## On soft theorems in multiflavour galileon theories

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ICHEP
Praha, 30-7-2020

Outline:

- Introduction to amplitude methods
- Role model: gluon amplitudes
- BCFW
- Effective Field Theories
- Soft bootstrap
- Galileon
- multi-Galileons
- Summary



## Introduction

- to understand the basic structure of elementary particles we use scattering experiments
- basic function: scattering cross section $\sigma$ : connected with probability for a given process of scattered particles as a function of their energy and momentum
-     + dependence on the angles $\rightarrow$ differential cross-section $d \sigma / d \Omega$
- theoretical prediction based on the quantum field theory
- basic object: scattering amplitude $A$
- due to translation invariance: scattering amplitudes are distributions

$$
\mathcal{A}_{n}=A_{n}\left(p_{1}^{\mu}, \ldots, p_{n}^{\mu}\right) \delta\left(\Sigma p_{i}^{\mu}\right) \Pi \delta\left(p_{i}^{2}=m_{i}^{2}\right)
$$

- $d \sigma / d \Omega \sim|A|^{2}$
- scattering amplitude can be calculated systematically as an expansion of a small parameter using the so-call Feynman diagrams


## Example: gluon amplitudes

standard method of calculating $n$-gluon scattering processes:

- dominated by pure-gluon interactions in QCD
- elementary 3pt and 4pt vertices

- construct all possible Feynman diagrams, e.g. 8pt tree-level:


$$
\begin{aligned}
& \text { O } 50008000 \\
& \&
\end{aligned}
$$

$$
+
$$

In total 34300 diagrams for the 8pts

- complicated already for the tree level diagrams even for small number of external legs


## History: gluon amplitude, tree-level

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- 5pt: calculated in '80, calculation blows up on several pages

total number: $25=\times 15$

$\times 10$
structure of the numerators, schematically:

$$
\begin{aligned}
& \text { double-propagator: }\left(p_{i} \cdot p_{j}\right)\left(p_{k} \cdot \epsilon\right)(\epsilon \cdot \epsilon)(\epsilon \cdot \epsilon), \\
& \text { single-propagator: }\left(p_{k} \cdot \epsilon\right)(\epsilon \cdot \epsilon)(\epsilon \cdot \epsilon)
\end{aligned}
$$

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\end{aligned}
$$

- 6pt: impossible by standard methods, but...


## History: gluon amplitude, tree-level, 6pt

 SSC approved in 1983 (to be cancelled 10 years later) motivated the following work
# THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON-GLUON FUSION 

Stephen J. PARKE and T.R. TAYLOR

Fermi Natonal Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985

The cross section for two-gluon to four-gluon scattering is given in a form suitable for fast numerical calculations.

Theoretical predictions for four-jet production at hadron colliders allow detailed tests of QCD. Moreover, at SSC energies, four jets become a serious background to many interesting processes which probe new physics, e.g. pair production of electroweak bosons [1]. Hence a detailed knowledge of four-jet event characteristics is crucial for good background rejection. Although some individual contributions to four-jet production have already been analysed (see e.g. ref. [2]), the two-gluon to four-gluon scattering, which is the dominant contribution for a wide range of subprocess energies, has remained beyond the scope of previous computational techniques. Here we outline our calculation of the cross section for this process, in the tree approximation of perturbative QCD. The final cross section is presented in a form suitable for fast numerical calculations.

Our calculation makes use of techniques developed in ref. [3], based on the application of extended supersymmetry. We adopt the convention that all particles

## History: gluon amplitude, tree-level, 6pt

## Parke and Taylor concluded:

Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

## History: gluon amplitude, tree-level, 6pt

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Indeed it was given a year later [Parke, Taylor '86] for the MHV:

$$
A_{n}(--+\ldots+)=\frac{\langle 12\rangle^{4}}{\langle 12\rangle\langle 23\rangle \ldots\langle n 1\rangle}
$$

## One line formula!

The so-called spinor-helicity formalism was introduced (reasonable variables for massless particles) cf. [Mangano,Parke, Xu '87]

$$
\langle i j\rangle=\sqrt{\left|2 p_{i} \cdot p_{j}\right|} \mathrm{e}^{i \phi_{i j}}
$$

Is there some better way to calculate?

## Example: gluon amplitudes

Important simplification at tree level:

- colour ordering $\rightarrow$ stripped amplitude


$$
M^{a_{1} \ldots a_{n}}\left(p_{1}, \ldots p_{n}\right)=\sum_{\sigma / Z_{n}} \operatorname{Tr}\left(t^{a_{\sigma(1)}} \ldots t^{a_{\sigma(n)}}\right) M_{\sigma}\left(p_{1}, \ldots, p_{n}\right)
$$

- $M_{\sigma}\left(p_{\sigma(1)}, \ldots, p_{\sigma(n)}\right)=M\left(p_{1}, \ldots, p_{n}\right) \equiv M(1,2, \ldots n)$
- propagators $\Rightarrow$ the only poles of $M_{\sigma}$
- thanks to ordering the only possible poles are:

$$
P_{i j}^{2}=\left(p_{i}+p_{i+1}+\ldots+p_{j-1}+p_{j}\right)^{2}
$$

## Pole structure

Weinberg's theorem (one-particle unitarity): on the factorization channel

$$
\lim _{P_{1 j}^{2} \rightarrow 0} M(1,2, \ldots n)=\sum_{h_{l}} M_{L}(1,2 \ldots j, I) \times \frac{1}{P_{1 j}^{2}} \times M_{R}(I, j+1, \ldots n)
$$



## BCFW relations, preliminaries

## [Britto, Cachazo, Feng, Witten '05]

Reconstruct the amplitude from its poles (in complex plane)

- shift in two external momenta

$$
p_{i} \rightarrow p_{i}+z q, \quad p_{j} \rightarrow p_{j}-z q
$$

- keep $p_{i}$ and $p_{j}$ on-shell, i.e.

$$
q^{2}=q \cdot p_{i}=q \cdot p_{j}=0
$$

- amplitude becomes a meromorphic function $A(z)$
- only simple poles coming from propagators $P_{a b}(z)$
- original function is $A(0)$

BCFW relations: factorization channels

Cauchy's theorem


$$
\frac{1}{2 \pi \mathrm{i}} \int \frac{d z}{z} A(z)=A(0)+\sum_{k} \frac{\operatorname{Res}\left(A, z_{k}\right)}{z_{k}}
$$

BCFW relations: factorization channels

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$$
0=\frac{1}{2 \pi \mathrm{i}} \int \frac{d z}{z} A(z)=A(0)+\sum_{k} \frac{\operatorname{Res}\left(A, z_{k}\right)}{z_{k}}
$$

If $A(z)$ vanishes for $z \rightarrow \infty$

$$
A=A(0)=-\sum_{k} \frac{\operatorname{Res}\left(A, z_{k}\right)}{z_{k}}
$$

## BCFW relations

$$
P_{a b}^{2}(z)=0 \quad \text { if one and only one } i(\text { or } j) \text { in }(a, a+1, \ldots, b)
$$

Suppose $i \in(a, \ldots, b) \not \supset j$

$$
\begin{aligned}
P_{a b}^{2}(z)=\left(p_{a}+\ldots+p_{i-1}+p_{i}+z q+p_{i+1}\right. & \left.+\ldots+p_{b}\right)^{2}= \\
& =P_{a b}^{2}+2\left(q \cdot P_{a b}\right) z=0
\end{aligned}
$$

solution

$$
z_{a b}=-\frac{P_{a b}^{2}}{2\left(q \cdot P_{a b}\right)} \quad \Rightarrow \quad P_{a b}^{2}(z)=-\frac{P_{a b}^{2}}{z_{a b}}\left(z-z_{a b}\right)
$$

and thus using the one-particle unitarity:

$$
\operatorname{Res}\left(A, z_{a b}\right)=\sum_{s} A_{L}^{-s}\left(z_{a b}\right) \times \frac{-z_{a b}}{P_{a b}^{2}} \times A_{R}^{s}\left(z_{a b}\right)
$$

(sum via allowed helicities)

## BCFW relations

Using Cauchy's formula, we have finally as a result

$$
A=\sum_{k, s} A_{L}^{-s_{k}}\left(z_{k}\right) \frac{1}{P_{k}^{2}} A_{R}^{s_{k}}\left(z_{k}\right)
$$

- based on two-line shift (convenient choice: adjacent $i, j$ )
- recursive formula (down to 3-pt amplitudes)
- number of terms small $=$ number of factorization channels
- all ingredients are on shell


## BCFW recursion relations: problems

We have assumed that

$$
A(z) \rightarrow 0, \quad \text { for } \quad z \rightarrow \infty
$$

More generally we have to include a boundary term in the Cauchy's theorem.

## Effective field theories

## Effective field theories: general picture

Now we have infinitely many unfixed " $\lambda$ " terms. Schematically

$$
\mathcal{L}=\frac{1}{2}(\partial \phi)^{2}+\lambda_{4}\left(\partial^{m_{4}} \phi\right)^{4}+\lambda_{6}\left(\partial^{m_{6}} \phi\right)^{6}+\ldots
$$

Example: 6pt scattering, Feynman diagrams


Corresponding amplitude:

$$
\mathcal{M}_{6}=\sum_{I=\text { poles }} \lambda_{4}^{2} \frac{\cdots}{P_{I}}+\lambda_{6}(\ldots)
$$

$\lambda_{6}$ part: not fixed by the pole behaviour.
Task: to find a condition in order to link these two terms

## Effective field theories: introduction

Usual steps:
Symmetry $\rightarrow$ Lagrangian $\rightarrow$ Amplitudes $\rightarrow$ physical quantities (cross-section, masses, decay constants, ...)
In our work - opposite direction:
Amplitudes $\rightarrow$ physical quantities $(\rightarrow$ Lagrangian $\rightarrow$ Symmetry)
Our aim: classification of interesting EFTs
works done in collaborations with Clifford Cheung, Jiri Novotny, Chia-Hsien Shen, Jaroslav Trnka and Congkao Wen

## Effective field theories: scalar theories, 3pt

As simple as possible: a spin-0 massless degree of freedom with a three-point interaction.

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As simple as possible: a spin-0 massless degree of freedom with a three-point interaction.

General formula for any three-particle amplitude

$$
A\left(1^{h_{1}} 2^{h_{2}} 3^{h_{3}}\right)= \begin{cases}\langle 12\rangle^{h_{3}-h_{1}-h_{2}}\langle 23\rangle^{h_{1}-h_{2}-h_{3}}\langle 31\rangle^{h_{2}-h_{3}-h_{1}}, & \Sigma h_{i} \leq 0 \\ {[12]^{h_{1}+h_{2}-h_{3}}[23]^{h_{2}+h_{3}-h_{1}}[31]^{h_{3}+h_{1}-h_{2}},} & \Sigma h_{i} \geq 0\end{cases}
$$

n.b. again the spinor-helicity notation, e.g. $p_{i} \cdot p_{j} \sim\langle i j\rangle[i j]$

## Effective field theories: scalar theories, 3pt

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$$

n.b. again the spinor-helicity notation, e.g. $p_{i} \cdot p_{j} \sim\langle i j\rangle[i j]$

For scalars $\left(h_{i}=0\right)$ this is a constant - corresponding to $\mathcal{L}_{\text {int }}=\lambda \phi^{3}$.
All derivatives can be removed by equations of motions (boxes)

$$
\mathcal{L}_{\text {int }}=\left(\partial_{\alpha} \ldots \partial_{\omega} \phi\right)\left(\partial^{\alpha} \ldots \partial^{\omega} \phi\right) \phi \quad \rightarrow \quad \mathcal{L}_{\text {int }}=(\square \phi)(\ldots)
$$

## Effective field theories: scalar theories, 4pt

We start with ( $m$ counts derivatives)

$$
\mathcal{L}_{\text {int }}=\lambda_{4} \partial^{m} \phi^{4}
$$

n.b. we want to connect this four-point vertex with the 6 -point contact terms

This rules out again the no-derivative terms, as the powercounting dictates:

$$
\partial^{m} \times \frac{1}{\partial^{2}} \times \partial^{m} \quad \rightarrow \quad \partial^{2 m-2} \phi^{6}
$$

and we have to start at least with $m=2$, i.e. two derivatives

## Simplest example: two derivatives, single scalar

$$
\mathcal{L}=\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi+\lambda_{4} \partial^{2} \phi^{4}+\lambda_{6} \partial^{2} \phi^{6}+\ldots
$$

How to connect $\lambda_{4}$ and $\lambda_{6}$ ?
Note that this Lagrangian, an infinite series, looks complicated, but it is not the case. It represents a free theory:

$$
\mathcal{L}=\frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi \underbrace{\left(1+\lambda_{4} \phi^{2}+\ldots\right)}_{F(\phi)}
$$

$F(\phi)$ can be removed by a field redefinition

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$F(\phi)$ can be removed by a field redefinition
Outcome so far: the non-trivial simplest cases:

- more than two derivatives
- more flavours ( $\phi \rightarrow \phi_{1}, \phi_{2}, \ldots$ )


## More flavours

## [KK,Novotny,Trnka'13]

$$
\mathcal{L}=\frac{1}{2} \partial_{\mu} \phi^{i} \partial^{\mu} \phi^{i}+\lambda_{i j k l} \partial_{\mu} \phi^{i} \partial^{\mu} \phi^{j} \phi^{k} \phi^{\prime}+\lambda_{i_{1} \ldots 1_{6}} \partial_{\mu} \phi^{i_{1}} \partial^{\mu} \phi^{i_{2}} \phi^{i_{3}} \ldots \phi^{i_{6}}+\ldots
$$

- Can be used for systematic studies of two species, three species, etc.
- Very complicated generally
- Assume some simplification, a group structure

$$
\phi=\phi^{a} T^{a}
$$

- motivated by the 'gluon case': flavour ordering

$$
A^{a_{1} \ldots a_{n}}=\sum_{p e r m} \operatorname{Tr}\left(T^{a_{1}} \ldots T^{a_{n}}\right) A\left(p_{1}, \ldots p_{n}\right)
$$

## More flavours: stripped amplitude

first non-trivial case 6 pt scattering:

power-counting is ok:

$$
\lambda_{4}^{2} p^{2} \frac{1}{p^{2}} p^{2}+\lambda_{6} p^{2}
$$

in order to combine the pole and contact term we need to consider some limit. The most natural candidate: we will demand soft limit, i.e.

$$
\begin{gathered}
A \rightarrow 0, \quad \text { for } \quad p \rightarrow 0 \\
\Rightarrow \quad \lambda_{4}^{2} \sim \lambda_{6}
\end{gathered}
$$

## Standard direction(s)

Assuming the shift symmetry

$$
\phi^{a} \rightarrow \phi^{a}+\epsilon^{a}
$$

$\Rightarrow$ Noether current

$$
A_{\mu}^{a}=\frac{\delta \mathcal{L}}{\delta \partial^{\mu} \phi^{a}}
$$

$\Rightarrow$ Ward identity $\Rightarrow$ LSZ

$$
\langle 0| A_{\mu}^{a}(x)\left|\phi^{b}(p)\right\rangle=i F \delta^{a b} p_{\mu} \mathrm{e}^{-i p x}
$$

$\Rightarrow$ Adler zero

$$
\lim _{p \rightarrow 0}\left\langle f \mid i+\phi^{a}(p)\right\rangle=0
$$

$\Rightarrow$ CCWZ: non-linear sigma model

$$
\mathcal{L}=\frac{F^{2}}{2} \operatorname{Tr}\left(\partial_{\mu} U^{\dagger} \partial^{\mu} U\right), \quad U=\mathrm{e}^{\frac{i}{F} \phi^{a} T^{a}}
$$

eg. [Weinber'66], [lan Low '14-'15], [Goon,Joyce,Trodden '14], [Bogers, Brauner '18], . . .

## Bottom $\rightarrow$ up construction

## Natural classification: $\sigma$ and $\rho$

Soft limit of one external leg of the tree-level amplitude

$$
A\left(t p_{1}, p_{2}, \ldots, p_{n}\right)=\mathcal{O}\left(t^{\sigma}\right), \quad \text { as } \quad t p_{1} \rightarrow 0
$$

Interaction term

$$
\mathcal{L}=\partial^{m} \phi^{n}
$$

Another natural parameter is (counts the homogeneity)

$$
\rho=\frac{m-2}{n-2} \quad \text { "averaging number of derivatives" }
$$

Non-trivial cases

$$
\text { for: } \mathcal{L}=\partial^{m} \phi^{n}: \quad m<\sigma n
$$

or

$$
\sigma>\frac{(n-2) \rho+2}{n}
$$

i.e.

| $\rho$ | $\sigma$ at least |
| :---: | :---: |
| 0 | 1 |
| 1 | 2 |
| 2 | 2 |
| 3 | 3 |

i.e. non-trivial regime for $\rho \leq \sigma$

## First case: $\rho=0$ (i.e. two derivatives)

Schematically for single scalar case

$$
\mathcal{L}=\frac{1}{2}(\partial \phi)^{2}+\sum_{i} \lambda_{4}^{i}\left(\partial^{2} \phi^{4}\right)+\sum_{i} \lambda_{6}^{i}\left(\partial^{2} \phi^{6}\right)+\ldots
$$

similarly for multi-flavour $\left(\phi_{i}: \phi_{1}, \phi_{2}, \ldots\right)$.
non-trivial case

$$
\sigma=1
$$

Outcome:

- single scalar: free theory
- multiple scalars (with flavour-ordering): non-linear sigma model n.b. it represents a generalization of [Susskind, Frye '70], [Ellis, Renner '70]


## Second case: $\rho=1, \sigma=2$ (double soft limit)

1. focus on the lowest combination and fix the form:

$$
\mathcal{L}_{\text {int }}=c_{2}(\partial \phi \cdot \partial \phi)^{2}+c_{3}(\partial \phi \cdot \partial \phi)^{3} \quad \text { condition: } c_{3}=4 c_{2}^{4}
$$



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$$



2. find the symmetry

$$
\phi \rightarrow \phi-b_{\rho} x^{\rho}+b_{\rho} \partial^{\rho} \phi \phi \quad \text { (again up to } 6 \mathrm{pt} \text { so far) }
$$

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$$

3. ansatz of the form

$$
c_{n}(\partial \phi \cdot \partial \phi)^{n}+c_{n+1}(\partial \phi \cdot \partial \phi)^{n} \partial \phi \cdot \partial \phi
$$

4. in order to cancel: $2(n+1) c_{n+1}=(2 n-1) c_{n}$
i.e. $c_{1}=\frac{1}{2} \Rightarrow c_{2}=\frac{1}{8}, c_{3}=\frac{1}{16}, c_{4}=\frac{5}{128}, \ldots$

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$$
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$$

solution:

$$
\mathcal{L}=-\sqrt{1-(\partial \phi \cdot \partial \phi)}
$$

This theory known as a scalar part of the Dirac-Born-Infeld [1934] - DBI action
Scalar field can be seen as a fluctuation of a 4-dim brane in five-dim Minkowski space


Remark: soft limit and symmetry are "equivalent"

## Third case: $\rho=2, \sigma=2$ (double soft limit)

Similarly to previous case we will arrive to a unique solution: the Galileon Lagrangian

$$
\begin{gathered}
\mathcal{L}=\sum_{n=1}^{d+1} d_{n} \phi \mathcal{L}_{n-1}^{\text {der }} \\
\mathcal{L}_{n}^{\operatorname{der}}=\varepsilon^{\mu_{1} \ldots \mu_{d}} \varepsilon^{\nu_{1} \ldots \nu_{d}} \prod_{i=1}^{n} \partial_{\mu_{i}} \partial_{\nu_{i}} \phi \prod_{j=n+1}^{d} \eta_{\mu_{j} \nu_{j}}=-(d-n)!\operatorname{det}\left\{\partial^{\nu_{i}} \partial_{\nu_{j}} \phi\right\} .
\end{gathered}
$$

It possesses the Galilean shift symmetry

$$
\phi \rightarrow \phi+a+b_{\mu} x^{\mu}
$$

and leads to EoM of second-order in field derivatives.

Galileon itself is a remarkable theory: can be connected with a local modification of gravity [Nicolis, Rattazzi, Trincherini '09].

## Surprise: $\rho=2, \sigma=3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])


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- we demanded $\mathcal{O}\left(p^{3}\right)$ behaviour
- we have verified: possible up to very high-pt order
- suggested a new theory: special galileon [Cheung,KK,Novotny, Trnka 1412.4095]


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- suggested a new theory: special galileon [Cheung,KK,Novotny, Trnka 1412.4095]
- symmetry explanation: hidden symmetry [K. Hinterbichler and A. Joyce 1501.07600]

$$
\phi \rightarrow \phi+s_{\mu \nu} x^{\mu} x^{\nu}-12 \lambda_{4} s^{\mu \nu} \partial_{\mu} \phi \partial_{\nu} \phi
$$

- theory appears also in the context of CHY-type formulation [Cachazo, He, Yuan 1412.3479, version 2]

Classification of EFTs (spin-0): result "soft bootstrap" $\Rightarrow$ Periodic table of scalar theories [Cheung, KK, Novotny, Shen, Trnka 2017]


## EFT: further avenues

- different spins studied, e.g. spin-1/2 Elvang et al.'18 or spin 1: Cheung,KK,Novotny,Shen, Trnka,Wen'18
- mixing different spins $\longrightarrow$ see Přeučil's talk
- going beyond $O\left(p^{2}\right)$ NLSM (cf. Bijnens,KK,Sjö '19)
- multiple flavours - especially without flavour ordering: only two-flavour case fully classified $\longrightarrow$ see following
- connection with CHY [Cachazo, He, Yuan] formalism
- role of masses
- role of dualities
- ...


## Galileons

Galileons popped out naturally from our classification. What are they? Motivation: dark energy problem may imply some modification of gravity on large scales. Nowadays there are various dynamical gravity models: $f(R)$ gravity, scalar-tensor theory, Dvali-Gabadadze-Porrati model, Galileon gravity,....
General relativity modified, but we need to reproduce the successful measurements at solar system scales $\Rightarrow$ screening mechanism, e.g.:

- Chameleon mechanism
- Vainshtein mechanism ['72] [for theories with second-derivatives of $\phi$, for short-length scales the non-linearity are large]
The (classical) Lagrangian density definition:
- invariant under the Galilean shift transformation $\phi \rightarrow \phi+a+b_{\mu} x^{\mu}$
- lead to equations of motions of second-order in field derivatives


## Galileons

Tree-level diagrams Up to 5-pt we have the following tree-level Feynman diagrams:


$$
\begin{aligned}
& \mathcal{M}(1,2,3)=6 d_{3} G(1,2)=\frac{3}{2} d_{3} p_{3}^{4}=0 \\
& \mathcal{M}(1,2,3,4)=12\left(2 d_{4}-9 d_{3}^{2}\right) G(1,2,3) \\
& \mathcal{M}(1,2,3,4,5)=-24\left(72 d_{3}^{3}-24 d_{3} d_{4}+5 d_{5}\right) \\
& \quad \times G(1,2,3,4)
\end{aligned}
$$

Surprisingly very simple results: cancellation between different contributions.
Explanation: galileon duality [P. Creminelli, M. Serone, G. Trevisan and E. Trincherini '14], [C. de Rham, L. Keltner and A. J. Tolley,'14], [KK and J. Novotny '14]

## Multi galileons

Amplitude methods suit well to a systematic study of multigalileon theories.
Work in progress (in collaboration with Jiri Novotny):
Discussion on:

- role of duality in multigalileon theories
- role of the 3-line vertices
- role of the new soft theorems
soft theorem with the "right-hand side" consequence of [KK, Novotny, Shifman, Trnka '19]
- definition of galileon theory itself
- complete classification


## Summary

Mainly presented a

- Short overview of Amplitudes for EFTs
- Very efficient way of studying new phenomena
- Briefly presented a new study on the multigalileon theories


## Summary

Mainly presented a

- Short overview of Amplitudes for EFTs
- Very efficient way of studying new phenomena
- Briefly presented a new study on the multigalileon theories

Thank you!

