Removal Energy and Optical Potentials in Quasielastic and Δ (1232) Resonance Production in Lepton-Nucleus Scattering.

Arie Bodek and Tejin Cai
Department of Physics and Astronomy
University of Rochester

Reference:

Arie Bodek and Tejin Cai, ``Removal Energies and Final State Interaction in Lepton Nucleus Scattering"
Eur. Phys. J. C79 (2019) 293. arXiv:1801.07975 [nucl-th] 2018.

Arie Bodek and Tejin Cai, `` Comparison of optical potential for nucleons and Δ resonances"
arXiv:2004.00087 [hep-ph] (Apr 2, 2020) published in
Eur. Phys. J. C (2020) 80:655
https://doi.org/10.1140/epjc/s10052-020-8236-8

Talk presented at ICHEP 2020 .
https://indico.cern.ch/event/868940/contributions/3816933/

Abstract

The modeling of the energy of final state particles in quasielastic (QE)-like events in neutrino scattering on bound nucleons requires knowledge of:

a) **The removal energy of the initial state bound nucleon ε**

b) **The average Coulomb potential V_{eff}**

c) **The average of the real part of the nuclear optical potentials U^{QE}** and **U^{Δ}** for final state **nucleons** and **Δ(1232) .**

The values **ε are** extracted from ee'P spectral functions. **V_{eff}** and **U^{QE}** and **U^{Δ}** are extracted as a function of final of final state baryon kinetic energy T from the peak positions in inclusive electron scattering data on nuclear targets. We compare **U^{QE}** to calculations by Cooper et. al.

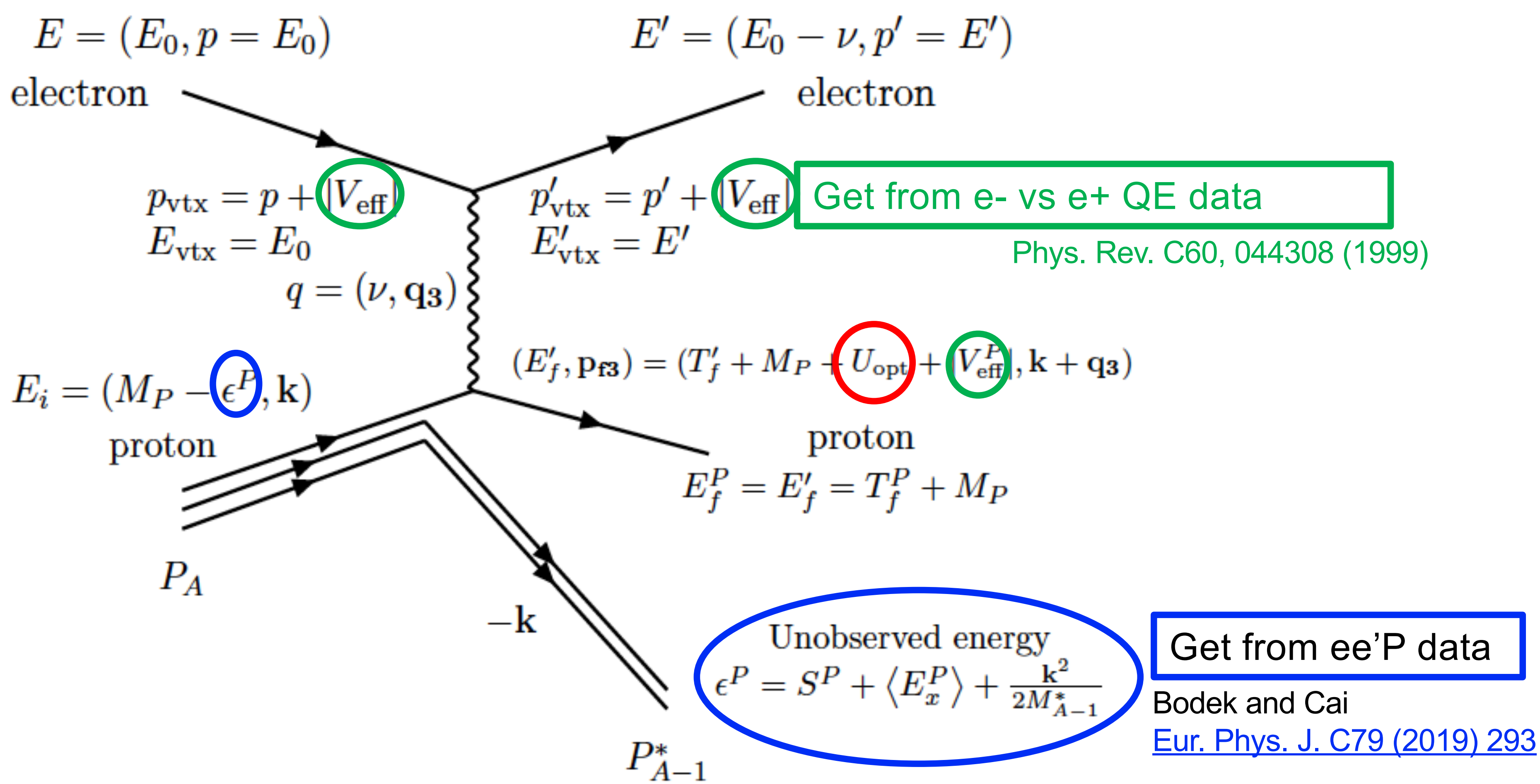
We find that U^{Δ} is more negative than U^{QE} with $U^{\Delta} \approx 1.5\ U^{QE}$ for Carbon. We also present results for Lithium, Aluminum, Calcium, Iron and Lead. This is different fron GiBUU which has $U^{\Delta} \approx 0.67\ U^{QE}$

.

Goal: Correctly Model the energy of final state particles in electron and neutrino-nucleus scattering

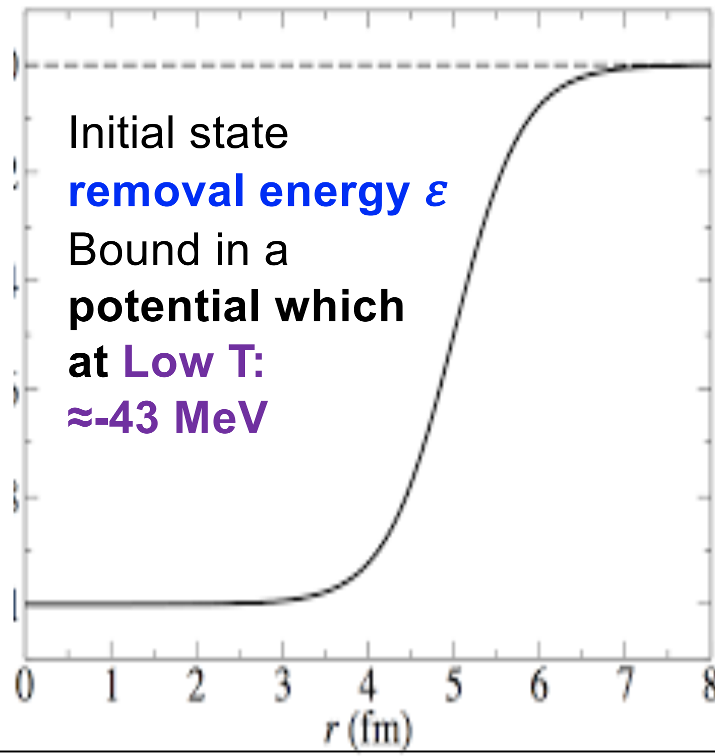
1. Nucleon has to be removed from the nucleus leaving nucleus at some excited state
2. Removal energy ***ε*** is due to nucleon bound in initial state potential (both nuclear and EM)
3. Initial and final state leptons experience Coulomb potential (V_{eff})
4. Final state hadrons experience both Coulomb V_{eff} and Nuclear **($U_{Optical}$)** potentials.

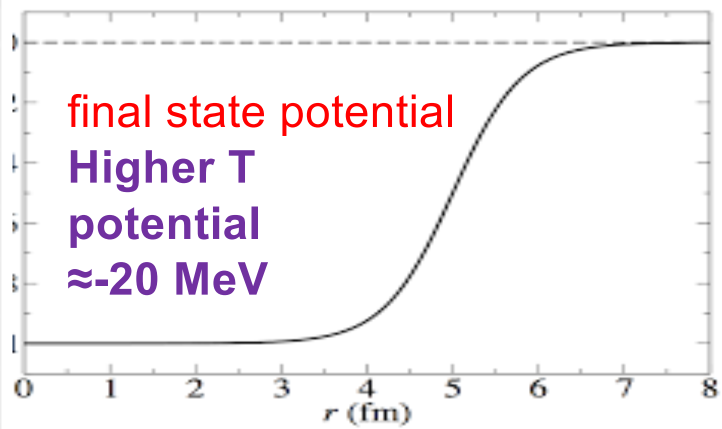
Electron scattering on proton **(QE scattering on bound protons)**



Nuclear Optical Potential has both **real** and imaginary part. Imaginary part is sometimes referred to as Final State Interaction (FSI) resulting in inelastic scattering of nucleon in final stste.

Here we only discuss **real part**, which is dependent on **density, radius and kinetic energy T.**. (Proton decelerated by Nucleon potential and accelerated by the Coulomb Veff)





Since final state hadron has higher kinetic energy T, its potential is lower, and as it leaves the nucleus it slows down

Estimate (not used) of U potential in initial state. Get about 43 MeV for Carbon 12

$$U^{P,N}(p_i^2) = \epsilon^{P,N} + T_i^{P,N}$$

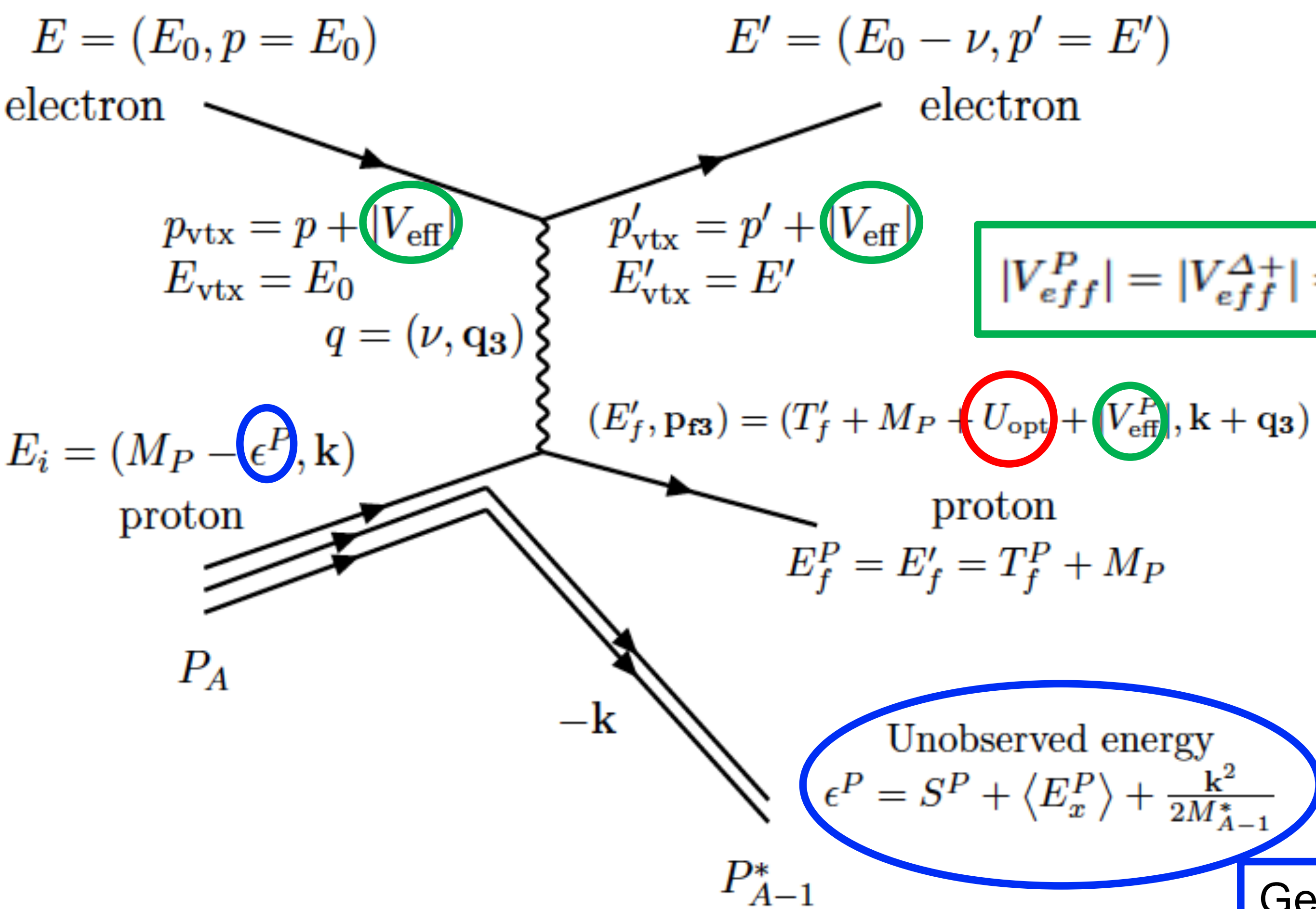
For neutrino MC, we need average nucleon potential $U_{optical}$ over radius and density.

Coulomb Potential Veff = 3.1 V for Carbon 12. It affects both charged leptons and charged hadrons. It also changes Q^2,

ACCOUNTING FOR COULOMB ENERGIES

$$Q^2 = 4(E_0 + |V_{eff}|)(E_0 - \nu + |V_{eff}|)\sin^2\frac{\theta}{2} \qquad (1)$$

$$\mathbf{q_3^2} = Q^2 + \nu^2$$



$$|V^P_{eff}| = |V^{\Delta+}_{eff}| = |V^{W+}_{eff}| = \frac{Z-1}{Z}|V_{eff}|$$

Get from ee'P data

$$\nu + (M_{P,N} - \epsilon^{P,N}) = E_f$$

$$E_f^P = \sqrt{(k+q_3)^2 + M_P^2} + U_{opt}^{QE} + |V_{eff}^P|$$

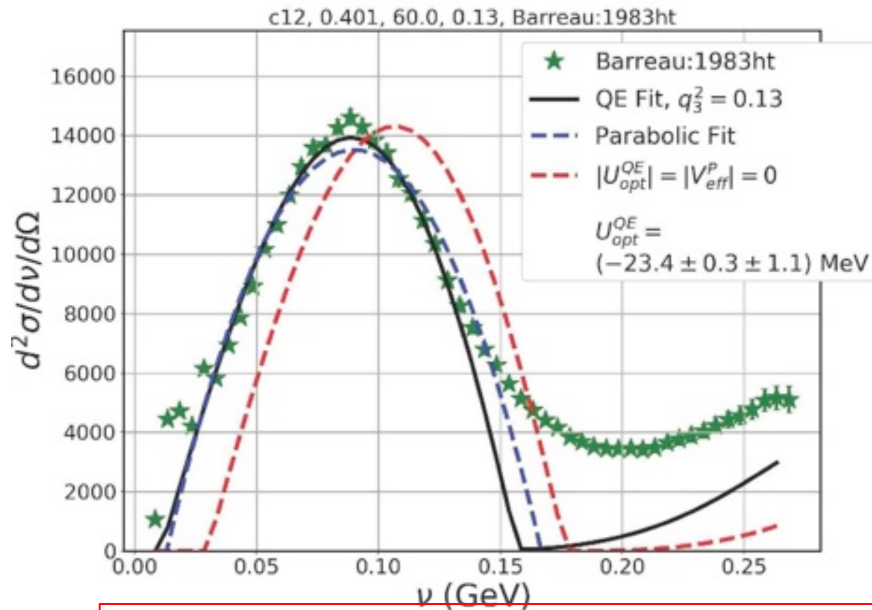
$$E_f^N = \sqrt{(k+q_3)^2 + M_N^2} + U_{opt}^{QE}$$

$$T^{P,N} = E_f^{P,N} - M_{P,N}$$

$$E_f^{\Delta+} = \sqrt{(k+q_3)^2 + W_{\Delta+}^2} + U_{opt}^{\Delta} + |V_{eff}^{\Delta+}|$$

$$E_f^{\Delta 0} = \sqrt{(k+q_3)^2 + W_{\Delta 0}^2} + U_{opt}^{\Delta}$$

$$T^{\Delta(+,0)} = E_f^{\Delta(+,0)} - W_{\Delta(+,0)},$$



Effect of QE nucleon optical potential

Red dashed lines are prediction with zero optical potential --------------

$$\epsilon^{P,N} = S^{P,N} + \langle E_x^{P,N} \rangle + \frac{k^2}{2M_{A-1}^*}.$$

For QE scattering: Use Relativistic Fermi gas (RFG) and fit to QE peak position (1/3 of the distribution) to extract optical potential

$U_{optical}$ (QE) (Solid line).

Compare to simple parabolic fit to obtain systematic error.

$$\nu + (M_{P,N} - \epsilon^{P,N}) = E_f$$

$$E_f^P = \sqrt{(k+q_3)^2 + M_P^2} + U_{opt}^{QE} + |V_{eff}^P|$$

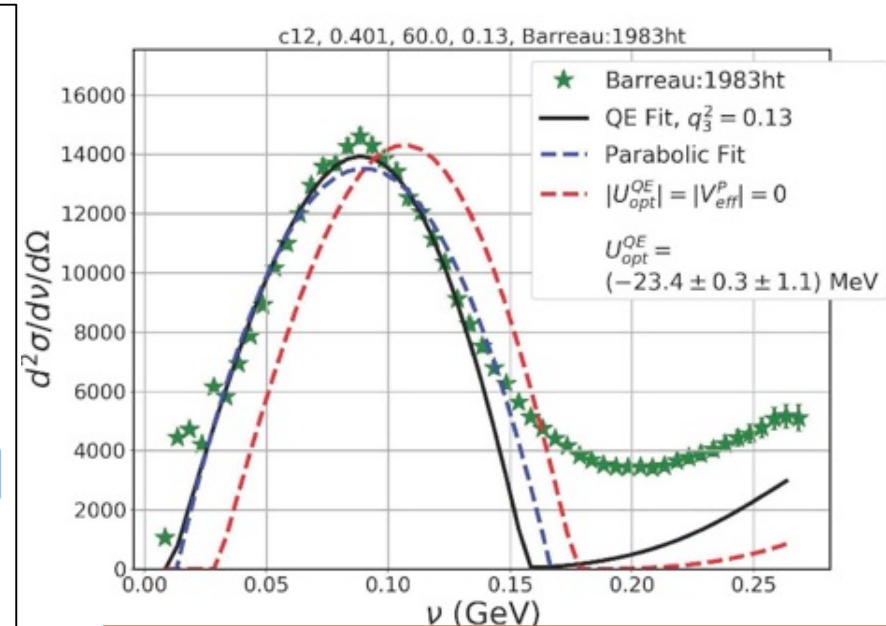
$$E_f^N = \sqrt{(k+q_3)^2 + M_N^2} + U_{opt}^{QE}$$

$$T^{P,N} = E_f^{P,N} - M_{P,N}$$

$$E_f^{\Delta+} = \sqrt{(k+q_3)^2 + W_{\Delta+}^2} + U_{opt}^{\Delta} + |V_{eff}^{\Delta+}|$$

$$E_f^{\Delta 0} = \sqrt{(k+q_3)^2 + W_{\Delta 0}^2} + U_{opt}^{\Delta}$$

$$T^{\Delta(+,0)} = E_f^{\Delta(+,0)} - W_{\Delta(+,0)},$$

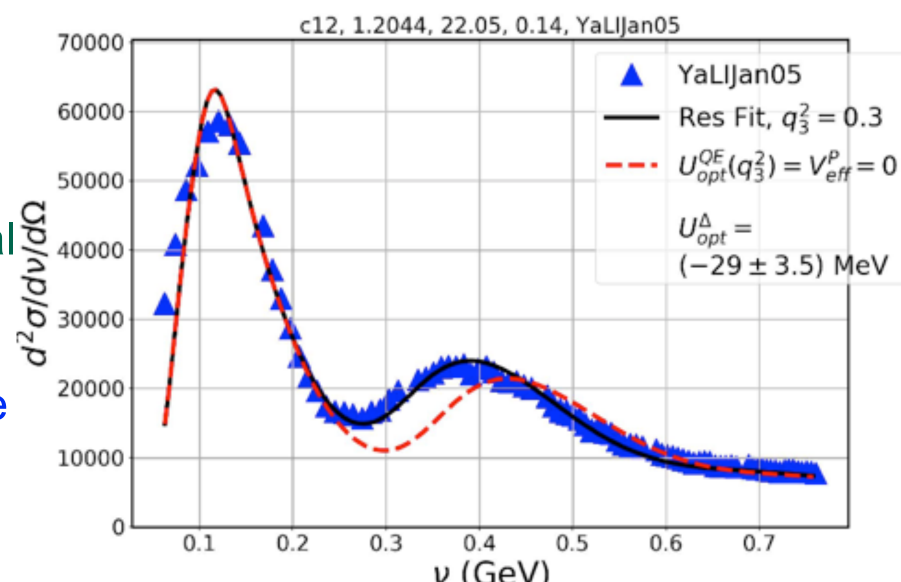


Effect of QE nucleon optical potential

Red dashed lines are prediction with zero optical potential --------------

For QE scattering: Use Relativistic Fermi gas (RFG) and fit to QE peak position to extract optical potential **$U_{optical}$** (QE) (Solid line)

For **Δ**(1232:) First model QE scattering with effective spectral function (**mimics ψ' scaling**) and smear resonance plus continuum fits for free nucleon with RFG. Fit **Δ** (1232) peak position to extract optical potential **$U_{optical}$ (Δ)** for **Δ**(1232)



Effect of **Δ** (1232) optical potential

Theoretical formalisms use **proton-nucleus scattering data** to fit to parametrizations of the nucleon optical potential.

One example: the parametrization of of Cooper, Hama, Clark and Cooper (**Cooper1993,Cooper2009**)

GiBUU follows a similar approach. Plot shows comparison between GiBUU and Cooper

Electron and neutrino interactions can occur at any location in the nucleus.

It is **the average value of the optical potential over the entire nucleus** that is the parameter that is needed for neutrino MC simulations such as GENIE and NEUT.

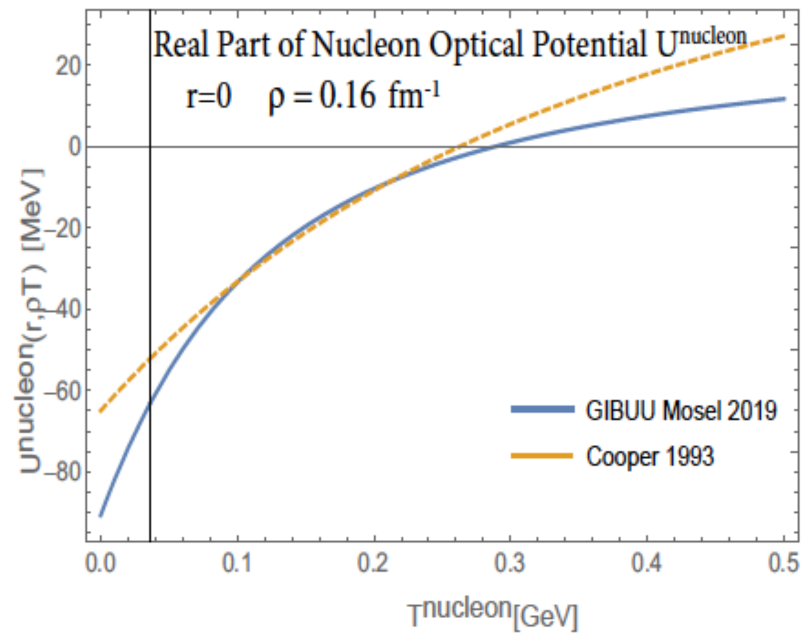
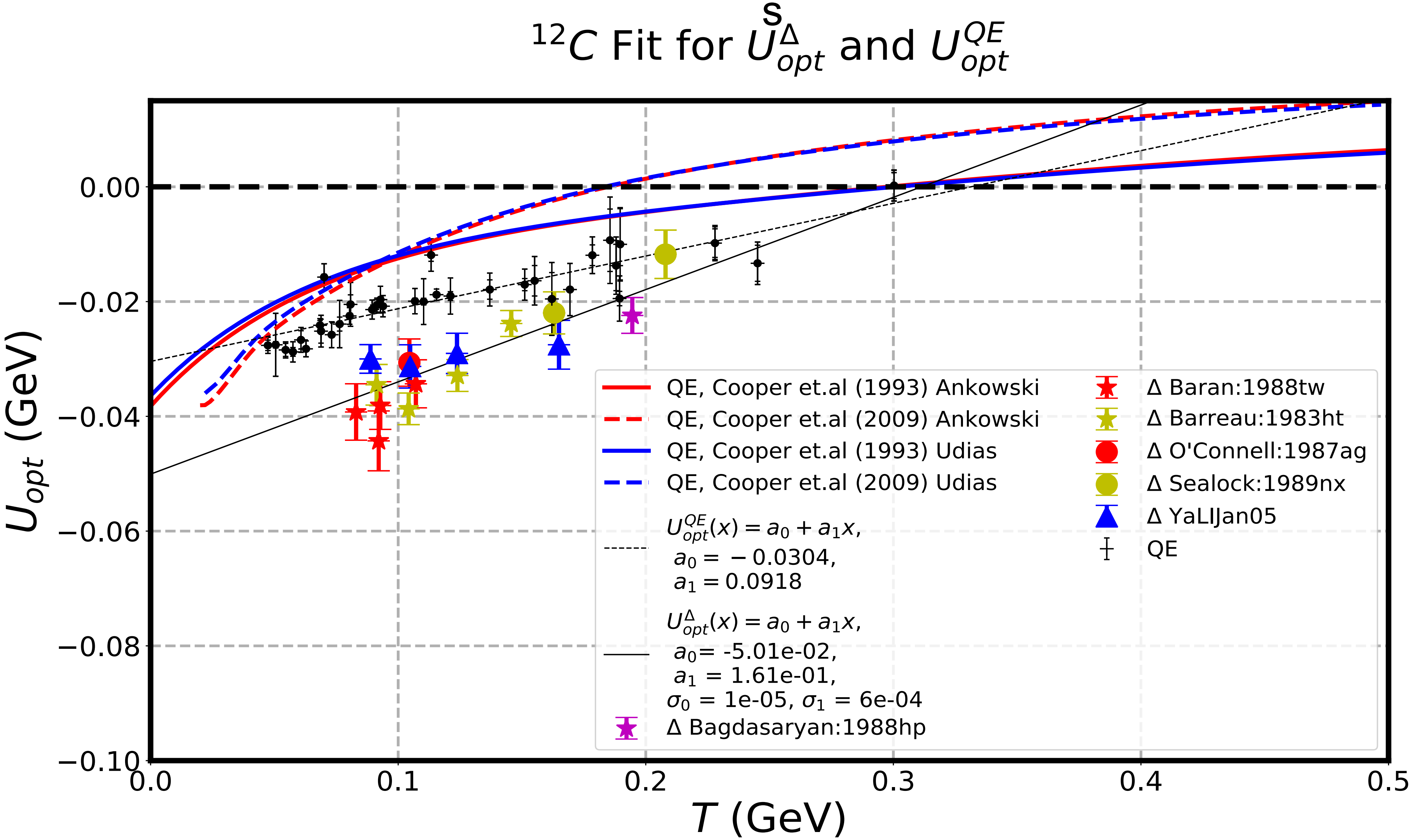


Fig. 6. A comparison of the real part of the nucleon optical potential ($U^{nucleon}(r, \rho, T)$) versus kinetic energy T at r=0 and nuclear density ρ= 0.16 fm^{-1} for GiBUU 2019 [41] as compared to the potential parametrized by Cooper 1993 [37] (curves from Ref. [41]). The two optical potentials are consistent with each other between nucleon kinetic energy between 0.1 and 0.3 GeV^2.

For C12 we find $U_{opt}^{\Delta} \approx 1.5\, U_{opt}^{QE}$ for $^{12}_{6}\mathbf{C}$. versus $U_{GiBUU}^{\Delta} = (2/3)\, U_{GiBUU}^{nucleon}$.



5.1 Interaction energy

In the off-shell formalism of Bodek and Ritchie [46], which is used in GENIE equations 3 and 4 can be written in terms of an energy dependent *interaction energy*:

OFF-SHELL FORMALISM

$$\nu + (M_{P,N} - \epsilon^{off-shell(P,N)}_{QE-interaction}) = \sqrt{p^2_{f3} + M^2_{P,N}} \quad (5)$$

$$\nu + (M_{P,N} - \epsilon^{off-shell(P,N)}_{\Delta-interaction}) = \sqrt{p^2_{f3} + W^2_{\Delta+,0}} \ ,$$

where

$$\epsilon^{off-shell(P,N)}_{QE-interaction} = \epsilon^{P,N} + U^{QE}_{opt}(p^2_{f3}) + |V^{P,N}_{eff}| \quad (6)$$

$$\epsilon^{off-shell(P,N)}_{\Delta-interaction} = \epsilon^{P,N} + U^{\Delta}_{opt}(p^2_{f3}) + |V^{\Delta+,0}_{eff}|.$$

Note that optical potential is negative, so at low Q it looks like a smaller removal energy which depends on the kinetic energy T of the final state baryon.

For electron scattering on a bound nucleon in $^{12}_{6}C$, our results imply that the *interaction energies* for the range of final state baryon kinetic energies between 0.05 and 0.3 GeV vary from 5 to 28 MeV for the nucleon and from 11 to 29 MeV for the Δ.

EFFECT ON THE DETERMINATION OF NEUTRINO ENERGY IN NEUTRINO OSCILLATIONS EXP.

In the on-shell formalism of Moniz et al. [45] (used in NEUT) the *Moniz interaction energies* are defined as:

$$\nu + (M_{P,N} + T_i^{P,N} - \epsilon^{Moniz(P,N)}_{QE-interaction}) = \sqrt{p_{f3}^2 + M_{P,N}^2}$$

$$\nu + (M_{P,N}) + T_i^{P,N} - \epsilon^{Moniz(P,N)}_{\Delta-interaction} = \sqrt{p_{f3}^2 + W_{\Delta+,0}^2} \; ,$$

$$\epsilon^{Moniz(P,N}_{QE-interaction} = \epsilon^{P,N} + T_i^{P,N} + U^{QE}_{opt}(p^2_{f3}) + |V^{P,N}_{eff}|$$

$$\epsilon^{Moniz(P,N)}_{\Delta-interaction} = \epsilon^{P,N} + T_i^{P,N} + U^{\Delta}_{opt}(p^2_{f3}) + |V^{\Delta+,0}_{eff}|.$$

For electron proton scattering on $^{12}_{6}C$, our results imply that the *Moniz interaction energies* for the range of final state baryon kinetic energies between 0.05 and 0.3 GeV vary from 21 to 44 MeV for the nucleon and from 4 to 44 MeV for the Δ.

However, in the analysis of Moniz et al. [45] the *interaction energies* $\epsilon^{Moniz(P,N)}_{QE-interaction}$ and $\epsilon^{Moniz(P,N)}_{\Delta-interaction}$ are assumed to be the same which is not correct. In addition, they are assumed to be constant, which is also not correct since they depends on both the initial state kinetic energy and on U_{opt} (which is a function of kinetic energy of the final state baryon).

ON-SHELL FORMALISM

Note, since kinetic energy is included in the Moniz expression the interaction energy needs to be to compensate for the initial stste kinetic energy on average.

Note: the Moniz interaction energy depends on both initial state and final state kinetic energies.

EFFECT ON THE DETERMINATION OF NEUTRINO ENERGY IN NEUTRINO OSCILLATIONS EXP.

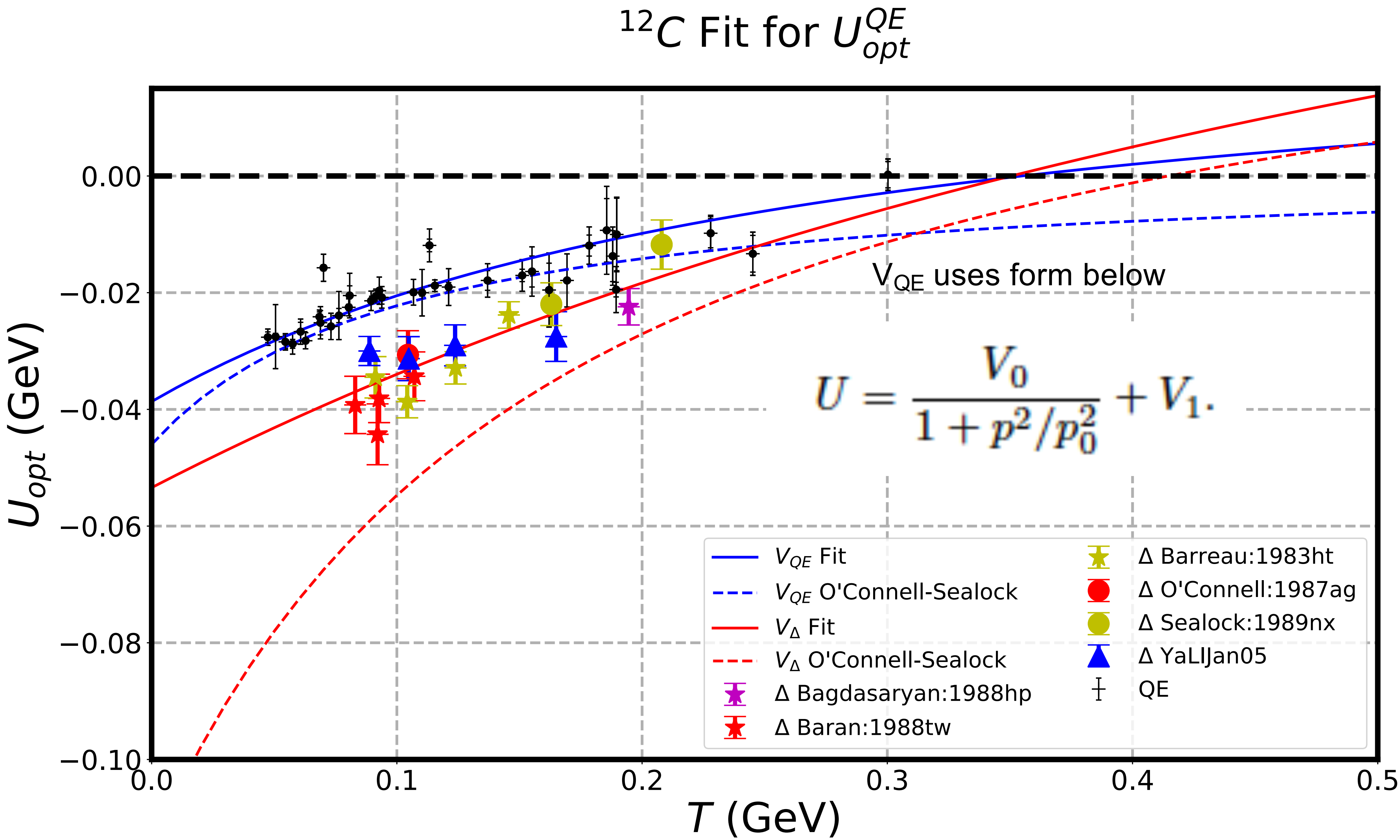
Discussion of Δ (1232)

We find $U^{\Delta}_{opt} \approx 1.5\, U^{QE}_{opt}$ for ${}^{12}_{6}\mathbf{C}$. versus $U^{\Delta}_{GiBUU} = (2/3)\, U^{nucleon}_{GiBUU}$.

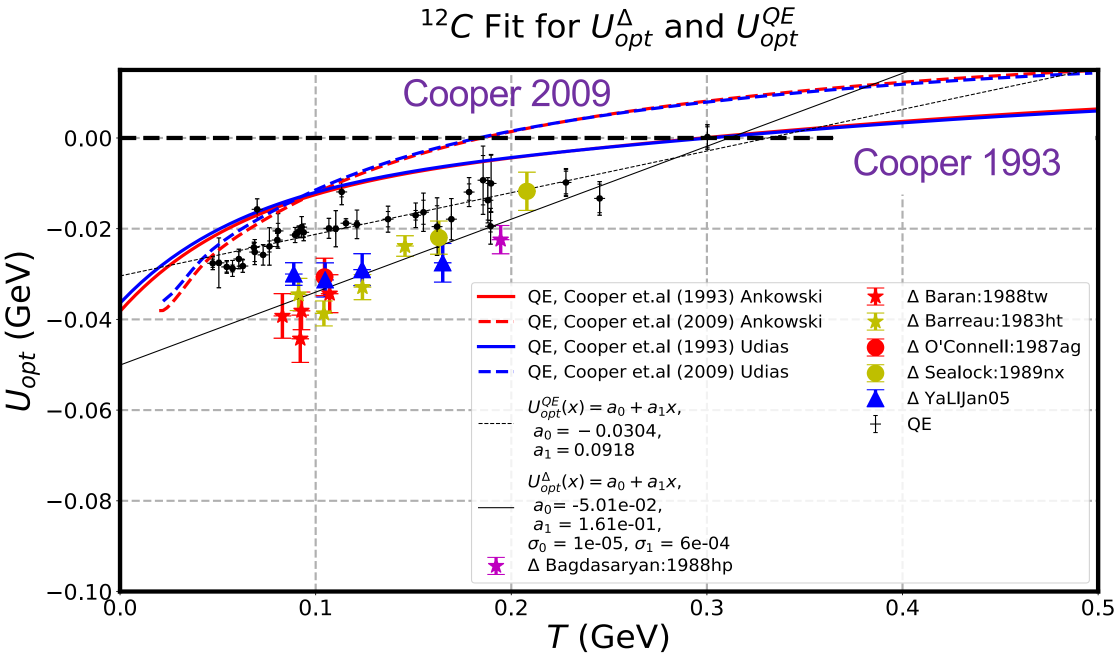
The **Δ**(1232) is lifetime is 5.6x10^{-24} seconds. For all values of the energy transfers in this paper the **Δ**(1232) decays inside the nucleus.

Consequently one would expect that the optical potential for the **Δ**(1232) should be the sum of the optical potentials for the nucleon and the pion in the final state (and therefore more negative).

Comparison with O'Connell-Sealock 1990 paper (fit using their form)



Summary of results for Carbon



Find U^{Δ} is more negative than U^{QE} with $U^{\Delta} \approx 1.5\, U^{QE}$

$$\nu + (M_{P,N} - \epsilon^{P,N}) = E_f$$

$$E_f^P = \sqrt{(k+q_3)^2 + M_P^2} + U_{opt}^{QE} + |V_{eff}^P|$$

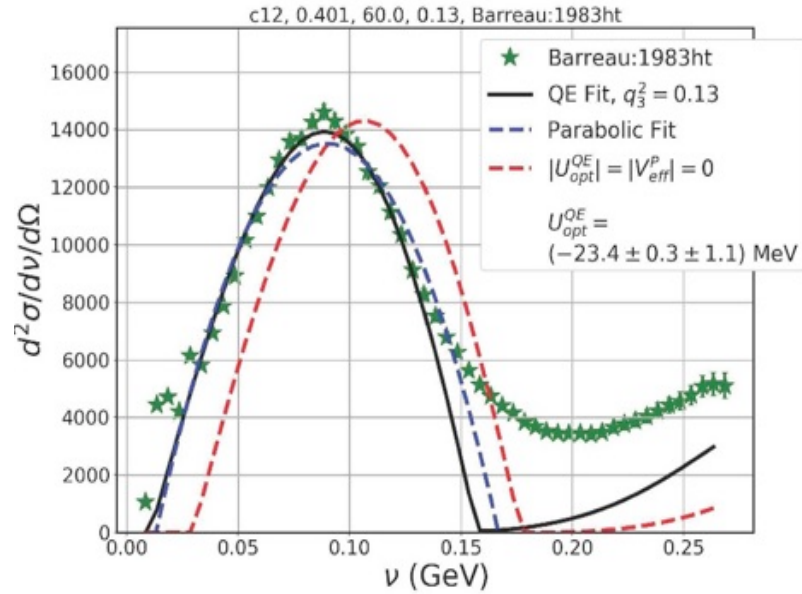
$$E_f^N = \sqrt{(k+q_3)^2 + M_N^2} + U_{opt}^{QE}$$

$$T^{P,N} = E_f^{P,N} - M_{P,N}$$

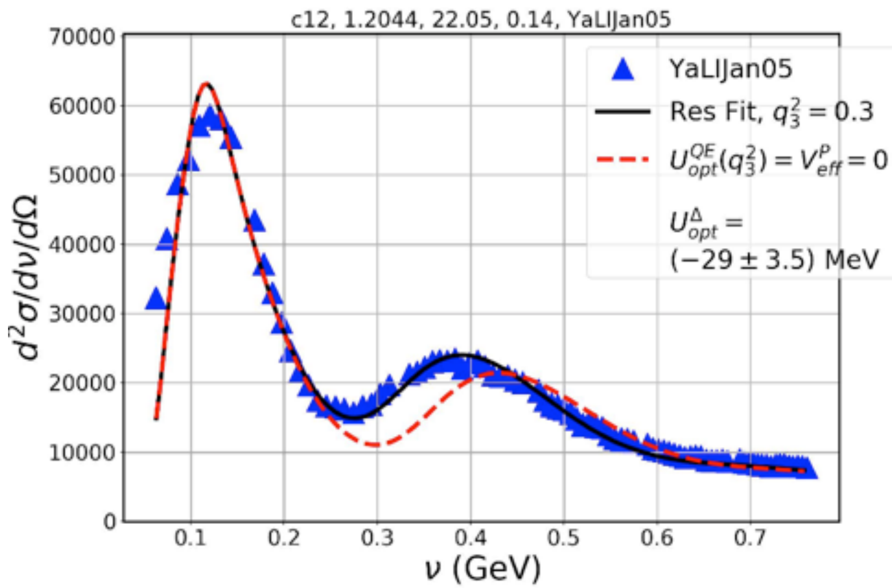
$$E_f^{\Delta +} = \sqrt{(k+q_3)^2 + W_{\Delta +}^2} + U_{opt}^{\Delta} + |V_{eff}^{\Delta +}|$$

$$E_f^{\Delta 0} = \sqrt{(k+q_3)^2 + W_{\Delta 0}^2} + U_{opt}^{\Delta}$$

$$T^{\Delta(+,0)} = E_f^{\Delta(+,0)} - W_{\Delta(+,0)},$$



Effect of QE nucleon optical potential



Effect of **Δ** (1232) optical potential

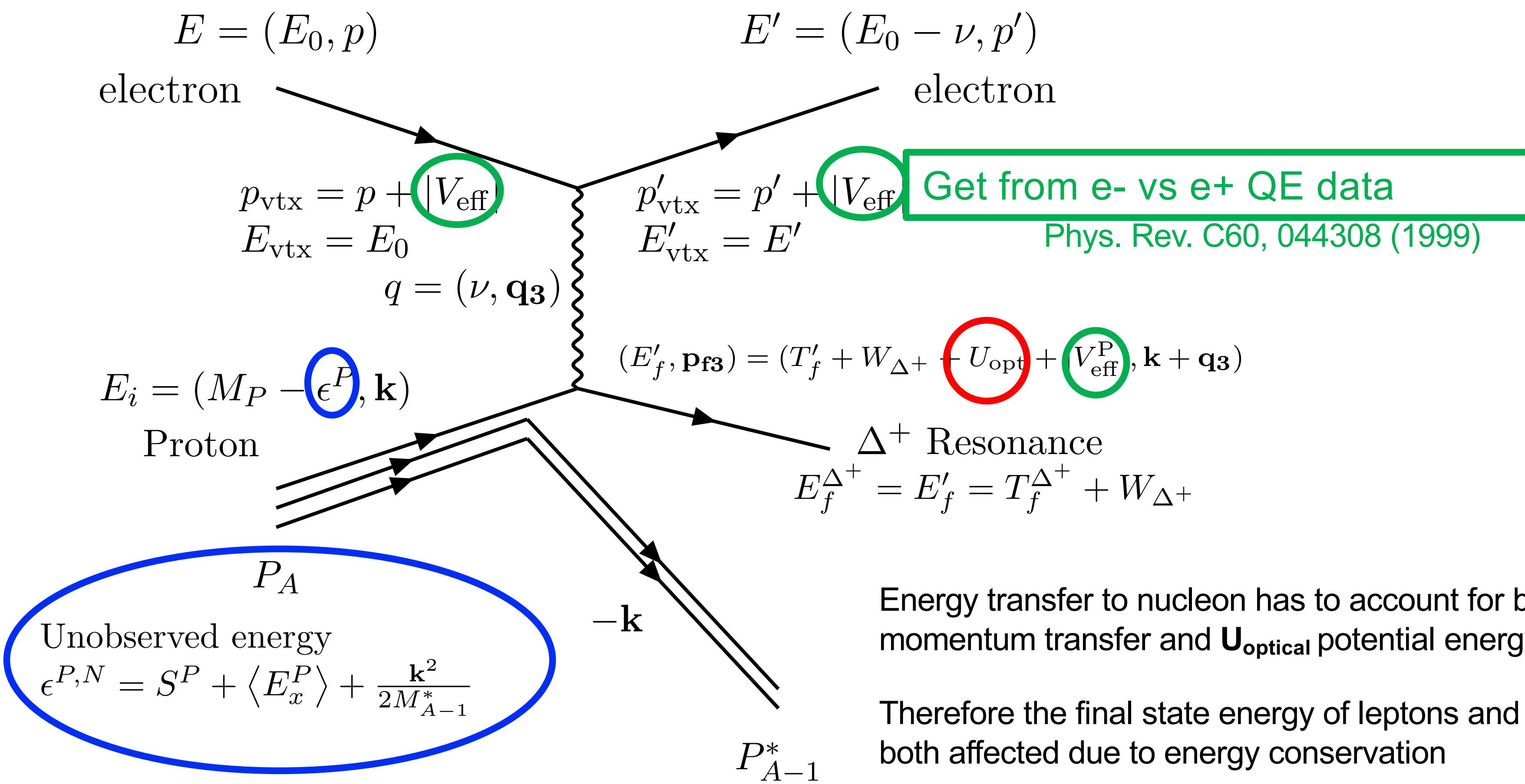
Red dashed lines are prediction with zero optical potential

Conclusion

- We have extracted the removal energies ε for a variety of nuclei from ee'P data.
- We also extracted the optical potential $U_{Optical}$ for the **nucleon** and for the **Δ**(1232) which depend on the kinetic energies of final state baryons.
- The final state lepton and hadron energies in electron and neutrino scattering depend on ε, V_{eff} and $U_{Optical}$. Our measurements reduce the uncertainties in these variables from 20 MeV to 5 MeV. Thus greatly reducing the dominant systematic error in the measurement of neutrino oscillations parameters.

Electron scattering on proton

$\Delta(1232)$ **production on bound protons)**
Different optical potential



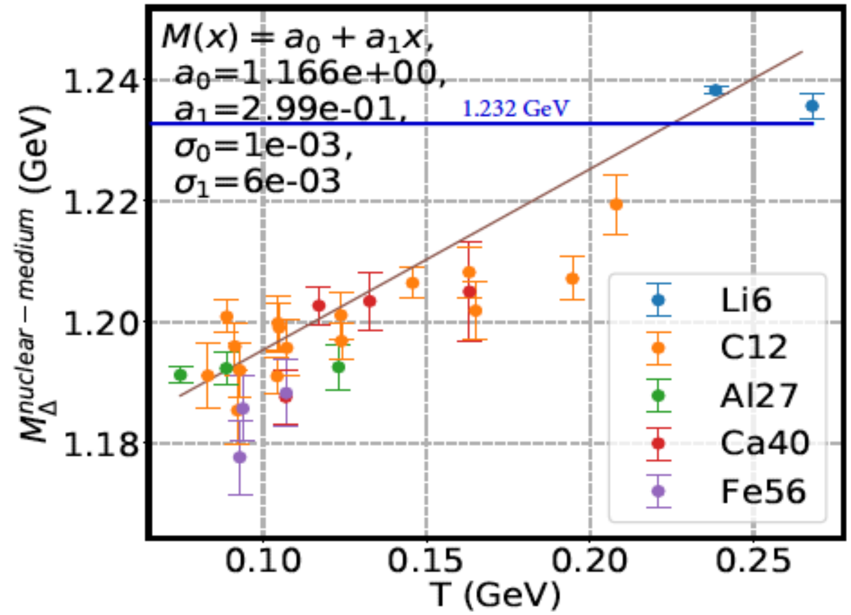
Energy transfer to nucleon has to account for both momentum transfer and $\mathbf{U_{optical}}$ potential energy.

Therefore the final state energy of leptons and hadrons are both affected due to energy conservation

Get from ee'P data

Bodek and Cai
Eur. Phys. J. C79 (2019) 293

Some authors [48] have cast the effect of the nuclear optical potential on the nucleon and $\Delta(1232)$ as an energy dependent change in their effective mass in the nuclear medium. Under this interpretation, both the nucleon and the Δ revert back to their free mass values after leaving the nucleus. For example, the distribution of the final state mass of the decay particles of Δ resonances produced in neutrino-(Propane/Freon) interactions [49] peaks around 1.232 GeV.



7.2 The effective mass of the nucleon and $\Delta(1232)$ in the nuclear medium

For purpose of comparison to other publications, we transform our results for the average optical potential for the Δ to an equivalent change of the effective mass of the Δ in the nucleus using the following expression with M_Δ^{free} = 1.232 GeV.

$$\sqrt{(k+q_3)^2 + (M_\Delta^{nuclear-medium})^2} = \sqrt{(k+q_3)^2 + (M_\Delta^{free})^2} + U_{opt}^{\Delta} + |V_{eff}|. \quad (7)$$

Effective mass interpretation is not useful for MC generators such as NEUT and GENIE.

It these MC, the only way to identify which free nucleon structure functions to use is by using W in the final state (which is in an optical potential). E.g. when W in the final state a nucleon, it is identified as a QE event, and nucleon form factors are used in the calculation of thje cross sections.