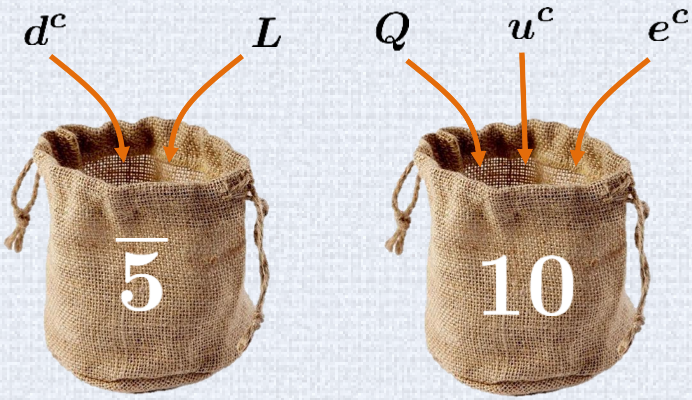
GUTs

A standard GUT(Grand Unified Theory): values of the gauge couplings can be explained

G_{SM} is a subgroup of some simple Lie group: SU(5), SO(10), E(6), ...

And fermions?

$SU(5)$



SM fermions are packed together in representations of the GUT group

Relations between SM Yukawa matrices are expected

$SO(10)$

E.g.

$$\boxed{Y_{ij}}\,\mathbf{16}_i\cdot\mathbf{16}_j\cdot\mathbf{10}$$

$$\Downarrow$$

$$\boxed{Y}=Y_U=Y_D=Y_E\,(=Y_\nu)$$

Flavor

There are 3 copies/generations/flavors/families of each fermion representation at low energies (we don't know why)

Flavor = property of fermions

SM

$$Y^U_{ij} Q_i u^c_j H + Y^D_{ij} Q_i d^c_j H^* + Y^E_{ij} L_i e^c_j H^*$$

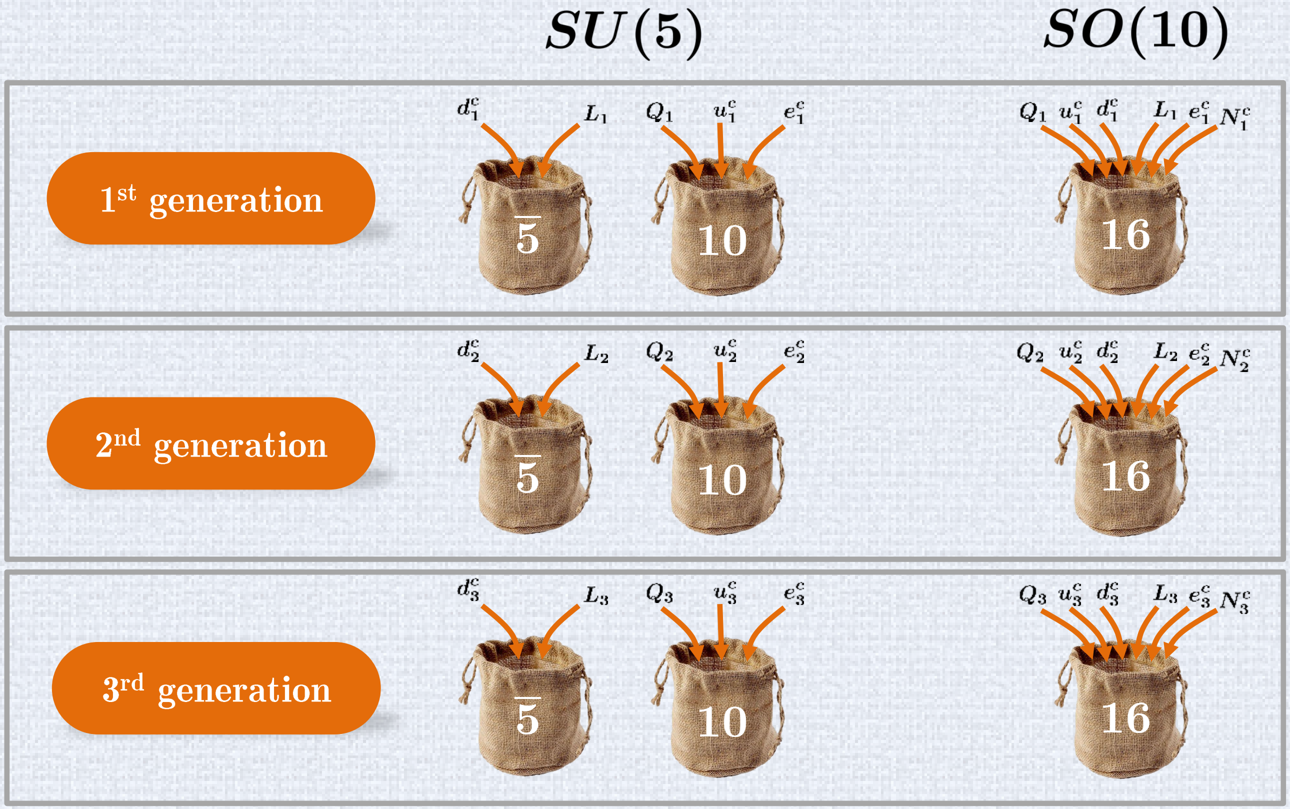
Yukawa couplings: 3 by 3 matrices

$$\begin{pmatrix} y^U_{11} & y^U_{12} & y^U_{13} \\ y^U_{21} & y^U_{22} & y^U_{23} \\ y^U_{31} & y^U_{32} & y^U_{33} \end{pmatrix} \begin{pmatrix} y^D_{11} & y^D_{12} & y^D_{13} \\ y^D_{21} & y^D_{22} & y^D_{23} \\ y^D_{31} & y^D_{32} & y^D_{33} \end{pmatrix} \begin{pmatrix} y^E_{11} & y^E_{12} & y^E_{13} \\ y^E_{21} & y^E_{22} & y^E_{23} \\ y^E_{31} & y^E_{32} & y^E_{33} \end{pmatrix} \quad M_\nu$$

$m_{u,c,t,d,s,b}$ **and** V_{CKM}

$m_{e,\mu,\tau,\nu_1,\nu_2,\nu_3}$ **and** V_{PMNS}

Flavor and GUTs



'Standard' GUTs:
Perfect family replication

Cannot explain flavor

$$\boxed{Y_{ij}} \mathbf{16}_i \cdot \mathbf{16}_j \cdot \mathbf{10}$$

3 by 3 matrix as the SM Yukawas

Family unification

The concept is old

[Gell-Mann, Ramond, Slansky, 1979] [Georgi 1979]
[Wilczek, Zee 1979, 1982] [Frampton 1979] …

But no realistic model until now

Why? Group G and representation R must be such that before EWSB only the SM fermions are be massless

(so must be careful with the '...' in the picture)

Family unification

[Georgi, "Towards a Grand Unified Theory of flavor" 1979]

4. Generalizations of one-family SU(5)

I now have the tools to generalize the SU(5) theory in a non-trivial way. I want to find a group and a representation for the LH fermions consistent with the rules set down in sect. 3 such that the ordinary mass fermions consist of several families of quarks and leptons, but I want to avoid a trivial solution in which a small representation is repeated several times, so I demand yet another principle.

The third law of grand unification: no irreducible representation should appear more than once in the representation of the LH fermions.
In the simplest SU(5), of course, this is just the restriction to one family.

One might hope to obtain more than one family without enlarging the gauge group by making use of more complicated representations of SU(5). Unfortunately, this does not work. Such representations, if they yield any ordinary mass fermions at all, predict ordinary mass fermions with peculiar SU(3) × SU(2) × U(1) properties *. Unless there are real surprises between presently accessible energies and 100 GeV, such representations are ruled out.

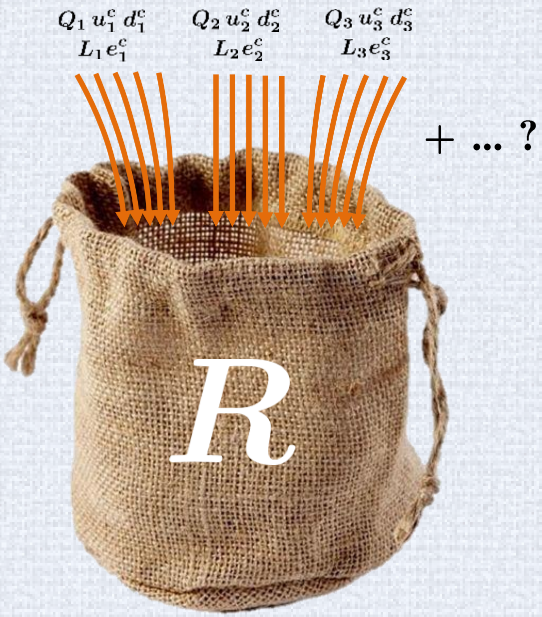
In enlarging the gauge group, I will restrict myself to groups which have the ordinary SU(5) theory as a subgroup. In fact, for reasons which will soon become clear, I will concentrate on larger unitary groups SU(N) into which SU(5) is em-

ld
[Georgi 1979]
ton 1979] …

ntil now

entation R
WSB only
massless
the picture)

Family unification



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(so must be careful with the '...' in the picture)

But in the last few years ...

Suggestion which might work:
$G=SU(19)$ and $R=171$

[RF 1504.03695]
[Yamatsu 1807.10855]

It might even be the only setup that works under some assumptions (4D-spacetime, no extra confining gauge interactions, ...)

A model

$SU(19)$

Fermions: $171 = (19 \times 19)_A$ ← 2-index anti-symmetric tensor

We assume one scalar representation only: $\Omega = \overline{3876}$ ← 4-index fully anti-symmetric tensor

$$y171_{ab}171_{cd}\Omega^{abcd}$$

A single number !

It alone controls all F-F-S interactions (normalization factor)

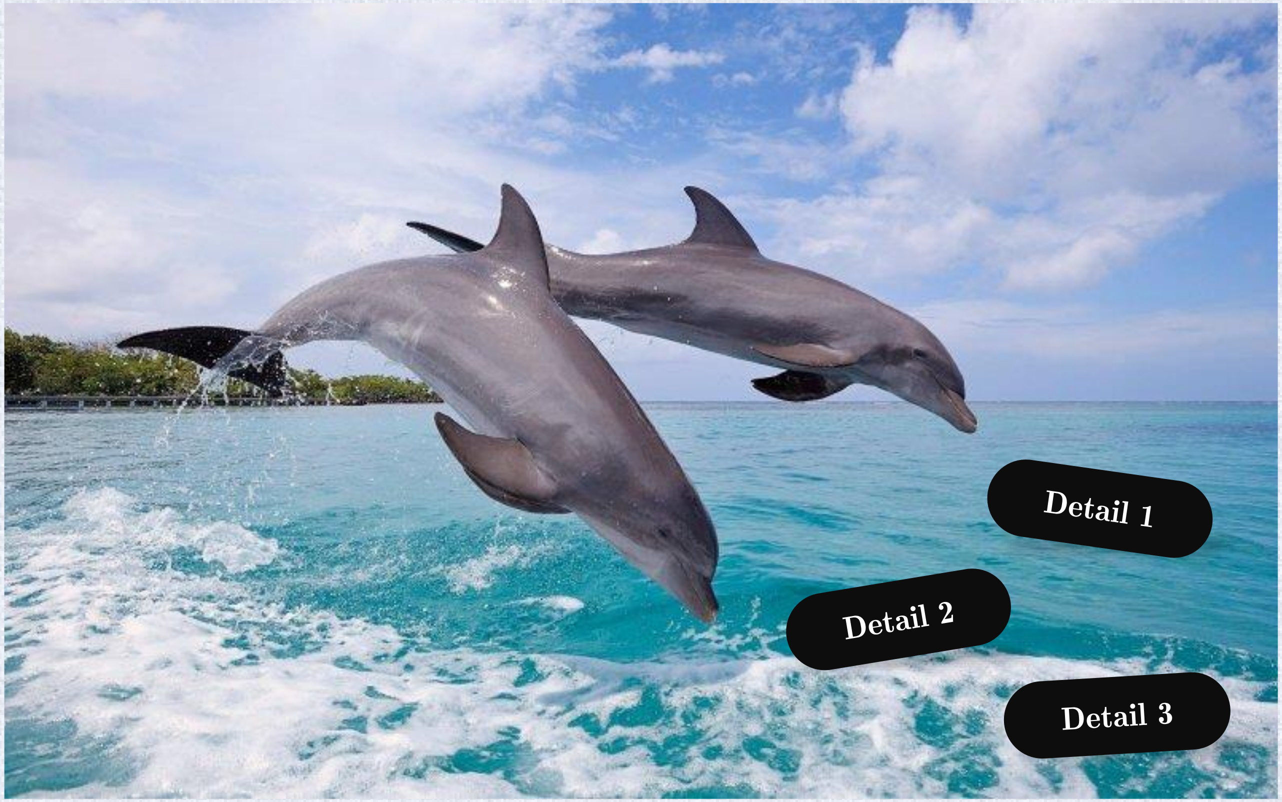
How is this even possible?

How do we get all SM flavor parameters at low energies?

Fundamentally the SM flavor structure is fully controlled by the scalar potential

Quick description of what is going on

1. The scalar field (Ω) contains several SM group singlets $(1,1,0)$ [S] and doublets $(1,2,\pm1/2)$ [H].
2. The VEVs of the S's control which fermions are light (=0 mass before EWSB)
3. One linear combination of the H's is the 125 GeV Higgs



Let's dive into (some) details

Fermions

Branching rules

There are many G_{SM} subgroups inside SU(19). We want ...

$$\underbrace{\mathbf{19}}_{SU(19)} \rightarrow \underbrace{\mathbb{Q} + \mathbb{u}^c + \mathbb{d}^c + \mathbb{L} + \mathbb{e}^c + \mathbb{N}_i^c}_{G_{SM}\times SU(4)_F}$$

With the usual G_{SM} quantum numbers

SU(4)

Why SU(4)? It is the **largest simple subgroup which commutes with G_{SM}** so we can **see it as a flavor group**

$\mathbb{N}_i^c$ are charged under SU(4) (subindex i: denotes a quadruplet)

Fermions

$$\mathbf{171} = \begin{array}{c} \\ \mathbb{d}^c \\ \mathbb{L} \\ \mathbb{Q} \\ \mathbb{u}^c \\ \mathbb{e}^c \\ \mathbb{N}_j^c \end{array} \begin{array}{c} \begin{array}{cccccc} \mathbb{d}^c & \mathbb{L} & \mathbb{Q} & \mathbb{u}^c & \mathbb{e}^c & \mathbb{N}_i^c \end{array} \\ \left(\begin{array}{cccccc} u & Q^c & L_1^c & d_2 & \cdot & d_i^c \\ \times & e & d_1 & \cdot & L_2^c & L_i \\ \times & \times & \cdot & L_5 & \cdot & Q_i \\ \times & \times & \times & \cdot & d_5^c & u_i^c \\ \times & \times & \times & \times & \cdot & e_i^c \\ \times & \times & \times & \times & \times & N_{ij}^c \end{array}\right) \end{array}$$

19×19 anti-symmetric matrix

Shown here in block form (6x6 blocks)

E.g.: Quantum numbers of block $\mathbf{171}_{\mathbb{L}\mathbb{N}_i^c}$:

$\mathbb{L}\times\mathbb{N}_i^c = (\mathbf{1},\mathbf{2},-\mathbf{1/2})$ so $\mathbf{171}_{\mathbb{L}\mathbb{N}_i^c} = L_i$

i subscript: quadruplet of SU(4)

$$4Q + 1Q^c + 4u^c + 1u + 5d^c + 2d + 5L + 2L^c + 4e^c + 1e + \text{vector fermions}$$
$$=$$
$$3\,(Q + u^c + d^c + L + e^c) + \text{vector fermions}$$

SM fermions only at low energies

!

Scalars

Low energy phenomenology is controlled by these scalars only

S

$(1, 1, 0)$

$$\overbrace{\mathbb{d}^c\mathbb{Q}\mathbb{Q}\mathbb{u}^c}^{S_L}, \overbrace{\mathbb{L}\mathbb{Q}\mathbb{u}^c\mathbb{e}^c}^{S_{DL}}, \overbrace{\mathbb{d}^c\mathbb{u}^c\mathbb{u}^c\mathbb{e}^c}^{S_D}, \overbrace{\mathbb{d}^c\mathbb{d}^c\mathbb{u}^c\mathbb{N}_i^c}^{S_{UD}^i},$$

$$\overbrace{\mathbb{d}^c\mathbb{L}\mathbb{Q}\mathbb{N}_i^c}^{S_{QDL}^i}, \overbrace{\mathbb{L}\mathbb{L}\mathbb{e}^c\mathbb{N}_i^c}^{S_{EL}^i}, \overbrace{\mathbb{N}_i^c\mathbb{N}_j^c\mathbb{N}_k^c\mathbb{N}_l^c}^{S_N}$$

SM scalar singlets

They control which combinations of fermions are light

E.g.: $4Q + 1Q^c$

A combination of the 4 Q's pairs with Q^c and is superheavy. This combination is controlled by S's.

H

$(1, 2, 1/2)$

$$\mathbb{L}\mathbb{d}^c\mathbb{d}^c\mathbb{u}^c, \mathbb{d}^c\mathbb{L}\mathbb{L}\mathbb{Q}, \overbrace{\mathbb{Q}\mathbb{u}^c\mathbb{N}_i^c\mathbb{N}_j^c}^{H_{QN}^{ij}}, \overbrace{\mathbb{L}\mathbb{N}_j^c\mathbb{N}_k^c\mathbb{N}_l^c}^{H_{N,i}}$$

$\widetilde{H}$

$(1, 2, -1/2)$

$$\mathbb{d}^c\mathbb{d}^c\mathbb{d}^c\mathbb{L}, \overbrace{\mathbb{Q}\mathbb{u}^c\mathbb{e}^c\mathbb{N}_i^c}^{\widetilde{H}_{DE}^i}, \overbrace{\mathbb{d}^c\mathbb{Q}\mathbb{N}_i^c\mathbb{N}_j^c}^{\widetilde{H}_D^{ij}}, \overbrace{\mathbb{L}\mathbb{e}^c\mathbb{N}_i^c\mathbb{N}_j^c}^{\widetilde{H}_E^{ij}}$$

SM Higgs

Linear combinations of all these doublets (requires fine-tunning)

Predicting the SM flavor structure

$$\mathbf{171}_{ab}\mathbf{171}_{cd}\Omega^{abcd}$$

$$=$$

$$\frac{1}{3}Q_iQ^cS^i_{QDL} + \frac{\sqrt{2}}{3}u^c_i u S^i_{UD} + \sqrt{\frac{2}{3}}e^c_i e S^i_{EL} + \frac{1}{4}\sqrt{\frac{2}{3}}\epsilon_{ijkl}N^c_{ij}N^c_{kl}S_N$$

$$+\begin{pmatrix} d^c_i \\ d^c_5 \end{pmatrix}^T \left[\frac{\sqrt{2}}{3}\begin{pmatrix} -S^i_{QDL} & \sqrt{2}S^i_{UD} \\ -S_{DL} & \sqrt{2}S_D \end{pmatrix}\right]\begin{pmatrix} d_1 \\ d_2 \end{pmatrix} + \begin{pmatrix} L_i \\ L_5 \end{pmatrix}^T \left[\frac{1}{\sqrt{3}}\begin{pmatrix} -S^i_{QDL} & \sqrt{2}S^i_{EL} \\ -\frac{2}{\sqrt{3}}S_L & S_{DL} \end{pmatrix}\right]\begin{pmatrix} L^c_1 \\ L^c_2 \end{pmatrix}$$

$$+Q_i\left[-\frac{2}{3}H^{ij}_{QN}\right]u^c_j + Q_i\left[\frac{\sqrt{2}}{3}\begin{pmatrix} -\sqrt{2}\widetilde{H}^{ij}_D & \widetilde{H}^i_{DE} \end{pmatrix}\right]\begin{pmatrix} d^c_j \\ d^c_5 \end{pmatrix} + \begin{pmatrix} L_i \\ L_5 \end{pmatrix}^T \left[\frac{1}{\sqrt{3}}\begin{pmatrix} -\epsilon_{ijkl}H_{N,l} \\ \sqrt{2}H^{jk}_{QN} \end{pmatrix}\right]N^c_{jk}$$

$$+\begin{pmatrix} L_i \\ L_5 \end{pmatrix}^T \left[\frac{\sqrt{2}}{3}\begin{pmatrix} -\sqrt{2}\widetilde{H}^{ij}_E \\ \widetilde{H}^j_{DE} \end{pmatrix}\right] e^c_j + \cdots$$

i,j=1,2,3,4 are *SU(4)* 'flavor' indices

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Integrates out heavy fermion, leaving only 3 SM families

$$+Q_i\left[-\frac{2}{3}H^{ij}_{QN}\right]u^c_j + Q_i\left[\frac{\sqrt{2}}{3}\begin{pmatrix} -\sqrt{2}\widetilde{H}^{ij}_D & \widetilde{H}^i_{DE} \end{pmatrix}\right]\begin{pmatrix} d^c_j \\ d^c_5 \end{pmatrix} + \begin{pmatrix} L_i \\ L_5 \end{pmatrix}^T\left[\frac{1}{\sqrt{3}}\begin{pmatrix} -\epsilon_{ijkl}H_{N,l} \\ \sqrt{2}H^{jk}_{QN} \end{pmatrix}\right]N^c_{jk}$$

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H $\widetilde{H}$

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H $\widetilde{H}$

Can the SM masses and angles be reproduced? Yes

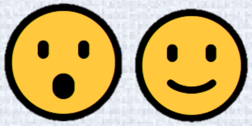
Predictions for SM parameters?

Not without looking at the scalar potential

Other comments

What about symmetry breaking: $SU(19) \rightarrow G_{SM}$?

I — Same scalar which interacts with fermions has a direction which can do it!



II — Potential: 1 mass + 3 quartic interactions. SM breaking VEV consistent with tadpole equations ...

III — ... but some masses are negative (?). Higher order terms and/or other scalars likely needed

IV — Gauge anomaly present. Gauge only part of SU(19) or add more fermions (which decouple)

Summary

First realistic model of family unification

All Yukawa interactions depend on 1 number

**Flavor = Fermion property? No.
Flavor originates from the scalar sector**

The various SM flavor parameters depend exclusively on ratios of VEVs

Thank you

Prague, 18/July/2020