Interplay of neutrino flavor, spin and collective oscillations in supernovae

Konstantin Kouzakov[1], **Yu-Feng Li**[2], **Vadim Shakhov**[1], **Konstantin Stankevich**[1], **Alexander I. Studenikin**[1,3], **Ziyi Yuan**[2]

1. Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russia
2. Institute of High Energy Physics and School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China
3. Joint Institute for Nuclear Research, Dubna 141980, Moscow Region, Russia

Correspondence: liyufeng@ihep.ac.cn, studenik@srd.sinp.msu.ru

1 Introduction

Future large liquid-scintillator detectors, such as JUNO, Hyper-Kamiokande and DUNE, open a new era in studies of supernovae and presupernovae neutrinos [1,2]. Astrophysical neutrinos would give important information for the explosion mechanism of collapse-driven supernovae. This paper is dedicated to the neutrino evolution in astrophysical environment. The neutrino collective oscillations is an important phenomenon that should be taken into account while considering neutrinos propagation in astrophysical media (see a review in [3]). The authors of [4,5] for the first time studied collective neutrino oscillation accounting for transitions between states with different helicity engendered by interaction of the neutrino magnetic moment with a magnetic field. One of the recent studies of effects of neutrino magnetic moments in collective oscillations is presented in [6]. Note that, as it was shown for the first time in [7], the neutrino spin oscillations can be engendered not only by the neutrino interaction with a magnetic field but also by neutrino interactions with matter in the case when there is a transversal matter current or matter polarization. The existence of these effects and their importance for astrophysical applications have been confirmed in [8]. A detailed study of neutrino spin and spin-flavour oscillations in transversal matter currents with standard and non-standard interactions has been presented in [9].

In [10] the effect of spin oscillations engendered by transversally moving matter was included in collective neutrino oscillation. In [11,12] collective neutrino oscillations were studied in the case of neutrino nonstandard interactions. Here below, we summarize all effects that can lead to neutrino spin oscillations. The results presented in this paper are as follows: a) the general neutrino evolution Hamiltonian accounting for neutrino flavour, spin and collective oscillations and the interplay of these effects; b) the case of arbitrary moving matter is considered within the collective neutrino oscillations.

2 Neutrino evolution in astrophysical environment

2.1 Master equations

Consider two flavor neutrinos with two possible helicities $\nu_f = (\nu_e^-, \nu_x^-, \nu_e^+, \nu_x^+)$, where ν_x stands for ν_μ or ν_τ. Then the neutrino system is described by density matrix (we follow the formalism developed in [4,5])

$$\rho = \begin{pmatrix} \rho_\nu & X \\ X^\dagger & \rho_{\bar\nu} \end{pmatrix}, \qquad (1)$$

where ρ and $\rho_{\bar\nu}$ are the usual 2×2 flavour density matrices. In case of Dirac neutrinos the matrices describe active and sterile neutrino, respectively. In case of Majorana neutrinos $\rho_{\bar\nu}$ describes antineutrino. External magnetic field and transversally moving matter leads to coupling between ρ and $\rho_{\bar\nu}$ (see [9] and references therein) and, therefore, we should also consider non-diagonal matrices X

$$X = \begin{pmatrix} \rho_{\nu_e\bar\nu_e} & \rho_{\nu_e\bar\nu_x} \\ \rho_{\nu_x\bar\nu_e} & \rho_{\nu_x\bar\nu_x} \end{pmatrix}. \qquad (2)$$

The evolution of ρ is governed by Liouville-von Neumann equation of motion

$$i\frac{d\rho}{dt} = [H, \rho], \qquad (3)$$

where H is a Hamiltonian that describes external environment. It consists of four parts

$$H = H_{vac} + H_{mat} + H_v + H_B + H_{\nu\nu}, \qquad (4)$$

where H_{vac} is a vacuum, Hamiltonian, H_{mat} is a matter potential for electron and neutron moving in arbitrary direction , H_B is the Hamiltonians that account for magnetic field. The neutrino-neutrino interaction potential, $H_{\nu\nu}$, is

$$H_{\nu\nu} = \sqrt{2}G_F n_\nu \int dE \left[G^\dagger(\rho(E) - \rho(E)^{c*})G + \frac{1}{2}G^\dagger \,\mathrm{Tr}\left[(\rho(E) - \rho(E)^{c*})G\right] \right], \qquad (5)$$

with G_F being the Fermi coupling constant and n_ν being neutrino density profile. The density matrix ρ^c is define in the same way as in [6]

$$\rho^c = \begin{pmatrix} \rho_{\bar\nu} & X^* \\ X^T & \rho_\nu \end{pmatrix}. \qquad (6)$$

Dimensionless matrix G is

$$G = \begin{pmatrix} +1 & 0 & 0 & 0 \\ 0 & +1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \qquad (7)$$

The explicit forms of Hamiltonians H_{vac}, H_{mat} and H_B depend on the neutrino nature. In the following sections we consider Dirac and Majorana neutrino.

2.2 The case of Dirac neutrinos

The Hamiltonian that accounts for neutrino magnetic moment interaction with parallel and perpendicular components of the magnetic field $B = B_{||} + B_\perp$ has the following form

$$H_B^D = \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & \left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & -\mu_{ee}B_\perp e^{i\phi} & -\mu_{ex}B_\perp e^{i\phi} \\ \left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & \left(\frac{\mu}{\gamma}\right)_{xx} B_{||} & -\mu_{ex}B_\perp e^{i\phi} & -\mu_{xx}B_\perp e^{i\phi} \\ -\mu_{ee}B_\perp e^{-i\phi} & -\mu_{ex}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ee} B_{||} & -\left(\frac{\mu}{\gamma}\right)_{ex} B_{||} \\ -\mu_{ex}B_\perp e^{-i\phi} & -\mu_{xx}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ex} B_{||} & -\left(\frac{\mu}{\gamma}\right)_{xx} B_{||} \end{pmatrix}, \qquad (8)$$

where ϕ is the angle between $\mathbf{v}_\perp$ and $\mathbf{B}_\perp$. Magnetic moments in flavour basis $\mu_{\alpha\beta}$ are expressed through magnetic moments μ_{ij} in mass states

$$\begin{aligned} \mu_{ee} &= \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta, \\ \mu_{ex} &= \mu_{12}\cos 2\theta + \frac{1}{2}(\mu_{22} - \mu_{11})\sin 2\theta, \\ \mu_{xx} &= \mu_{11}\sin^2\theta + \mu_{22}\cos^2\theta - \mu_{12}\sin 2\theta, \end{aligned} \qquad (9)$$

for neutrino interaction with $B_\perp$ and

$$\begin{aligned} \left(\frac{\mu}{\gamma}\right)_{ee} &= \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \\ \left(\frac{\mu}{\gamma}\right)_{ex} &= \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta, \\ \left(\frac{\mu}{\gamma}\right)_{xx} &= \frac{\mu_{11}}{\gamma_{11}}\sin^2\theta + \frac{\mu_{22}}{\gamma_{22}}\cos^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \end{aligned} \qquad (10)$$

for neutrino interaction with $B_{||}$. Here we have introduced the following notifications

$$\gamma_\alpha^{-1} = \frac{m_\alpha}{E_\alpha}, \quad \gamma_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} + \gamma_\beta^{-1}\right), \quad \tilde\gamma_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} - \gamma_\beta^{-1}\right). \qquad (11)$$

The Hamiltonian that accounts for electron and neutron matter is

$$H_{mat}^D = \frac{G_F}{2\sqrt{2}} \begin{pmatrix} 2(2n_e - n_n)\left(1 - v_{||}\right) & 0 & (2n_e - n_n)v_\perp\left(\frac{\eta}{\gamma}\right)_{ee} & (2n_e - n_n)v_\perp\left(\frac{\eta}{\gamma}\right)_{ex} \\ 0 & -2n_n\left(1 - v_{||}\right) & -n_n v_\perp\left(\frac{\eta}{\gamma}\right)_{ex} & -n_n v_\perp\left(\frac{\eta}{\gamma}\right)_{xx} \\ (2n_e - n_n)v_\perp\left(\frac{\eta}{\gamma}\right)_{ee} & -n_n v_\perp\left(\frac{\eta}{\gamma}\right)_{ex} & 0 & 0 \\ (2n_e - n_n)v_\perp\left(\frac{\eta}{\gamma}\right)_{ex} & -n_n v_\perp\left(\frac{\eta}{\gamma}\right)_{xx} & 0 & 0 \end{pmatrix}, \qquad (12)$$

where n_n and n_e are the neutron and electron density profiles, $v = v_{||} + v_\perp$ is the neutron velocity and

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \frac{\cos^2\theta}{\gamma_{11}} + \frac{\sin^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{xx} = \frac{\sin^2\theta}{\gamma_{11}} + \frac{\cos^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{ex} = \frac{\sin 2\theta}{\tilde\gamma_{21}}. \qquad (13)$$

2.3 The case of Majorana neutrinos

The derivation of the neutrino evolution Hamiltonian accounting for the effects of a magnetic field and interactions with arbitrary moving matter in the case of Majorana neutrinos can be made in a similar way as it was done for Dirac neutrinos in [9]. For the Majorana neutrinos the diagonal magnetic moments are zero in both flavour and mass basis (see, for instance, [14]). Therefore, it is convenient to introduce the magnetic moment matrix

$$\mu_{\alpha\beta} = \begin{pmatrix} 0 & i\mu \\ -i\mu & 0 \end{pmatrix}. \qquad (14)$$

Then the Hamiltonian for Majorana neutrino in the magnetic field is expressed as

$$H_B^M = i\mu\cos 2\theta \begin{pmatrix} 0 & \frac{1}{\gamma_{12}}B_{||} & 0 & -B_\perp e^{i\phi} \\ -\frac{1}{\gamma_{12}}B_{||} & 0 & B_\perp e^{i\phi} & 0 \\ 0 & -B_\perp e^{-i\phi} & 0 & -\frac{1}{\gamma_{12}}B_{||} \\ B_\perp e^{-i\phi} & 0 & \frac{1}{\gamma_{12}}B_{||} & 0 \end{pmatrix}. \qquad (15)$$

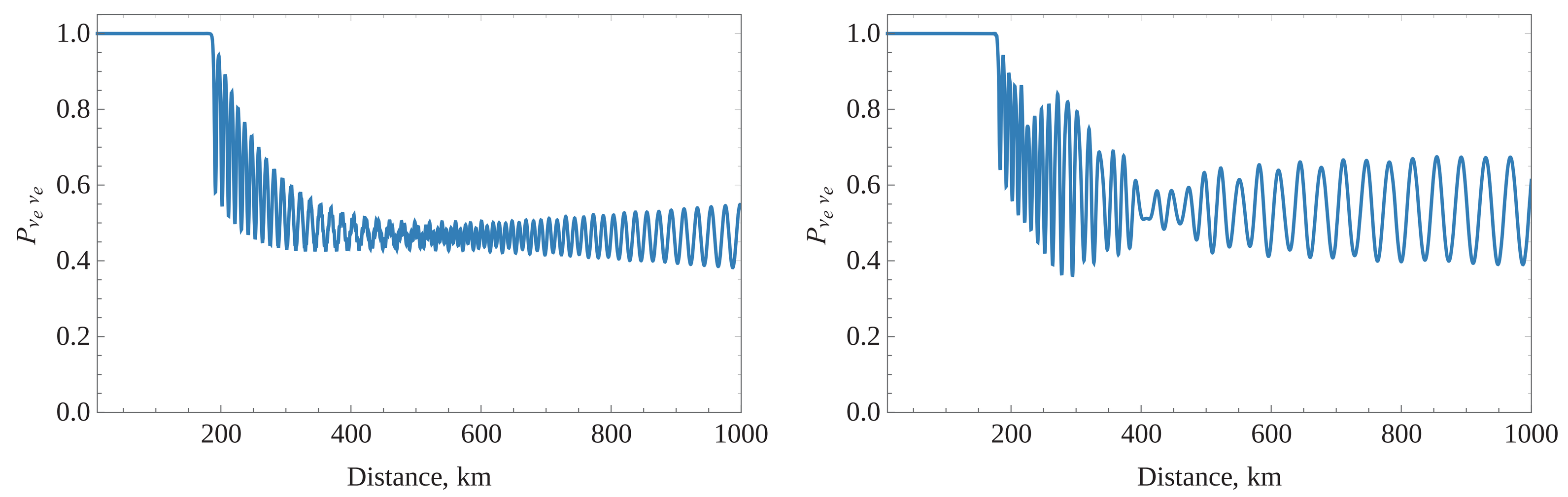
The Majorana neutrino Hamiltonian accounting for arbitrary moving matter is expressed by almost the same formula as for the Dirac neutrino $H_{mat}^M = 2H_{mat}^D$.

3 Interplay

It is easy to see, that in the case of Dirac neutrinos the transversally moving matter plays the same role as perpendicular magnetic filed. Therefore, the neutrino interaction with transversally moving matter can be expressed as an effective shift of neutrino magnetic moment. In the case of Majorana neutrinos the effect of transversally moving matter can be taking into account by shifting of non-diagonal magnetic moments and by introducing effective diagonal magnetic moments. The parallel moving matter can be expressed as an effective mass at rest. Therefore, it useful to introduce the effective Hamiltonian accounting for external environment that Hamiltonian can be used for the analytical analyse of collective neutrino oscillations. The full and comprehensive analyse one can find in [15].

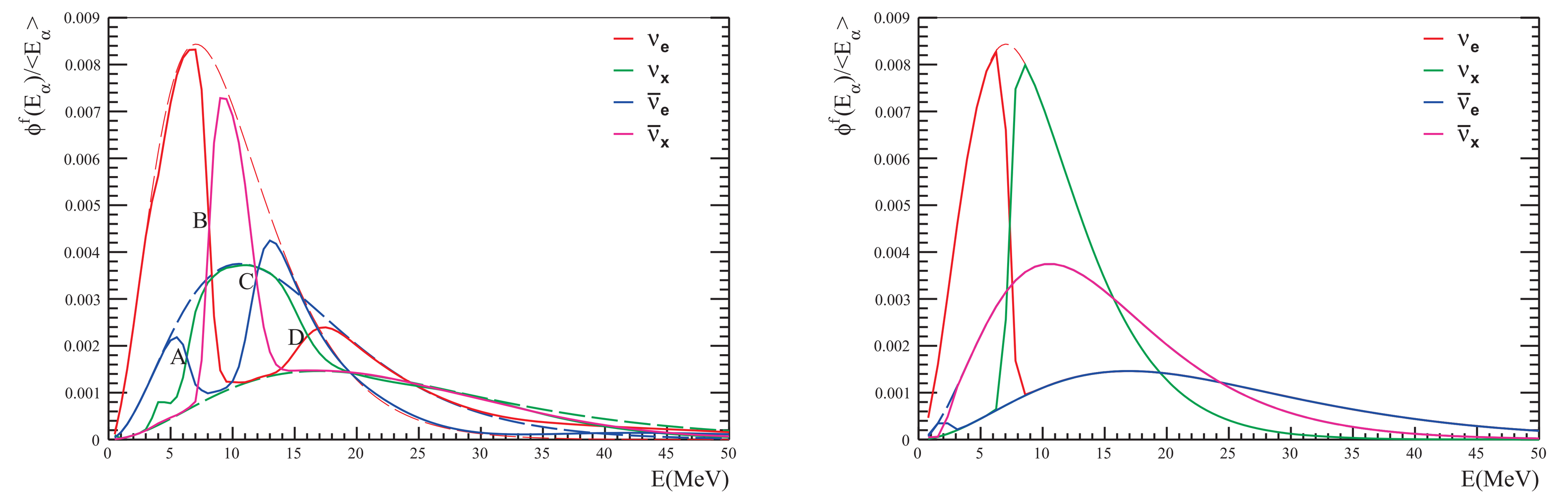
4 Numerical solutions for collective oscillations

We present our numerical calculations for some specific cases in order to highlight the effects of moving matter. The parallel moving matter can shift the MSW-resonance that can significantly change the neutrino oscillation probability [12,16]. Using the supernova model introduced in [12] we plot the oscillations probability for $v_{||} = 0$ and $v_{||} = 0.9$ (see fig.1). Numerical solution for the collective neutrino oscillations accounting for transversal matter current is presented in fig. 2.



a) b)

Figure 1: The survival probability of electron neutrino in the absence moving matter (a) and for the case $v_{||} = 0.9$.



a) b)

Figure 2: Initial (dashed, $r = 50$ km) and final (solid, $r = 200$ km) supernova neutrino spectra for different neutrino species as a function of the neutrino energy for the normal (left) and inverted (right) mass ordering. The average energies of ν_e, $\bar\nu_e$ and ν_x are taken as 10 MeV, 15 MeV, and 24 MeV, respectively. Note that the bulb model for supernova neutrino emission and single-angle approximation have been used for the numerical calculation, and A, B, C, D in the left panel are cross points for illustration. The transverse matter effects are taken from Ref. [9] in the case of Majorana neutrinos.

This work was supported by the Russian Foundation for Basic Research under Grant No. 20-52-53022-GFEN-a, and by the joint project of the National Natural Science Foundation of China, and the Russian Foundation for Basic Research.

References

[1] T. Totani, K. Sato, H. Dalhed and J. Wilson, Astrophys. J. **496** (1998), 216-225.
[2] H. L. Li, Y. F. Li, L. J. Wen and S. Zhou, JCAP **05** (2020), 049.
[3] H. Duan, G. M. Fuller and Y. Z. Qian, Ann. Rev. Nucl. Part. Sci. **60** (2010), 569-594
[4] A. de Gouvea and S. Shalgar, JCAP **10** (2012) 027.
[5] A. de Gouvea and S. Shalgar, JCAP **04** (2013) 018.
[6] S. Abbar, Phys. Rev. D **101** (2020) 103032.
[7] A.Studenikin, *Phys. Atom. Nucl.* **67** (2004) 993.
[8] A. Dobrynina, A. Kartavtsev and G. Raffelt, Phys. Rev. D **93** (2016) no.12, 125030.
[9] P. Pustoshny and A. Studenikin, Phys. Rev. D **98** (2018) 113009.
[10] A. Chatelain and C. Volpe, Phys. Rev. D **95** (2017) 043005.
[11] A. Chatelain and M. C. Volpe, Phys. Rev. D **97** (2018) 023014.
[12] C. J. Stapleford, D. J. Vnnen, J. P. Kneller, G. C. McLaughlin, B. T. Shapiro, Phys. Rev. D **94** (2016) 093007.
[13] D. Vaananen and C. Volpe, Phys. Rev. D **88** (2013) 065003.
[14] C.Giunti, A.Studenikin, Rev. Mod. Phys. **87** (2015) 531.
[15] K. Kouzakov, Y.-F. Li, V. Shakhov, K. Stankevich, A. Studenikin, Z. Yuan, *to be published*.
[16] D. Vaananen, G. McLaughlin, Phys. Rev. D **93** (2016) no.10, 105044