Georgi-Machacek Vacua

Duarte Azevedo[1]
Collaboration with: R. Santos[1,2,3], P. Ferreira[1,2], H. Logan[4]

[1]CFTC-ULisboa
[2]ISEG-Ulisboa
[3]LIP, Lisboa
[4]Carleton University, Canada

CFTC-UL, UA and LIP meeting - Experiment vs. Theory
Braga - Feb. 2020

LISBOA

CFTC

Introduction

$\rightarrow$ **Scalar particle discovered in 2012 with mass of $\sim$ 125 GeV at the Large Hadron Collider (LHC)**.
ATLAS, Phys.Lett. B716 (2012), and CMS, Phys. Lett. B 716 (2012).

The SM is complete. No experimental result that strongly deviates from the predictions. Some mentionable exceptions:

- Muon's anomalous magnetic moment.
- B meson decay rates.

Still we know it cannot be the whole story.

- **Gravity.**
- **Neutrinos' masses.**
- **Enough CP-violation:** to support Sakharov's condition for baryogenesis.
- **Dark Matter:** Several indirect evidence: Galaxy rotation curves, Gravitational lensing, Cosmic microwave background, etc.
- **...**

Group's representation

Different representations (group theory context) of the fields in the Lagrangian lead to different phenomenology!

Standard nomenclature: singlets, doublets, triplets $\rightarrow$ Weak isospin symmetry group $SU(2)_L$. Doublets and triplets might carry Hypercharge $U(1)_Y$.

$\rightarrow$ We need at least a doublet with hypercharge (as in SM) for the Higgs mechanism.

Theory vs. Experiment (Phenomenology)

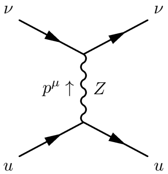
$$\begin{aligned} \text{Singlets} &\Longleftrightarrow \text{More neutral Higgs bosons (e.g. DM), strong first-order phase transitions.} \\ \text{Doublets} &\Longleftrightarrow \text{More neutral/charged Higgs bosons, CP-violation, FCNC.} \\ \text{Triplets} &\Longleftrightarrow \text{Same as previous (no FCNC), no Yukawa int., } \underline{\text{larger Gauge-Higgs couplings than SM}} \end{aligned} \tag{1}$$

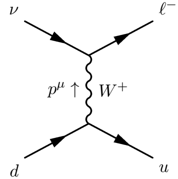
$$\begin{aligned} (D_\mu X)^\dagger (D^\mu X) \supset X^\dagger \Big[& \frac{g^2}{2} W_\mu^+ W^{-\mu} (T(T+1) - Y^2) + g^2 W_\mu^3 W^{3\mu} (T^3)^2 \\ & + g'^2 B_\mu B^\mu (Y)^2 - 2gg' B\mu W^{3\mu} (T^3)(Y) \Big] X \end{aligned} \tag{2}$$

The ρ parameter

The ratio of strengths between charged and neutral currents at low energies ($q \to 0$) when

$$\frac{-ig^{\mu\nu}}{q^2 - M_V^2} \to \frac{-ig^{\mu\nu}}{M_V^2} \tag{3}$$





$$\begin{aligned} i\Delta\mathcal{L}_Z &= \frac{-ig}{c_W}\left(\bar{\nu}\gamma_\mu\hat{\Omega}\nu\right)\frac{ig^{\mu\nu}}{M_Z^2}\frac{-ig}{c_W}\left(\bar{u}\gamma_\nu\hat{\Omega}u\right) \\ &= \frac{-ig^2}{c_W^2 M_Z^2}\left(\bar{\nu}\gamma_\mu\hat{\Omega}\nu\right)\left(\bar{u}\gamma^\mu\hat{\Omega}u\right) \\ &\equiv \frac{-ig^2}{c_W^2 M_Z^2} J_{Z\mu}J_Z^\mu \end{aligned} \tag{4}$$

$$\begin{aligned} i\Delta\mathcal{L}_W &= \frac{ig}{\sqrt{2}}(\bar{e}_L\gamma_\mu\nu_L)\frac{ig^{\mu\nu}}{M_W^2}\frac{ig}{\sqrt{2}}(\bar{u}_L\gamma_\nu d_L) \\ &= \frac{-ig^2}{M_W^2}\frac{1}{\sqrt{2}}(\bar{e}_L\gamma_\mu\nu_L)\frac{1}{\sqrt{2}}(\bar{u}_L\gamma^\mu d_L) \\ &\equiv \frac{-ig^2}{M_W^2} J^-_{W\mu}J_W^{+\mu}, \end{aligned} \tag{5}$$

The ρ parameter is the ratio between the strengths of neutral to charged currents

$$\rho = \left(\frac{g^2}{c_W^2 M_Z^2}\right) \times \left(\frac{g^2}{M_W^2}\right)^{-1} = \frac{M_W^2}{c_W^2 M_Z^2} = 1 \text{ (in the SM at tree-level).} \tag{6}$$

Current exp value: $\rho = 1.00039 \pm 0.00019$ [global fit - PDG 2019]

Custodial symmetry

The SM gauge boson's mass matrix in the (W^1, W^2, W^3, B) basis

$$M^2 = \frac{v^2}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix}. \tag{7}$$

If hypercharge interactions are *turned off*, $g' \to 0$

- $M_Z = M_W$
- $\theta_W = 0$

$\to$ Rotation symmetry between $W^1 \leftrightarrow W^2 \leftrightarrow W^3$ called **custodial symmetry** (Global $SU(2)$).

Preserves $\rho = 1$ to all orders of perturbation theory.

In a general model with N multiplets, X_i, and neutral component VEV, v_i, we have

$$\rho = \frac{\sum_i c_i[T_i(T_i+1) - Y_i^2/4]v_i^2}{\sum_i Y_i^2 v_i^2/4} \tag{8}$$

which for complex doublets $T = \frac{1}{2}$ with $Y = 1$ is identically 1.

$\to$ Curiosity: Another solution are septets with $T = 3$ and $Y = 4$.

Georgi-Machacek model

Georgi-Machacek (GM) model

1985

First proposed by Georgi and Machacek, and later (same year) by Chanowitz and Golden in 1985.

$\rightarrow$ Elegant way to induce $\rho = 1$ at tree-level in a triplet model by exploiting $SU(2)_R$ symmetry (already present in the scalar potential of the SM).

Consists of usual doublet Φ ($T = 1/2$) plus two triplets ($T = 1$) written in bi-tuplet representation

Real Ξ (Y=0) and **complex** χ with Y=2

$$\Phi = \begin{pmatrix} \phi^{0*} & \phi^{+} \\ -\phi^{+*} & \phi^{0} \end{pmatrix}, \qquad X = \begin{pmatrix} \chi^{0*} & \xi^{+} & \chi^{++} \\ -\chi^{+*} & \xi^{0} & \chi^{+} \\ \chi^{++*} & -\xi^{+*} & \chi^{0} \end{pmatrix} \tag{9}$$

where the $SU(2)_L \times SU(2)_R$ transformation can be written as

$$(\Phi' \text{ or } X') = \exp(iT^a\theta_L^a)(\Phi \text{ or } X)\exp(-iT^b\theta_R^b). \tag{10}$$

Most general potential for $SU(2)_L \times SU(2)_R$ global symmetry

$$\begin{aligned} V(\Phi, X) \quad = \quad & \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) \\ & +\lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \\ & -M_1\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)(UXU^\dagger)_{ab} - M_2\mathrm{Tr}(X^\dagger t^a X t^b)(UXU^\dagger)_{ab}. \end{aligned} \tag{11}$$

Vacua structure in a $\mathbb{Z}_2$ symmetric potential $X \to -X$

Potential

$$\begin{aligned} V(\Phi, X) &= \frac{\mu_2^2}{2}\mathrm{Tr}(\Phi^\dagger\Phi) + \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 + \frac{\mu_3^2}{2}\mathrm{Tr}(X^\dagger X) + \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(X^\dagger X) \\ &\quad +\lambda_3\mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4[\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5\mathrm{Tr}(\Phi^\dagger\tau^a\Phi\tau^b)\mathrm{Tr}(X^\dagger t^a X t^b) \end{aligned}$$

Goal: Find all possible vacua, properties and height relationships.

Motivation: Some unphysical vacuum can be the global minimum at a given parameter space point and coexist with a physical one $\Rightarrow$ Theory constraints on the parameter space.

Given the symmetries the most general vacuum expectation value (VEV) for the fields reads

Two $SU(2)_L \times SU(2)_R$ transformations, first with $\theta_L = \theta_R$ and second $\theta_L = -\theta_R$

$$\Phi = \begin{pmatrix} v_1 & 0 \\ 0 & v_1 \end{pmatrix}, \qquad X = \begin{pmatrix} v_8 - i v_9 & v_6 & 0 \\ -v_{10} + i v_{11} & v_5 & v_{10} + i v_{11} \\ 0 & -v_6 & v_8 + i v_9 \end{pmatrix}. \tag{12}$$

There are at least two possible vacua:

- **Dark Matter vacuum** - $(v_1 \neq 0 \wedge v_i = 0) \Rightarrow \rho = 1$ at and $\mathbb{Z}_2$ symmetry is not spontaneously broken.
- **Custodial vacuum** - $(v_1 \neq 0 \wedge v_5 = v_8 \wedge v_i = 0) \Rightarrow \rho = 1$ at tree-level.

$\rightarrow$ We considered first just the real part of the VEV matrix.

Dark matter vacuum

There is no mixing of fields

$$m_h^2 = 8\lambda_1 v^2 \qquad m_{\chi^0}^2 = \mu_3^2 + \left(2\lambda_2 - \frac{1}{2}\lambda_5\right) v^2$$

$$m_{\xi^0}^2 = m_{\xi^+}^2 = m_{\chi^+}^2 = \mu_3^2 + 2\lambda_2 v^2 \qquad m_{\chi^{++}}^2 = \mu_3^2 + \left(2\lambda_2 + \frac{1}{2}\lambda_5\right) v^2\,, \tag{13}$$

DM candidate is χ^0

Custodial vacuum

- A custodial five-plet $(H_5^{++}, H_5^{+}, H_5^{--}, H_5^{-}, H_5^{0})$, triplet $(H_3^{+}, H_3^{-}, H_3^{0})$ and singlets (h, H).

Each multiplet is degenerate in mass

$$\begin{aligned} m_5^2 &= \frac{M_1}{4v_\chi} v_\phi^2 + 12 M_2 v_\chi + \frac{3}{2}\lambda_5 v_\phi^2 + 8\lambda_3 v_\chi^2, \\ m_3^2 &= \frac{M_1}{4v_\chi}(v_\phi^2 + 8v_\chi^2) + \frac{\lambda_5}{2}(v_\phi^2 + 8v_\chi^2) = \left(\frac{M_1}{4v_\chi} + \frac{\lambda_5}{2}\right) v^2 \\ m_{h,H}^2 &= \frac{1}{2}\left[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \mp \sqrt{\left(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2\right)^2 + 4\left(\mathcal{M}_{12}^2\right)^2}\right]. \end{aligned} \tag{14}$$

Unphysical vacua

Charge-breaking (5 solutions)

- $v_5 = v_8 = 0$
- $v_1 = v_5 = v_8 = 0 \wedge v_6 = \pm v_{10}$
- $v_5 = v_8 = 0 \wedge v_6 = \pm v_{10}$

Corresponding gauge bosons' mass matrix (W^1, W^2, W^3, B)

$$M^2 = \begin{pmatrix} \frac{1}{4}g^2\left(v_1^2 + 4v_{10}^2\right) & 0 & 0 & 0 \\ 0 & \frac{1}{4}g^2\left(v_1^2 + 4\left(v_6^2 + v_{10}^2\right)\right) & 0 & 0 \\ 0 & 0 & \frac{1}{4}g^2\left(v_1^2 + 4v_6^2\right) & -\frac{1}{4}v_1^2 gg' \\ 0 & 0 & -\frac{1}{4}v_1^2 gg' & \frac{1}{4}g'^2\left(v_1^2 + 4v_{10}^2\right) \end{pmatrix} \tag{15}$$

There is no null eigenvalue $\rightarrow$ photon is massive!

Wrong-Electroweak (Hypercharge is not spontaneously broken, 2 solutions)

- $v_1 = v_8 = v_{10} = 0$
- $v_1 = v_5 = v_8 = v_{10} = 0$

Corresponding gauge bosons' mass matrix (W^1, W^2, W^3, B)

$$M^2 = \begin{pmatrix} g^2 v_5^2 & 0 & g^2 v_5 v_6 & 0 \\ 0 & g^2\left(v_5^2 + v_6^2\right) & 0 & 0 \\ g^2 v_5 v_6 & 0 & g^2 v_6^2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{16}$$

There are two null eigenvalue $\rightarrow$ Only two gauge bosons are massive!

We also found vacua with three massive and one massless gauge bosons. Example:

$$v_1 = v_6 = v_8 = 0 \tag{17}$$

Corresponding gauge bosons' mass matrix (W^1, W^2, W^3, B)

$$\begin{pmatrix} g^2\left(v_5^2 + v_{10}^2\right) & 0 & 0 & 0 \\ 0 & g^2\left(v_5^2 + v_{10}^2\right) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & v_{10}^2 g'^2 \end{pmatrix} \tag{18}$$

These are most likely unphysical but it ask for reevaluation of the charge operator definition, which in the SM is solely a convention.

Bilinear formalism

Because of the $\mathbb{Z}_2$ symmetry of the potential $\rightarrow$ there are no cubic VEV terms.

$$V = A^T X + \frac{1}{2} X^T B X \tag{19}$$

with A, B a vector and a matrices, respectively, of the parameters of the potential and X a vector of VEVs.

This formalism allows for easy comparison of potential heights between different minima!

(To be continued...)

Conclusions

Conclusions

Theory side

- The model is interesting due to capacity fixing $\rho = 1$ at tree-level.
- Characterizing the vacua and their phenomenology.
- Check if existence of unphysical vacua might constrain the parameter space.
- Work in progress...

Experimentally speaking it serves as a benchmark for collider study of triplets.

- Higher Gauge-Higgs couplings might be interesting to study and look for (some papers do already exist).
- Dark matter phase is still unexplored terrain.