# Why matter effects are important for medium baseline reactor neutrino experiments (ICHEP 2020, PRAGUE)

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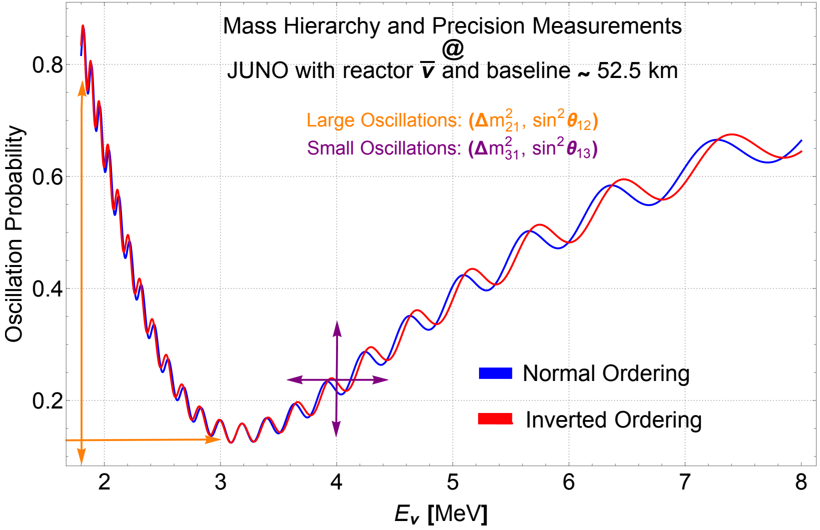
MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

MANITOP
Massive Neutrinos: Investigating their Theoretical Origin and Phenomenology

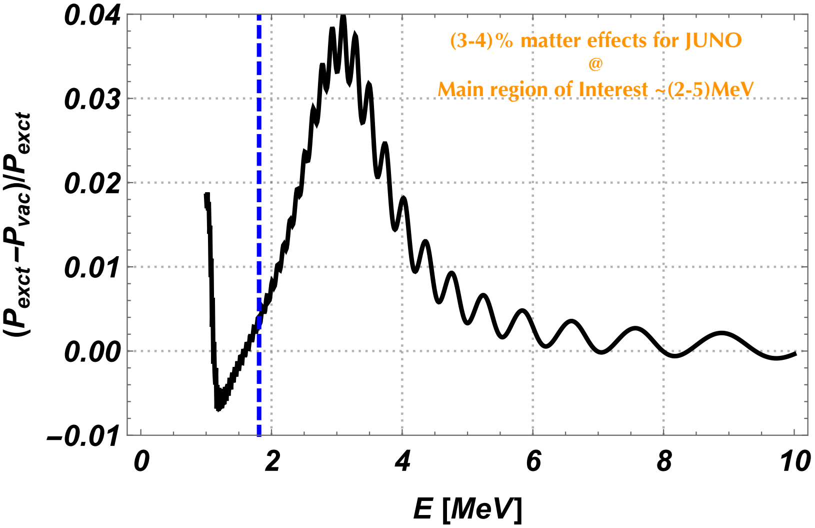
# Outline

- Discuss the approximate oscillation probability including matter effects for Medium Baseline Experiments (MBL)
- Estimates matter effects on mixing parameters, oscillation probability and differential event energy spectrum
- Disagreement between the expected and fitted values matter solar parameters, $\theta_{12}$ and $\Delta m^2_{21}$, (Shift Issue)
- Independent (Analytical) check of the "Shift Issue"
- Conclusion

# Oscillation Probability for a MBL Experiment



# Matter effects on oscillation probability for JUNO



# Oscillation Probability with Matter Effects

$$\widetilde{P}_{ee} = 1 - \cos^4\widetilde{\theta}_{13}\sin^2 2\widetilde{\theta}_{12}\sin^2\widetilde{\Delta}_{21} - \sin^2 2\widetilde{\theta}_{13}[\cos^2\widetilde{\theta}_{12}\sin^2\widetilde{\Delta}_{31} + \sin^2\widetilde{\theta}_{12}\sin^2\widetilde{\Delta}_{32}],$$

where $\widetilde{\Delta}_{ij} \equiv \Delta\widetilde{m}^2_{ij}L/4E$.

The mass-squared differences ($\Delta\widetilde{m}^2_{ij}$) and mixing angles ($\widetilde{\theta}_{ij}$) in matter can be calculated in any power of $(a/\Delta m^2_{ee})$ and $(a/\Delta m^2_{21})$ from exact the expressions found in [3]. We find:

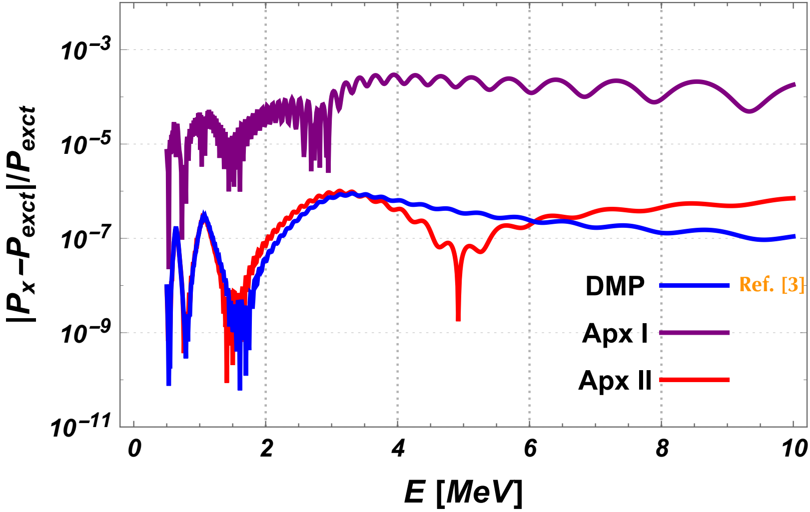
$$\Delta\widetilde{m}^2_{21} = \Delta m^2_{21} - \Delta m^2_{21}[(1-2s^2_{12})\frac{c^2_{13}a}{\Delta m^2_{21}} - 2s^2_{12}c^2_{12}(\frac{c^2_{13}a}{\Delta m^2_{21}})^2]....$$

$$\Delta\widetilde{m}^2_{31} = \Delta m^2_{31} + \Delta m^2_{ee}[(s^2_{13} - c^2_{12}c^2_{13})\frac{a}{\Delta m^2_{ee}}]...., \quad a \equiv \pm 2\sqrt{2}G_F N_e E$$

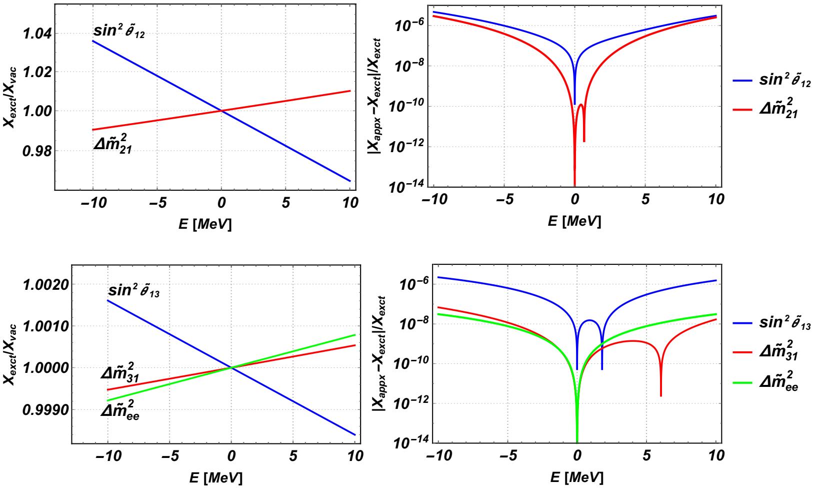
$$\sin^2\widetilde{\theta}_{13} = s^2_{13} + 2s^2_{13}c^2_{13}\frac{a}{\Delta m^2_{ee}}...., \quad \Delta m^2_{ee} \equiv c^2_{12}\Delta m^2_{31} + s^2_{12}\Delta m^2_{32}$$

$$\sin^2\widetilde{\theta}_{12} = s^2_{12} + 2s^2_{12}c^2_{12}\frac{c^2_{13}a}{\Delta m^2_{21}} + 3s^2_{12}c^2_{12}\cos 2\theta_{12}(\frac{c^2_{13}a}{\Delta m^2_{21}})^2......$$

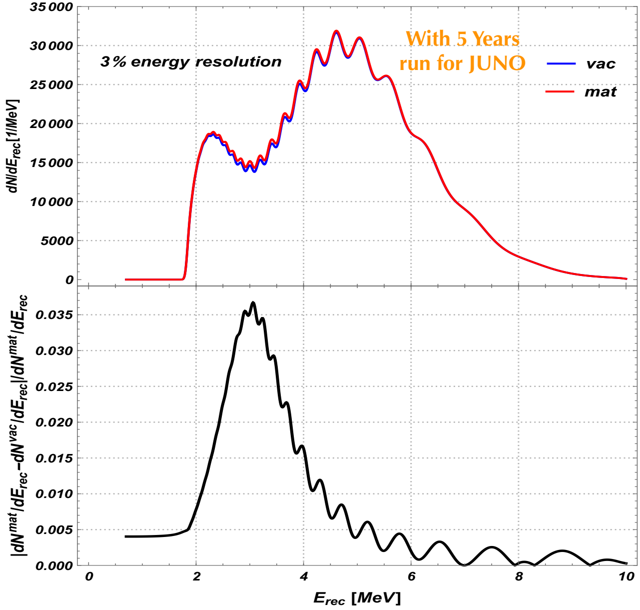
# Accuracy of our calculations



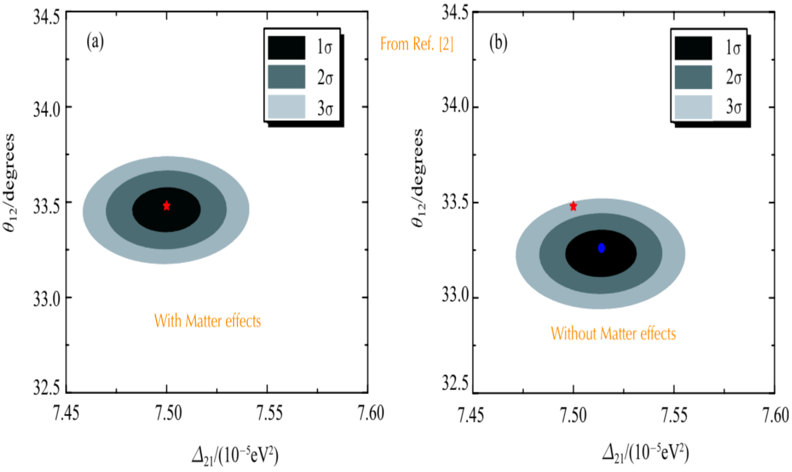
# Matter Effects on Mixing Parameters



# Matter Effects on Event Energy Spectrum



# Shifts of Solar Parameters due to the Matter Effects



# The "Shift Issue"

From analysis (and also ref.[2]) we find, assuming the constant matter density 2.6 g/cm$^3$, the fractional shifts for the solar parameters are

$$\left(\frac{\delta(\sin^2\theta_{12})}{\sin^2\theta_{12}}, \frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}}\right)\Bigg|_{\text{purely numerical } \chi^2} \simeq (-1.1, \quad 0.19)\%. \qquad (1)$$

While we find the expected shifts (above formulas) at fixed neutrino energy, $E$, are

$$\left(\frac{\delta(\sin^2\theta_{12})}{\sin^2\theta_{12}}, \frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}}\right)\Bigg|_{\text{naive expectation}} \simeq \begin{cases} (-1.1, \quad 0.30)\% & \text{using } E = 3 \text{ MeV}, \\ (-0.74, \quad 0.21)\% & \text{using } E = 2 \text{ MeV}. \end{cases}$$

# Independent Check of the Shift

Expected event number distribution $N(E)$ as function of the neutrino energy $E$ simply written as

$$N(E) = N_0(E)\ \tilde{P}_{ee}(E),$$

where

$$N_0(E) \quad \equiv \quad \text{event number distribution without oscillations}$$

$$\tilde{P}_{ee}(E) \approx P_{ee}^{\text{vac}}(E) + \delta P_{ee}\left(\frac{\delta(\sin^2 2\theta_{12})}{\sin^2 2\theta_{12}}, \frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}}\right),$$

where the first term

$$P_{ee}^{\text{vac}}(E) \ \simeq \ 1 - c_{13}^4 \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{21} L}{4E}\right) - \frac{1}{2}\sin^2 2\theta_{13},$$

# Independent check of the shift

while the second term is the correction due to matter effects as

$$\delta P_{ee}\left(\frac{\delta(\sin^2 2\theta_{12})}{\sin^2 2\theta_{12}}, \frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}}\right) = -c^4_{13}\,\sin^2 2\theta_{12}\sin\Delta_{21}$$
$$\times\left[\sin\Delta_{21}\,\frac{\delta(\sin^2 2\theta_{12})}{\sin^2 2\theta_{12}} + 2\Delta_{12}\cos\Delta_{12}\,\frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}}\right],$$

where

$$\frac{\delta(\sin^2 2\theta_{12})}{\sin^2 2\theta_{12}} = -2\alpha(\rho)E$$
$$\frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}} = \alpha(\rho)E$$

$$\alpha(\rho) \equiv \frac{2\sqrt{2}G_F N_e c^2_{13}\cos 2\theta_{12}}{\Delta m^2_{21}}$$

# New event energy spectrum

$$N^{\rm obs}(E) \approx N_0^{\rm tot}\lambda(E)[P_{ee}^{\rm vac}(E) + \delta P_{ee}(-2\alpha(\rho)E, \alpha(\rho)E)],$$

where

$$N_0^{\rm tot} \equiv \text{Expected total number of events without Osc.}$$

$$\lambda(E) \equiv (S(E) \times \sigma_{IBD}(E))/N_0^{\rm tot}$$

Assume the event rate distribution is best fitted with a vacuum oscillation probability, but with a slight dimensionless shifts $x$ and $y$ in the solar mixing parameters as

$$\begin{aligned} \sin^2 2\theta_{12}^{\rm fit} &= \sin^2 2\theta_{12}(1+x) \\ \Delta m_{21}^{2\,\rm fit} &= \Delta m_{21}^2(1+y), \end{aligned}$$

# New event energy spectrum

Then as if there is no matter effects,

$$N^{\rm fit}(E) \approx N_0^{\rm tot}\lambda(E)[P_{ee}^{\rm vac}(E) + \delta P_{ee}(x,y)],$$

where

$$\delta P_{ee}(x,y) = -c_{13}^4 \sin^2 2\theta_{12} \sin\Delta_{21}\left[\frac{\sin\Delta_{21}}{\sin^2 2\theta_{12}} x + \frac{2\Delta_{12}\cos\Delta_{12}}{\Delta m_{21}^2} y\right]$$

The Task, x, y = ?
Define a $\chi^2$ function:

$$\chi^2(x,y) = \int\left(\frac{N^{\rm obs}(E) - N^{\rm fit}(E)}{\sqrt{N^{\rm obs}(E)}}\right)^2 dE,$$

$$= N_0^{\rm tot}\int \frac{[\delta P_{ee}(-2\alpha(\rho)E, \alpha(\rho)E) - \delta P_{ee}(x,y)]^2}{\tilde{P}_{ee}(E)}\lambda(E)dE$$

# Minimizing the $\chi^2$

The minimization condition

$$\frac{\partial \chi^2(x,y)}{\partial x} = \frac{\partial \chi^2(x,y)}{\partial y} = 0$$

leads to coupled linear equations for the two unknowns, x and y,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix},$$

where $a, b, c, d, f$ and $g$ are derived as

$$\begin{aligned} a &\equiv \int (\sin^2 \Delta_{21})\, h(E)\lambda(E) dE \approx 2.95, \\ b &\equiv \int (\Delta_{21} \sin 2\Delta_{21})\, h(E)\lambda(E) dE \approx 0.823, \\ c &\equiv b/2 \approx 0.411, \\ d &\equiv 2 \int (\Delta_{21}^2 \cos^2 \Delta_{21})\, h(E)\lambda(E) dE \approx 1.17, \end{aligned}$$

# Minimizing the $\chi^2$

$$h(E) \equiv (\sin^2 \Delta_{21})/P_{ee}^{\mathrm{vac}}(E)$$

$$\begin{aligned} f(\rho) &\equiv -\alpha(\rho) \int (2\sin^2 \Delta_{21} - \Delta_{21} \sin 2\Delta_{21}) h(E) \lambda(E) E dE \\ &\approx -1.75 \times 10^{-2} \left( \frac{\rho}{2.6 \text{ g/cm}^3} \right) \cos^2 \theta_{13} \end{aligned}$$

$$\begin{aligned} g(\rho) &\equiv -\alpha(\rho) \int (\Delta_{21} \sin 2\Delta_{21} - 2\Delta_{21}^2 \cos^2 \Delta_{21}) h(E) \lambda(E) E dE \\ &\approx -3.38 \times 10^{-4} \left( \frac{\rho}{2.6 \text{ g/cm}^3} \right) \cos^2 \theta_{13} \,, \end{aligned}$$

# x=?, y=?

For $\rho = 2.6$ g/cm$^3$ and $\sin^2\theta_{13} = 0.0214$, we have
x = -0.32%, y = 0.20%
Thus, we find,

$$\left(\frac{\delta(\sin^2\theta_{12})}{\sin^2\theta_{12}}, \frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}}\right)\Bigg|_{\rm Analytical} = (-1.1, \quad 0.20)\%$$

which agrees with

$$\left(\frac{\delta(\sin^2\theta_{12})}{\sin^2\theta_{12}}, \frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}}\right)\Bigg|_{\chi^2} = (-1.1, \quad 0.19)\%,$$

$$\left(\frac{\delta(\sin^2\theta_{12})}{\sin^2\theta_{12}}, \frac{\delta(\Delta m^2_{21})}{\Delta m^2_{21}}\right)\Bigg|_{\rm expected}$$

$$\simeq \begin{cases} (-1.1, \quad 0.30)\% & \text{using } E = 3 \text{ MeV}, \\ (-0.74, \quad 0.21)\% & \text{using } E = 2 \text{ MeV}. \end{cases}$$

# Summary on the "Shift Issue"

Thus, for the small matter effects, the true and fitted parameters are analytically related as

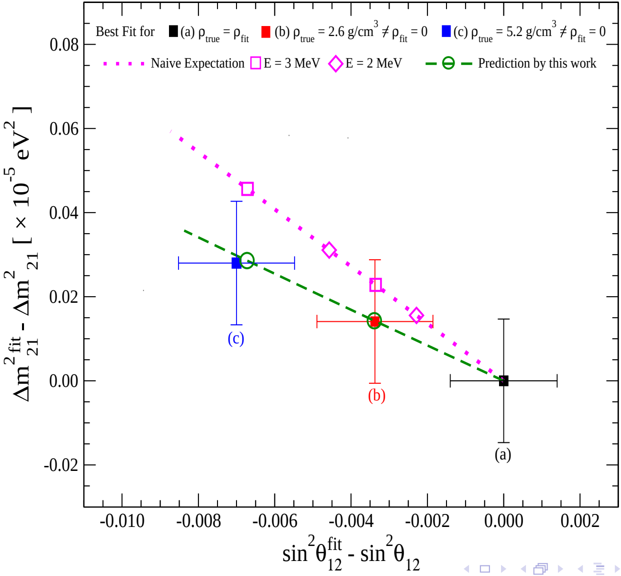
$$\sin^2\theta_{12}^{\text{true}} \approx \sin^2\theta_{12}^{\text{fit}}\left[1-\left(\frac{d.f(\Delta\rho)-b.g(\Delta\rho)}{a.d-b.c}\right)\left(\frac{\cos^2\theta_{12}}{\cos 2\theta_{12}}\right)\right]$$

and

$$\Delta m_{21}^{2\ \text{true}} \approx \Delta m_{21}^{2\ \text{fit}}\left[1+\left(\frac{a.g(\Delta\rho)-c.f(\Delta\rho)}{a.d-b.c}\right)\right],$$

where $\Delta\rho \equiv (\rho_{\text{true}}-\rho_{\text{fit}})$

# Analytical shifts agree with the numerical $\chi^2$?



# Conclusions

- We give simple oscillation probability formula and perturbative expansions for oscillation parameters, $\sin^2\theta$'s and $\Delta m^2$'s.
- We find that the maximum matter effects occur at the solar minimum of around (3-4)%.
- Argued that solar parameter shifts are in conflict with full $\chi^2$ (Shift Issue)
- The "Shift Issue" was independently confirmed analytically.
- Could have some connection with the $2\sigma$ mismatch on $\Delta m^2_{21}$'s solar and KamLAND measurements (See @MariamTortola Talk slide 9), NSI or something else, yet to know.
- **Thanks**!

# References

[1] e-Print: arXiv: 1910.12900 [hep-ph],
[2] e-Print: arXiv: 1605.00900 [hep-ph],
[3] e-Print: arXiv: 1604.08167 [hep-ph].