Summary of the Latest 3-Flavor Neutrino Oscillation Results from the NOvA Experiment

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Neutrino Physics

Unanswered *ν* Questions:

Neutrino mixing can be described by the PMNS mixing matrix (3 real rotation angles + one CP violating phase factor) and 2 squared mass splittings.

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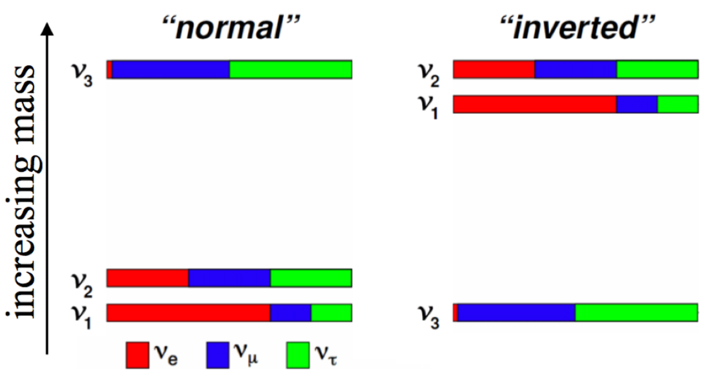
Hierarchy?



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- Is $m_3 > m_1$ or $m_3 < m_1$?

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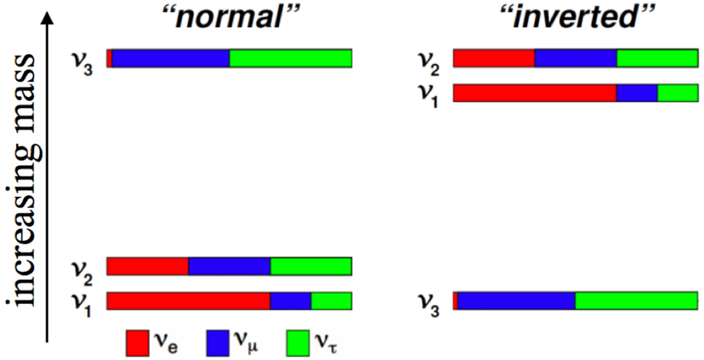
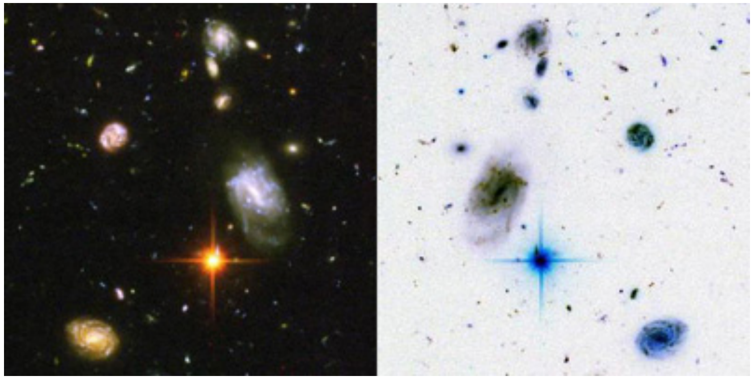


δ_{CP}?



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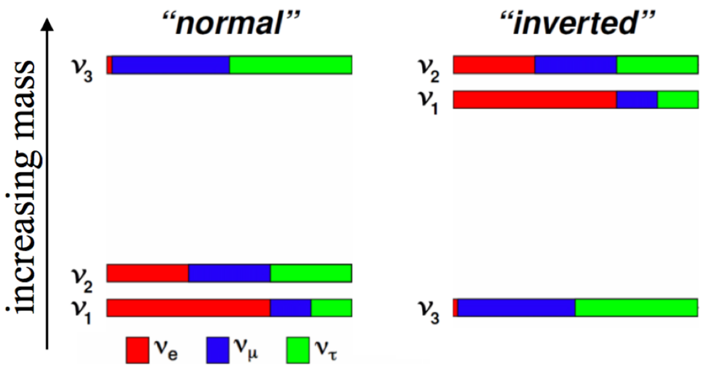
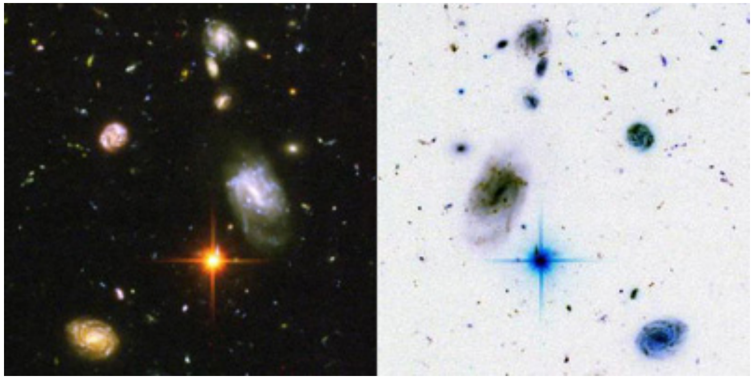


There still are many unanswered questions in neutrino physics today.

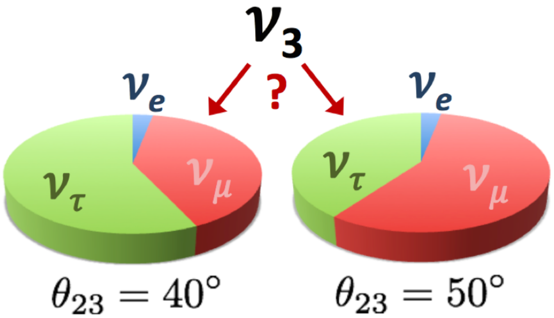
Just to list a few:

- Is $m_3 > m_1$ or $m_3 < m_1$?
- Do neutrinos exhibit CP violation?
- What is the underlying texture of the PMNS mixing matrix? What is the "octant" of $\boldsymbol{\theta}_{23}$?

$\boldsymbol{\delta}_{CP}$?



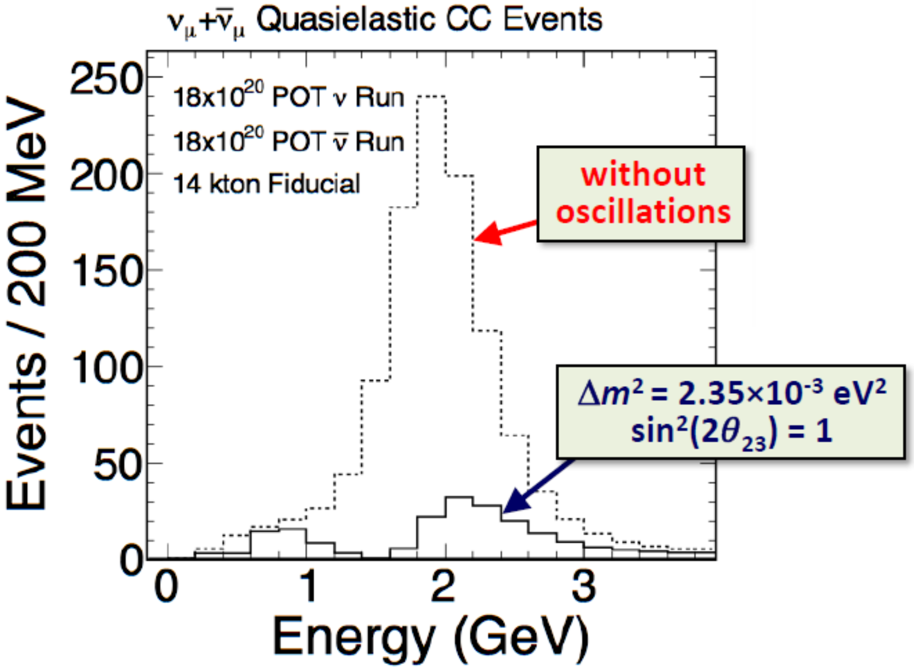
Octant?



Neutrino Oscillations:

ν_μ survival probability:

- Depends on θ_{23}, Δm_{32}^2, and energy.

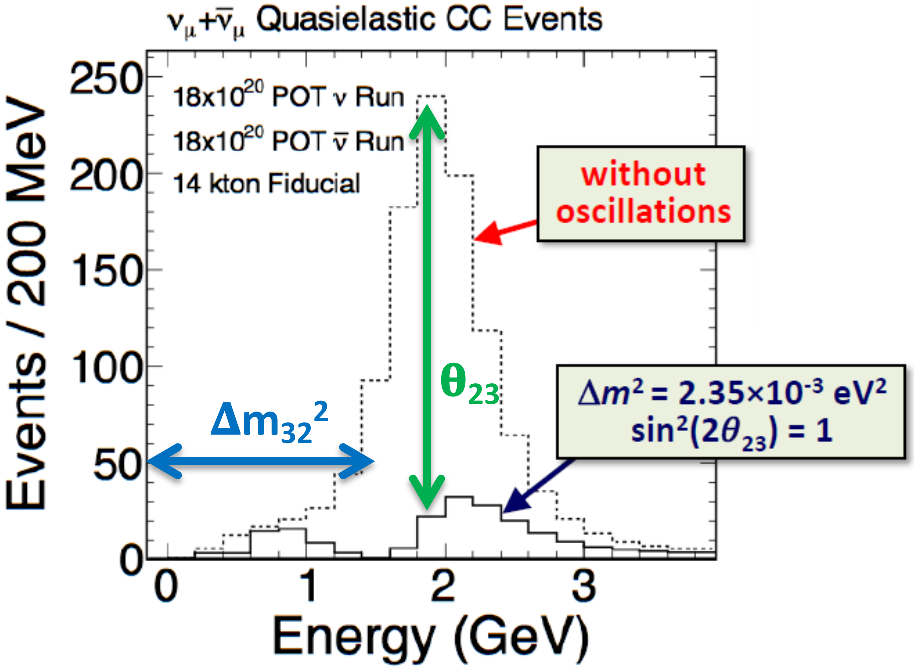


Neutrino Oscillations:

ν_μ survival probability:

- Depends on θ_{23}, Δm_{32}^2, and energy.

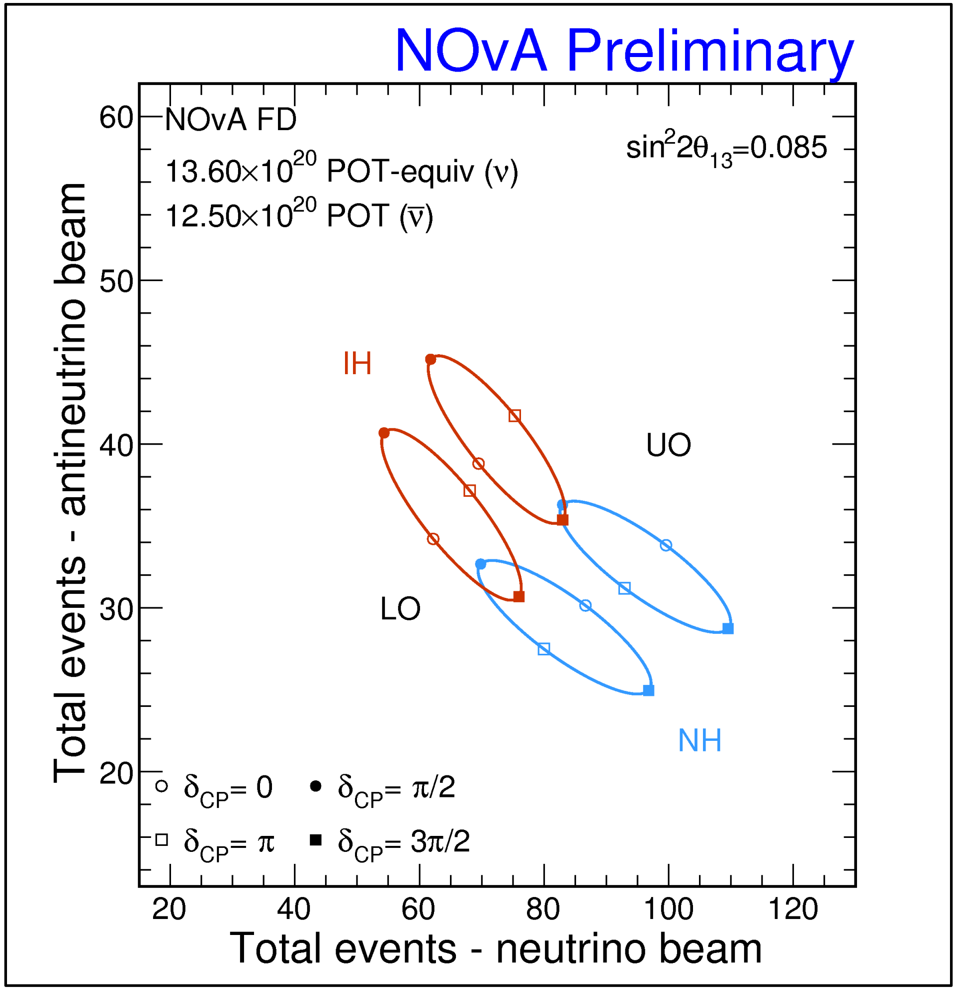
- By measuring the energy spectrum of ν_μ candidate events in our FD, we can extract information about θ_{23} and Δm_{32}^2.



Neutrino Oscillations:

ν_e appearance probability:

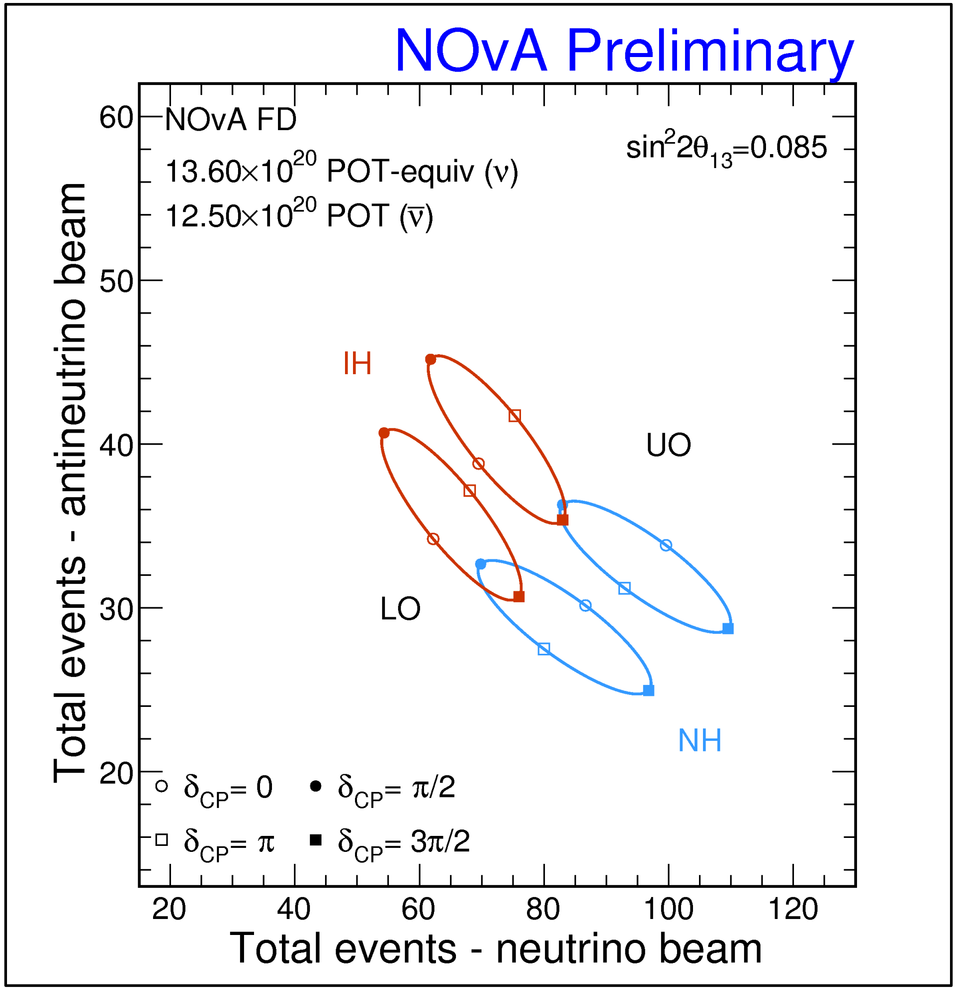
- Gives us access to the questions of hierarchy, δ_{CP}, and the octant of θ_{23}.



Neutrino Oscillations:

ν_e appearance probability:

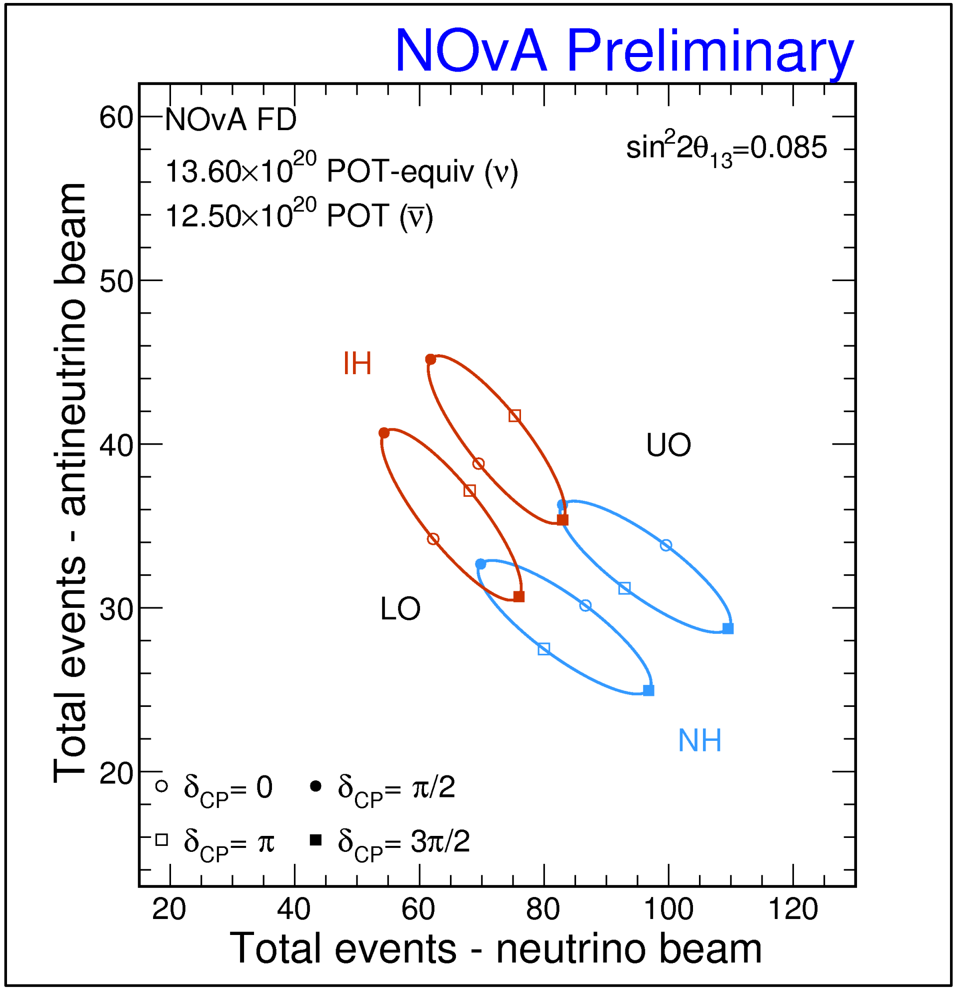
- Gives us access to the questions of hierarchy, δ_{CP}, and the octant of θ_{23}.
- The 4 ellipses represent the 4 combinations of hierarchy and octant.
- Points on a single ellipse represent the possible values of δ_{CP} [0, 2π].



Neutrino Oscillations:

ν_e appearance probability:

- Gives us access to the questions of hierarchy, $\boldsymbol{\delta}_{CP}$, and the octant of $\boldsymbol{\theta}_{23}$.
- The 4 ellipses represent the 4 combinations of hierarchy and octant.
- Points on a single ellipse represent the possible values of $\boldsymbol{\delta}_{CP}$ $[0, 2\boldsymbol{\pi}]$.
- Different combinations of these 3 factors *can* produce asymmetries in the ν_e / $\overline{\nu_e}$ appearance rates.



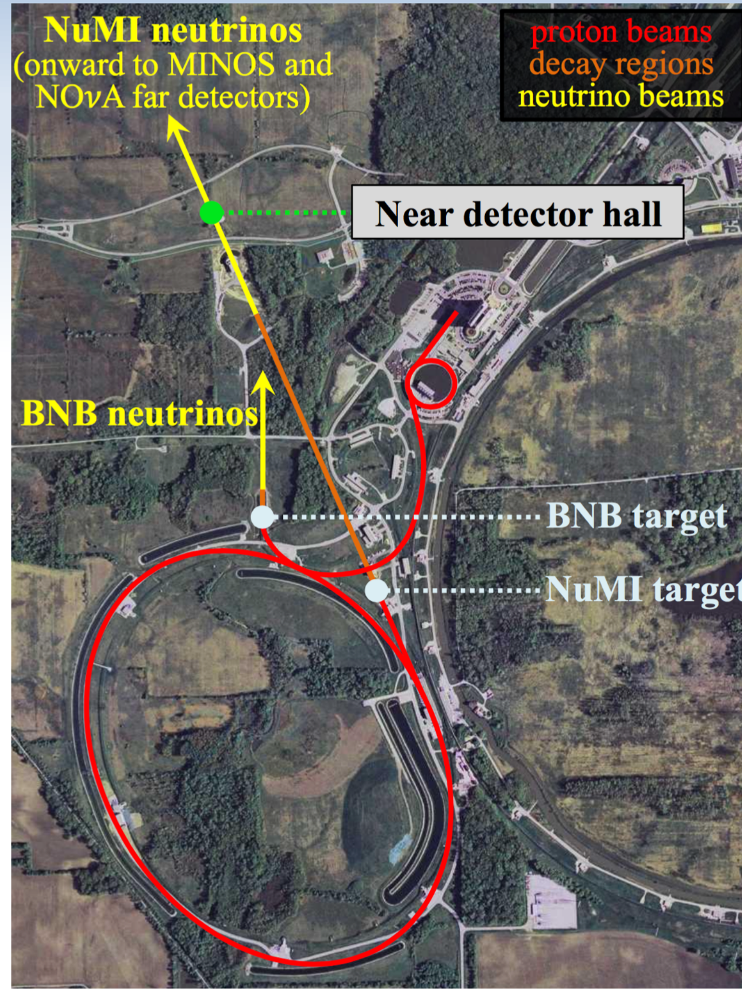
NOvA performs a simultaneous fit to the ν_μ, ν_e, $\overline{\nu_\mu}$, and $\overline{\nu_e}$ spectra measured at the far detector.

The NOvA Experiment

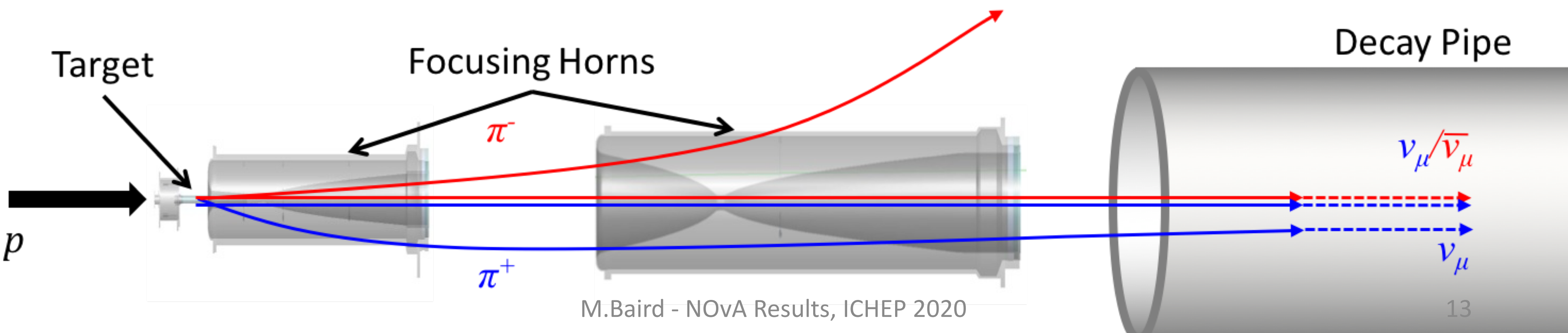
The NOvA Experiment:

NuMI - Neutrinos at the Main Injector

- Provides a 10 μsec beam pulse every ~1.4 sec
- Focusing horns can select 95% pure ν_μ or 93% pure $\overline{\nu}_\mu$ beam.
- Running at > 700 kW since 2017, FNAL is currently working on upgrades towards 900+ kW.



This talk shows results adding ~50% more neutrino-beam mode data over 2019 results.

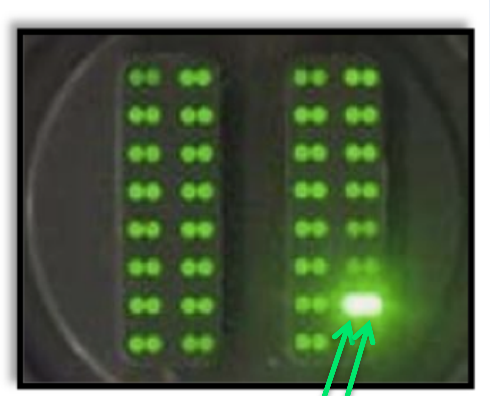


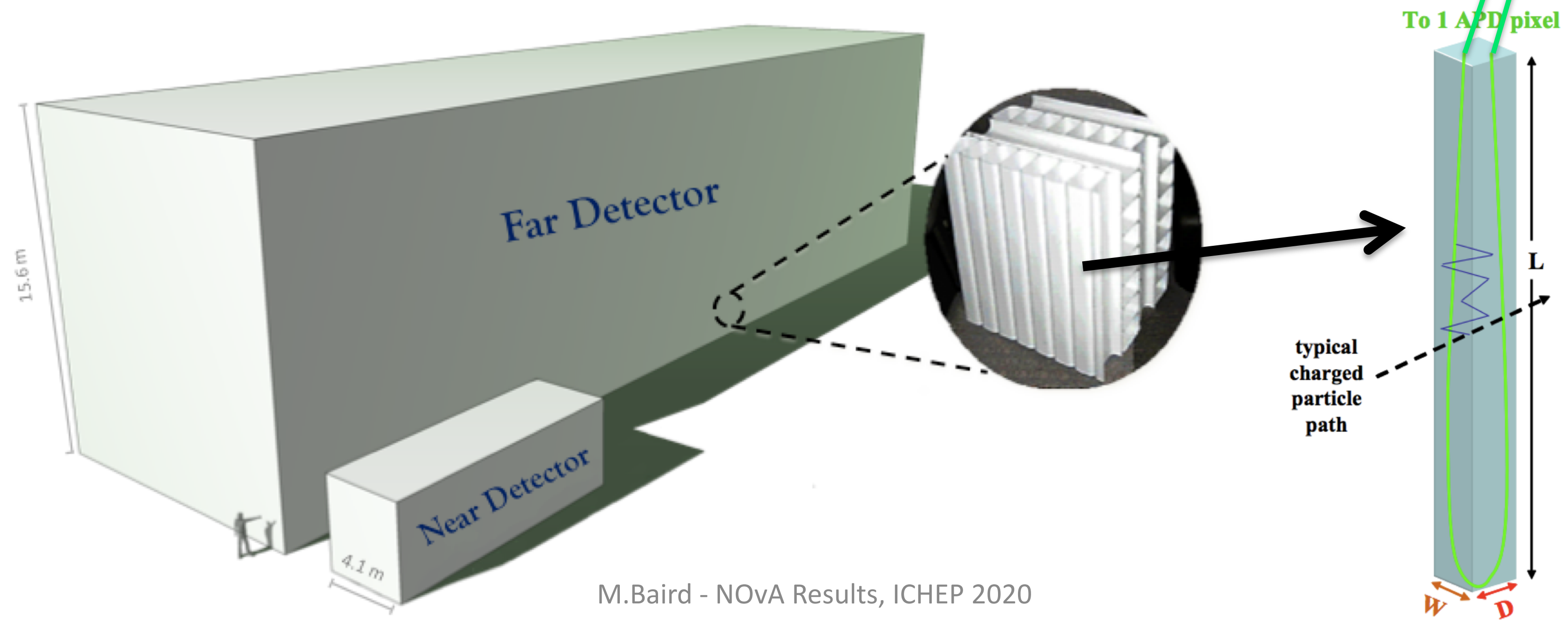
The NOvA Experiment: Detectors

Two functionally identical detectors:

extruded PVC, mineral oil as scintillator, avalanche photo-diodes for light collection

- **Near:** 300 ton, 1 km from source, 105 m underground
- **Far:** 14 kton, 810 km from source, on the surface





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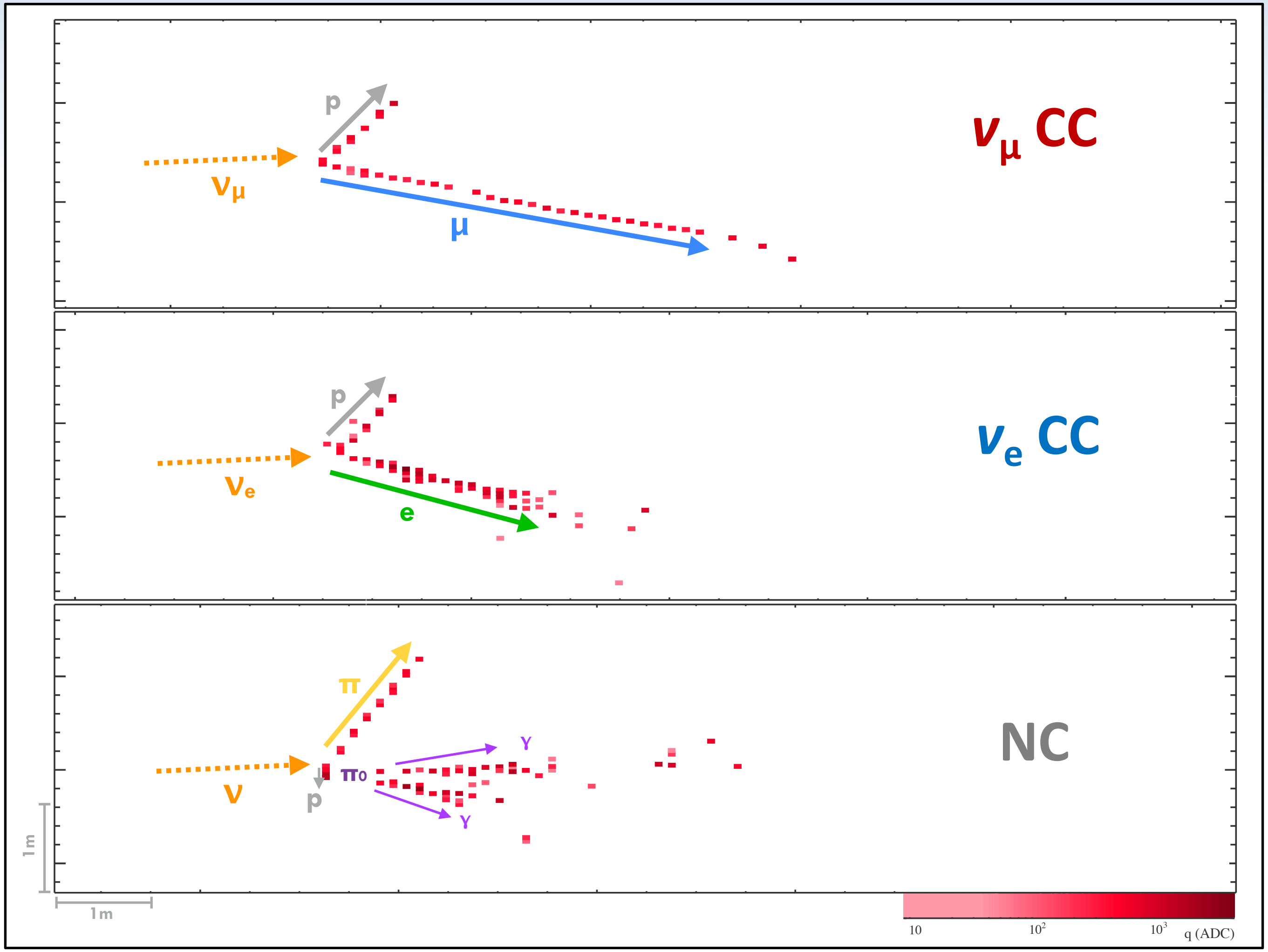


The NOvA 3-flavor analysis requires us to identify neutrinos by flavor and reconstruct their energies before fitting to the final spectra.

Event Selection:

- We classify events using a convolutional neural network called the Convolutional Visual Network (CVN) yielding an increase in exposure of ~30% over traditional PIDs.

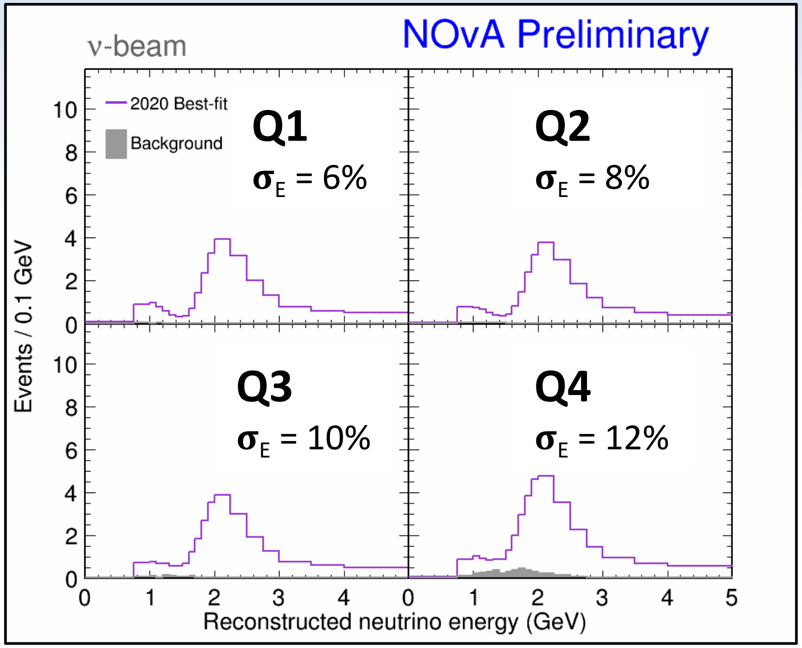
- NOvA was the first to use this technique in a particle physics analysis (JINST 11 P09001 (2016)).

- We have implemented a variety of new developments for this analysis leading to a > x10 increase in training and evaluation speed.

Can you beat CVN at neutrino classification?

New developments:
- Improved training samples (removed ν_τ and tighter cosmic preselection)
- Incorporating systematics into training
- Switched to MobileNet architecture
- Using HDF5 file format and keras/tensorflow for training.

Analysis Samples:



ν_μ events:

- Sensitivity depends primarily on the shape of the energy spectrum.
- Separate the events into quantiles by hadronic energy fraction (energy resolution.)
- Most backgrounds end up in the fourth quantile.

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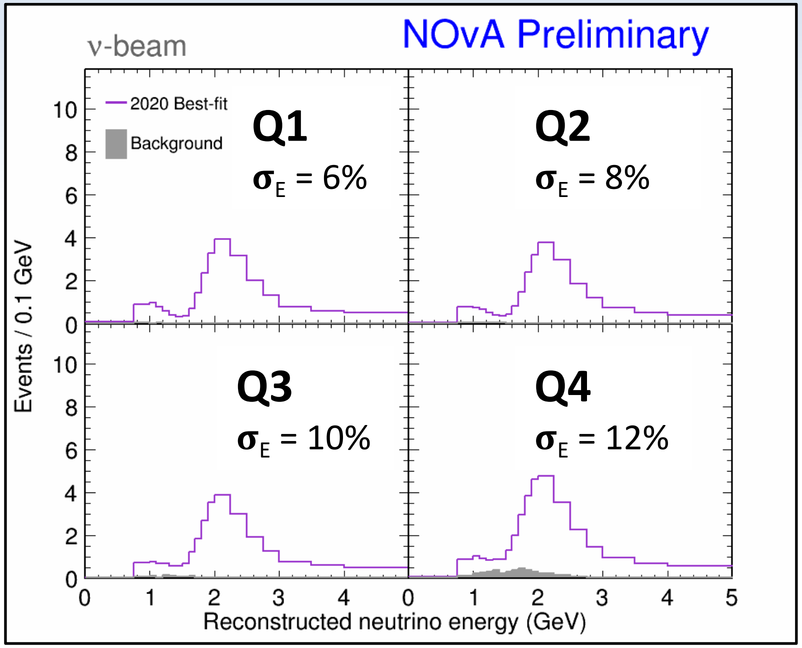

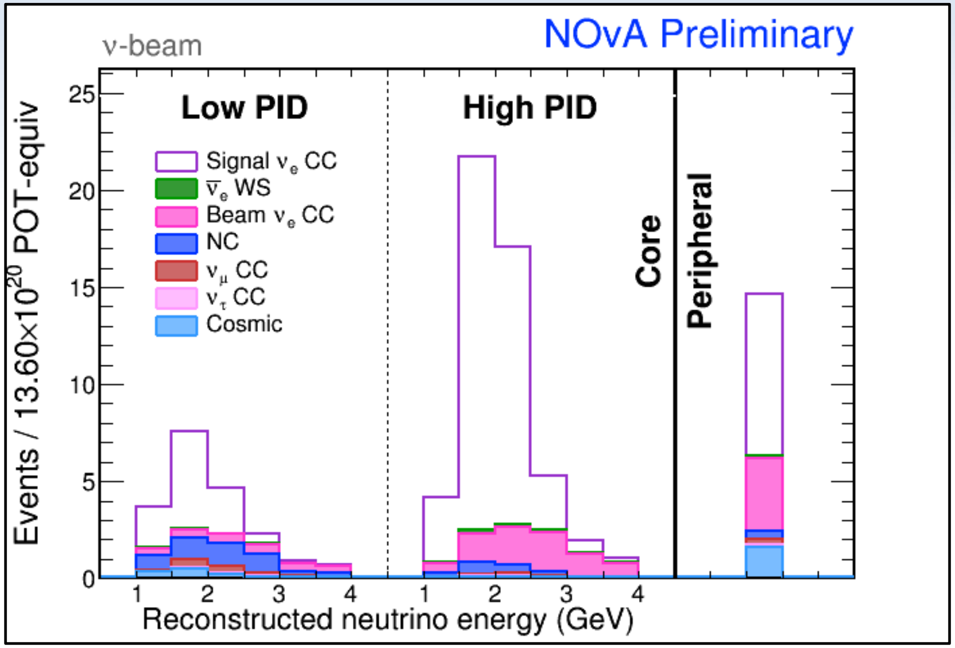

ν_μ events:

- Sensitivity depends primarily on the shape of the energy spectrum.
- Separate the events into quantiles by hadronic energy fraction (energy resolution.)
- Most backgrounds end up in the fourth quantile.

ν_e events:

- Sensitivity depends primarily on the ability to separate signal from backgrounds.
- Separate contained events by high/low PID to isolate a higher purity sample.
- Allow for a third “peripheral” sample of uncontained but high-PID events (no energy binning.)

Extrapolation:

- We continue to use an extrapolation technique in the analysis:
  - modify our FD prediction based on ND data/MC difference
  - significantly reduces correlated uncertainties.

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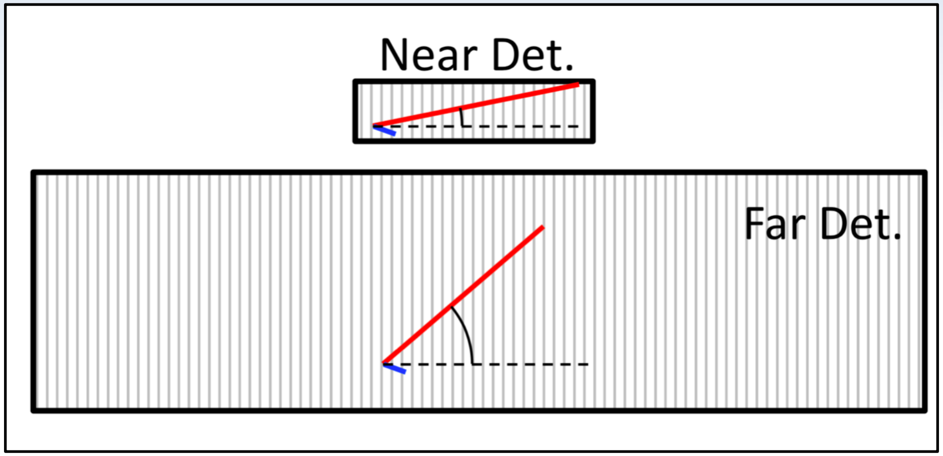
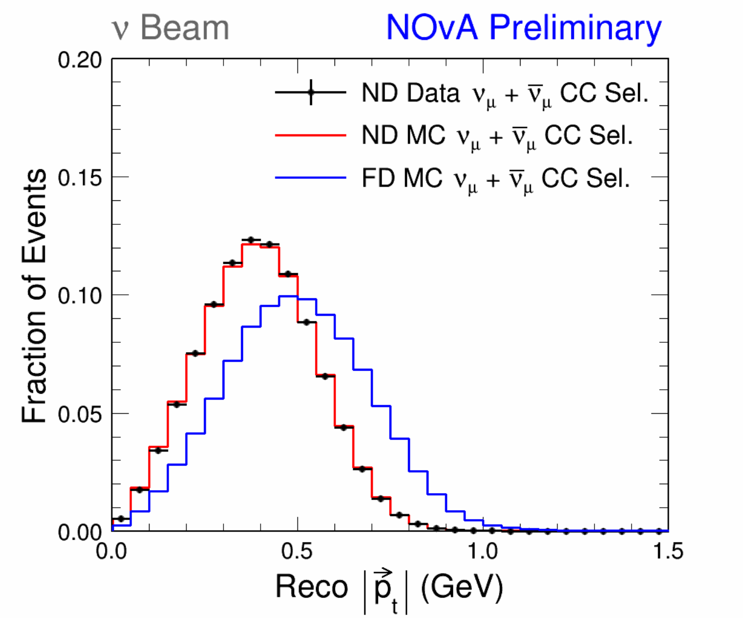


- ND containment limits the range of accepted lepton angles more than in the FD
  - Mitigate this by extrapolating in bins of transverse lepton momentum, p_t



Extrapolation:

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    - modify our FD prediction based on ND data/MC difference
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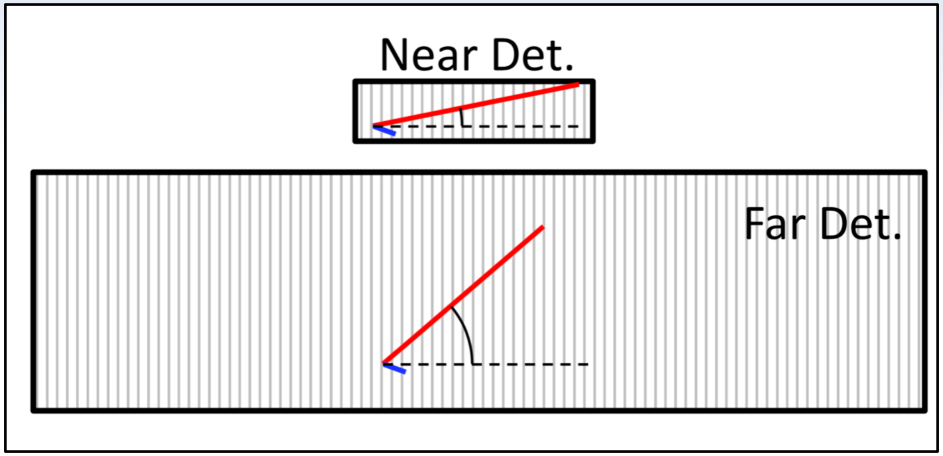
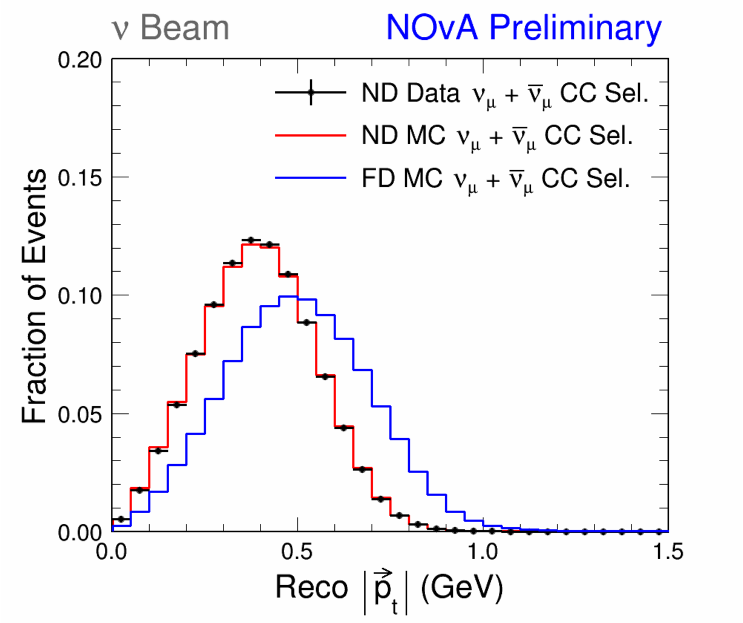
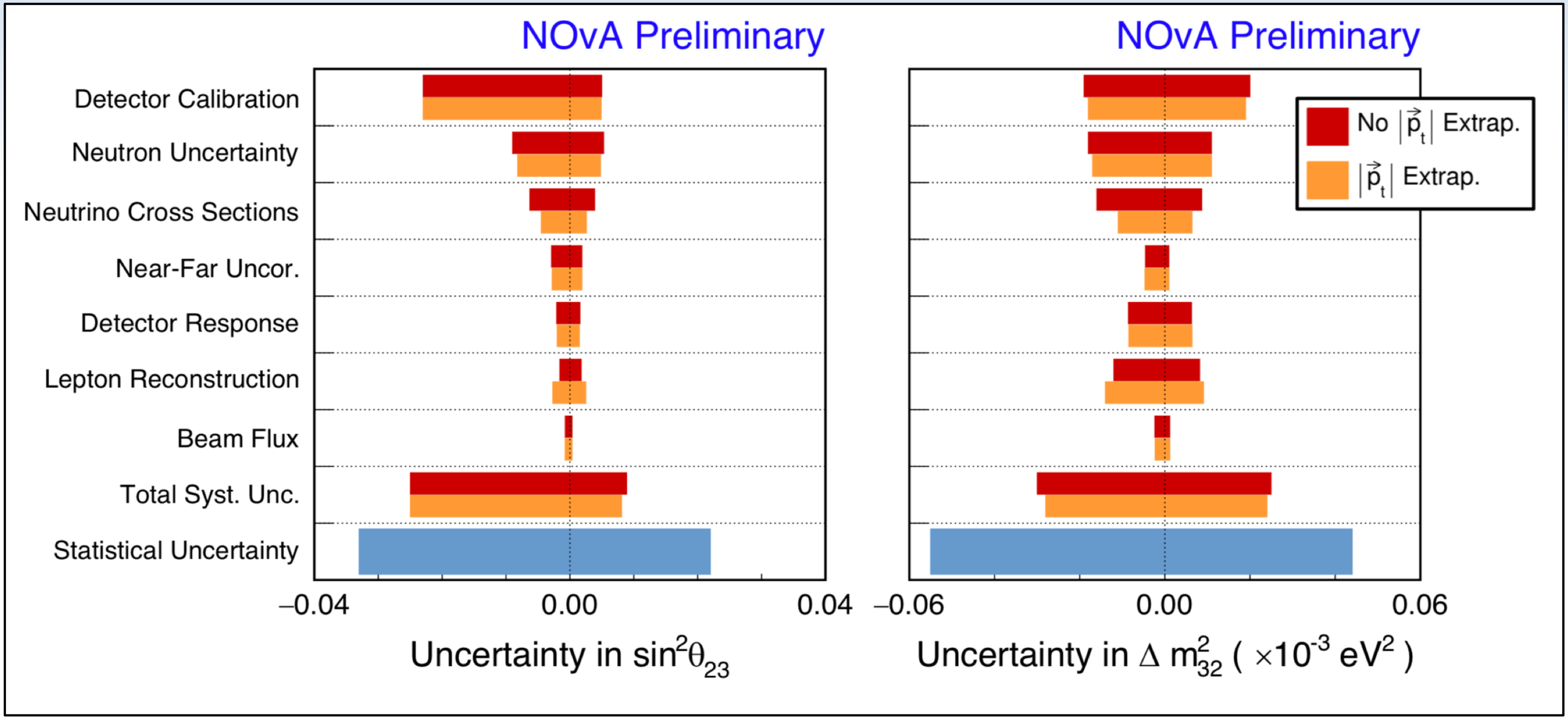
- ND containment limits the range of accepted lepton angles more than in the FD
    - Mitigate this by extrapolating in bins of transverse lepton momentum, p_t
- Split ND sample into 3 bins of p_t and extrapolate each separately to the FD
    - p_t bins are resumed before fitting
    - Effectively "rebalances" the kinematics to better match between the detectors





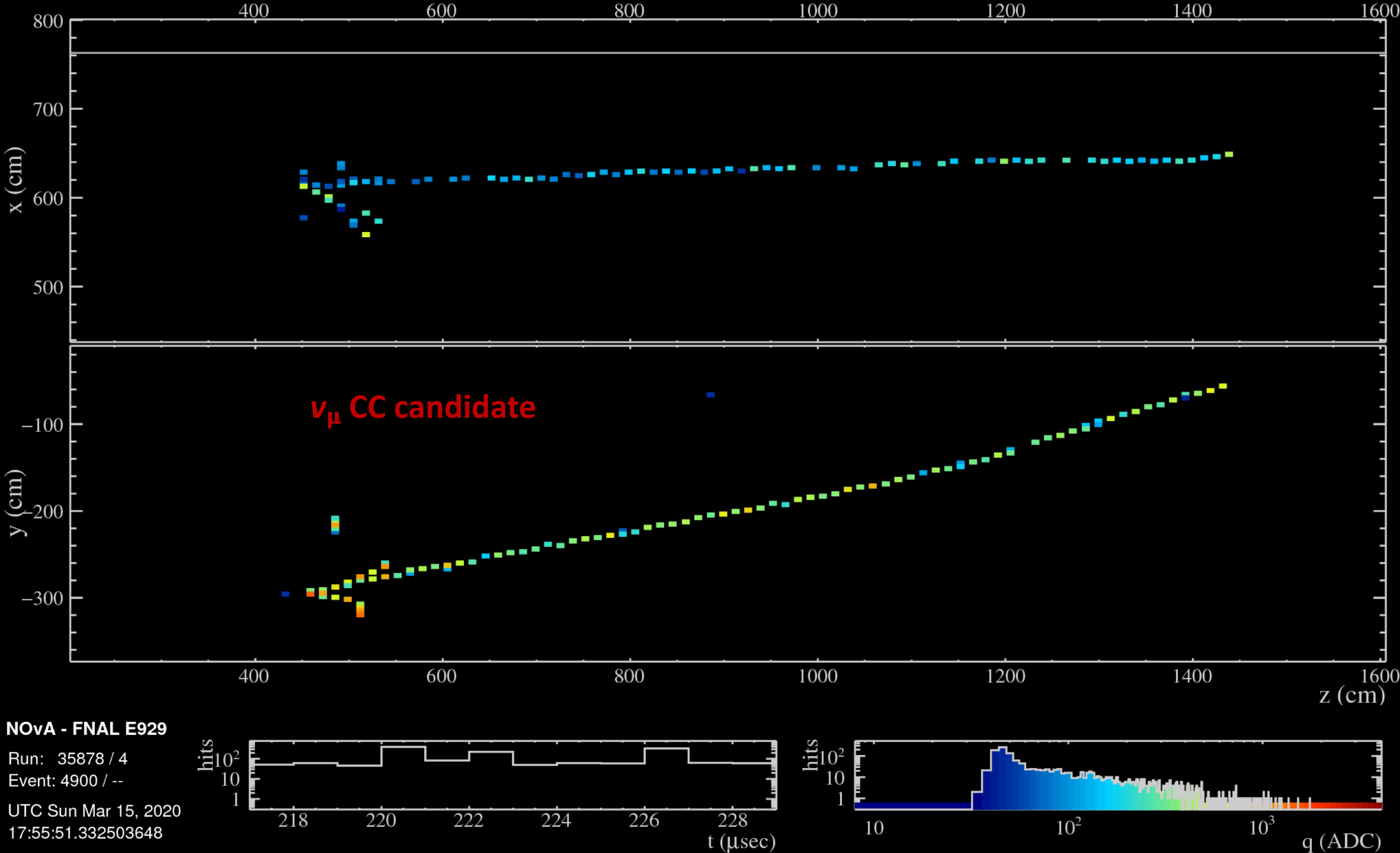
Extrapolation & Systematics:



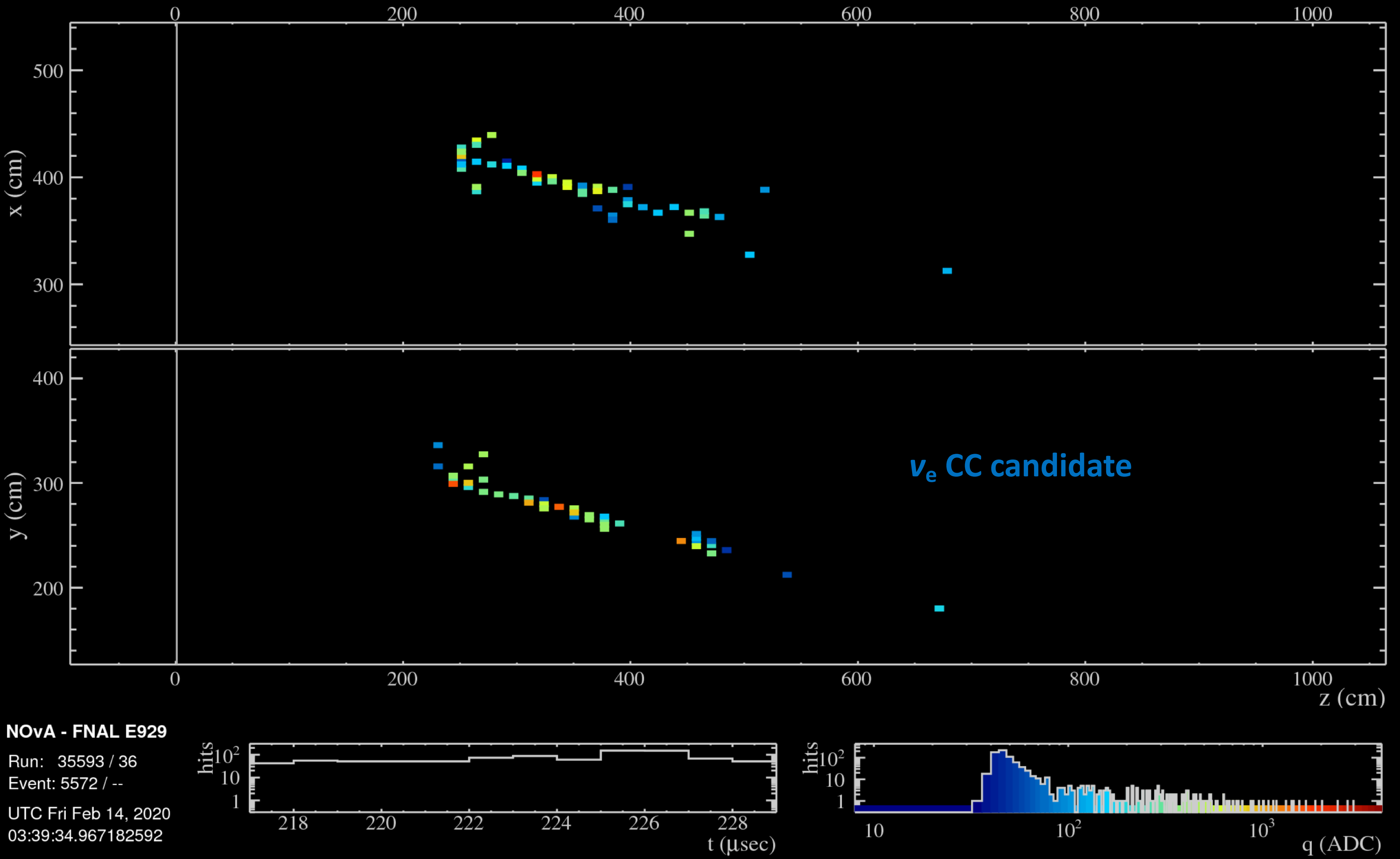
- **The result of this new p_t extrapolation technique is that we:**
  - slightly increase sensitivity to well-understood lepton reconstruction
  - increase analysis robustness against "unknown unknowns"
  - reduce cross-section uncertainties by **30%** and total uncertainties by **5-10%**

Results:

Events:

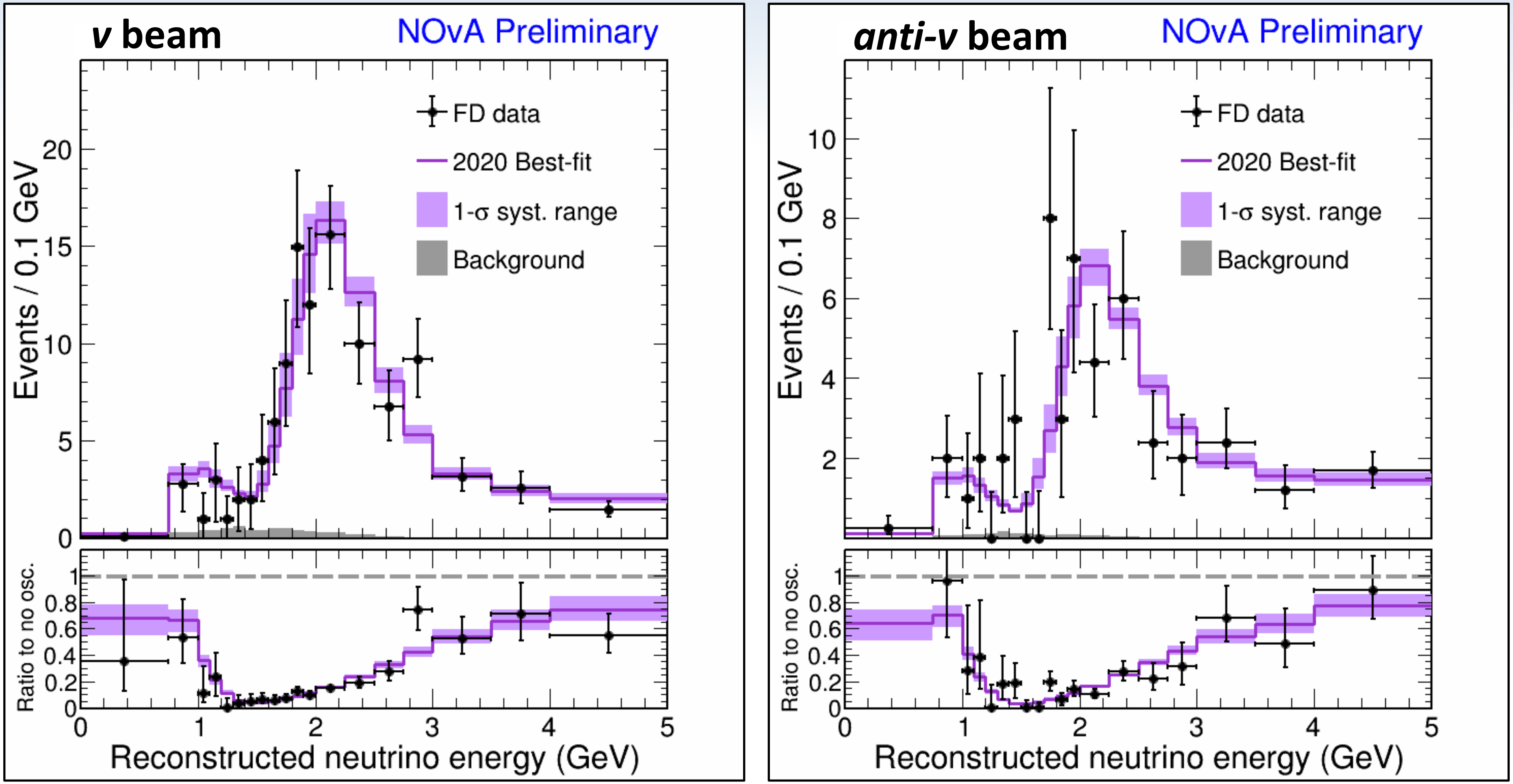

ν_μ CC candidate
x (cm)
y (cm)
z (cm)
NOvA - FNAL E929
Run: 35878 / 4
Event: 4900 / --
UTC Sun Mar 15, 2020
17:55:51.332503648
hits
t (μsec)
q (ADC)

Events:



x (cm)
y (cm)
z (cm)
ν_e CC candidate
NOvA - FNAL E929
Run: 35593 / 36
Event: 5572 / --
UTC Fri Feb 14, 2020
03:39:34.967182592
hits
t (μsec)
q (ADC)

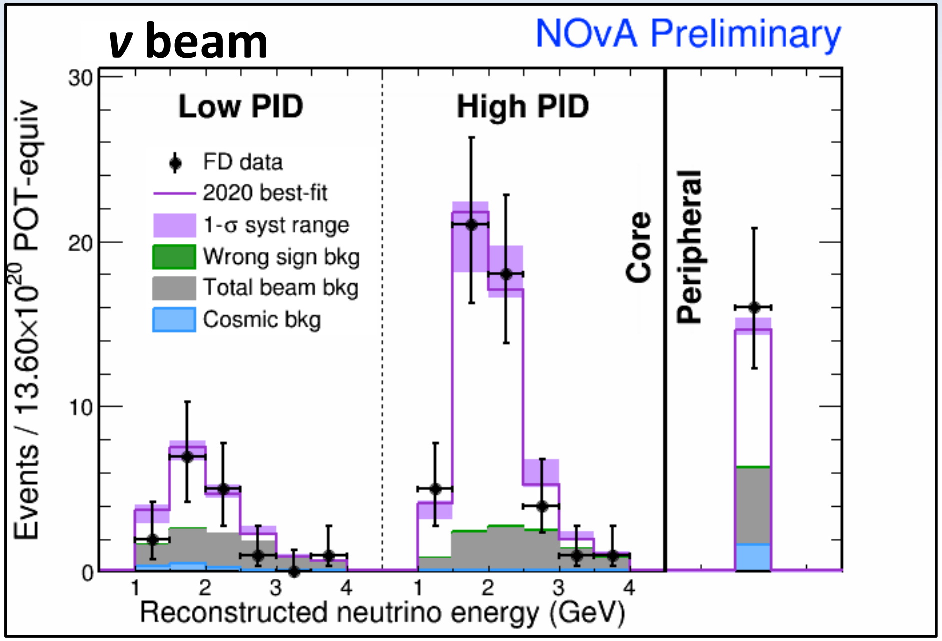
ν_μ Spectra & Numbers:

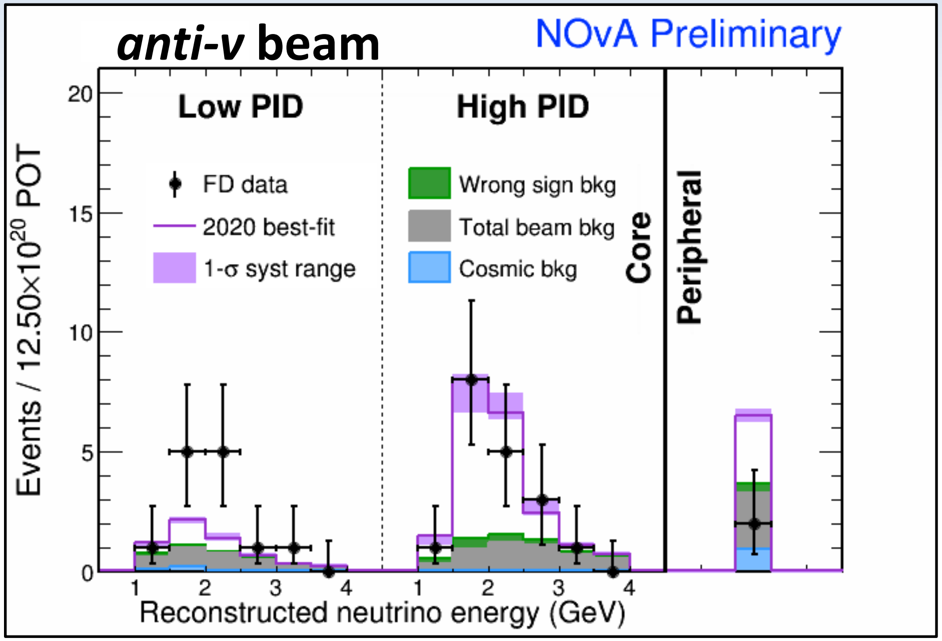


Observed: 211 events
Prediction: 222 ± 23 (8.2 bkg)

Observed: 105 events
Prediction: 105 ± 10 (2.1 bkg)

ν_e Spectra & Numbers:



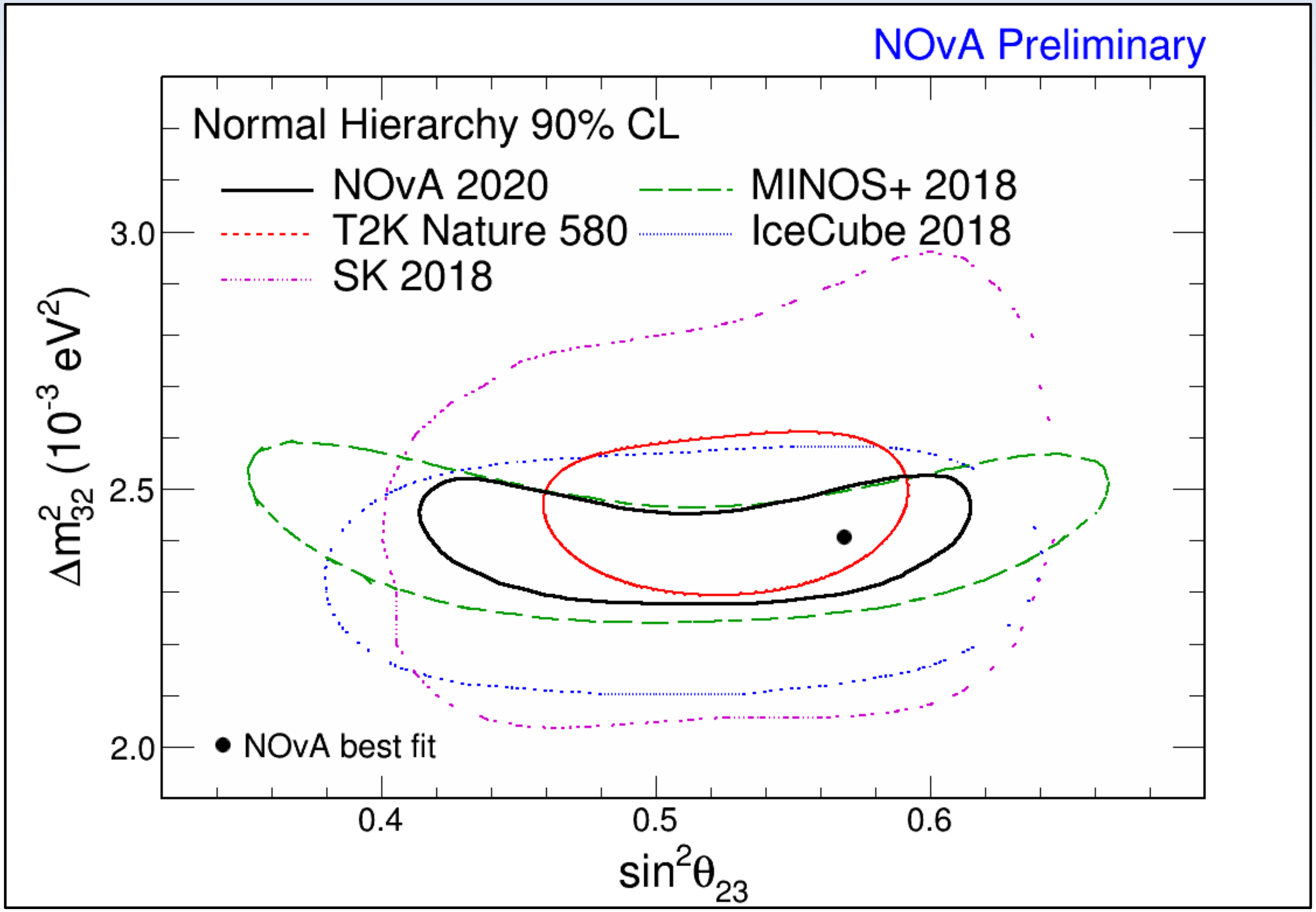


Total Observed	82	Range
Total Prediction	85.8	52-110
Wrong-sign	1.0	0.6-1.7
Beam Bkgd.	22.7	
Cosmic Bkgd.	3.1	
Total Bkgd.	26.8	26-28

Total Observed	33	Range
Total Prediction	33.2	25-45
Wrong-sign	2.3	1.0-3.2
Beam Bkgd.	10.2	
Cosmic Bkgd.	1.6	
Total Bkgd.	14.0	13-15

Strong (> 4 σ) evidence for $\bar{\nu}_e$ appearance.

Fit Results:



Best Fit:

NOTE:

The above does not include new results from other experiments shown at Neutrino-2020 a few weeks ago.

Δm^2_{32} = (2.41 ± 0.07) x 10^{-3} eV^2

$\sin^2(\theta_{23})$ = 0.57 $^{+0.04}_{-0.03}$ (49°)

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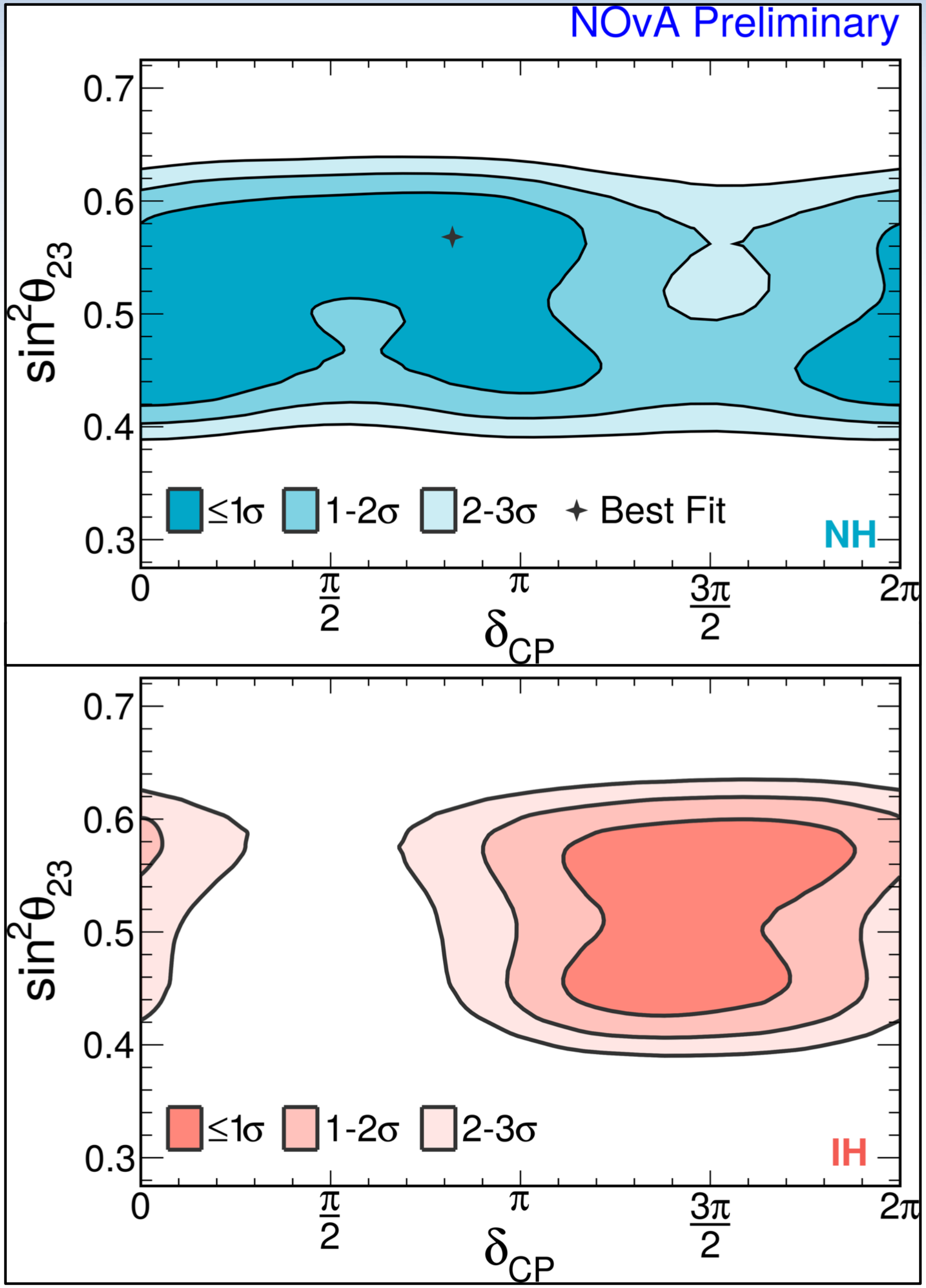
$\sin^2(\theta_{23}) = 0.57\ ^{+0.04}_{-0.03}$ (49°)

B.F. for $\delta_{CP} = 0.83\pi$



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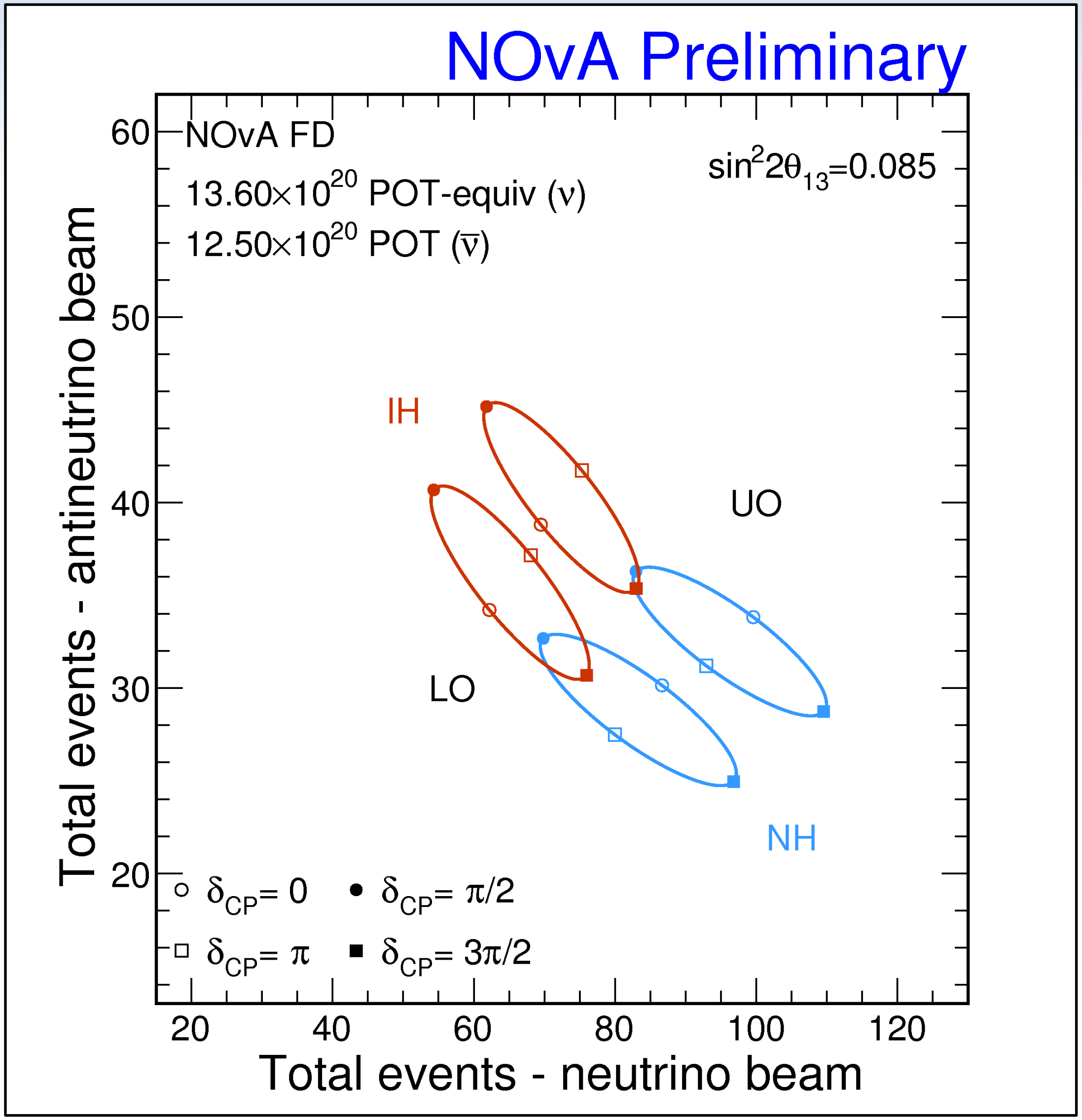
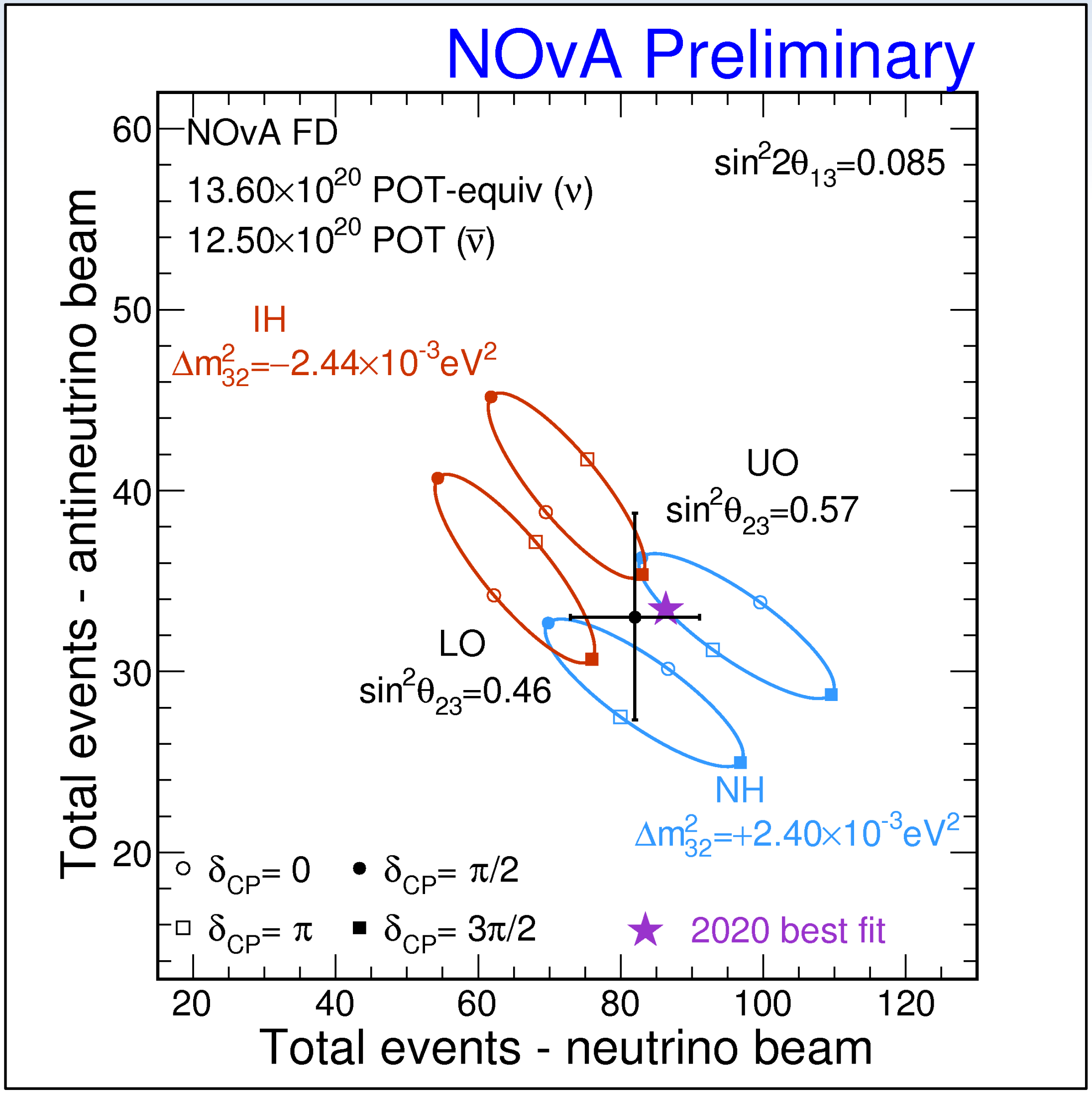


NOvA Preliminary
NOvA FD
13.60×10^20 POT-equiv (ν)
12.50×10^20 POT (ν̄)
sin²2θ13=0.085
IH
UO
LO
NH
δCP= 0
δCP= π/2
δCP= π
δCP= 3π/2
Total events - antineutrino beam
Total events - neutrino beam
20
30
40
50
60
20
40
60
80
100
120

Fit Results:



We see no strong asymmetry in the ν_e / $\overline{\nu_e}$ appearance rates.

Disfavor hierarchy-δ_{CP} combinations that would produce that asymmetry.

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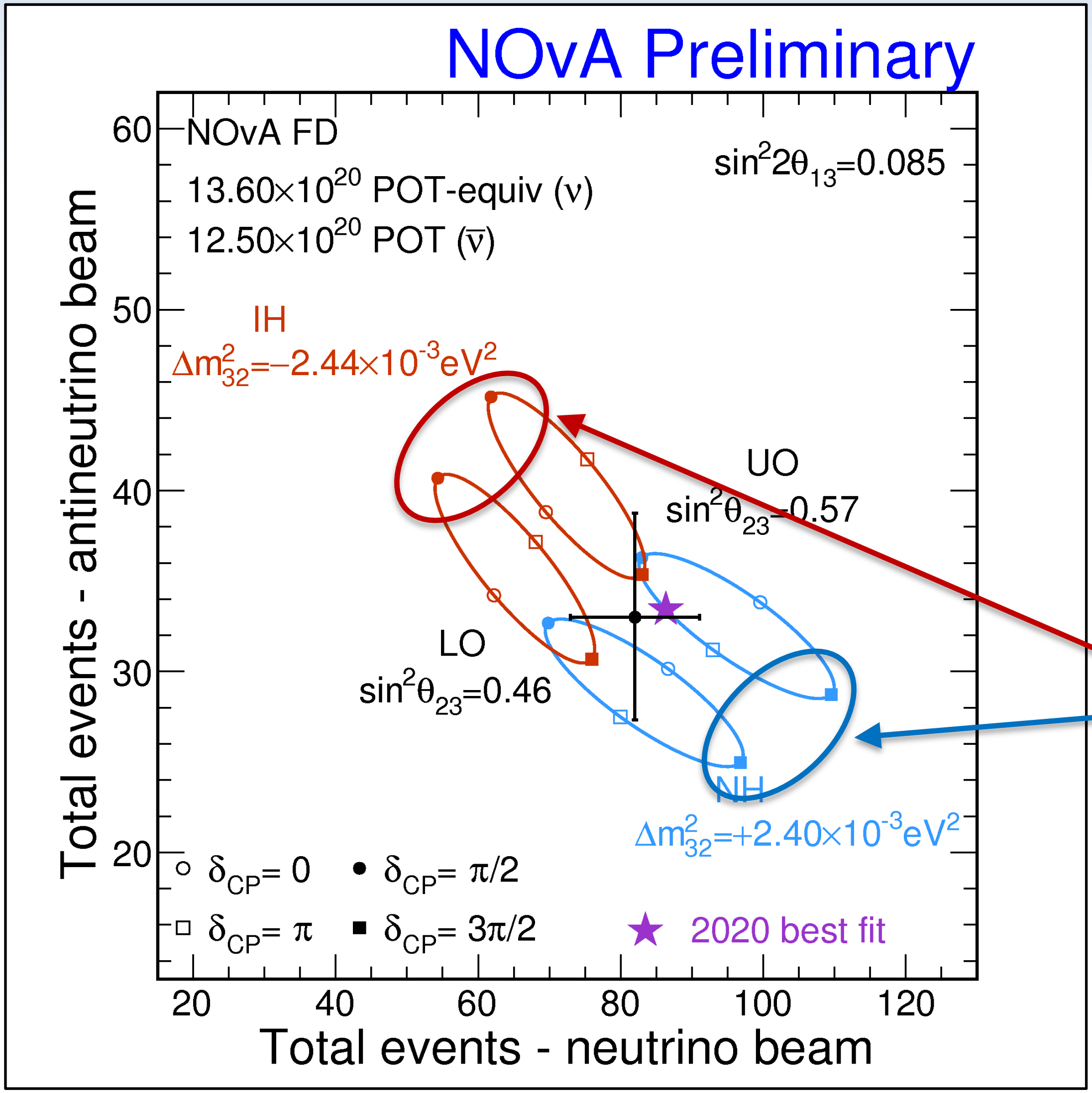


We see no strong asymmetry in the ν_e / $\overline{\nu_e}$ appearance rates.

Disfavor hierarchy-δ_{CP} combinations that would produce that asymmetry.

Exclude IH $\delta_{CP} = \pi/2$ at > 3 σ

Disfavor NH $\delta_{CP} = 3\pi/2$ at ~2 σ

Slightly prefer:

Normal Hierarchy at 1.0 σ

Upper Octant at 1.2 σ

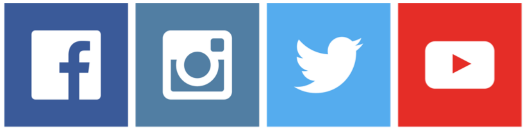
Conclusions:

- **NOvA has performed a joint analysis combining ν_μ / $\bar{\nu}_\mu$ disappearance and ν_e / $\bar{\nu}_e$ appearance results with roughly half of our expected final data set:**
  - prefer the Normal Hierarchy at ~1σ
  - prefer non-maximal mixing in the upper octant at 1.2 σ
  - exclude IH, δ = π/2 at > 3 σ and disfavors NH δ = 3π/2 at ~2 σ

- **We are looking forward to continuing our work with our T2K colleagues towards a joint analysis.**

- **An exciting future is on the horizon with more data and more analyses from NOvA's broad physics program, including input from our test beam program (see the next talk.)**

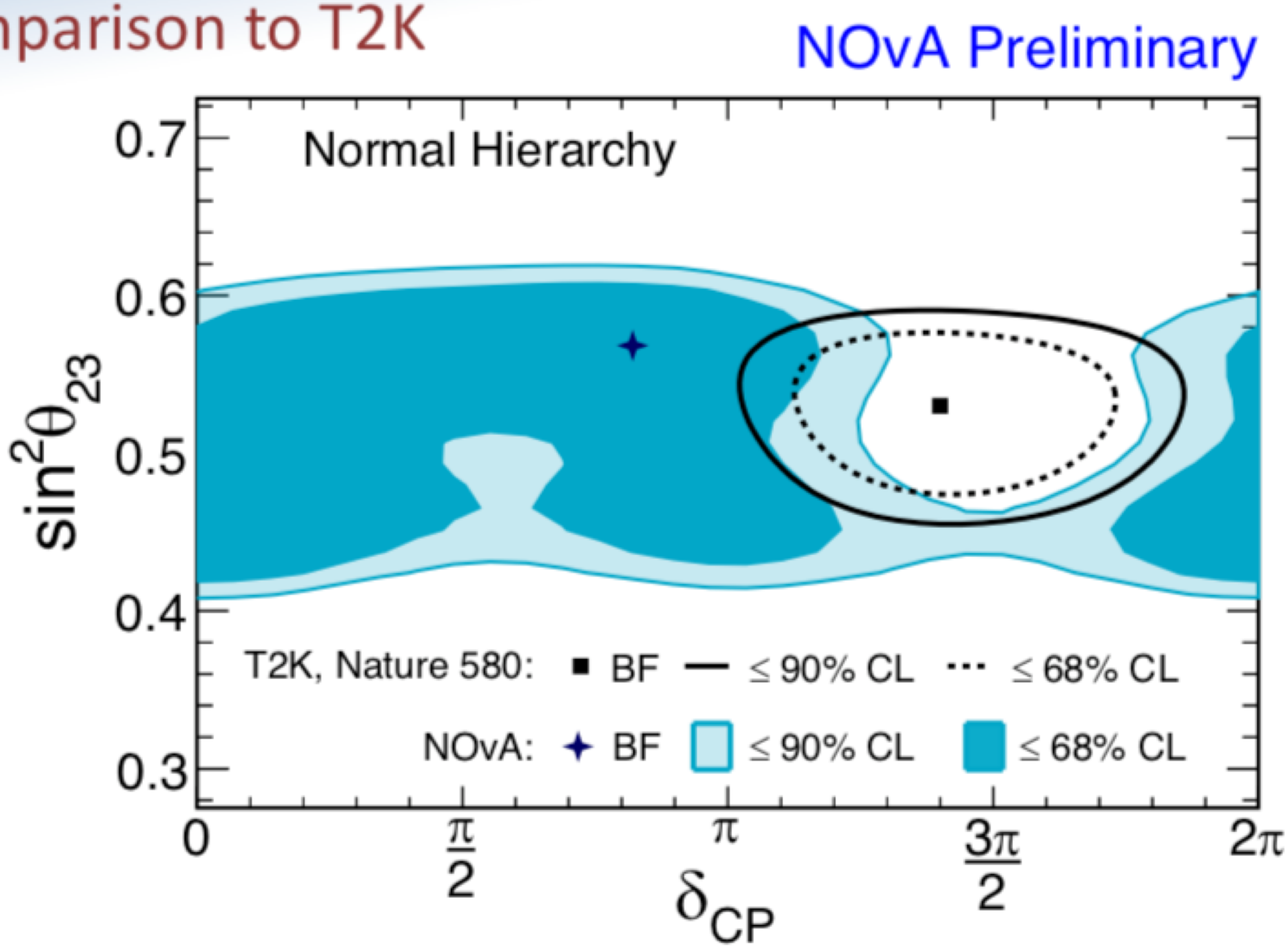
Thank you!



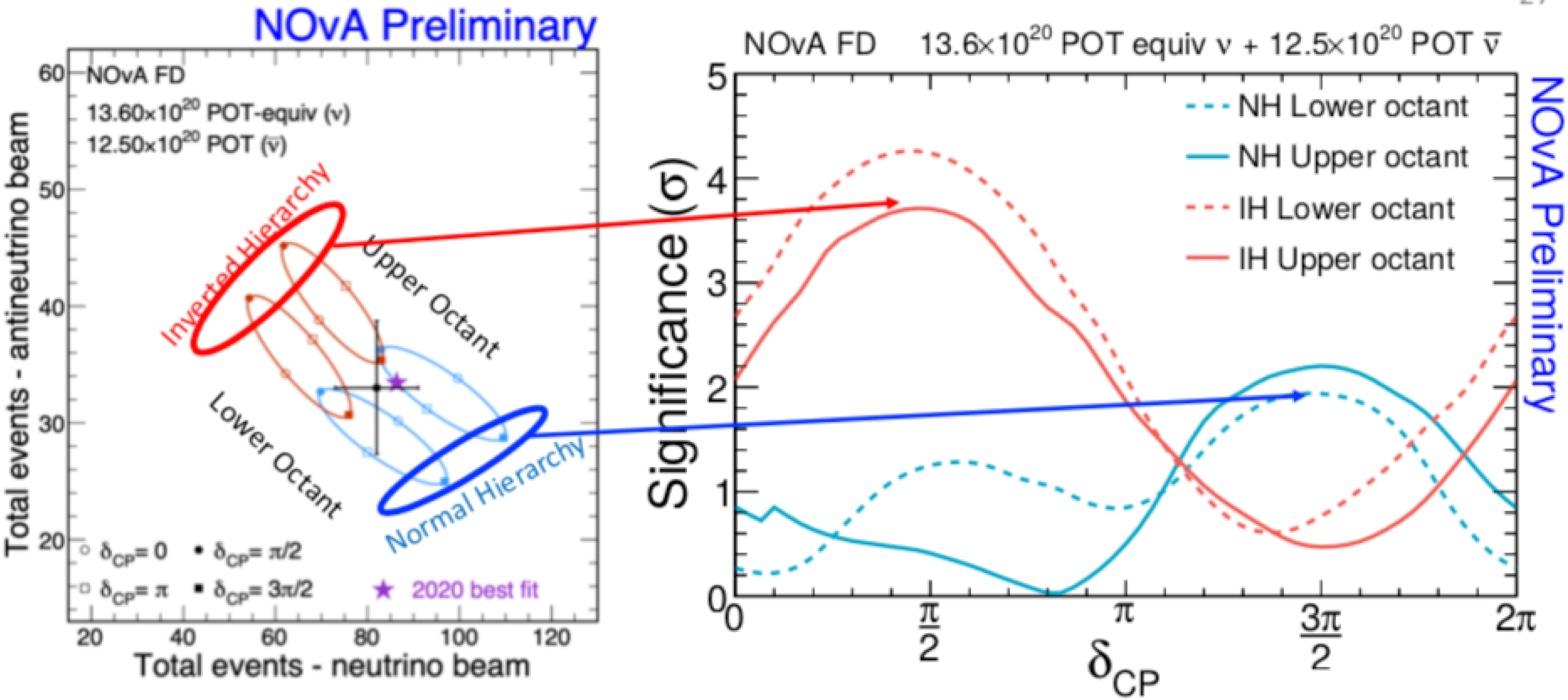
http://novaexperiment.fnal.gov

Backups:

Comparison to T2K



- Clear tension with T2K's preferred region.
- Quantifying consistency requires a joint fit of the data from the two experiments, which is already in the works.
  - Semi-annual workshops, regular joint group meetings, and a signed joint agreement.

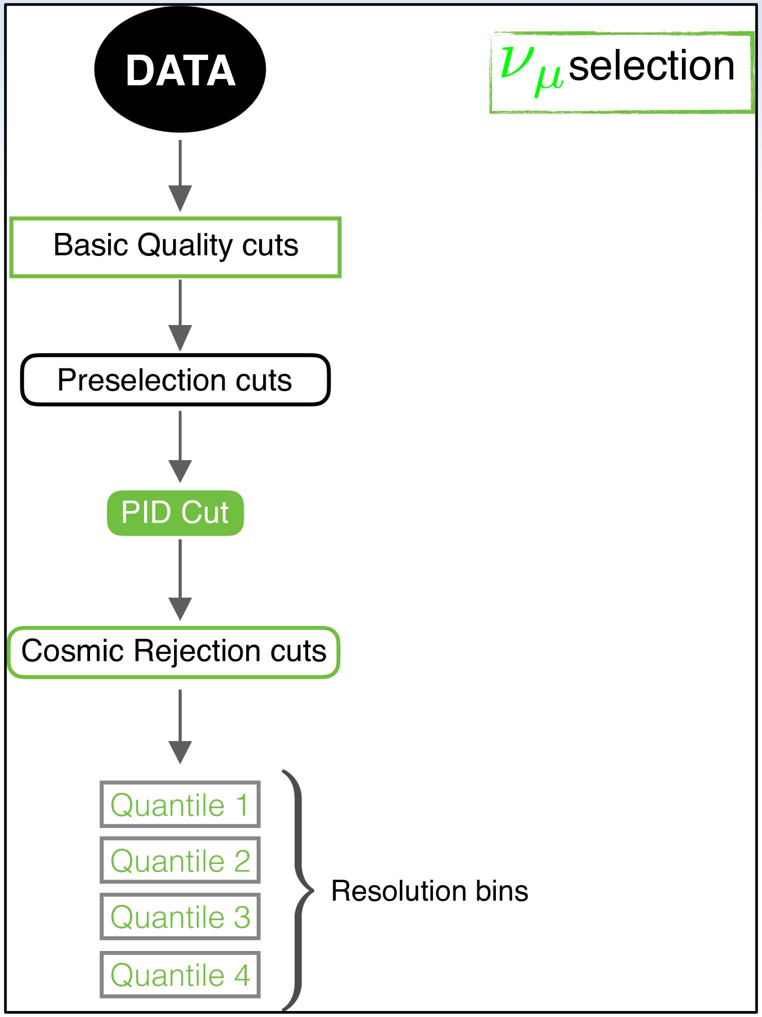


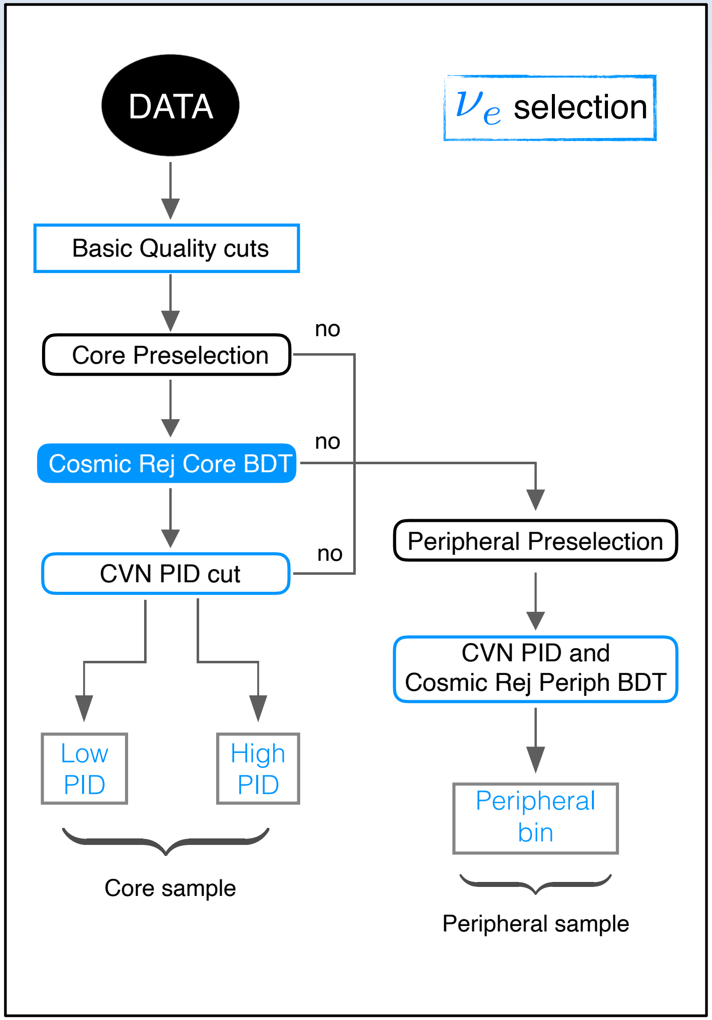
- We see no strong asymmetry in the rates of appearance of ν_e and $\bar{\nu}_e$
- Disfavor hierarchy-δ combinations which would produce that asymmetry

Exclude IH $\delta = \pi/2$ at $>3\sigma$

Disfavor NH $\delta = 3\pi/2$ at $\sim 2\sigma$

Analysis Flow Charts:

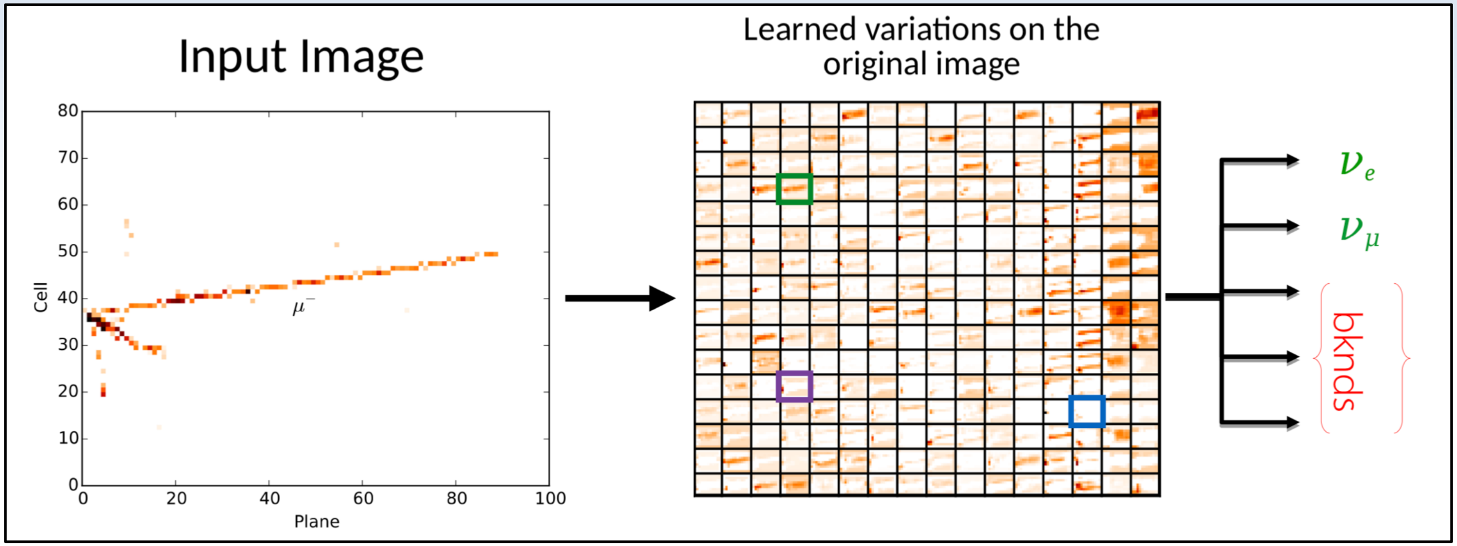


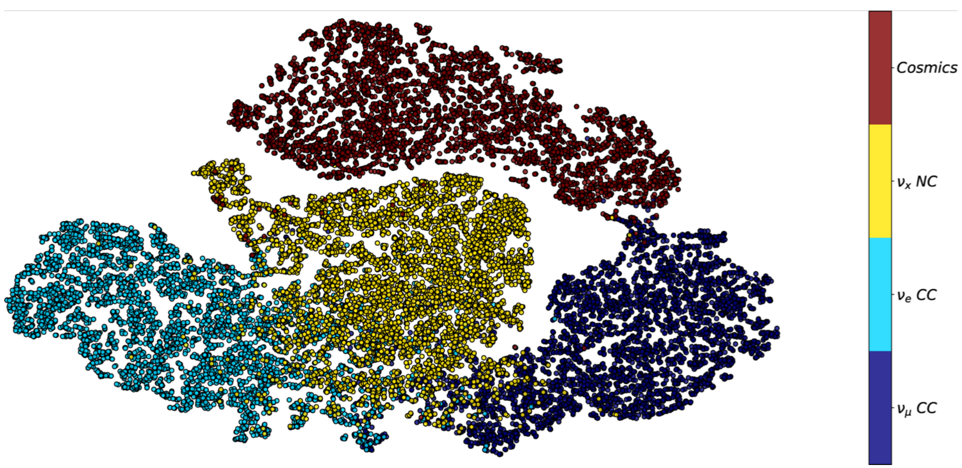


ν_μ events are separated into quantiles by hadronic energy fraction, isolating higher quality events.

ν_e events are separated into high/low PID bins and a peripheral sample, isolating a higher purity ν_e sample and separating out poorly reconstructed events.

Event Selection:





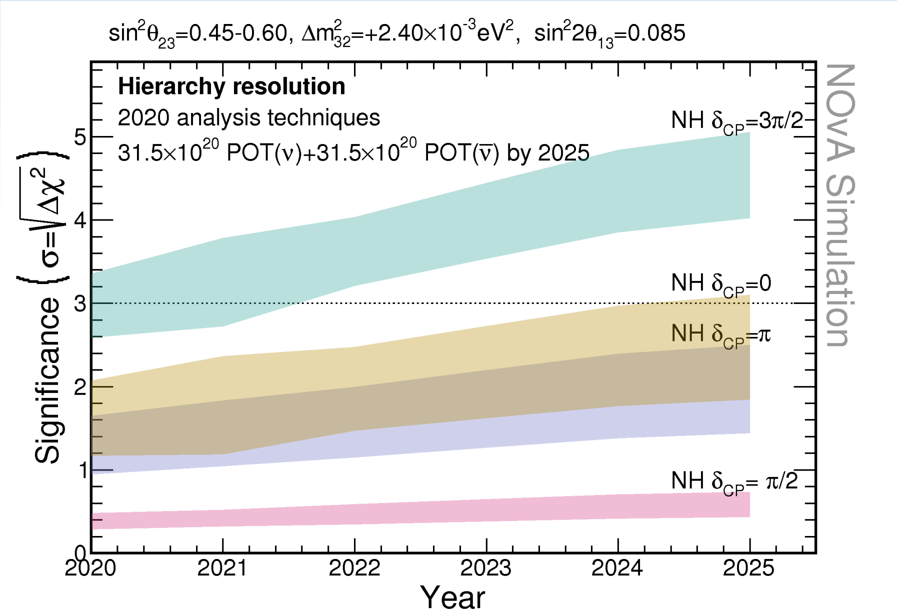
Can you beat CVN at neutrino classification?

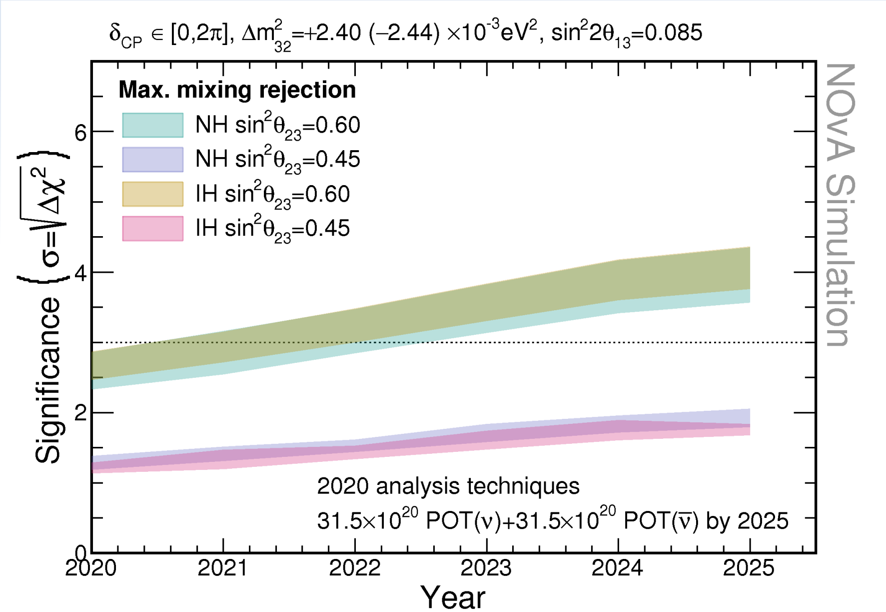
New developments:

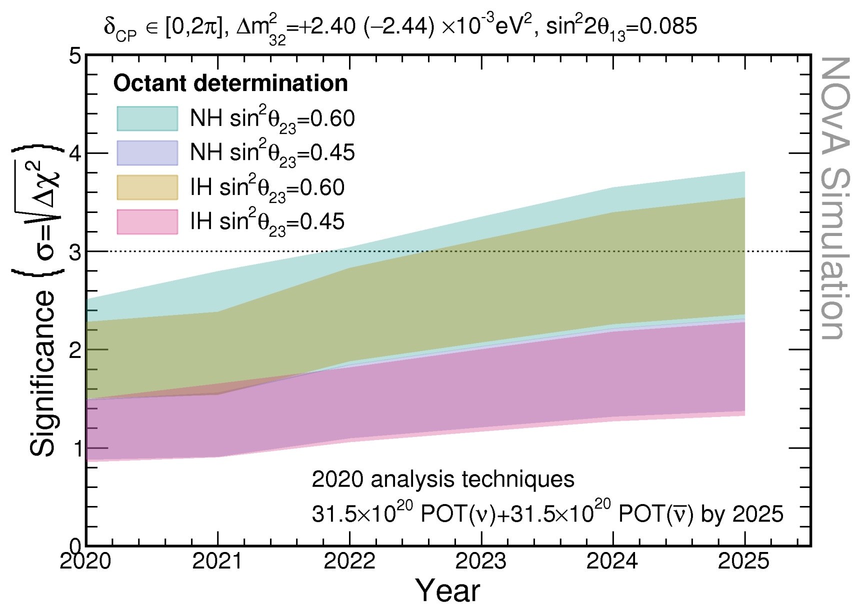
- Improved training samples (removed ν_τ and tighter cosmic preselection)
- Incorporating systematics into training
- Switched to MobileNet architecture
- Using HDF5 file format and keras/tensorflow for training

All together, we have boosted training and evaluation speed by > x10, opening up more possibilities for exploring our PIDs in the future.

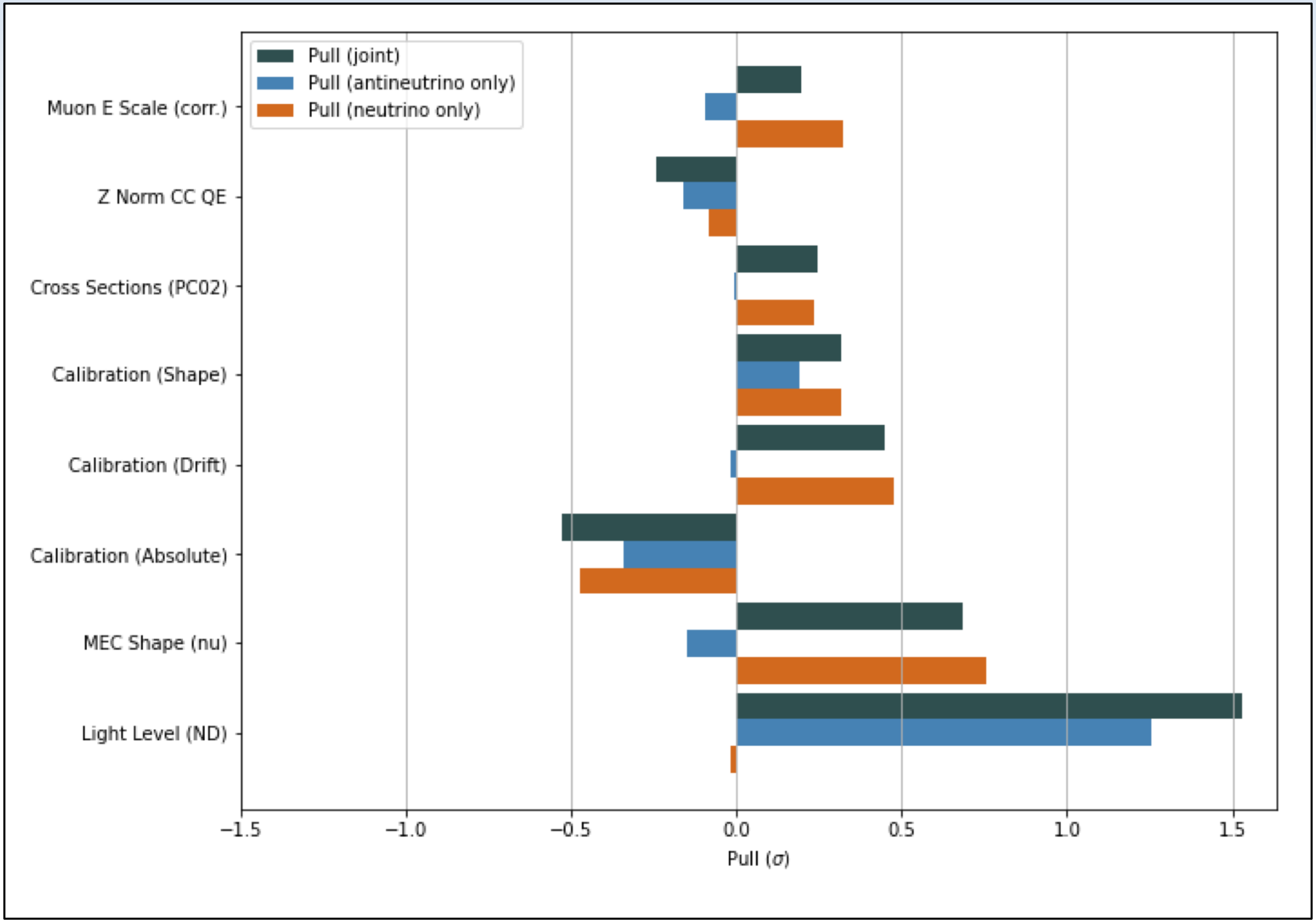
Future Reach:







Systematic Pulls in the Final Fit:



Systematic Pulls in the Final Fit:

Table 1: Top ten systematic pulls in the joint fit with neutrino and antineutrino beam data, ordered by absolute size of the pull. The systematic pulls are produced by running a one point fit with oscillation parameters fixed at the following values: $\delta_{CP} = 0.82\pi$, $\sin^2(\theta_{23}) = 0.568$, $\sin^2(2\theta_{13}) = 0.085$, $\Delta m^2_{32} = 2.40 \times 10^{-3}$ eV^2, which correspond to the best fit point in the fit to beam data, with all systematics included.

Systematic	Pull	Pull (neutrino only)	Pull (antineutrino only)
Light_Level_ND	1.532	-0.017	1.258
MECShape2020Nu	0.684	0.757	-0.146
Calibration	-0.529	-0.474	-0.339
CalibDrift	0.451	0.478	-0.019
CalibShape	0.318	0.318	0.193
genie_small_pc02	0.247	0.234	-0.008
ZNormCCQE	-0.244	-0.084	-0.159
CorrMuEScaleSyst2020	0.200	0.322	-0.096
hNFSI_MFP_2020	-0.169	-0.147	-0.020
NormHornCorr	-0.140	-0.128	0.006

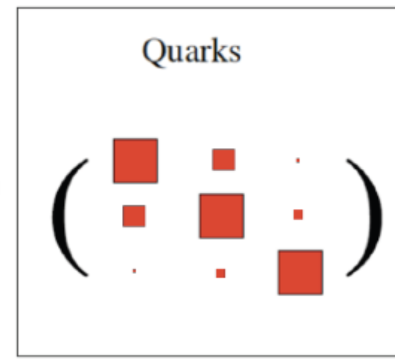
Neutrino Oscillations:

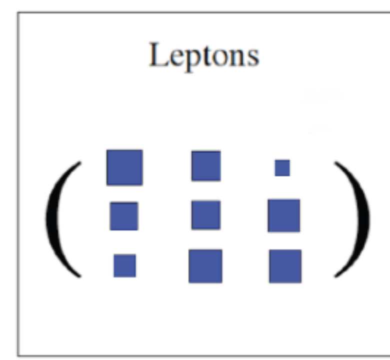
Neutrino Mixing:

$$\begin{pmatrix} \upsilon_e \\ \upsilon_\mu \\ \upsilon_\tau \end{pmatrix} = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}e^{i\delta} & C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta} & S_{23}C_{13} \\ S_{12}S_{23} - C_{12}C_{23}S_{13}e^{i\delta} & -C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta} & C_{23}C_{13} \end{bmatrix} \begin{pmatrix} \upsilon_1 \\ \upsilon_2 \\ \upsilon_3 \end{pmatrix}$$

$$S_{ij} \equiv Sin(\theta_{ij}) \quad C_{ij} \equiv Cos(\theta_{ij})$$

- Neutrino mixing is described by 3 real rotation angles and a CP violating phase factor δ, plus 2 squared mass splittings.
- All three rotation angles have been measured, but we don't yet know (very well) what delta is.
- The mixing is very different in the quark and lepton sectors!





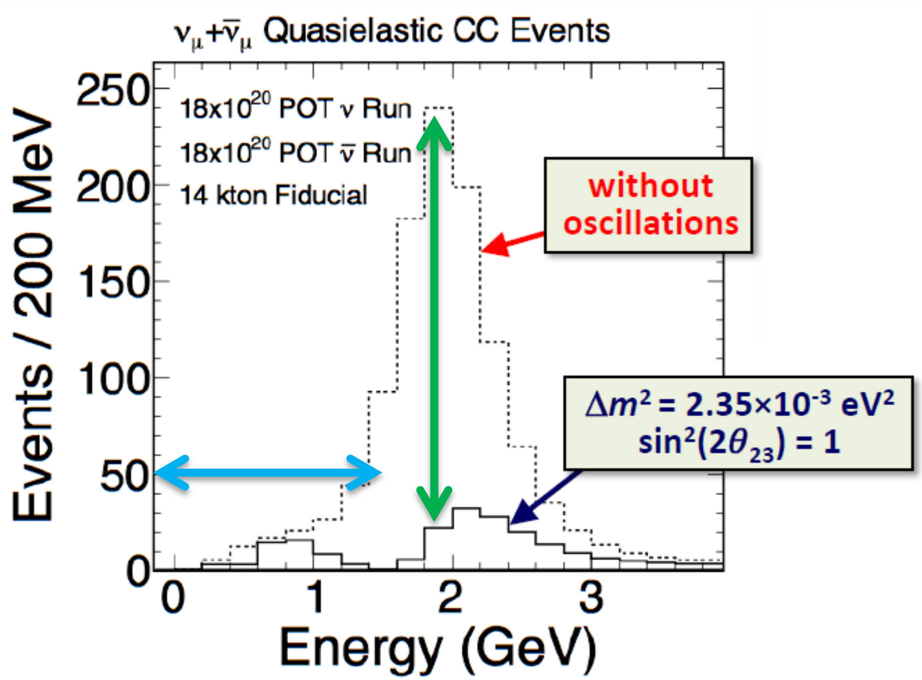
Neutrino Oscillations:

ν_μ survival probability:

$$P(\nu_\mu \to \nu_\mu) \approx 1 - sin^2(2\theta_{23})sin^2\left(\frac{1.27\Delta m_{32}{}^2 L}{E}\right)$$

$$\Delta m_{ij}{}^2 \equiv m_i{}^2 - m_j{}^2$$

Δ_{23}



Neutrino Oscillations:

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$$\Delta m_{ij}{}^2 \equiv m_i{}^2 - m_j{}^2$$

Δ_{23}

ν_e appearance probability:

$$P(\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e) \approx P_{atm} + P_{sol} + 2\sqrt{P_{atm}P_{sol}}[\cos(\Delta_{32})\cos(\delta) \mp \sin(\Delta_{32})\sin(\delta)]$$

$$P_{atm} \equiv \sin^2(\Theta_{23})\sin^2(2\Theta_{13})\frac{\sin^2(\Delta_{31} \mp aL)}{(\Delta_{31} \mp aL)^2}(\Delta_{31})^2$$

$$P_{sol} \equiv \cos^2(\Theta_{23})\sin^2(2\Theta_{12})\frac{\sin^2(\mp aL)}{(\mp aL)^2}(\Delta_{21})^2$$

$$"-" = neutrinos$$

$$"+" = anti-neutrinos$$

$$a \equiv G_F N_e/\sqrt{2}$$

$$N_e = electron\ density\ in\ Earth$$

octant

Is $\theta_{23} > 45°$ or $\theta_{23} < 45°$?

hierarchy

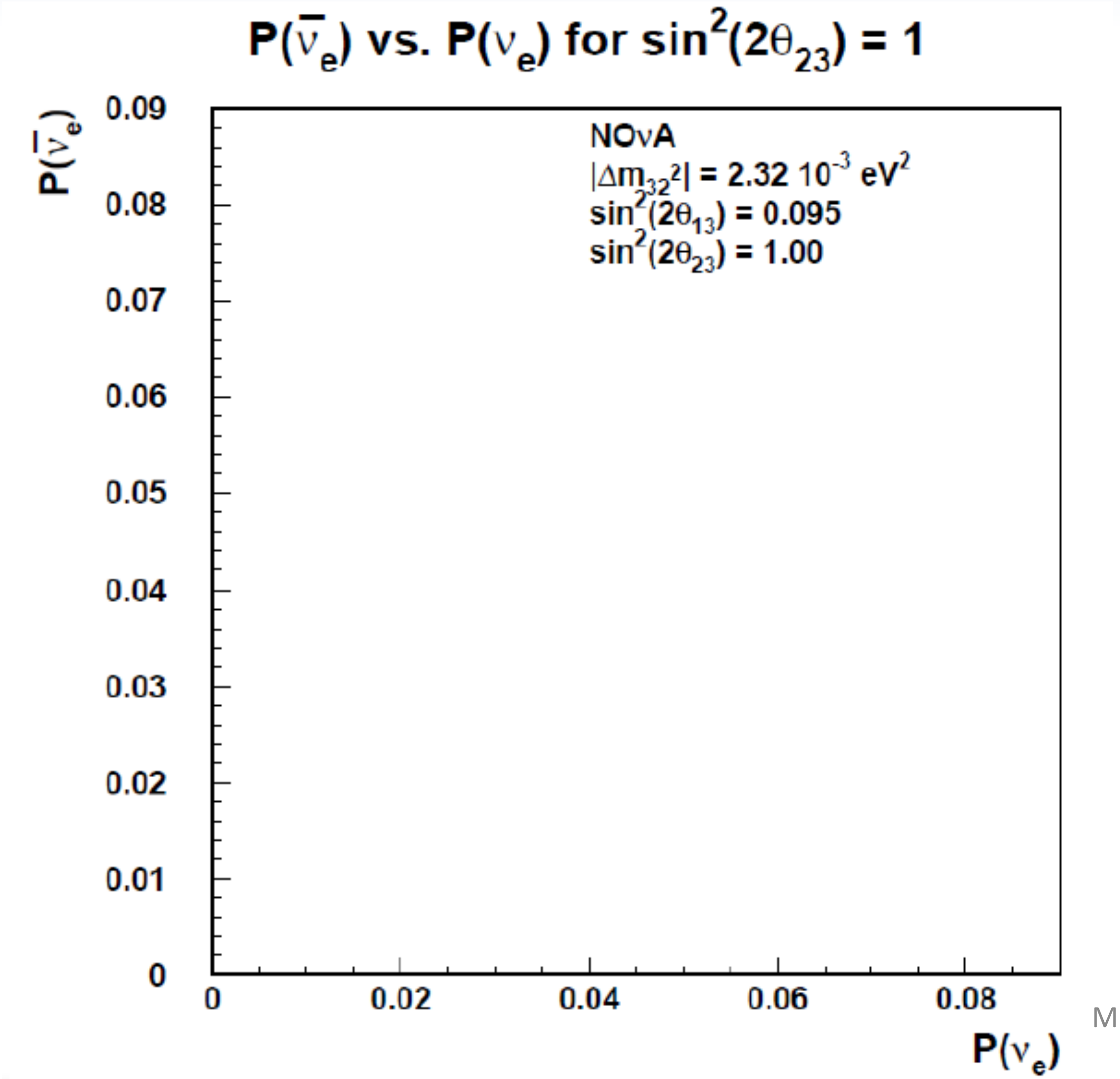
Is $m_3 > m_1$ or is $m_3 < m_1$?

CP violation

Is $\delta \neq 0$?

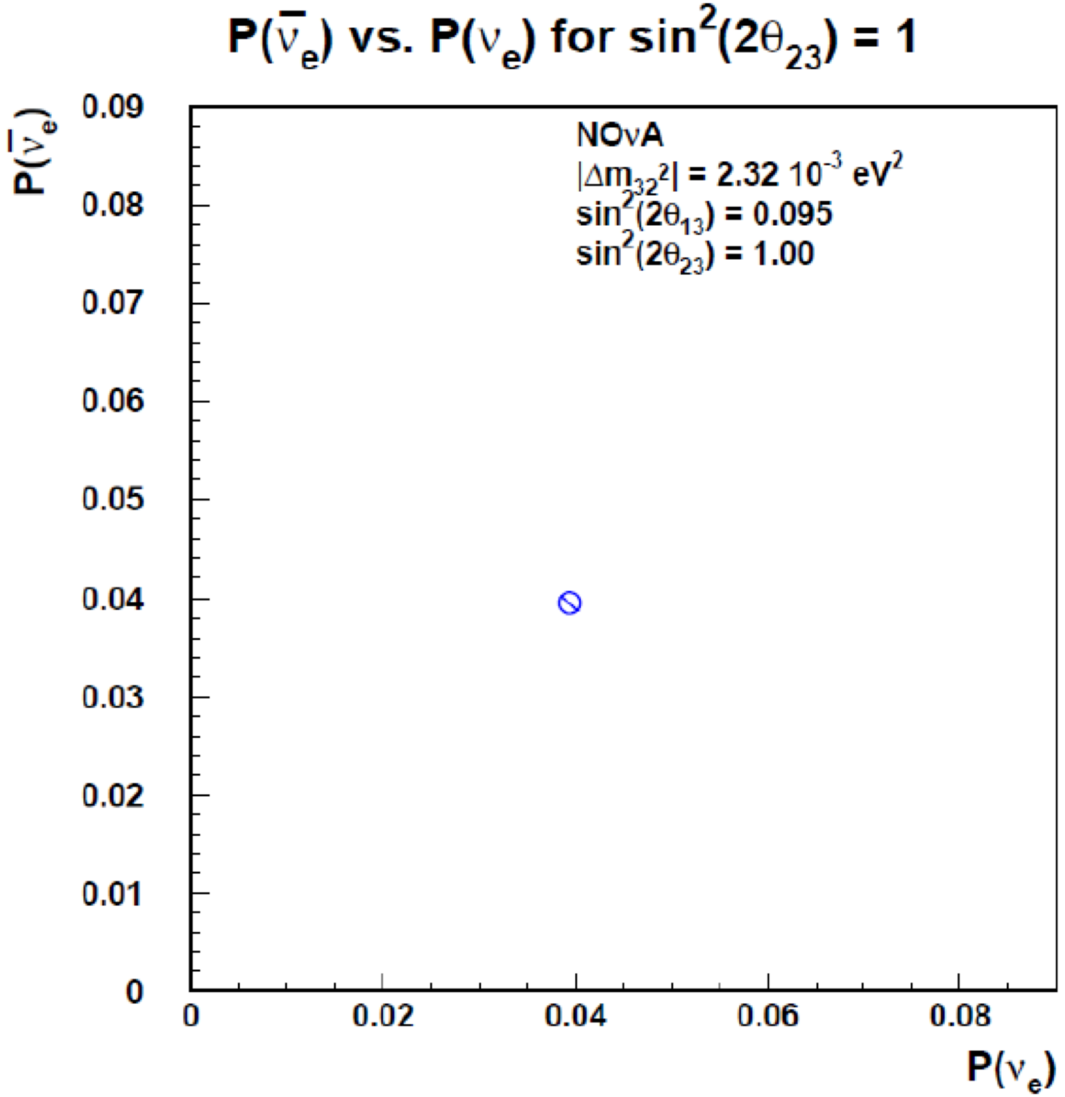
NOνA performs a simultaneous fit to the ν_μ, ν_e, $\overline{\nu_\mu}$, and $\overline{\nu_e}$ spectra measured at the far detector.

The “Bi-Probability” Plot:

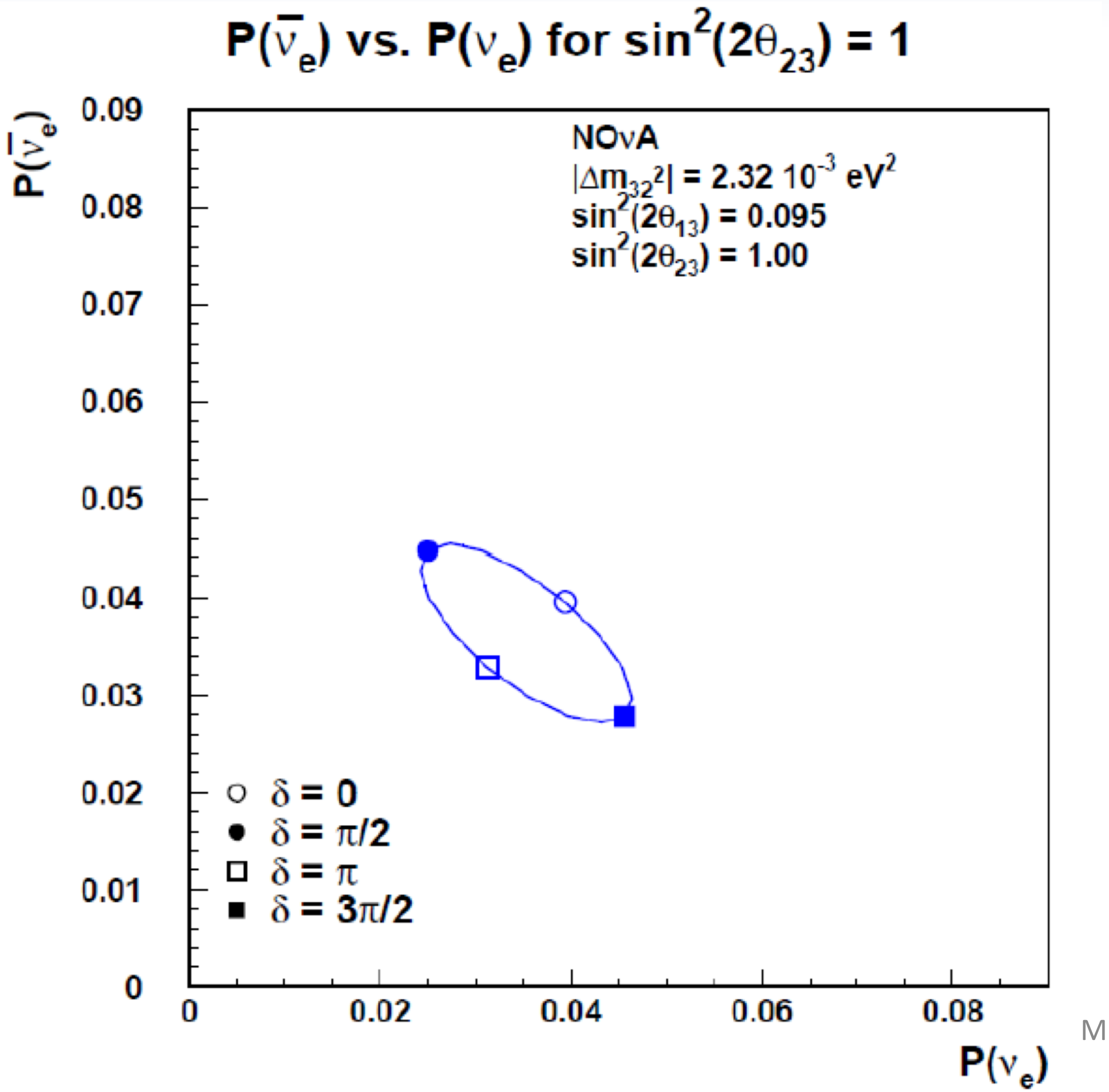


Measure a single point in $P(\nu_e)$ appearance, $P(\bar{\nu}_e)$ appearance space…

The "Bi-Probability" Plot:

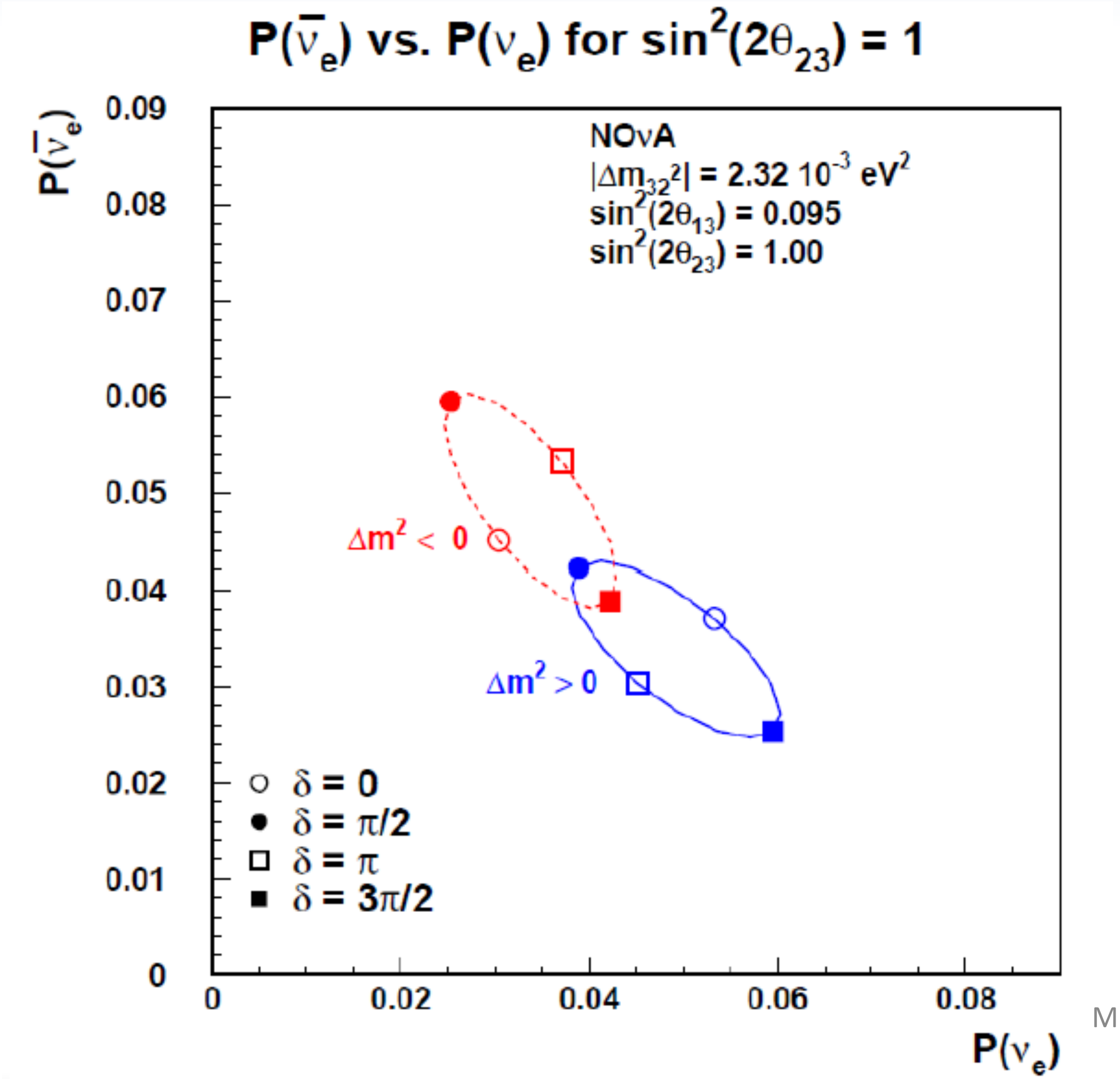


The "Bi-Probability" Plot:



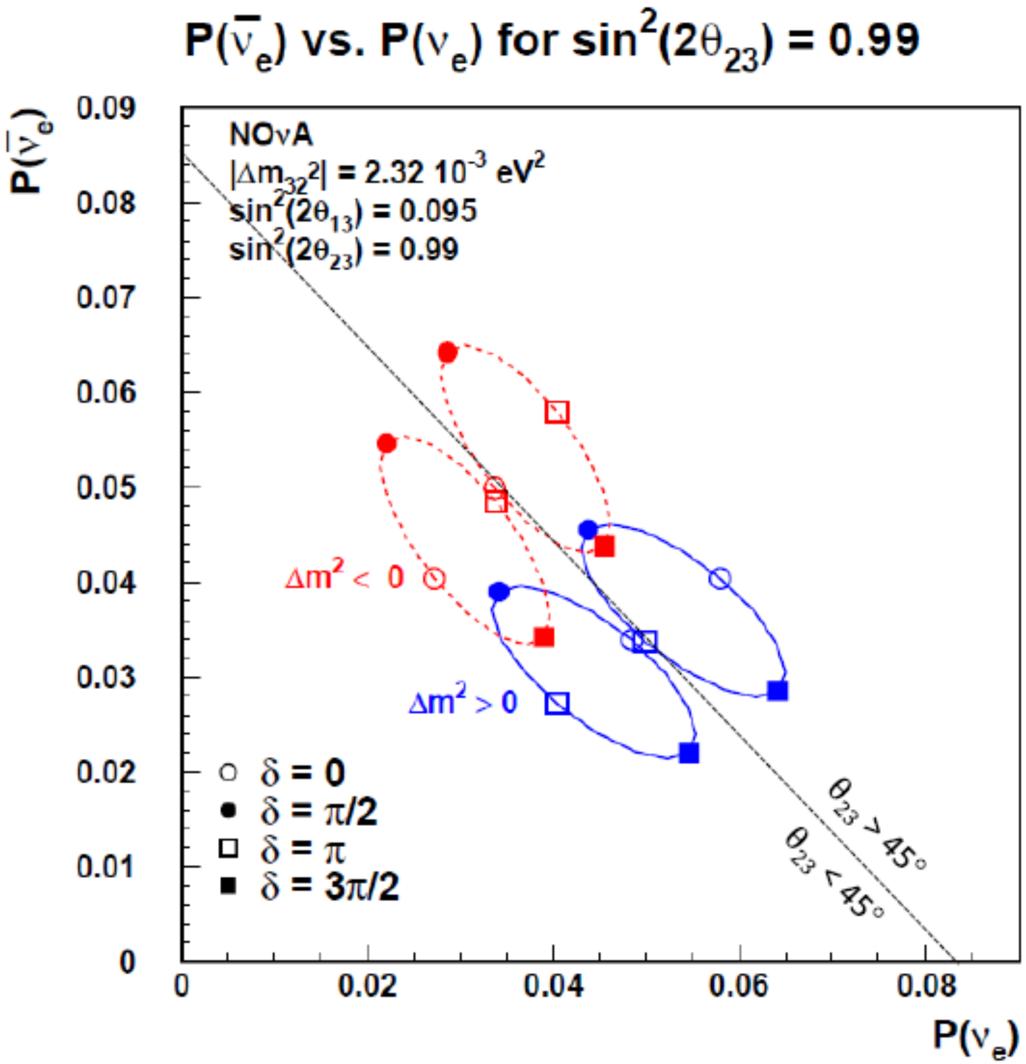
- **Including CP violation**

The "Bi-Probability" Plot:



- **Including CP violation**
- **Including the matter effect**

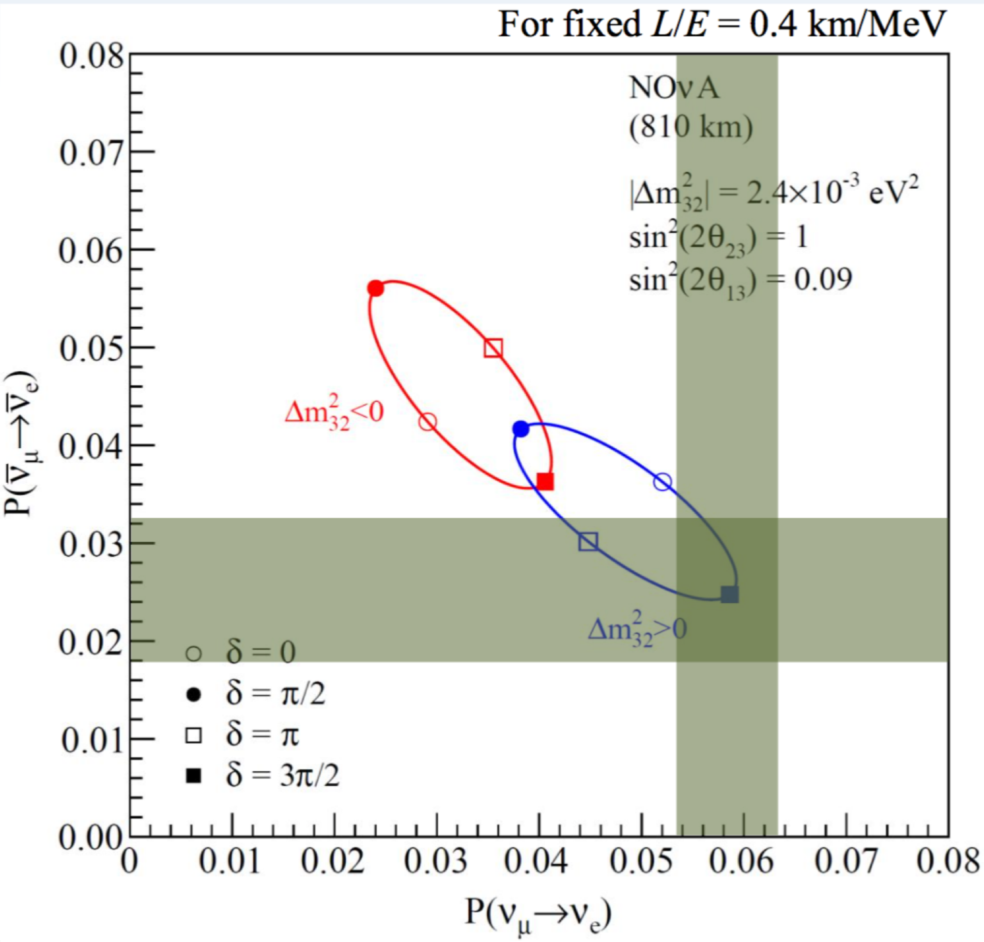
The "Bi-Probability" Plot:



- **Including CP violation**
- **Including the matter effect**
- **Including non-maximal Θ_{23}**

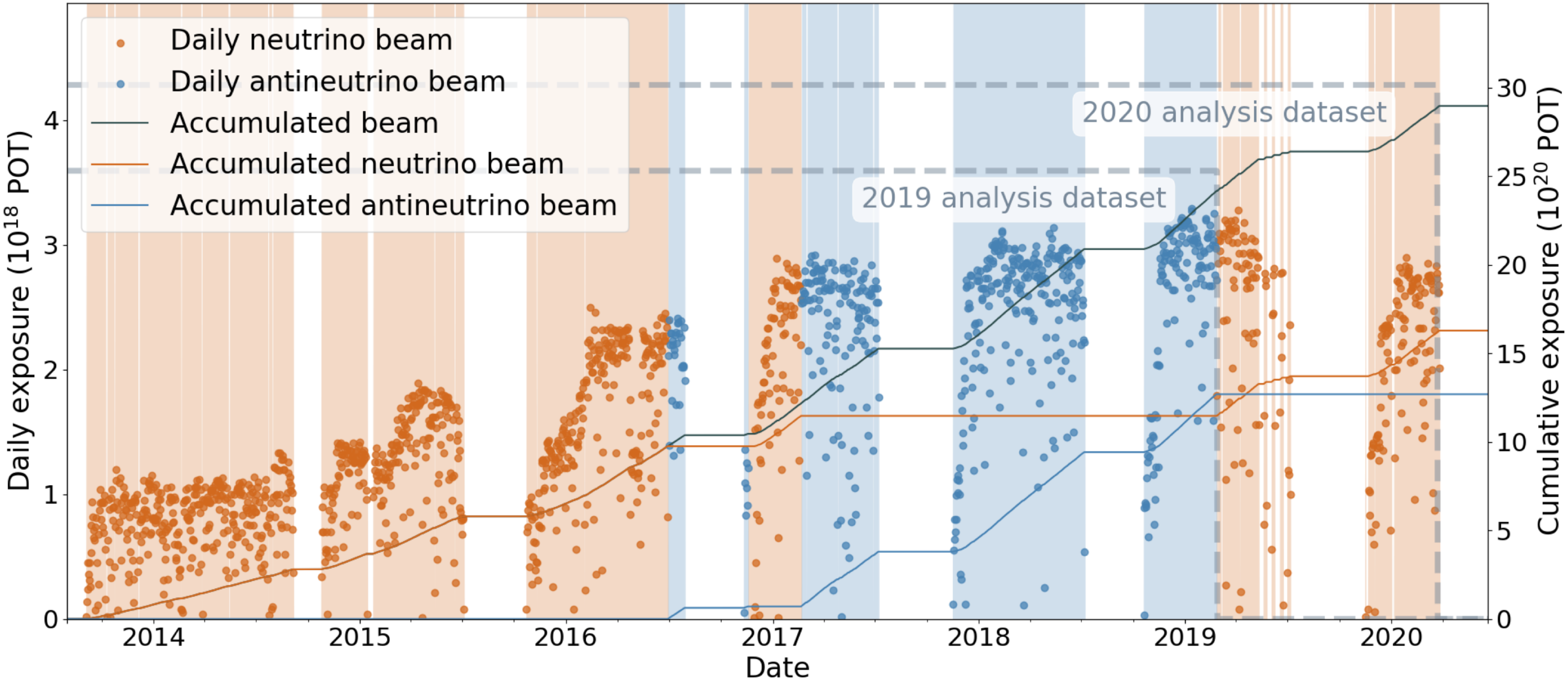
The "Bi-Probability" Plot:

A simultaneous measurement of ν_e appearance and $\overline{\nu}_e$ appearance will help us answer these open questions!



The NOvA Experiment:

NuMI - Neutrinos at the Main Injector



FHC = neutrino beam

RHC = anti-neutrino beam


Far Detector site


Block construction

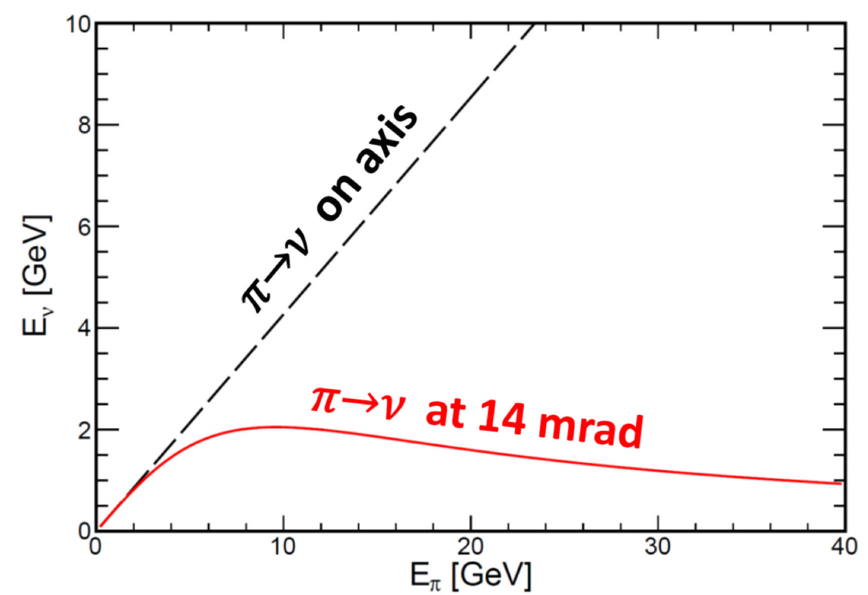

Outfitted Far Detector

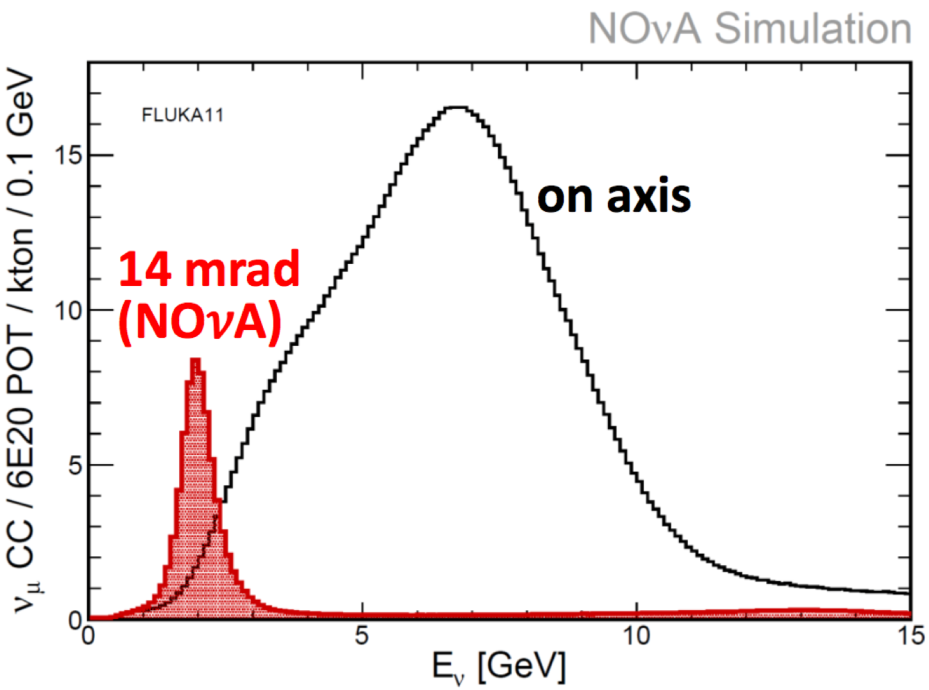

Near Detector

The NOvA Experiment: NuMI Beam

The NOvA experiment is 14 mrad off-axis:

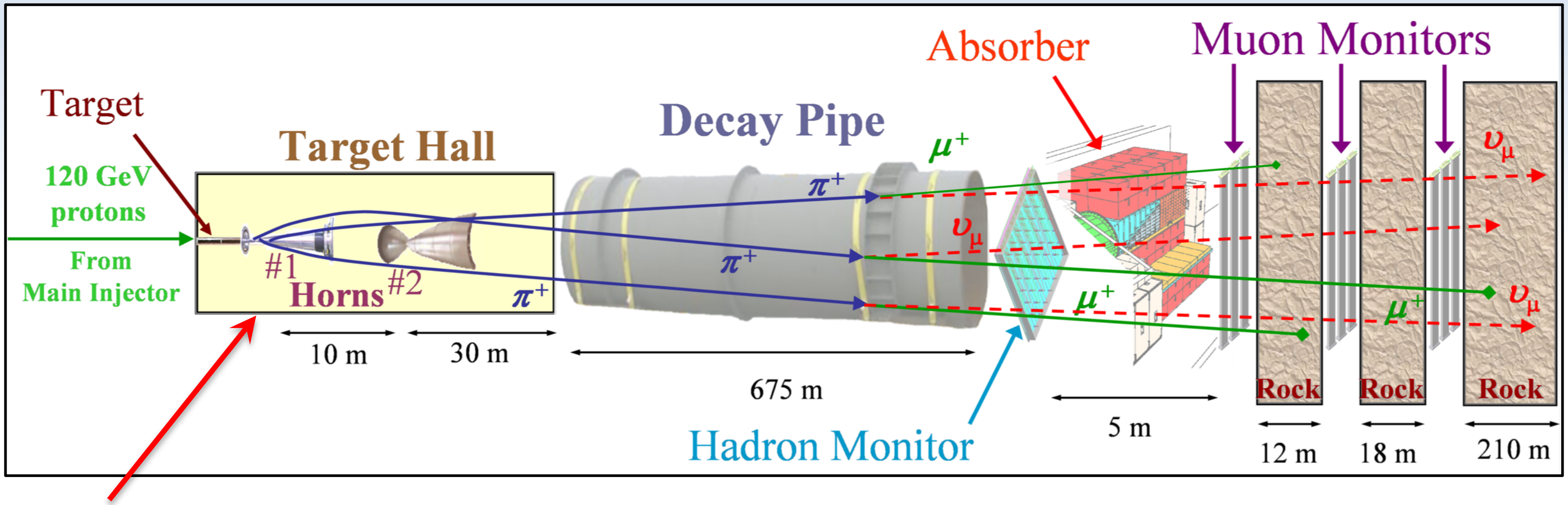
- gives us a narrowly peaked *ν* energy spectrum at 2 GeV
- 2 GeV = oscillation max for 810 km
- helps reduce NC backgrounds





$$E_\nu = \frac{0.43 E_\pi}{1 + \gamma^2 \theta^2}$$

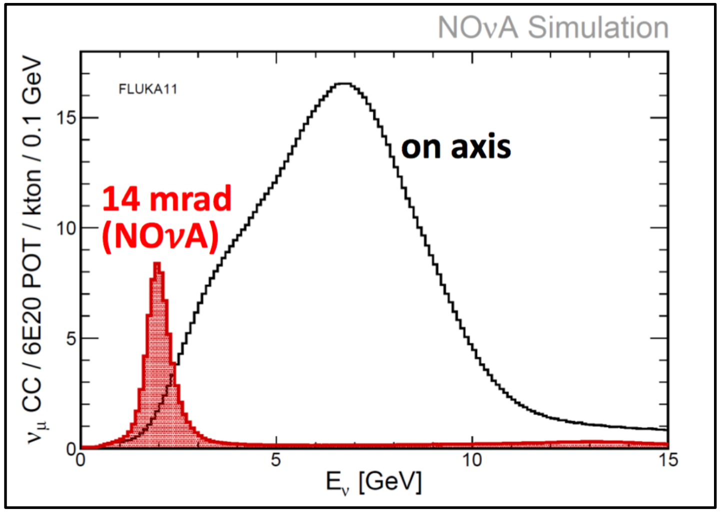
The NOvA Experiment: NuMI Beam



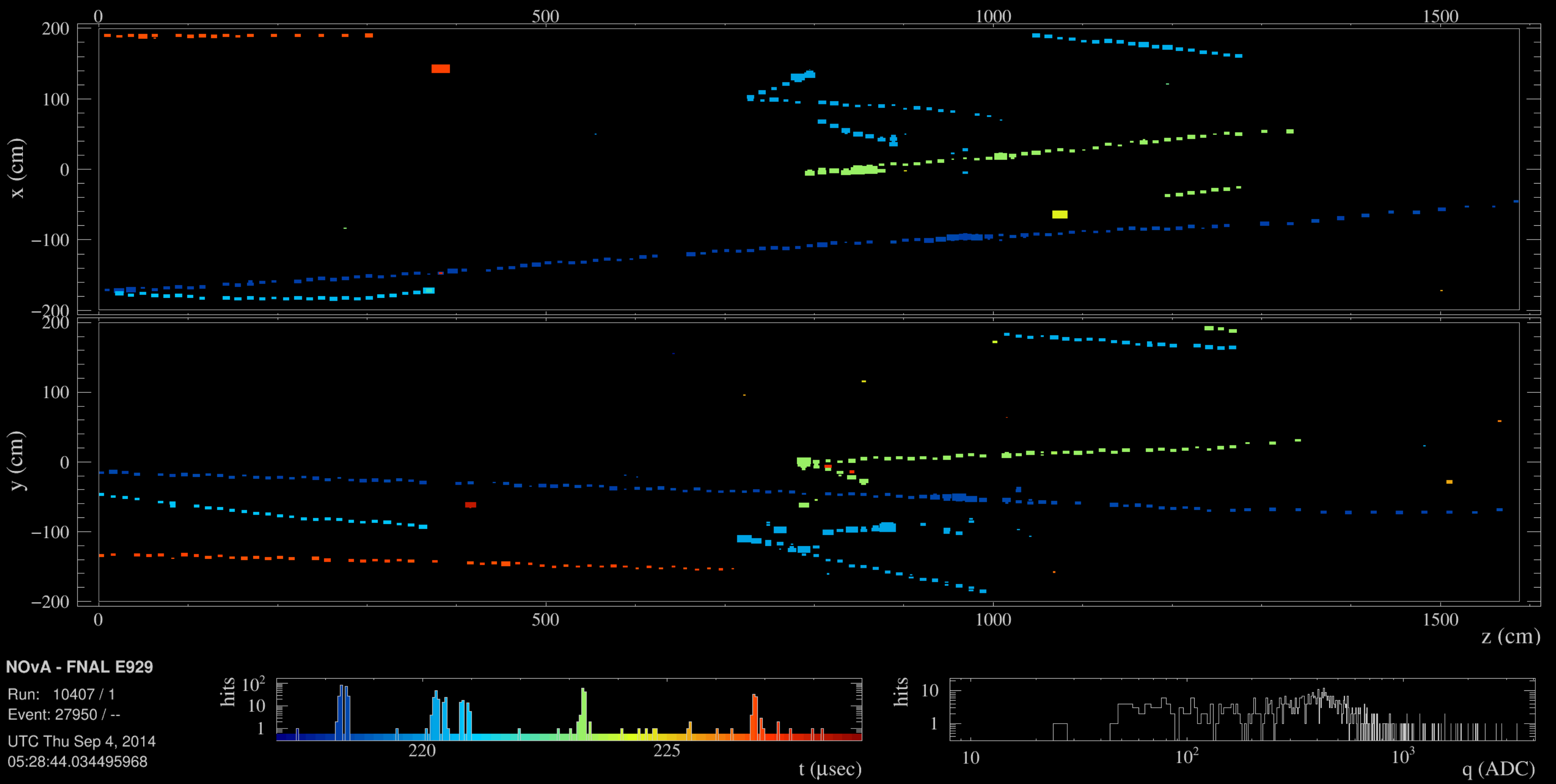
The horns can run in forward current (FHC) or reverse current (RHC) modes to create a neutrino or anti-neutrino beam respectively.

The NOvA experiment is 14 mrad off-axis:

- gives us a narrowly peaked *ν* energy spectrum at 2 GeV
- 2 GeV = oscillation max for 810 km
- helps reduce NC backgrounds

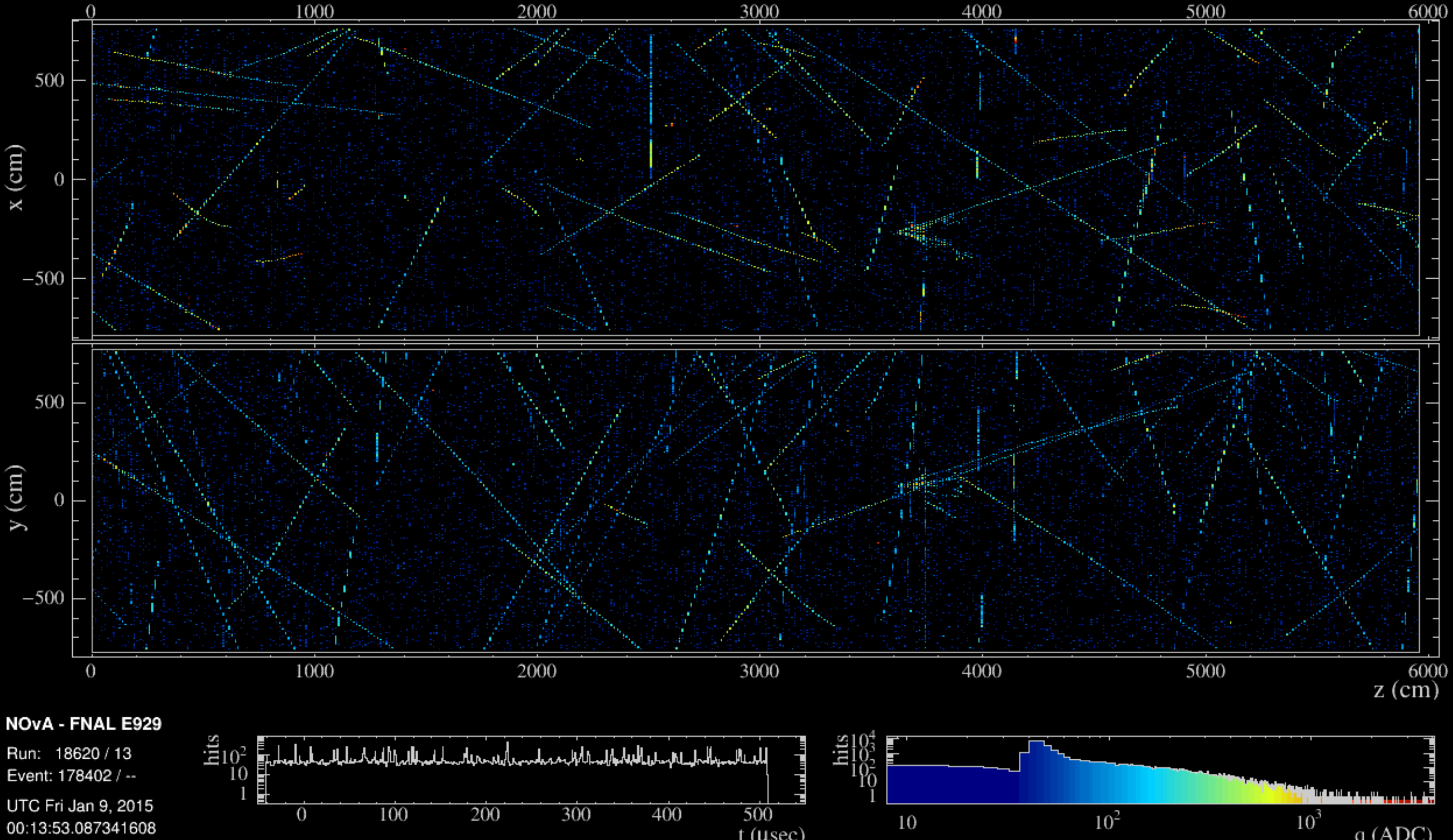


Near Detector Event Display



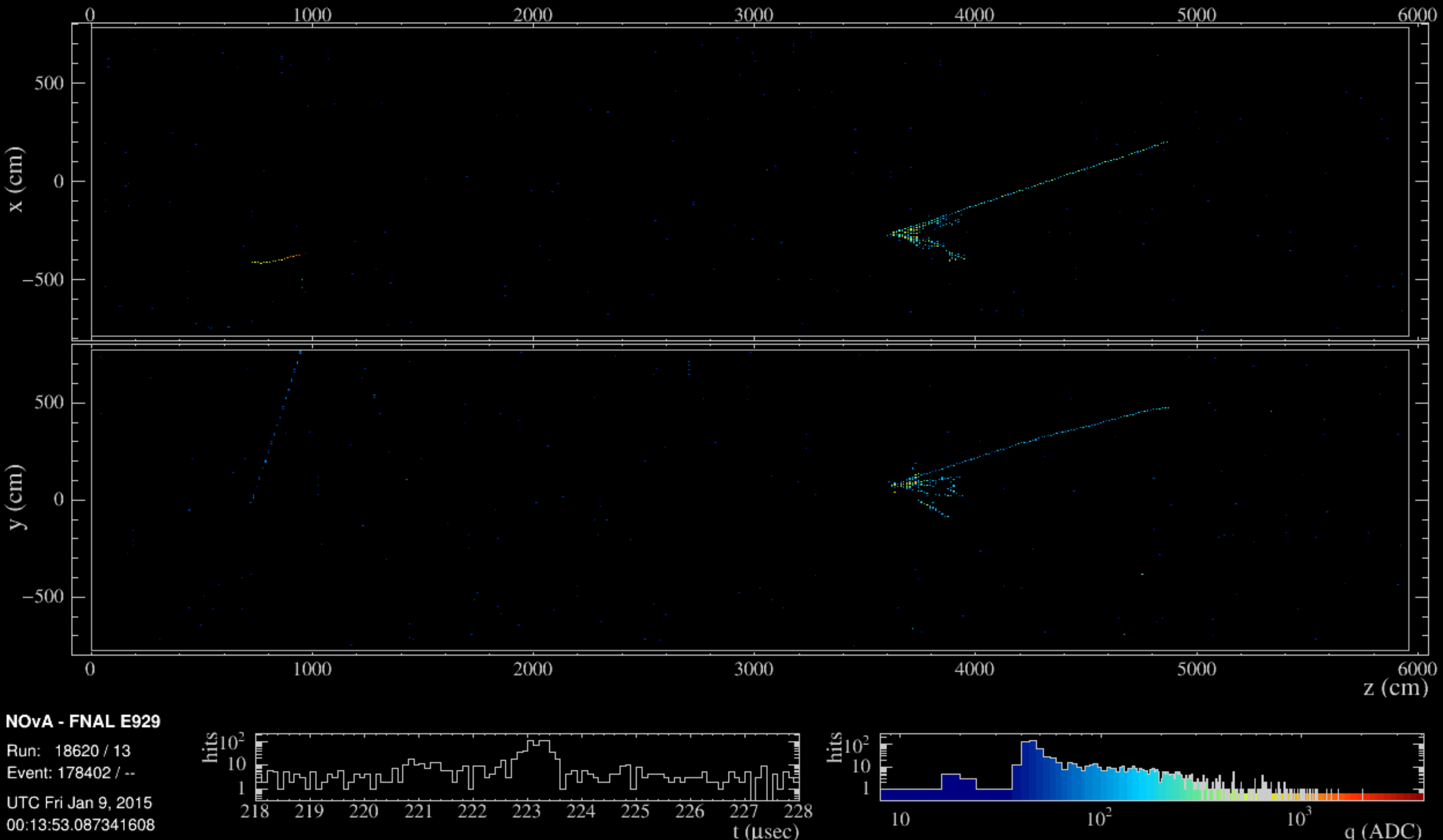
(colors show hit times)

Far Detector Event Display



(colors show charge)

Far Detector Event Display

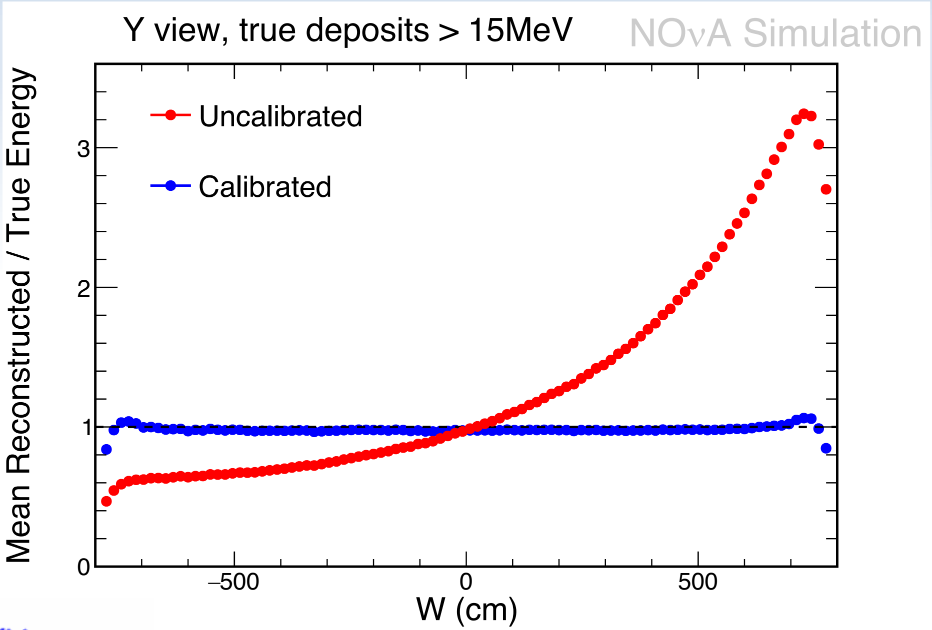


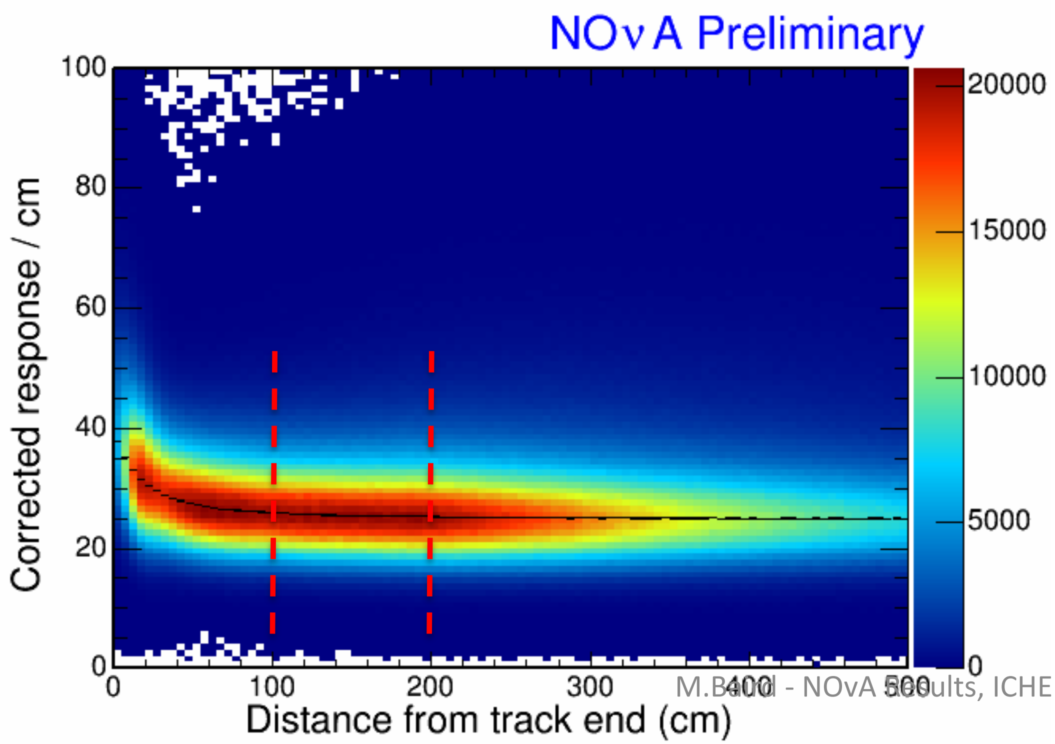
(colors show charge)

zoomed in on beam window

Calibration:

- Compute the attenuation curve for each fiber individually using through-going cosmic muons.
- This puts all fibers and cells on an equal footing.

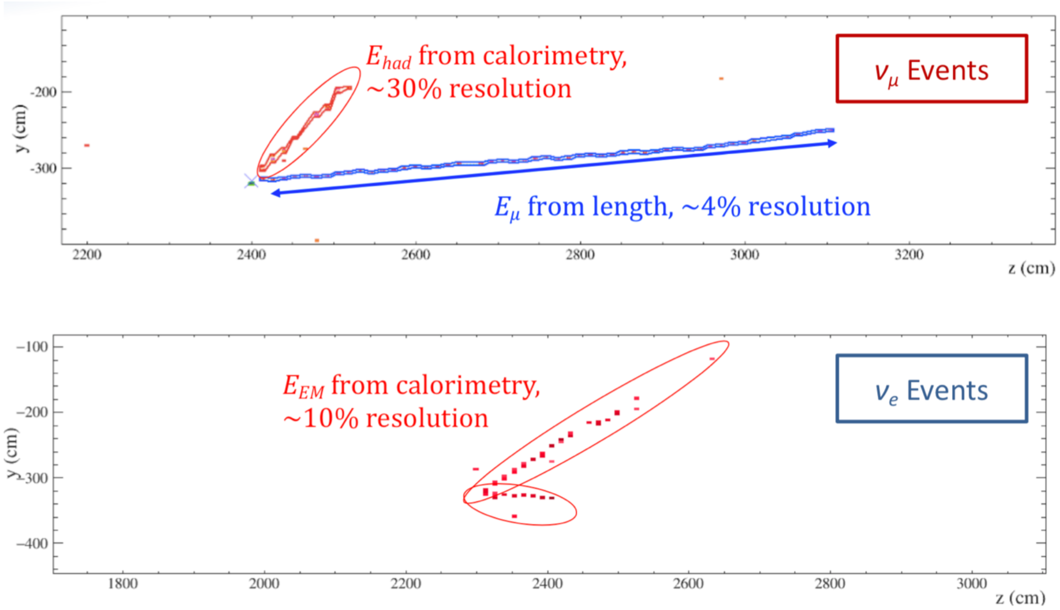


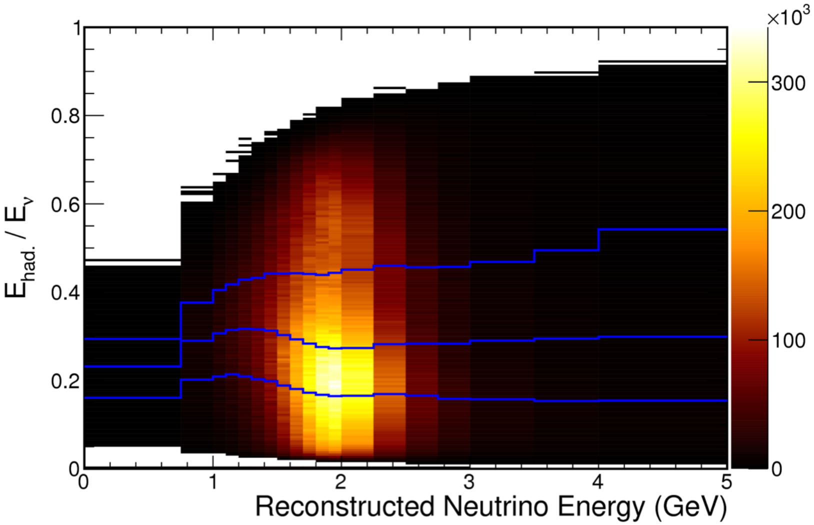


- Compute the absolute energy scale for the whole detector using stopping cosmic muons.
- Look for "good" hits in the MIP region of the track.

Energy Reconstruction:

- Muon energy reco done with track length, EM and hadronic energy converted from calorimetry.
- Numu events broken into 4 quantiles by hadronic energy fraction, separating events by energy resolution and background contamination.

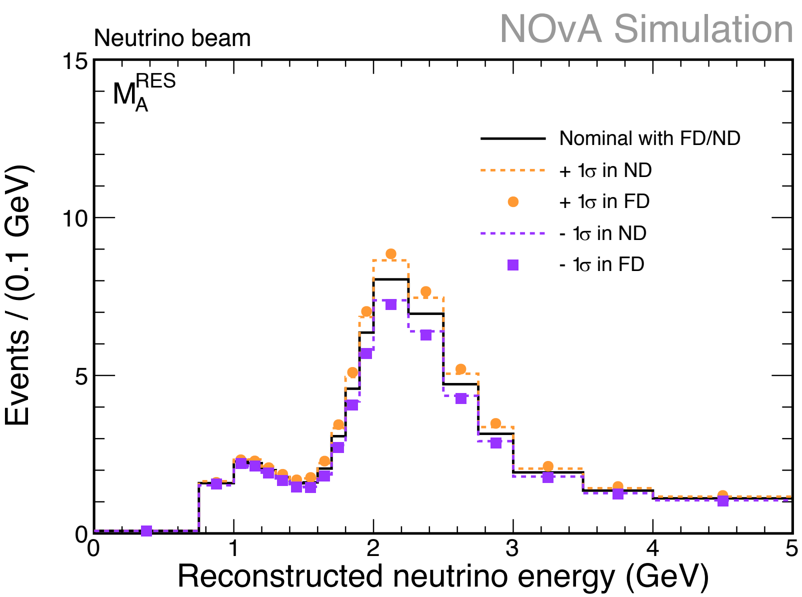


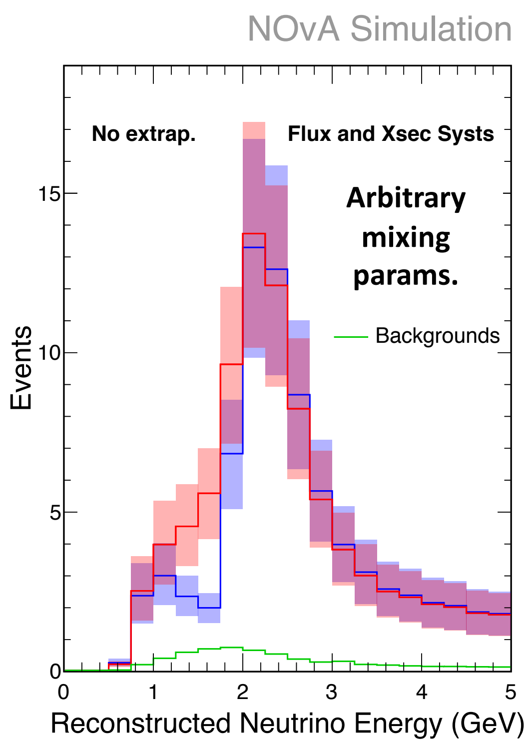


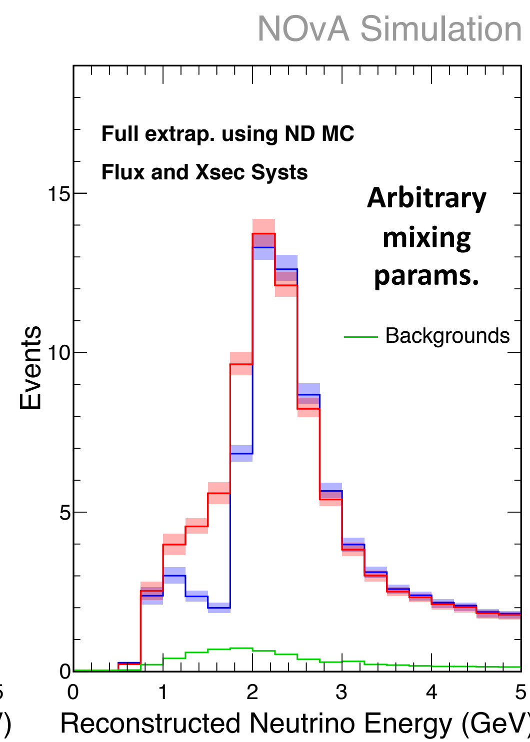
Far Detector Prediction:

- The near detector data is used to adjust the far detector prediction via an extrapolation procedure. Any shape/normalization differences between the ND data and MC will be translated to the FD.

- This helps reduce some of our major detector-correlated systematics such as flux and cross sections.







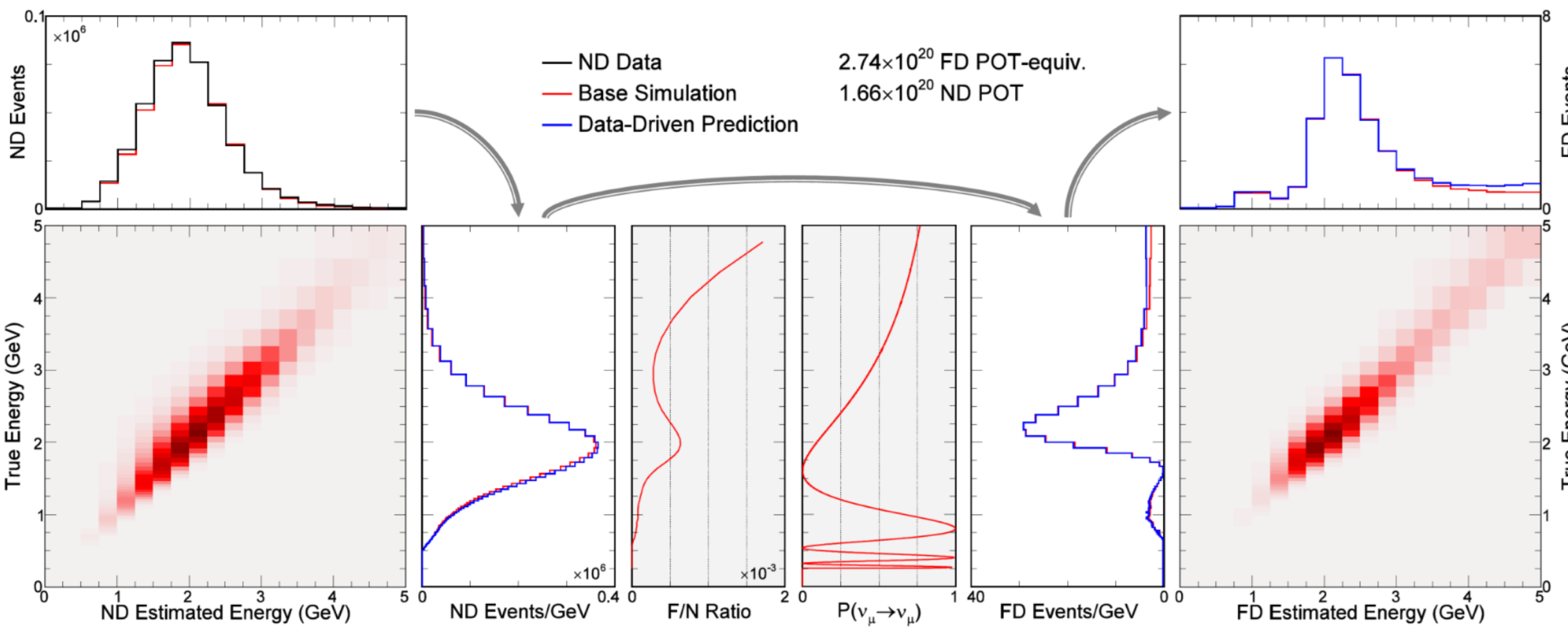
Far Detector Prediction:

The far detector prediction is generated using a near to far extrapolation process.

This extrapolation is repeated separately for each numu quantile.

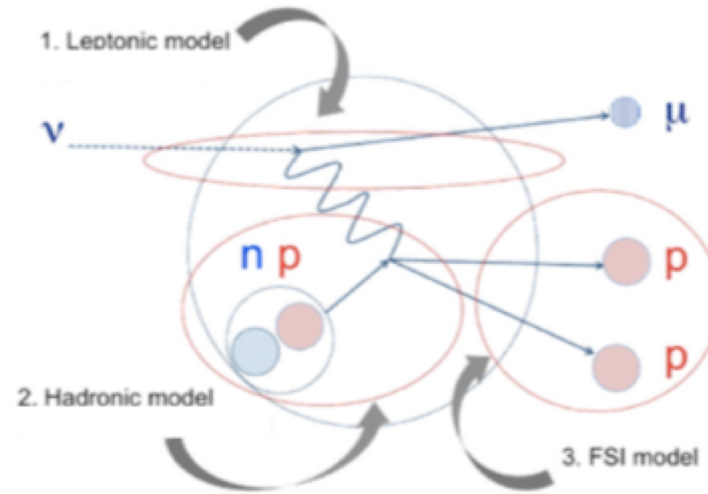
ND reconstructed E spectra

FD predicted E spectra



Neutrino Interaction Model

- Constantly evolving understanding of ν interactions.
- Upgrade to GENIE 3.0.6 → freedom to choose models
- Chose the most "theory-driven" set of models plus GENIE's re-tune of some parameters*.
- Some custom tuning is still required.
  - Substantially less than was needed with GENIE 2.12.2, which required tweaks to most models.



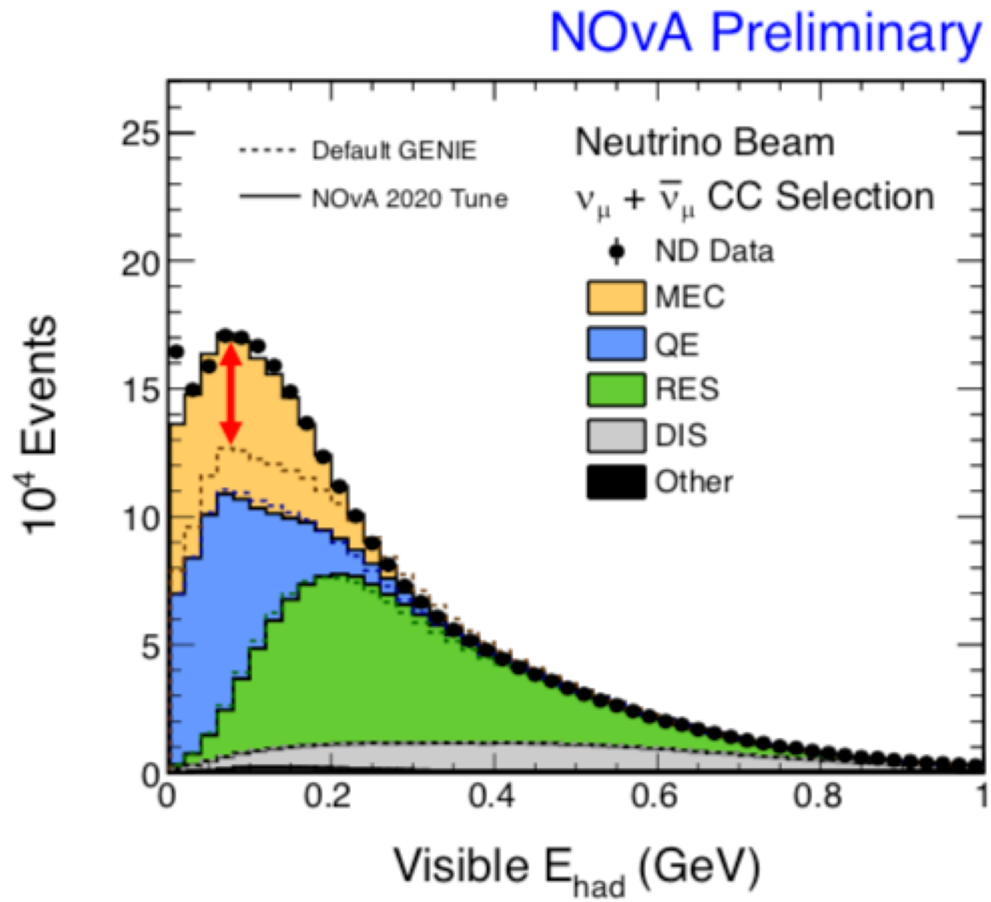
Process	Model	Reference
Quasielastic	Valencia 1p1h	J. Nieves, J. E. Amaro, M. Valverde, Phys. Rev. C 70 (2004) 055503
Form Factor	Z-expansion	A. Meyer, M. Betancourt, R. Gran, R. Hill, Phys. Rev. D 93 (2016)
Multi-nucleon	Valencia 2p2h	R. Gran, J. Nieves, F. Sanchez, M. Vicente Vacas, Phys. Rev. D 88 (2013)
Resonance	Berger-Sehgal	Ch. Berger, L. M. Sehgal, Phys. Rev. D 76 (2007)
DIS	Bodek-Yang	A. Bodek and U. K. Yang, NUINT02, Irvine, CA (2003)
Final State Int.	hN semi-classical cascade	S. Dytman, Acta Physica Polonica B 40 (2009)

* We call our tune N1810j_0211a, and it is built by starting with G1810b_0211a and substituting the Z-expansion form factor for the dipole one. This combination was not available in the 3.0.6 release, but it may be available in future versions.

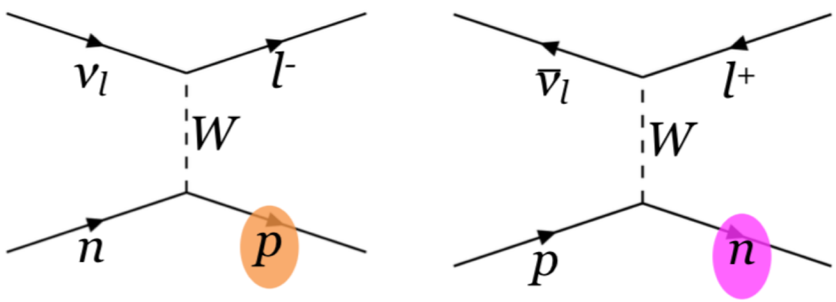
Fig: Teppei Katori, "Meson Exchange Current (MEC) Models in Neutrino Interaction Generators" AIP Conf.Proc. 1663 (2015) 030001

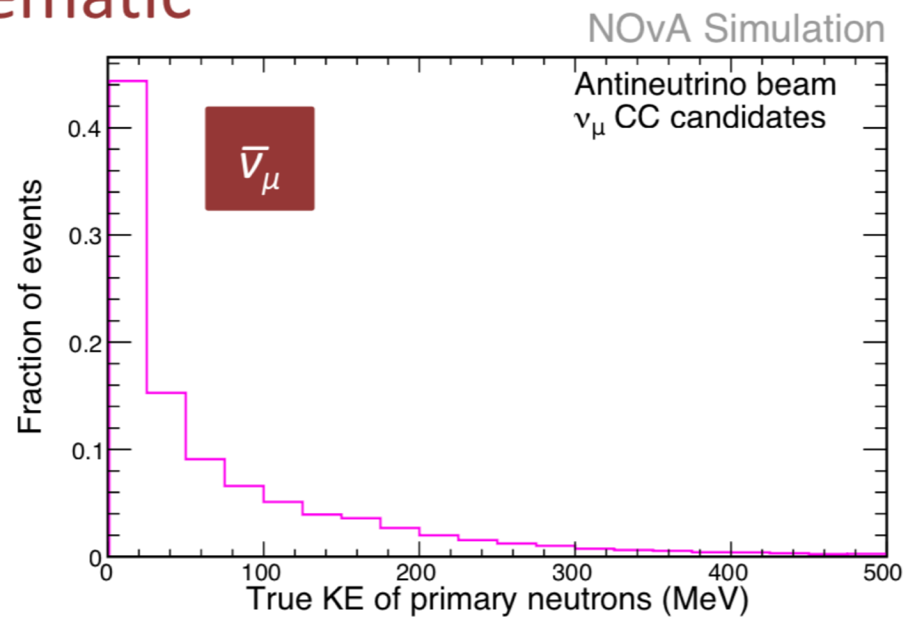
Neutrino Interaction Model

- 2p2h *or* Meson Exchange Current *or* Multi-nucleon Interactions:
  - Disagreement of models with multiple experiments well-known
  - Tuned to **NOvA ND data** with two 2D gaussians in q_0-$|\vec{q}|$ space.
  - Generous systematics covering normalization and kinematic shape
- Final State Interactions
  - Used **external π-scattering data** primarily to set uncertainties
  - Required adjusting central value, change in overall xsec was small.

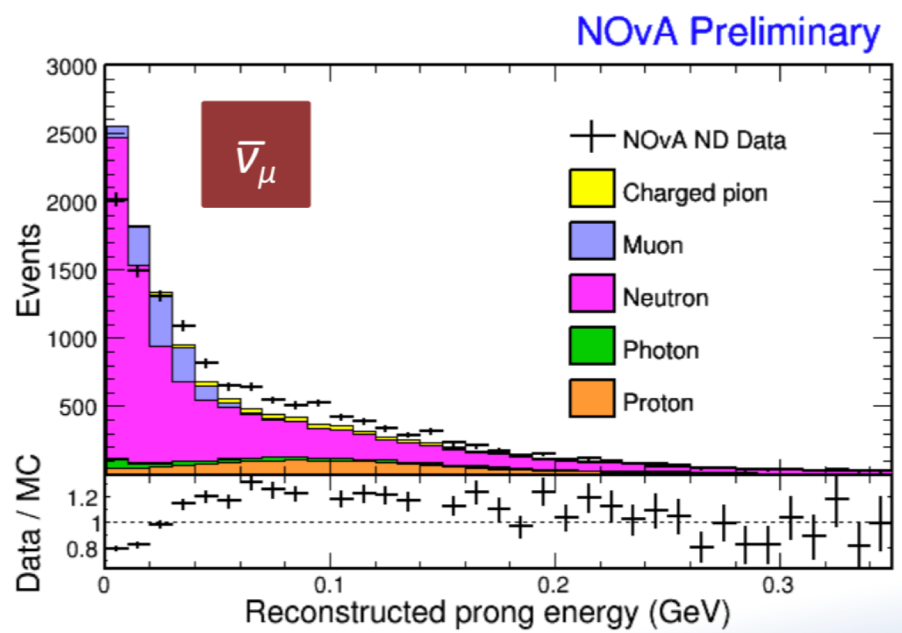


New neutron response systematic



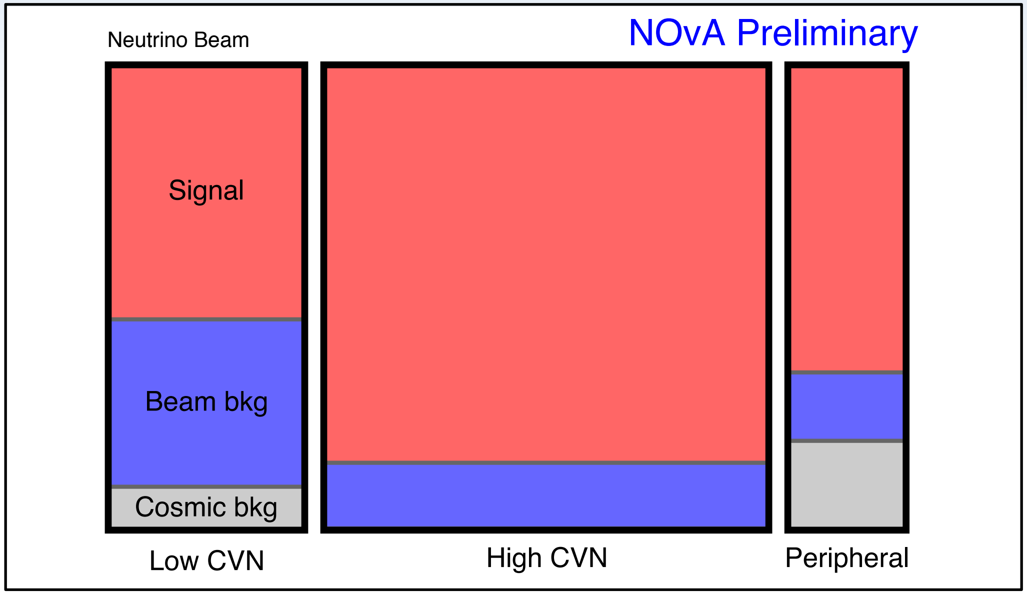


- $\bar{\nu}$'s have neutrons where ν's have protons.
  - Often several hundred MeV of energy.
  - Modeling these fast neutrons is known to be challenging.
- See some discrepancies in an enriched sample of neutron-like prongs.
- New systematic introduced:
  - Scales the amount of deposited energy of some neutrons to cover the low-energy discrepancy.
- Shifts the mean ν_μ energy by 1% in the antineutrino beam and 0.5% in the neutrino beam.
  - Negligible impact was seen on selection efficiencies.



ν_e Background Decomposition:

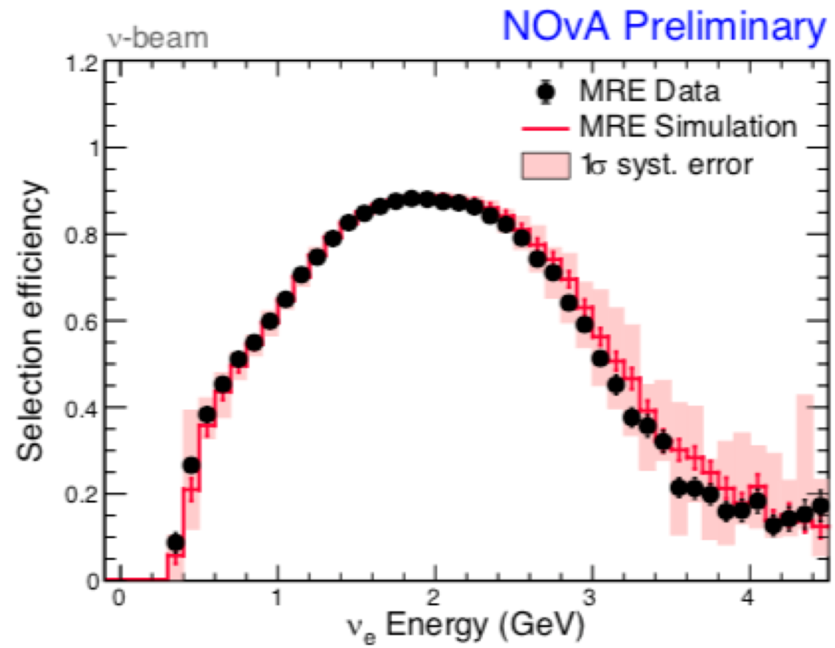
We expect more backgrounds in the ν_e event sample, and each component extrapolates differently to the FD.

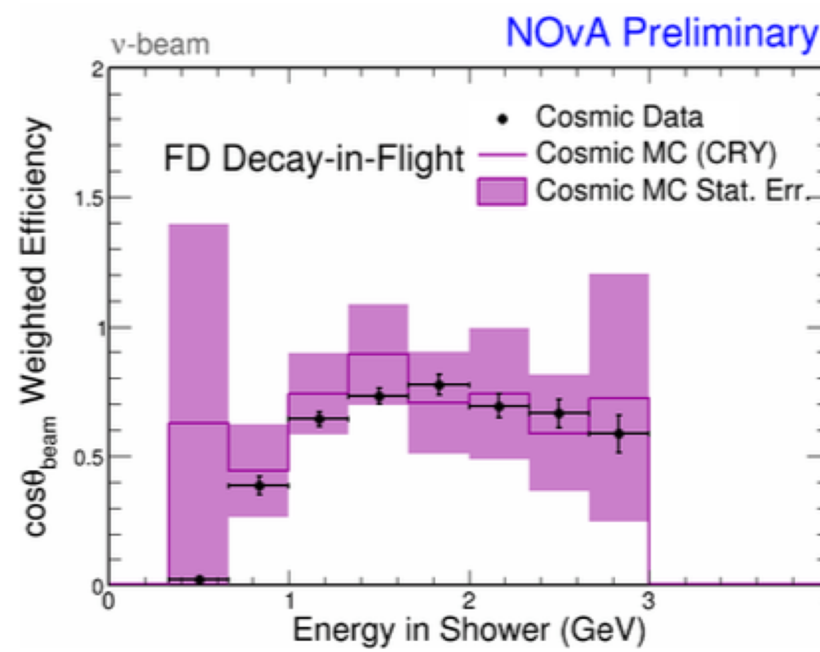


- We can improve our background estimates using statistical techniques (tracking ν_e events to parent pions, michele e^- tagging) by comparing ND data and MC.
- Results in adjustments to expected ND background components
- Stats are too low in RHC data, so we fall back on a proportional decomposition method.

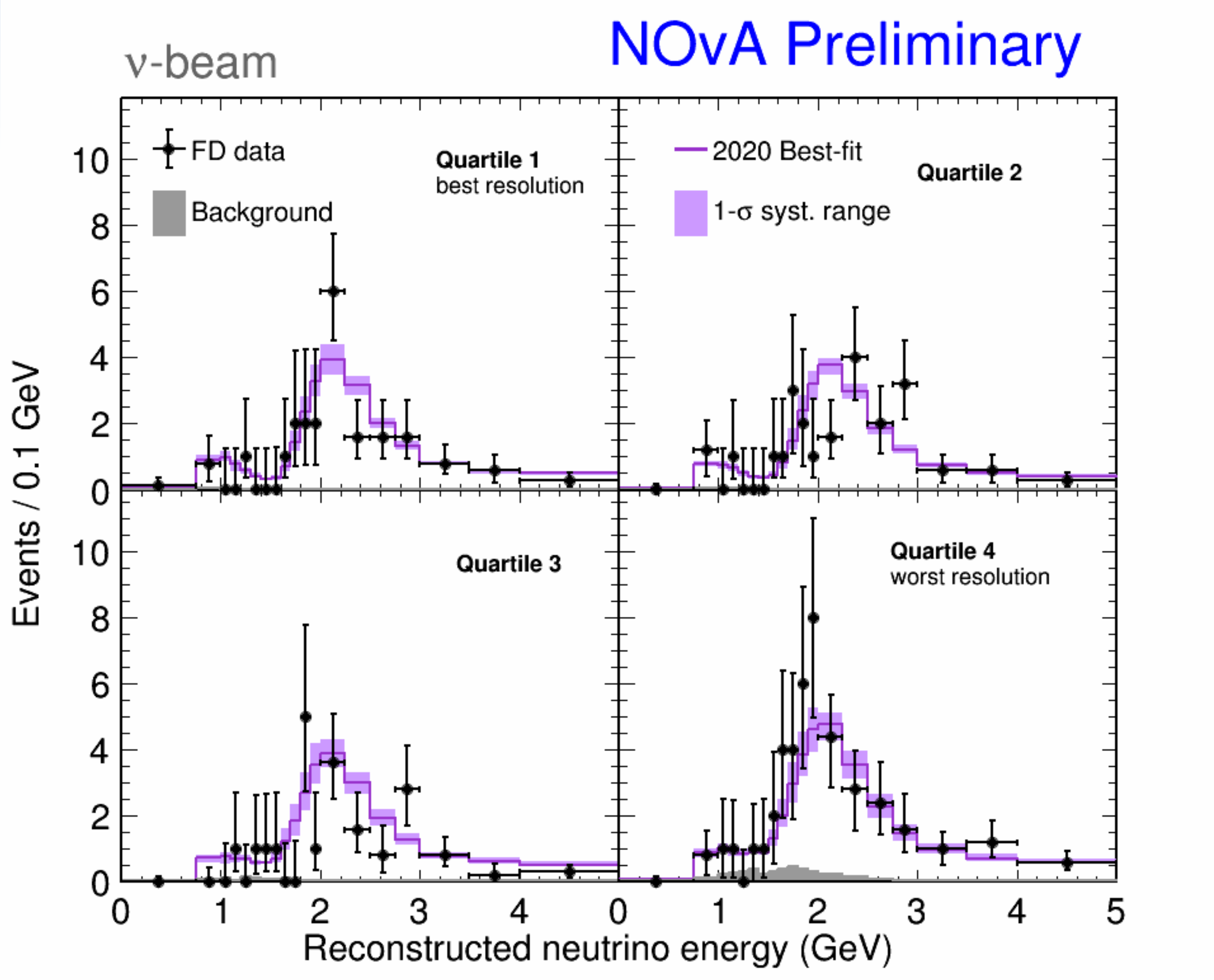
Selection: Validating Performance

- Examine PID efficiency relative to pre-selection.
  - Specifically target the behavior of the PID.
- ND: mixed data-MC sample
  - Mix simulated electrons and real hadronic showers
- FD: decay-in-flight electrons
  - Real electron showers from cosmic muons which decay

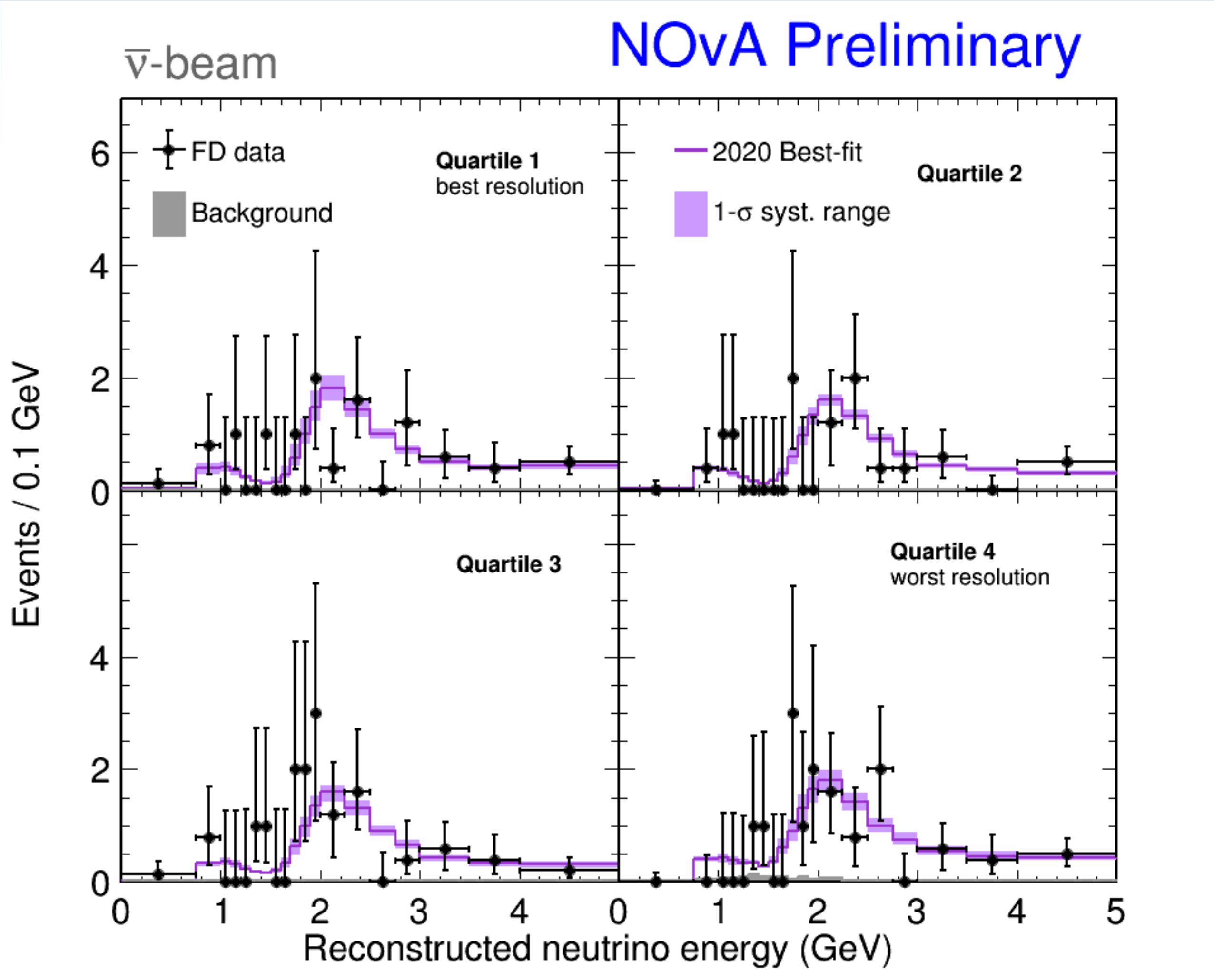




Numu Quantlies



Numu Quantlies

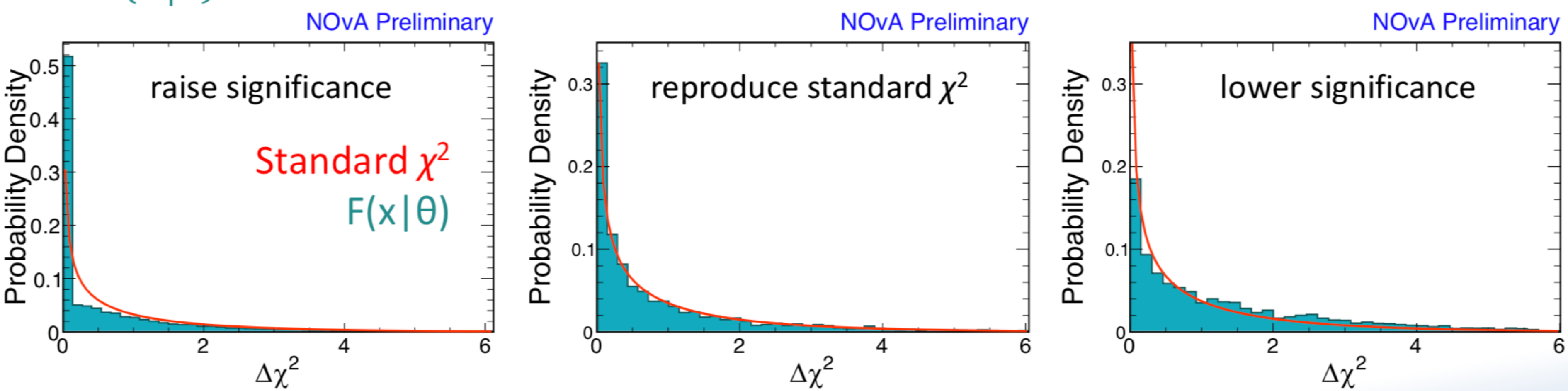


Statistical Approach: Feldman-Cousins

- Replace the standard χ^2 with an empirical distribution, $F(x|\theta)$:

$$F(x|\theta) = \text{Fraction of } N \text{ experiments where } [\, \chi^2(\text{fixed } \theta) - \chi^2(\text{best fit}) = x \,]$$

- Pseudo-experiments are generated from the data profile at θ.
  - i.e. fit all other parameters to data holding θ fixed at a particular value.
  - This procedure gives proper coverage while minimizing over-coverage.*
- A point θ is inside the (1-α) confidence interval if less than (1-α) experiments are more extreme than the data.
  - i.e. if the integral of $F(x|\theta)$ up to the observed $\Delta\chi^2$ at θ is < (1-α).
- $F(x|\theta)$ can…



* Test coverage using method from: R. L. Berger and D. D. Boos, J. Amer. Statist. Assoc., 89, 1012 (1994)