Loop Amplitudes Induced by Tensor Fermionic Current in Constant Homogeneous Electromagnetic Fields

Alexander Parkhomenko[1], Alexandra Dobrynina[1,2],
Ilya Karabanov[1] & Lubov Vassilevskaya[3]

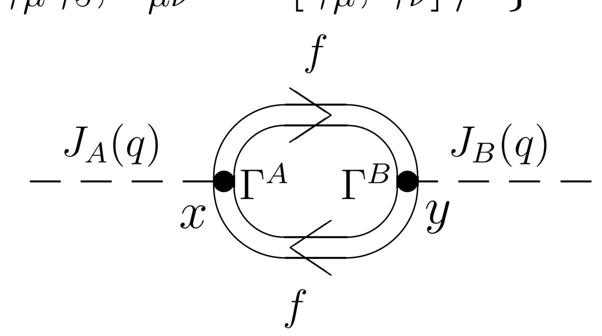
[1]P. G. Demidov Yaroslavl State University, Russia & [2]University of Hamburg, Germany & [3]Fulda University of Applied Sciences, Germany

General Case of Two-Point Correlator

Lagrangian density of local fermion interaction

$$\mathcal{L}_{\rm int}(x) = \left[\bar{f}(x)\Gamma^A f(x)\right] J_A(x)$$

J_A — generalized current (photon, neutrino current, etc.), Γ_A — any of γ-matrices from the set $\{1,\, \gamma_5,\, \gamma_\mu,\, \gamma_\mu\gamma_5, \sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2\}$



Two-point correlation function of general form

$$\Pi_{AB} = \int d^4X\, \mathrm{e}^{-i(qX)}\, \mathrm{Sp}\left\{S_{\rm F}(-X)\,\Gamma_A\, S_{\rm F}(X)\,\Gamma_B\right\}$$

$S_{\rm F}(X)$ — Lorentz-invariant part of exact fermion propagator in the Fock-Schwinger representation
Correlations of scalar, pseudoscalar, vector and axial-vector currents were studied [1]. We consider correlations of a tensor current with the other ones.

Basic Tensors in Presence of Magnetic Field

- Minkowski space filled with external magnetic field is divided into two subspaces

$$g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu}$$

$\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}$, $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$

- Dimensionless tensor of the external magnetic field and its dual

$$\varphi_{\alpha\beta} = F_{\alpha\beta}/B\,, \qquad \tilde{\varphi}_{\alpha\beta} = \varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}/2$$

- Arbitrary four-vector $a^\mu = (a_0, a_1, a_2, a_3)$ can be decomposed into two orthogonal components

$$a_\mu = \tilde{\Lambda}_{\mu\nu}a^\nu - \Lambda_{\mu\nu}a^\nu = a_{\parallel\mu} - a_{\perp\mu}$$

- For the scalar product of two four-vectors one has

$$(ab) = (ab)_\parallel - (ab)_\perp = (a\tilde{\Lambda}b) - (a\Lambda b)$$

Orthogonal Basis Motivated by Magnetic Field

Correlators having rank non-equal to zero, should be decomposed in some orthogonal set of vectors

$$b_\mu^{(1)} = (q\varphi)_\mu, \qquad b_\mu^{(2)} = (q\tilde{\varphi})_\mu$$
$$b_\mu^{(3)} = q^2\,(\Lambda q)_\mu - (q\Lambda q)\, q_\mu, \quad b_\mu^{(4)} = q_\mu$$

Third-rank tensor $T_{\mu\nu\rho}$ can be decomposed

$$T_{\mu\nu\rho} = \sum_{i,j,k=1}^{4} T_{ijk}\,\frac{b_\mu^{(i)} b_\nu^{(j)} b_\rho^{(k)}}{(b^{(i)}b^{(i)})\,(b^{(j)}b^{(j)})\,(b^{(k)}b^{(k)})},$$

$$T_{ijk} = T^{\mu\nu\rho} b_\mu^{(i)} b_\nu^{(j)} b_\rho^{(k)}.$$

Correlator of Vector and Tensor Currents

Vector-current conservation and anti-symmetry of the tensor current reduce the number of independent coefficients in the basis decomposition to 18. Of them, four coefficients only are non-trivial.

$$\Pi_{ijk}^{\rm (VT)}(q^2, q_\perp^2, \beta) = \frac{1}{4\pi^2}\int\limits_0^\infty \frac{dt}{t}\int\limits_0^1 du\, Y_{ijk}^{\rm (VT)}(q^2, q_\perp^2, \beta; t, u)$$
$$\times \exp\left\{-i\left[m_f^2 t - q_\parallel^2\, t\,(1-u^2)/4 + q_\perp^2\,\frac{\cos(\beta t u) - \cos(\beta t)}{2\beta\sin(\beta t)}\right]\right\}$$

$$Y_{114}^{\rm (VT)} = -Y_{141}^{\rm (VT)} = -m_f\, q_\perp^2\, q^2\, \frac{\beta t\cos(\beta t u)}{\sin(\beta t)}$$

$$Y_{223}^{\rm (VT)} = -Y_{232}^{\rm (VT)} = \frac{m_f q_\perp^2 (q_\parallel^2)^2 \beta t}{\sin(\beta t)}\left[\cos(\beta t) - \cos(\beta t u)\right]$$

$$Y_{224}^{\rm (VT)} = -Y_{242}^{\rm (VT)} = \frac{m_f q_\parallel^2 \beta t}{\sin(\beta t)}\left[q_\perp^2\cos(\beta t) - q_\parallel^2\cos(\beta t u)\right]$$

$$Y_{334}^{\rm (VT)} = -Y_{343}^{\rm (VT)} = -m_f\, q_\perp^2\, q_\parallel^2\, (q^2)^2\,\frac{\beta t\cos(\beta t u)}{\sin(\beta t)}$$

$Y_{4jk}^{\rm (VT)}$ vanish in chosen basis
$t = s_1 + s_2$, $u = (s_1 - s_2)/(s_1 + s_2)$
$q_\parallel^2 = q^2 + q_\perp^2$, $\beta = eB|Q_f|$

Photon Polarization Operator

Photon polarization operator $\mathcal{P}^{\mu\nu}(q)$ is related with $\gamma \to \gamma$ transition matrix element and, consequently, with the **correlator of two vector currents**

$$\mathcal{M}_{\gamma\to\gamma} = -i\,\varepsilon_\mu^{\prime *}(q)\,\mathcal{P}^{\mu\nu}(q)\,\varepsilon_\nu$$

Photon dispersion relations follow from the equations

$$q^2 - \Pi^{(\lambda)}(q) = 0 \quad (\lambda = 1, 2, 3)$$

$\Pi^{(\lambda)}(q)$ are eigenvalues of the photon polarization operator (electron loop is considered only)

$$\mathcal{P}_{\mu\nu}(q) = \sum_{\lambda=1}^{3}\frac{b_\mu^{(\lambda)} b_\nu^{(\lambda)}}{(b^{(\lambda)})^2}\,\Pi^{(\lambda)}(q)$$

As in vacuum, there are two physical eigenmodes of $\mathcal{P}_{\mu\nu}(q)$ in an external constant homogeneous magnetic field

$$\varepsilon_\mu^{(1)} = b_\mu^{(1)}/\sqrt{q_\perp^2}, \quad \varepsilon_\mu^{(2)} = b_\mu^{(2)}/\sqrt{q_\parallel^2}$$

Photon Polarization Operator in Magnetic Field

In the magnetic field, $\Pi^{(\lambda)}(q)$ contains both vacuum and field-induced parts

$$\Pi^{(\lambda)}(q) = -i\,\mathcal{P}(q^2) - \frac{\alpha}{\pi}\,Y_{VV}^{(\lambda)}$$

Details about $Y_{VV}^{(\lambda)}$ can be found in [2].
Models beyond the Standard Model can give rise to effective operators and the Pauli Lagrangian density

$$\mathcal{L}_{\rm AMM}(x) = -\frac{\mu_B a_e}{2}\left[\bar{\psi}(x)\sigma_{\mu\nu}\psi(x)\right] F^{\mu\nu}(x)$$

$\mu_B = e/(2m_e)$ is Bohr magneton, a_e is electron anomalous magnetic moment (AMM)

- $\mathcal{L}_{\rm AMM}$ gives additional contribution to the polarization operator
- **Contribution linear in the electron AMM is related with correlator of vector and tensor currents** $\Pi_{\mu\nu\rho}^{(VT)}$

AMM Contribution to Photon Polarization Operator

Field-induced part of $\Pi^{(\lambda)}(q)$ is modified

$$\Pi^{(\lambda)}(q) = -i\,\mathcal{P}(q^2) - \frac{\alpha}{\pi}\,Y_{VV}^{(\lambda)} + \frac{\alpha}{\pi}\,a_e\,Y_{VT}^{(\lambda)}$$

$$Y_{VT}^{(\lambda)} = \int_0^\infty\frac{dt}{t}\int_0^1 du\left\{\frac{\beta t}{\sin(\beta t)}\,y_{VT}^{(\lambda)}\,e^{-i\Omega} - q^2\,e^{-i\Omega_0}\right\}$$

$$y_{VT}^{(1)} = y_{VT}^{(3)} = q^2\cos(\beta t u)$$

$$y_{VT}^{(2)} = q_\parallel^2\cos(\beta t u) - q_\perp^2\cos(\beta t)$$

Notations are from [2]. Part independent on the field is subtracted. **For the electron, $a_e \sim \alpha$ and the AMM correction is small.**

Conclusions

Two-point correlators in presence of constant homogeneous external magnetic field are considered. Study of correlators of tensor fermionic current with the others allows to investigate effects of the fermion AMM in the one-loop approximation. Field-induced contribution to the photon polarization operator linear in electron anomalous magnetic moment is calculated and gives a rather small contribution in the case of an electron.

References

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