Sufficient and Necessary Conditions for CP Conservation with Majorana Neutrinos

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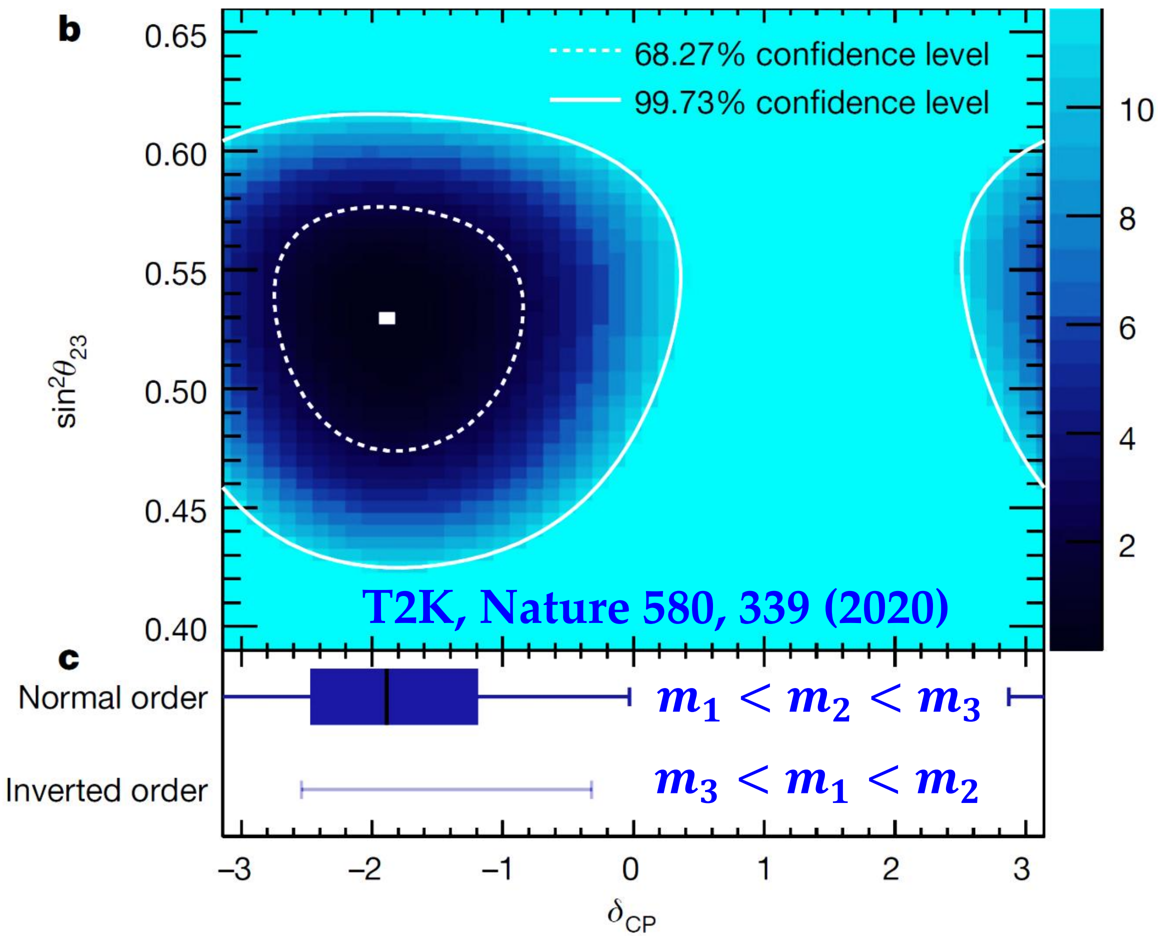
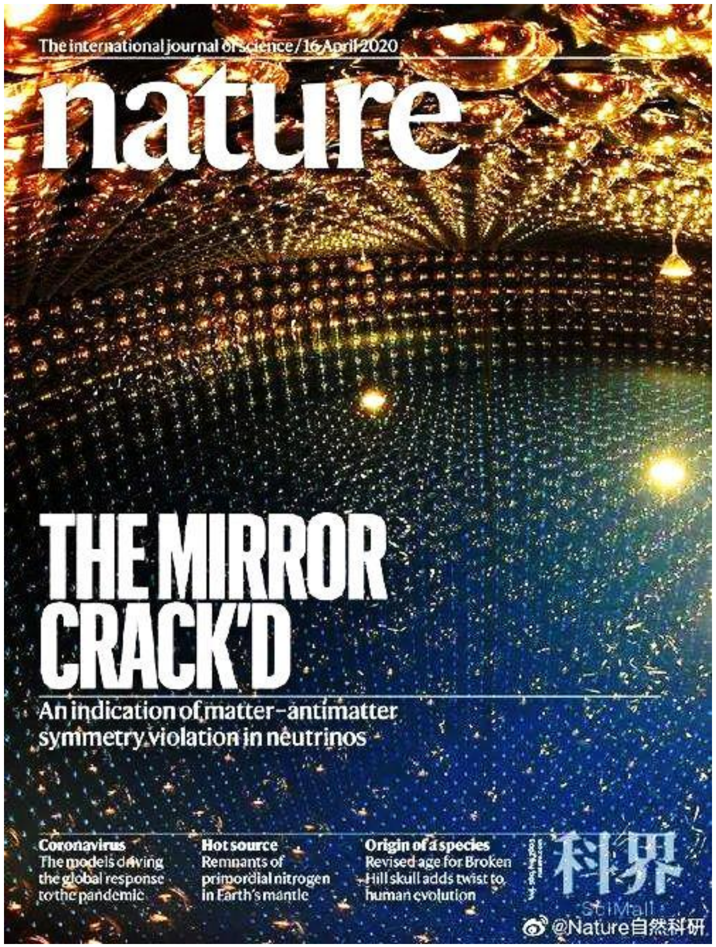
- CP violation (CPV) desirable for a dynamical generation of the matter-antimatter asymmetry (Sakharov, 1967)
- CPV discovered in the quark sector, and expected in the lepton sector as well (Xing, Phys. Rept., 2020)





CPV with Majorana Neutrinos

➢ **Extend the SM with massive Majorana neutrinos**

Lepton mass terms

$$-\mathcal{L}_{\rm mass} = \overline{l_{\rm L}} M_l l_{\rm R} + \frac{1}{2}\overline{\nu_{\rm L}} M_\nu \nu_{\rm L}^{\rm C} + {\rm h.c.}$$

$$\nu_{\rm L}^{\rm C} \equiv \mathcal{C}\overline{\nu_{\rm L}}^{\rm T}$$
$$\mathcal{C} \equiv {\rm i}\gamma^2\gamma^0$$

Charged-lepton mass matrix

Majorana neutrino mass matrix

At this moment, the CC-interaction is diagonal in three lepton flavors

➢ **Transform into the mass basis**

$$U_{l\rm L}^\dagger M_l U_{l\rm R} = \widehat{M}_l$$

$$U_{\nu\rm L}^\dagger M_\nu U_{\nu\rm L}^* = \widehat{M}_\nu$$

Leptonic CC interaction

$$\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\overline{\hat{l}_{\alpha\rm L}}\gamma^\mu V_{\alpha i}\hat{\nu}_{i\rm L} W_\mu^- + {\rm h.c.}$$

$$V = U_{l\rm L}^\dagger U_{\nu\rm L}$$

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-{\rm i}\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{{\rm i}\delta} & +c_{12}c_{23} - s_{12}s_{13}s_{23}e^{{\rm i}\delta} & c_{13}s_{23} \\ +s_{12}s_{23} - c_{12}s_{13}c_{23}e^{{\rm i}\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{{\rm i}\delta} & c_{13}c_{23} \end{pmatrix} \cdot \begin{pmatrix} e^{{\rm i}\rho} & 0 & 0 \\ 0 & e^{{\rm i}\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

CP violation stems from three nontrivial phases in the PMNS matrix *V*

Sufficient & Necessary Conditions-I

- **In the mass basis, one can verify that the sufficient and necessary condition for CP conservation is**

$$V = V^* \quad \Leftrightarrow \quad V_{\alpha i} = V^*_{\alpha i}$$

CP transformations:

$$\hat{l}_\alpha \Rightarrow \mathcal{C}\hat{l}^*_\alpha \qquad \hat{\nu}_i \Rightarrow \mathcal{C}\hat{\nu}^*_i \qquad W^-_\mu \Rightarrow -(-1)^{\delta_{0\mu}} W^+_\mu$$

While lepton mass terms are invariant, the CC-interaction term becomes

$$\overline{\hat{l}_{\alpha \mathrm{L}}} \gamma^\mu V_{\alpha i} \hat{\nu}_{i\mathrm{L}} W^-_\mu \quad \Rightarrow \quad \overline{\hat{\nu}_{i\mathrm{L}}} \gamma^\mu V_{\alpha i} \hat{l}_{\alpha \mathrm{L}} W^+_\mu$$

$$(\mathrm{h.c.}) = \overline{\hat{\nu}_{i\mathrm{L}}} \gamma^\mu V^*_{\alpha i} \hat{l}_{\alpha \mathrm{L}} W^+_\mu$$

CP invariance requires all CP phases in the PMNS matrix V to be vanishing or physically trivial.

Sufficient & Necessary Conditions-I

- **All the CP phases in the PMNS matrix *V* come from the complex lepton mass matrices (i.e., Yukawa couplings)**

$$U_{l\mathrm{L}}^{\dagger} M_l U_{l\mathrm{R}} = \widehat{M}_l \qquad U_{\nu\mathrm{L}}^{\dagger} M_\nu U_{\nu\mathrm{L}}^{*} = \widehat{M}_\nu \qquad \boxed{V = U_{l\mathrm{L}}^{\dagger} U_{\nu\mathrm{L}}}$$

which should be given explicitly in a neutrino mass model

- **Obviously, if we require M_l and M_ν to be real, then CP symmetry is preserved in the lepton sector. However, this is a sufficient but NOT necessary condition.**

For example, rotate lepton doublets in the flavor space by a unitary matrix *U*, and diagonalize the matrices M_l' & M_ν'

$$M_l' = U^{\dagger} M_l \qquad M_\nu' = U^{\dagger} M_\nu U^{*} \qquad \boxed{V = U_{l\mathrm{L}}^{\dagger} U U^{\dagger} U_{\nu\mathrm{L}} = V}$$

Question: what are sufficient & necessary conditions for CP conservation in an arbitrary flavor basis?

Weak-Basis Invariants

CPC vs. CPV: Any deviations from sufficient & necessary conditions for CP conservation imply CP violation.

We follow the approach of weak-basis invariants (WBI) in the lepton sector (Branco, Lavoura & Rebelo, 1986)

Generalized CP transformations:

$$l_{\rm L} \Rightarrow U_{\rm L} \mathcal{C} l_{\rm L}^{*} \qquad \nu_{\rm L} \Rightarrow U_{\rm L} \mathcal{C} \nu_{\rm L}^{*} \qquad l_{\rm R} \Rightarrow U_{\rm R} \mathcal{C} l_{\rm R}^{*} \qquad W_{\mu}^{-} \Rightarrow -(-1)^{\delta_{0\mu}} W_{\mu}^{+}$$

Under the above transformations, the theory is invariant if the following identities are satisfied:

$$U_{\rm L}^{\dagger} M_{\nu} U_{\rm L}^{*} = -M_{\nu}^{*} \; , \quad U_{\rm L}^{\dagger} M_{l} U_{\rm R} = M_{l}^{*}$$

Define $H_l \equiv M_l M_l^{\dagger} \qquad H_{\nu} \equiv M_{\nu} M_{\nu}^{\dagger} \qquad G_{l\nu} \equiv M_{\nu} H_l^{*} M_{\nu}^{\dagger}$

$$U_{\rm L}^{\dagger} H_l U_{\rm L} = H_l^{*} \qquad U_{\rm L}^{\dagger} H_{\nu} U_{\rm L} = H_{\nu}^{*} \qquad U_{\rm L}^{\dagger} G_{l\nu} U_{\rm L} = G_{l\nu}^{*}$$

Sufficient & Necessary Conditions-II

$$U_{\rm L}^{\dagger} H_l U_{\rm L} = H_l^* \qquad U_{\rm L}^{\dagger} H_\nu U_{\rm L} = H_\nu^* \qquad U_{\rm L}^{\dagger} G_{l\nu} U_{\rm L} = G_{l\nu}^*$$

It is straightforward to construct an infinite series of WBIs

$$\mathcal{I}^{abc}_{def} \equiv \mathrm{Im}\left\{\mathrm{Tr}\left[H_l^a H_\nu^b G_{l\nu}^c H_l^d H_\nu^e G_{l\nu}^f \cdots\right]\right\}$$

such that $I^{abc}_{def} = 0$ guarantees CP conservation.

Given non-degenerate lepton masses, four sufficient and necessary conditions have been found (Branco et al, 1986):

$$\widehat{\mathcal{I}}_1 \equiv \mathrm{Im}\left\{\mathrm{Tr}\left[H_l H_\nu G_{l\nu}\right]\right\} = 0$$

$$\widehat{\mathcal{I}}_2 \equiv \mathrm{Im}\left\{\mathrm{Tr}\left[H_l H_\nu^2 G_{l\nu}\right]\right\} = 0$$

$$\widehat{\mathcal{I}}_3 \equiv \mathrm{Im}\left\{\mathrm{Tr}\left[H_l H_\nu^2 G_{l\nu} H_\nu\right]\right\} = 0$$

$$\widehat{\mathcal{I}}_4 \equiv \mathrm{Im}\left\{\mathrm{Det}\left[G_{l\nu} + H_l H_\nu\right]\right\} = 0$$

Question: there are ONLY three CP-violating phases $\{\delta, \rho, \sigma\}$, why do we need four conditions?

Indeed a minimal set of three conditions have been given to ensure CP conservation (Dreiner et al, 2007):

$$\mathcal{I}_1 \equiv \mathrm{Tr}\left\{[H_\nu, H_l]^3\right\} = 0$$

$$\mathcal{I}_2 \equiv \mathrm{Im}\left\{\mathrm{Tr}\left[H_l H_\nu G_{l\nu}\right]\right\} = 0$$

$$\mathcal{I}_3 \equiv \mathrm{Tr}\left\{[G_{l\nu}, H_l]^3\right\} = 0$$

Problem: **one can show that these conditions are actually not sufficient for leptonic CP conservation!**

A counter example: $\mathcal{I}_1 = -6\mathrm{i}\Delta_{21}\Delta_{31}\Delta_{32}\Delta_{e\mu}\Delta_{\mu\tau}\Delta_{\tau e}\mathcal{J}$

$$\Delta_{ij} \equiv m_i^2 - m_j^2 \qquad \Delta_{\alpha\beta} \equiv m_\alpha^2 - m_\beta^2 \qquad \mathcal{J} \equiv s_{12}c_{12}s_{23}c_{23}s_{13}c_{13}^2 \sin\delta$$

First condition $I_1 = 0$ really leads to $\sin\delta = 0$ or $\delta = 0$ or π

$$I_2 = 0 \Rightarrow \quad 0 = f_1 \sin(2\rho) + f_2 \sin(2\sigma) + f_3 \sin(2\rho - 2\sigma)\ ,$$

$$I_3 = 0 \Rightarrow \quad 0 = g_1 \sin(2\rho) + g_2 \sin(2\sigma) + g_3 \sin(2\rho - 2\sigma) + g_4 \sin(2\rho + 2\sigma) + g_5 \sin(2\rho - 4\sigma) + g_6 \sin(2\sigma - 4\rho)$$

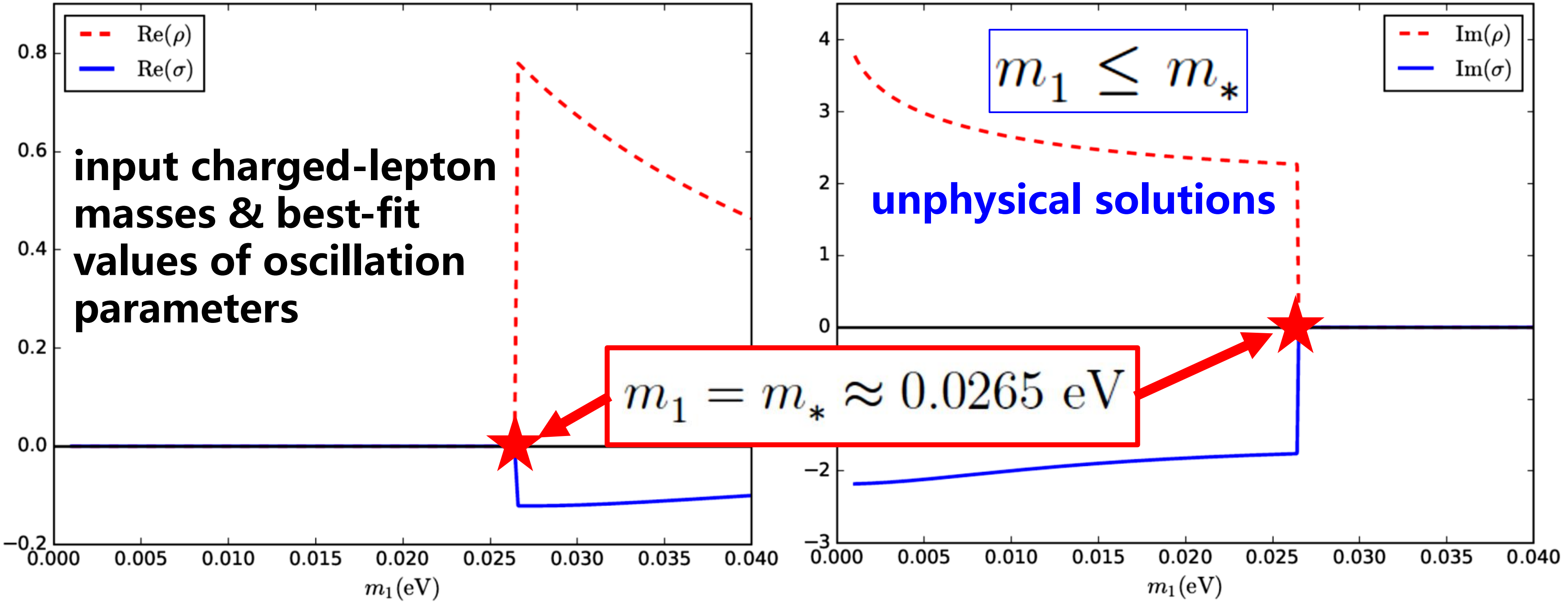
Sufficient & Necessary Conditions-II

$$0 = f_1 \sin(2\rho) + f_2 \sin(2\sigma) + f_3 \sin(2\rho - 2\sigma) \, ,$$
$$0 = g_1 \sin(2\rho) + g_2 \sin(2\sigma) + g_3 \sin(2\rho - 2\sigma) + g_4 \sin(2\rho + 2\sigma) + g_5 \sin(2\rho - 4\sigma) + g_6 \sin(2\sigma - 4\rho)$$

$\Rightarrow$ $\sin(2\rho) = 0$, $\sin(2\sigma) = 0$

Any other solutions?

Here f_i $(i = 1, 2, 3)$ and g_j $(j = 1, 2, \ldots, 6)$ are functions of all six lepton masses and three mixing angles

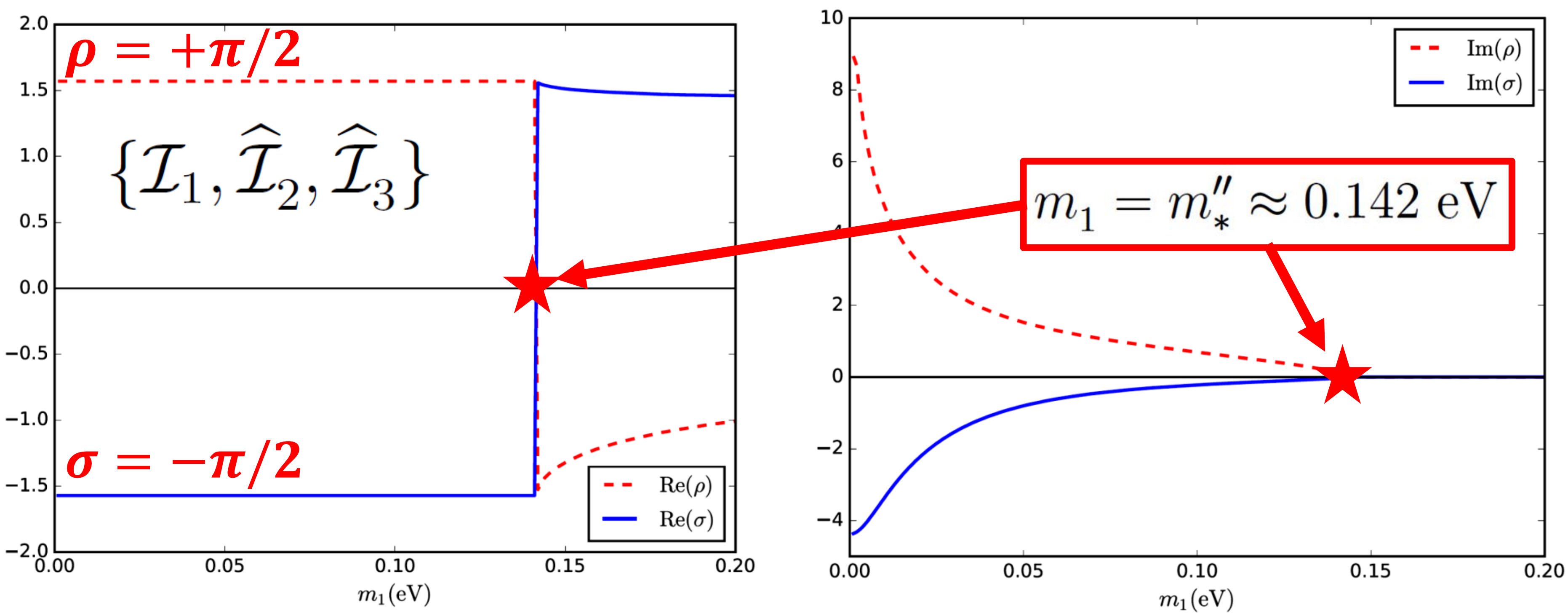


Sufficient & Necessary Conditions-II

What we learn form the counter example:

- **For $0 \leq m_1 < m_* \approx 0.0265$ eV, $I_1 = I_2 = I_3 = 0$ serve as three sufficient and necessary conditions for CPC**
- **For $m_1 > m_* \approx 0.0265$ eV, $I_1 = 0$ leads to $\sin\delta = 0$, but $I_2 = I_3 = 0$ have nontrivial solutions of $\{\rho, \sigma\}$, implying possible CPV**

New sets of WBIs (by no means unique)



Summary

- We have reexamined sufficient & necessary conditions for leptonic CP conservation with Majorana neutrinos
- The number of the minimal set of conditions depends on the yet unknown lightest neutrino mass m_1
- We give a new set of conditions for CP conservation for $m_1 < 0.142\ \mathrm{eV}$, which is large enough given the bound on neutrino masses from cosmological observations
- For arbitrary lightest neutrino masses, four conditions are required

$$\mathcal{I}_1 \equiv \mathrm{Tr}\left\{[H_\nu, H_l]^3\right\} = 0$$
$$\mathcal{I}_2 \equiv \mathrm{Im}\left\{\mathrm{Tr}\left[H_l H_\nu G_{l\nu}\right]\right\} = 0$$
$$\widehat{\mathcal{I}}_2 \equiv \mathrm{Im}\left\{\mathrm{Tr}\left[H_l H_\nu^2 G_{l\nu}\right]\right\} = 0$$
$$\widehat{\mathcal{I}}_3 \equiv \mathrm{Im}\left\{\mathrm{Tr}\left[H_l H_\nu^2 G_{l\nu} H_\nu\right]\right\} = 0$$

- These conditions will be useful for model building of neutrino masses, flavor mixing and leptonic CPV

Thanks a lot!