Neutrino oscillations in a magnetic field: The three-flavor case

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1 Introduction

We further develop our studies of neutrino oscillations in a magnetic field introduced in [1] and extend it to the case of three neutrino generations. Recently a new approach to description of neutrino spin and spin-flavor oscillations in the presence of an arbitrary constant magnetic field has been developed in [2,3]. Within the new approach the exact quantum stationary states are used for classification of neutrino spin states, rather than the neutrino helicity states that have been used for this purpose within the customary approach in many published papers (see [1] for the corresponding references). Recall that the helicity states are not stationary in the presence of a magnetic field. It has been shown [1,4] in particular, that in the presence of the transversal magnetic field $B_\perp$ for a given choice of parameters (the energy and magnetic moments of neutrinos and strength of the magnetic field) the amplitude of the flavour oscillations $\nu_e^L \Leftrightarrow \nu_\mu^L$ at the vacuum frequency $\omega_{vac} = \frac{\Delta m^2}{4p}$ is modulated by the magnetic field frequency $\omega_B = \mu B_\perp$:

$$P^{(B_\perp)}_{\nu_e^L\to\nu_\mu^L}(t) = \left(1 - \sin^2(\mu B_\perp t)\right)\sin^2 2\theta \sin^2\frac{\Delta m^2}{4p}t = \left(1 - P^{cust}_{\nu_e^L\to\nu_e^R}\right)P^{cust}_{\nu_e^L\to\nu_\mu^L}, \quad (1)$$

here μ is the effective magnetic moment of the electron neutrino and it is supposed that the following relations between diagonal and transition magnetic moments in the neutrino mass basis are valid: $\mu_1 = \mu_2, \quad \mu_{ij} = 0,\ i \neq j$. The customary expression

$$P^{cust}_{\nu_e^L\to\nu_\mu^L}(t) = \sin^2 2\theta \sin^2\frac{\Delta m^2}{4p}t \quad (2)$$

for the neutrino flavour oscillation probability in vacuum in the presence of the transversal field $B_\perp$ is modified by the factor $1 - P^{cust}_{\nu_e^L\to\nu_e^R}$. Since the transition magnetic moment in the flavour basis is absent in the case $\mu_1 = \mu_2$, the process $\nu_e^L \to \nu_e^R$ is the only way for spin flip, and then $1 - P^{cust}_{\nu_e^L\to\nu_e^R}$ should be interpreted as the probability of not changing the neutrino spin polarization. And consequently, this multiplier subtracts the contribution of neutrinos ν_e^R with the opposite polarization providing the survival of the only contribution from the direct neutrino flavour oscillations $\nu_e^L \Leftrightarrow \nu_\mu^L$. In [1,4] within the two-flavour and two-spin neutrino states approach it has been shown the interplay between different types of oscillations that gives rise to interesting phenomena:
1) the amplitude modulation of the probability of flavour oscillations $\nu_e^L \to \nu_\mu^L$ in the transversal magnetic field with the magnetic frequency $\omega_B = \mu B_\perp$ (in the case $\mu_1 = \mu_2$) and more complicated dependence on the harmonic functions with ω_B for $\mu_1 \neq \mu_2$;
2) the dependence of the spin oscillation probability $P_{\nu_e^L\to\nu_e^R}$ on the mass square difference Δm^2;
3) the appearance of the spin-flavour oscillations in the case $\mu_1 = \mu_2$ and $\mu_{12} = 0$, the transition goes through the two-step processes $\nu_e^L \to \nu_\mu^L \to \nu_\mu^R$ and $\nu_e^L \to \nu_e^R \to \nu_\mu^R$.
As a result, we predict modifications of the neutrino oscillation patterns that might provide new important phenomenological consequences in case of neutrinos propagation in extreme astrophysical environments where magnetic fields are present. Similar results on the important influence of the transversal magnetic field on amplitudes of various types of neutrino oscillations were obtained earlier [5] on the basis of the exact solution of the effective equation for neutrino evolution in the presence of a magnetic field, which accounts for four neutrino species corresponding to two different flavor states with positive and negative helicities.
In this paper we generalize our approach to studies of the interplay of neutrino flavour and spin oscillations in a constant transversal magnetic field [1] to the case of three neutrino flavours. The obtained results are of interest in applications to neutrino oscillations under the influence of extreme astrophysical environments, for example peculiar to magnetars and supernovas, as well as in studying neutrino propagation in interstellar magnetic fields (see [5]).

2 Neutrino oscillations in a magnetic field

Following [1], we use the approach based on the Dirac equation to describe the process of neutrino oscillations in a magnetic field. Below we concisely describe the used approach.

The wave function ν_i^s $(s = \pm 1)$ of a massive neutrino that propagates along $\mathbf{n_z}$ direction in the presence of a constant and homogeneous arbitrary orientated magnetic field can be found as the solution of the Dirac equation

$$(\gamma_\mu p^\mu - m_i - \mu_i \mathbf{\Sigma B})\nu_i^s(p) = 0, \quad (3)$$

where μ_i are the neutrino magnetic moments, $i = 1, 2, 3$ and $s = \pm 1$ is a spin number.
This equation can be rewritten in the equivalent Hamiltonian form

$$\hat{H}_i \nu_i^s = E\nu_i^s, \quad (4)$$

where the Hamiltonian is given by

$$\hat{H}_i = m_i\gamma_0 + \gamma_0\boldsymbol{\gamma p} + \mu_i\gamma_0\boldsymbol{\Sigma B}. \quad (5)$$

Given the ultrarelativistic approximation, for the neutrino energy spectrum we obtain

$$E_i^s \approx p + \frac{m_i^2}{2p} + \mu_i s B_\perp. \quad (6)$$

Since the oscillations frequencies are determined by the expression $E_i^s - E_k^{s'}$, we conclude that the probabilities of neutrino oscillations in a magnetic field are certain combinations of the oscillations on the vacuum frequencies Δm_{ik}^2 and magnetic field frequencies $\mu_i B_\perp$.
To classify neutrino stationary states in a magnetic field we use the generalized spin operator. The spin operator that commutes with the Hamiltonian (5) can be chosen in the form

$$\hat{S}_i = \frac{m_i}{\sqrt{m_i^2\boldsymbol{B}^2 + \boldsymbol{p}^2 B_\perp^2}}\left[\boldsymbol{\Sigma B} - \frac{i}{m_i}\gamma_0\gamma_5[\boldsymbol{\Sigma}\times\boldsymbol{p}]\boldsymbol{B}\right]. \quad (7)$$

For the neutrino stationary states the following relations are valide:

$$\hat{S}_i\left|\nu_i^s\right\rangle = s\left|\nu_i^s\right\rangle,\ s = \pm 1, \quad (8)$$

and

$$\left\langle\nu_i^s|\nu_k^{s'}\right\rangle = \delta_{ik}\delta_{ss'}. \quad (9)$$

Now in order to solve the problem of the neutrino flavour, spin and spin-flavour oscillations in the magnetic field we expand the neutrino chiral states over the neutrino stationary states

$$\nu_i^L(t) = c_i^+\nu_i^+(t) + c_i^-\nu_i^-(t), \quad (10)$$
$$\nu_i^R(t) = d_i^+\nu_i^+(t) + d_i^-\nu_i^-(t), \quad (11)$$

where $c_i^\pm$ and $d_i^\pm$ do not depend on time. For the coefficients $c_i^{+(-)}$ and $d_i^{+(-)}$ it is just straightforward

$$|c_i^\pm|^2 = \langle\nu^L|\hat{P}_i^\pm|\nu^L\rangle \quad (12)$$

$$|d_i^\pm|^2 = \langle\nu^R|\hat{P}_i^\pm|\nu^R\rangle \quad (13)$$

where

$$\hat{P}_i^\pm = \frac{1 \pm \hat{S}_i}{2} \quad (14)$$

are the projection operators.
In the ultrarelativistic case as the first-order approximation we choose

$$\nu^L = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}, \quad \nu^R = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \quad (15)$$

and then calculate the coefficients of interest:

$$|c_i^\pm|^2 = \langle\nu^L|\hat{P}_i^\pm|\nu^L\rangle \approx \frac{1}{2}, \quad (16)$$

$$|d_i^\pm|^2 = \langle\nu^R|\hat{P}_i^\pm|\nu^R\rangle \approx \frac{1}{2}. \quad (17)$$

It is easy then to derive the final expressions for the probabilities of neutrino flavour and spin oscillations in a magnetic field:

$$P_{\nu_e^L\to\nu_i^L}(x) = \left|\langle\nu_i^L(0)|\nu_e^L(x)\rangle\right|^2 = \Big|\left(|c_1^+|^2e^{-iE_1^+x} + |c_1^-|^2e^{-iE_1^-x}\right)U_{11}U_{i1}^* +$$
$$\left(|c_2^+|^2e^{-iE_2^+x} + |c_2^-|^2e^{-iE_2^-x}\right)U_{12}U_{i2}^* + \left(|c_3^+|^2e^{-iE_3^+x} + |c_3^-|^2e^{-iE_3^-x}\right)U_{13}U_{i3}^*\Big|^2, \quad (18)$$
$$P_{\nu_e^L\to\nu_i^R}(x) = \left|\langle\nu_i^R(0)|\nu_e^L(x)\rangle\right|^2 = \Big|\left(|d_1^+|^2e^{-iE_1^+x} + |d_1^-|^2e^{-iE_1^-x}\right)U_{11}U_{i1}^* +$$
$$\left(|d_2^+|^2e^{-iE_2^+x} + |d_2^-|^2e^{-iE_2^-x}\right)U_{12}U_{i2}^* + \left(|d_3^+|^2e^{-iE_3^+x} + |d_3^-|^2e^{-iE_3^-x}\right)U_{13}U_{i3}^*\Big|^2,$$

where $i = e, \mu, \tau$ are the neutrino flavours and U is the PMNS matrix.

3 Cosmic neutrino oscillations

Consider ultrahigh-energy cosmic neutrinos propagation through the interstellar space. The two-flavour case was considered in [5]. It was measured by the Interstellar Boundary Explorer that the strength of the interstellar magnetic field is approximately $2.9\ \mu$G. For simplicity we assume that $\mu_1 = \mu_2 = \mu_3$. We consider two possible neutrino energies: 1 ZeV and 10 ZeV.
Fig. 1 shows the probabilities of $\nu_e \to \nu_e$, $\nu_e \to \nu_\mu$ and $\nu_e \to \nu_\tau$ oscillations as a function of distance in parsec for the case of neutrino energy equal to 10 ZeV. The probabilities now exhibit a complicated interplay of oscillations on the following three characteristic scales:

1. The vacuum oscillations length $L_{12} = 4\pi p/\Delta m_{12}^2$
2. The vacuum oscillations length $L_{13} = 4\pi p/\Delta m_{13}^2$
3. The oscillation length associated with a magnetic field $L_B = 2\pi/\mu B$

For the values of neutrino energy and magnetic field the oscillation lengths are $L_B \approx 0.4$ pc, $L_{13} \approx 3$ pc and $L_{12} \approx 100$ pc. Depending on the value of neutrino energy, five qualitatively different regimes of oscillations are possible: 1) $L_{12} \gg L_{13} \gg L_B$, 2) $L_{12} \gg L_B \gg L_{13}$, 3) $L_B \gg L_{12} \gg L_{13}$, 4) $L_{12} \gg L_{13} \sim L_B$ and 5) $L_{12} \sim L_B \gg L_{13}$. Fig. 1 shows the third case. Fig. 2 presents same, but for the case $p = 1$ ZeV (i.e. $L_{12} \sim L_B \gg L_{13}$).
In our calculations we supposed that the neutrino masses are normally ordered (i.e. $m_3 > m_2 > m_1$) and the CP violating phase δ is zero. Note, however, that the oscillations probabilities (18) contain the terms that account for nonzero values of δ and the inverted mass hierarchy. These terms are proportional to the small mixing angle $\sin^2\theta_{13}$ and the corresponding effects are to subtle to be actually measured.

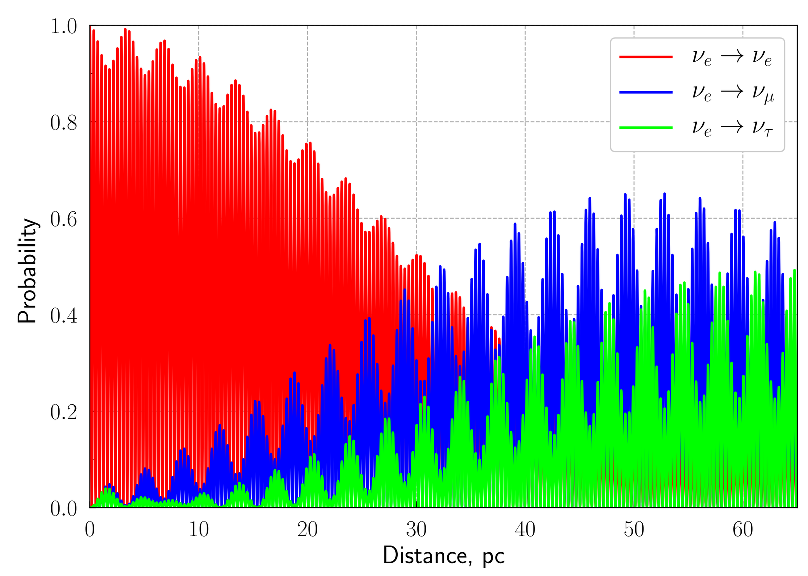
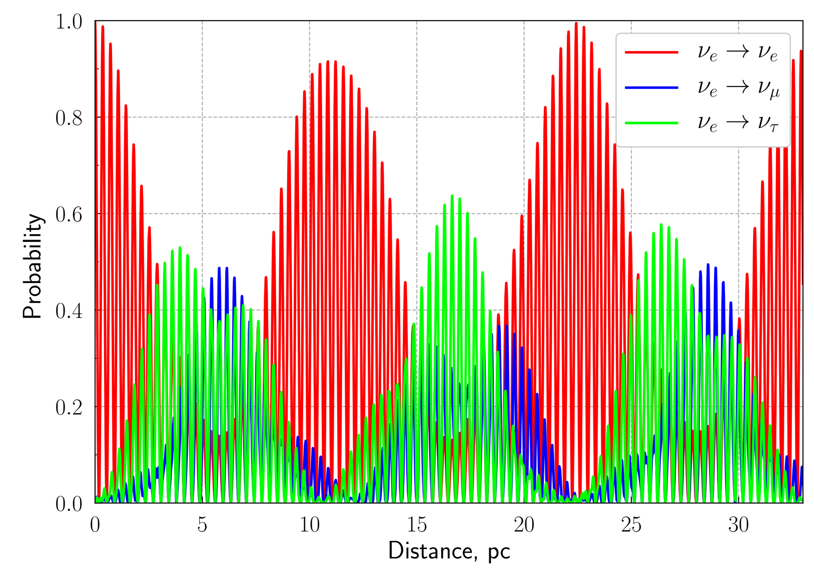


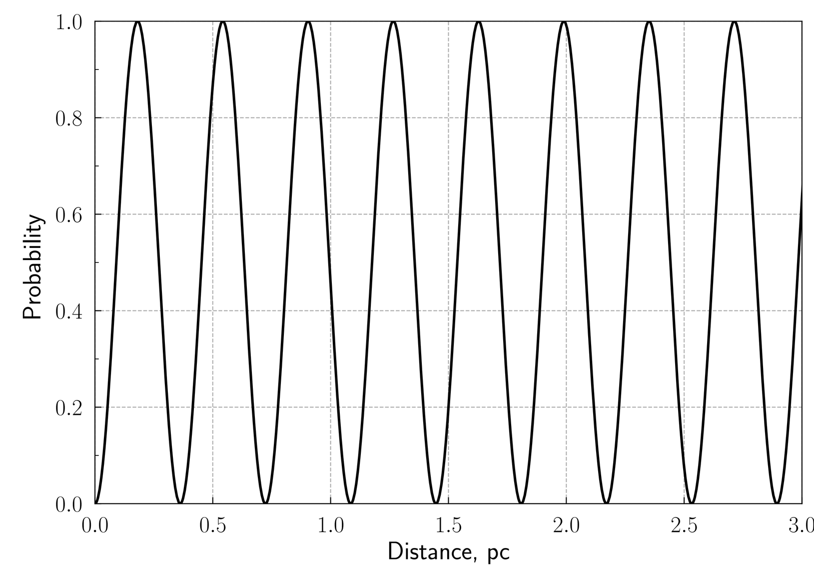
Fig. 3 shows the probability of oscillations into a righthanded neutrino state $\nu^R = \nu_e^R + \nu_\mu^R + \nu_\tau^R$ as a function of distance in parsec. Unlike the probabilities of flavour transitions, the probability of spin oscillations is described by a simple expression

$$P_{\nu_e^L\to\nu^R} = \sin^2\mu Bx, \quad (19)$$

where x is a distance.



Since cosmic neutrinos travel large distances, we can expect that due to the wavepacket separation effects the actual observable are the probabilities averaged over the distance. The effects of a magnetic field induce additional modes in the oscillations probabilities and clearly modify the averaged probabilities. Thus, for the case of large enough neutrino magnetic moments we predict a significant modification of the neutrino fluxes at the terrestrial detectors.



4 Conclusion

We extend our previous results on the neutrino flavour and spin oscil-lations in a magnetic field for the case of the two-falvours [1] to the case of the three-flavour neutrino mixing. It is shown that the probabilities of neutrino flavour oscillations in a magnetic field exhibit quiet complicated dependence on both the vacuum and magnetic field frequencies. The investigated effect also takes place for much lower neutrino energies than those considered above. The discussed phenomenon of neutrino oscillations in a magnetic field can significantly modify neutrino fluxes detected in experiments. A more detailed study of the effect for the three-flavour neutrino oscillations in a magnetic field will be presented in a forthcoming paper [9].

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