

Accuracy improvement of the atmospheric neutrino flux prediction using observed muon spectra at mountain altitude

Jan. 13, 2021, M. Honda
(Based on the talk at TAUP2019)

1. Atmospheric neutrino and muon in Hadronic interaction Phase Space.
2. The correlation of the variation of atmospheric neutrino and muon fluxes.
3. The error of atmospheric neutrino flux calculation when accurately measured muon flux is well reconstructed.
4. Other possible source of the error.

Gaisser Formula for the illustration

(by T.K.Gaisser at Takayama, 1998)

$$\Phi_{\nu} = \Phi_{primary} \otimes R_{cut} \otimes Y_{\nu}$$

$$\Phi_{\mu} = \Phi_{primary} \otimes R_{cut} \otimes Y_{\mu}$$

Where

$\Phi_{primary}$

: Primary Cosmic Ray Flux

$$R_{cut} = R_{cut}(R_{cr}, latt., long., \theta, \phi)$$

: Geomagnetic field

$$Y_{\nu} = Yield_{\nu}(h, \theta)$$

$$Y_{\mu} = Yield_{\mu}(h, \theta)$$

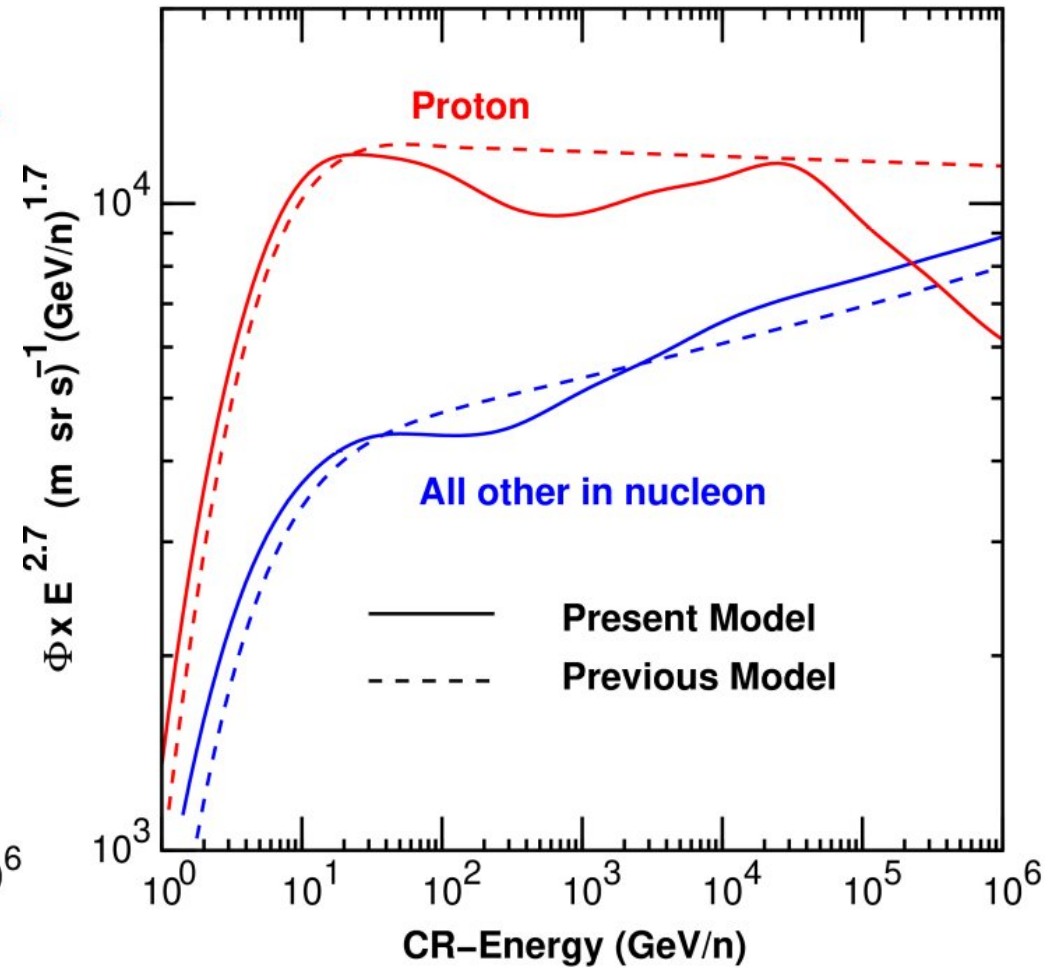
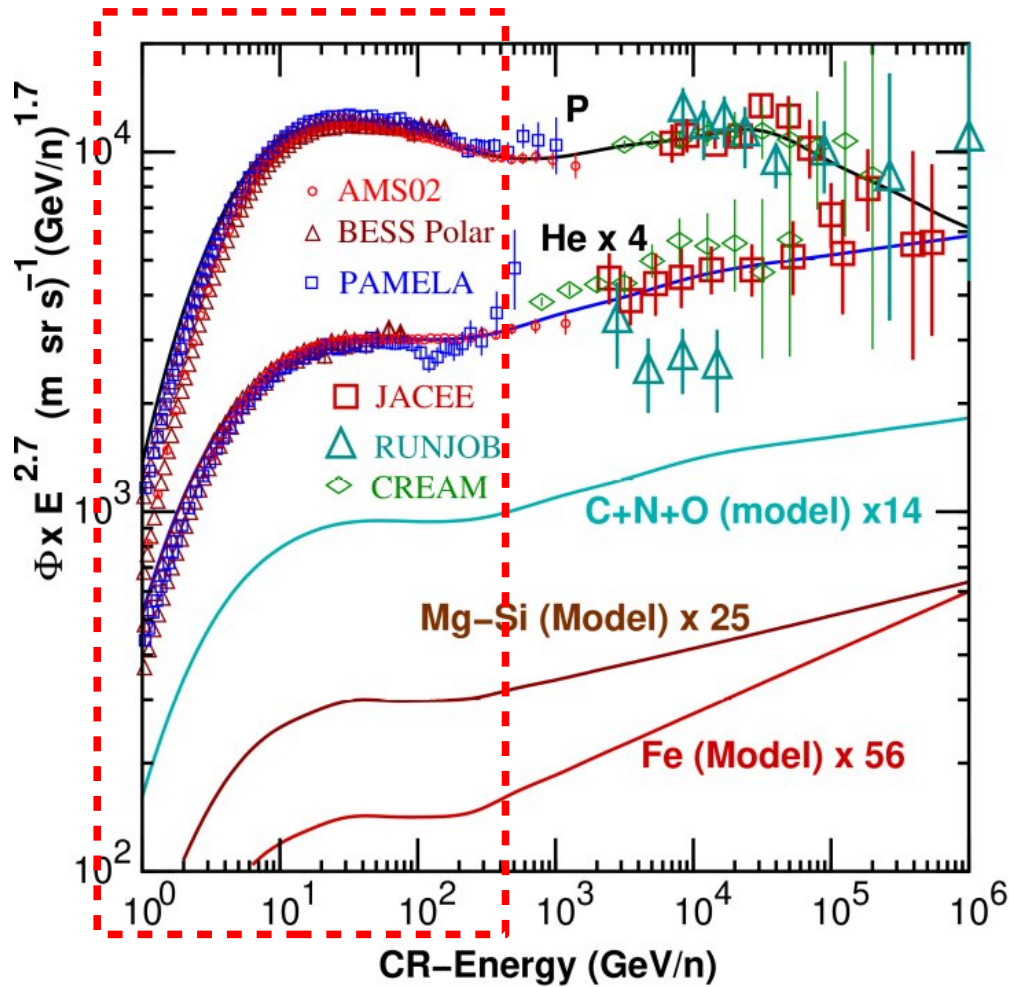
: Hadronic Interaction Model,

Air Profile, and meson-muon decay

: Hadronic Interaction Model

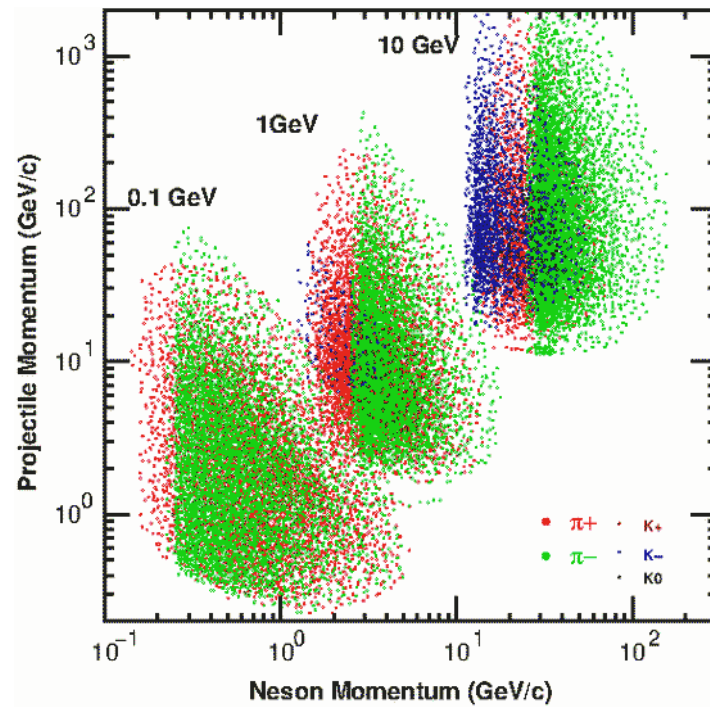
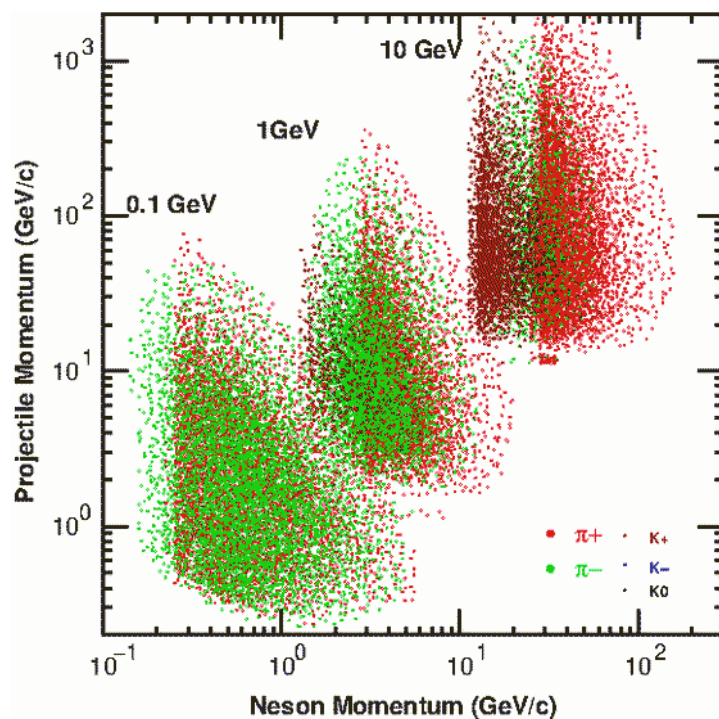
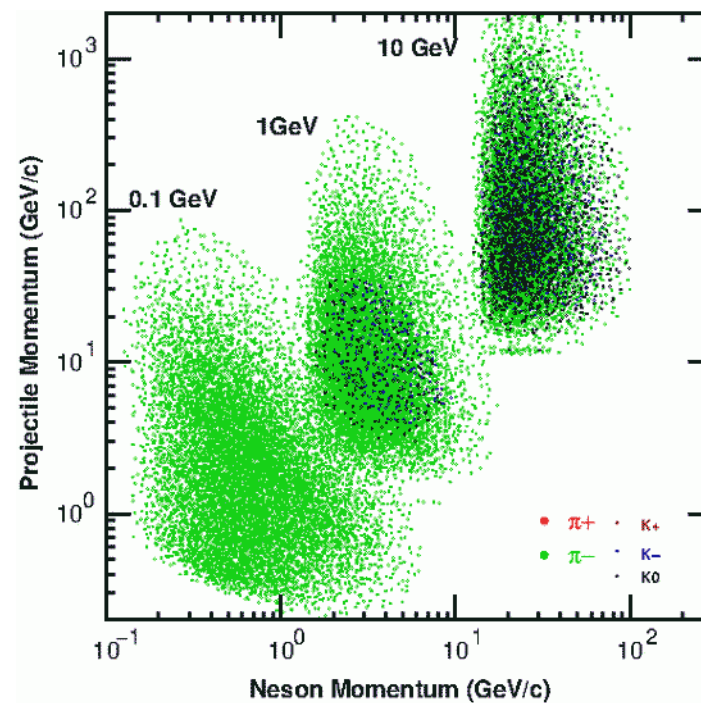
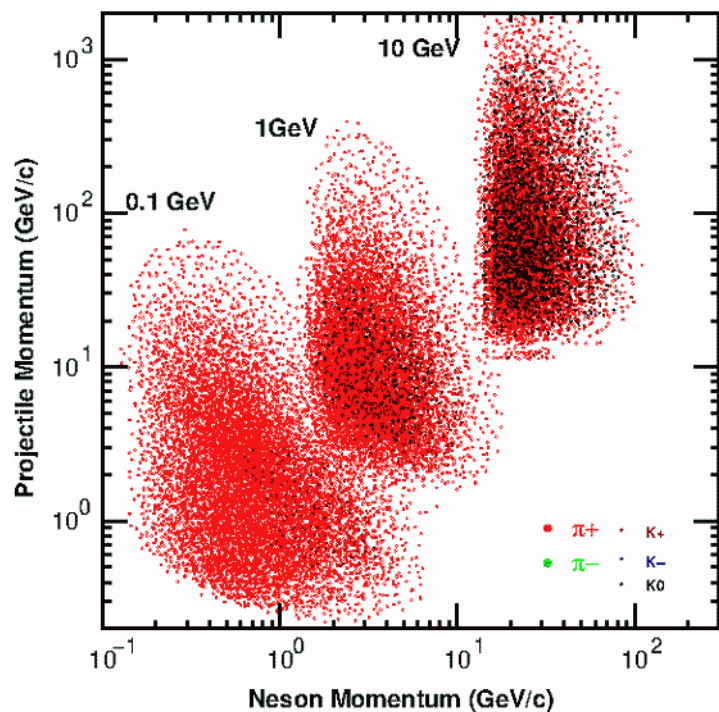
Cosmic Ray Spectra Model Based on AMS02 Observation (2017 1ry model)

Fixed within the error of 5%

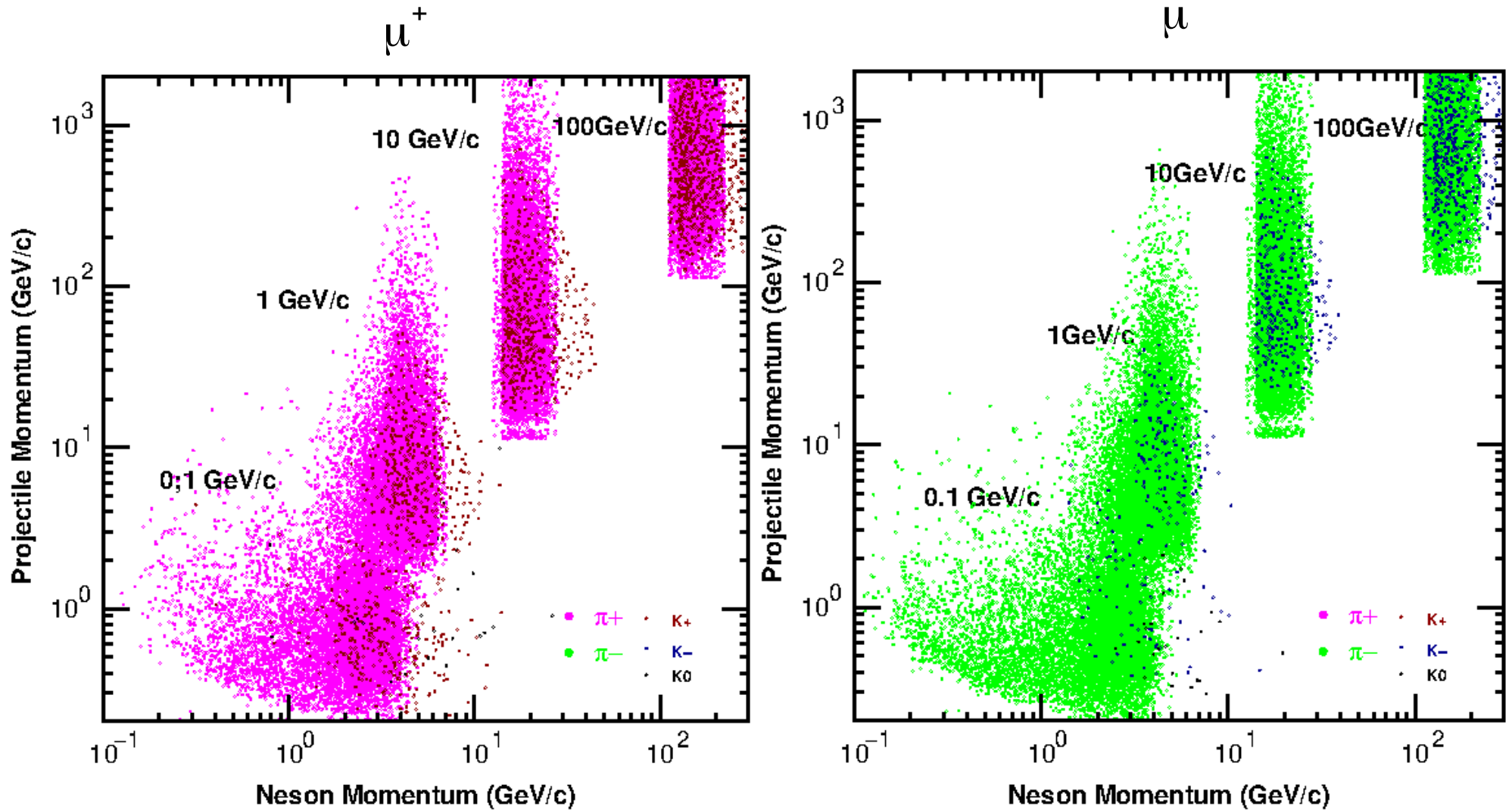


Looking forward to hearing from CALET and ISS-CREAM

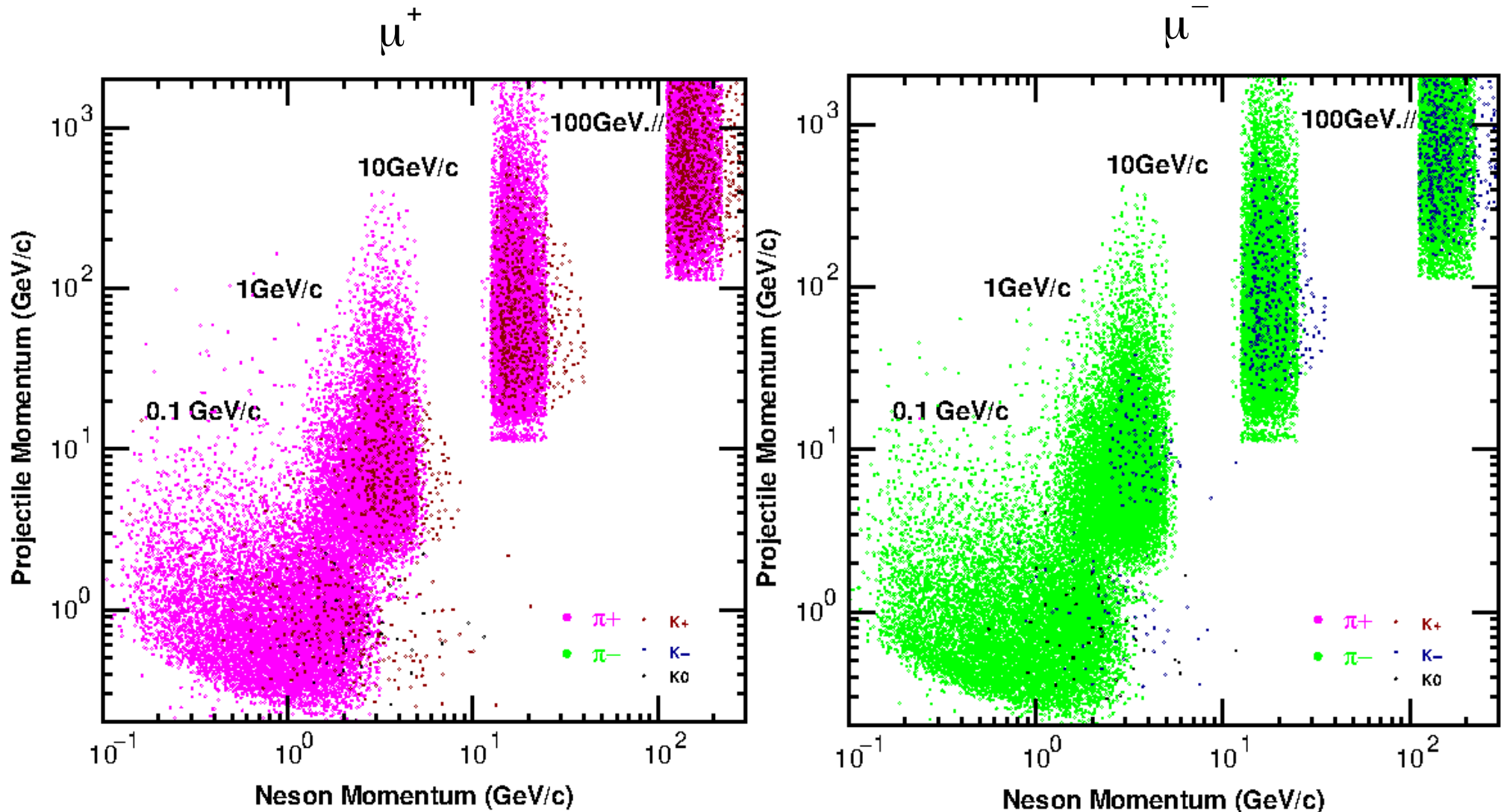
Neutrino in Phase Space of Hadronic Interaction



Muon observed at Tsukuba (sea level) in Phase Space of Hadronic interaction.



Muon observed at Mt. Norikura (2770m a.s.l.) in Phase Space of Hadronic Interaction.



Semi-analytic expression for atmospheric lepton flux calculation

$$\Phi_{L^{obs}}(p_L^{obs}, x^{obs}) = \sum_{N^{proj}} \sum_{M^{brn}} \int \int D(N^{proj}, p_N^{proj}, M^{brn}, p_M^{brn}, L^{obs}, p_L^{obs}, x^{obs}) dp_M^{brn} dp_N^{proj}$$

$$\begin{aligned} & D(N^{proj}, p_N^{proj}, M^{brn}, p_M^{brn}, L^{obs}, p_L^{obs}, x^{obs}) \\ & \equiv \int M \ 2 \ L(M^{brn}, p_M^{brn}, x^{int}, L^{obs}, p_L^{obs}, x^{obs}) \\ & \quad \times H_{int}(N^{proj}, p_N^{proj}, M^{brn}, p_M^{brn}) \\ & \quad \times \sigma_{prod}(N_{proj}, p_{proj}) \cdot \rho(x^{int}) \\ & \quad \times \Phi_{proj}(N^{proj}, p^{proj}, x^{int}) dx^{int} \end{aligned}$$

$M \ 2 \ L(M^{brn}, p_M^{brn}, x^{int}, L^{obs}, p_L^{obs}, x^{obs})$: Probability of a meson found at x^{int} create a lepton observed at x^{obs} .

$H_{int}(N^{proj}, p_N^{proj}, M^{brn}, p_M^{brn})$: Probability of a projectile particle create the meson in the hadronic interaction with air.

$\sigma_{prod}(N_{proj}, p_{proj})$: Production cross section of the projectile particle with Air nucleus.

$\rho(x^{int})$: Air density.

$\Phi_{proj}(N^{proj}, p^{proj}, x^{int})$: Projectile particle flux at x^{int} .

Variation of atmospheric lepton flux by the variation of hadronic interaction

$$\Phi_{L^{obs}}(p_L^{obs}, x^{obs}) = \sum_{N^{proj}} \sum_{M^{brn}} \int \int D(N^{proj}, p_N^{proj}, M^{brn}, p_M^{brn}, L^{obs}, p_L^{obs}, x^{obs}) dp_M^{brn} dp_N^{proj}$$

$$\begin{aligned} \tilde{\Phi}_{L^{obs}}(p_L^{obs}, x^{obs}) &= \sum_{N^{proj}} \sum_{M^{brn}} \int \int D(N^{proj}, p_N^{proj}, M^{brn}, p_M^{brn}, L^{obs}, p_L^{obs}, x^{obs}) \\ &\quad \times \left(1 + \Delta_{int}(N^{proj}, p_N^{proj}, M^{brn}, p_M^{brn}) \right) dp_M^{brn} dp_N^{proj} \end{aligned}$$

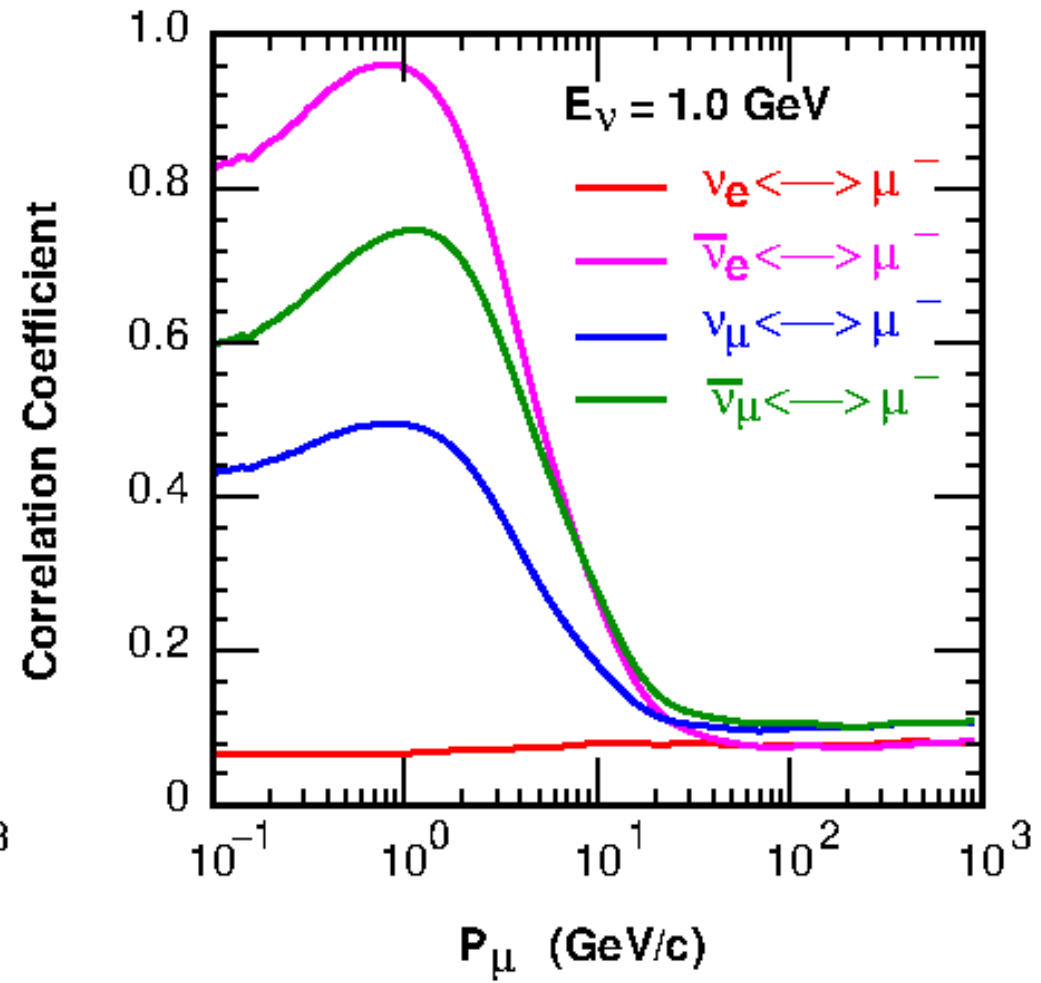
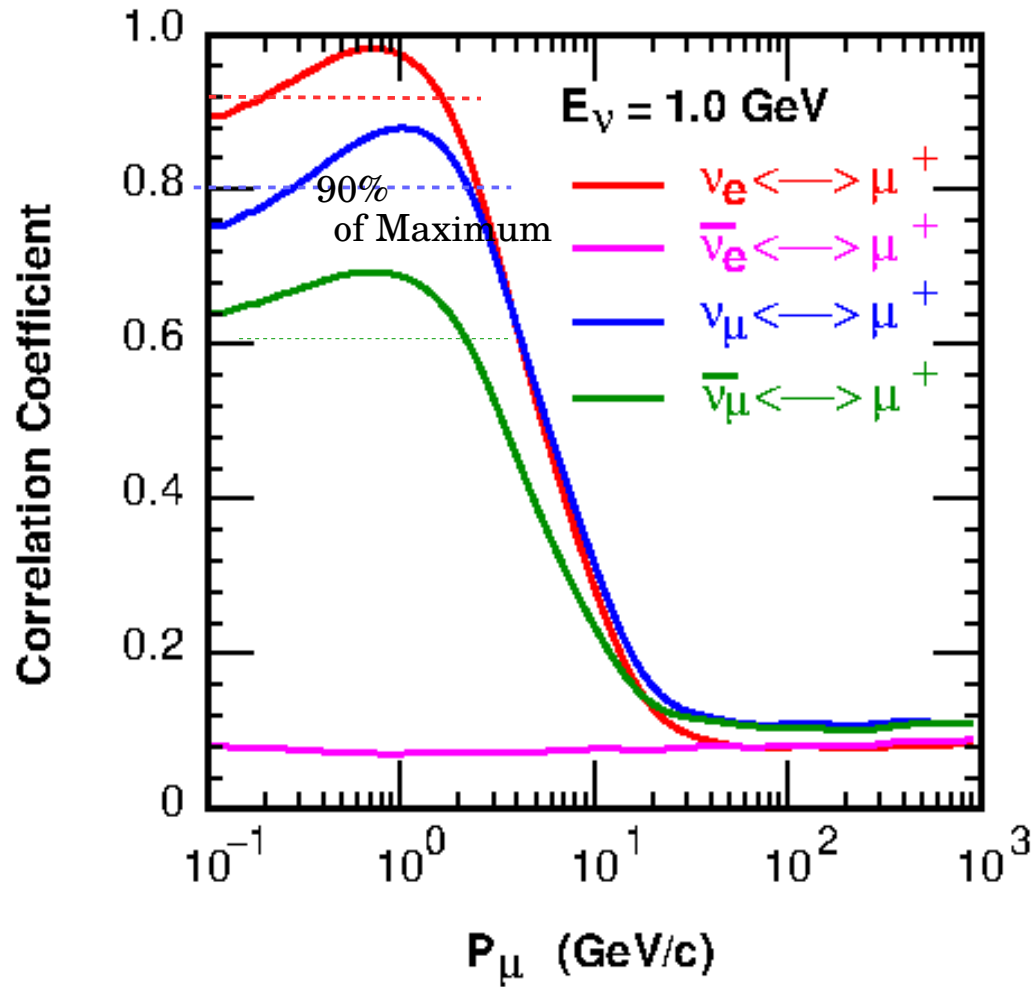
$$\Delta_{int}(N^{proj}, p_N^{proj}, M^{brn}, p_M^{brn}) = \delta \cdot \sum_{i,j} R_{N^{proj}, M^{brn}}^{i,j} B_{\Delta}^i(\log p_M^{brn}) B_{\Delta}^j(\log p_N^{proj})$$

$$B_{\Delta}^i(x) = b\left(\frac{x - i \cdot \Delta}{\Delta} - x_0\right)$$

$$b(t) = \begin{cases} \frac{1}{6}(3|t|^3 - 6|t|^2 + 4) & (|t| \leq 1) \\ -\frac{1}{6}(|t| - 2)^3 & (1 \leq |t| \leq 2) \\ 0 & (|t| \geq 2) \end{cases},$$

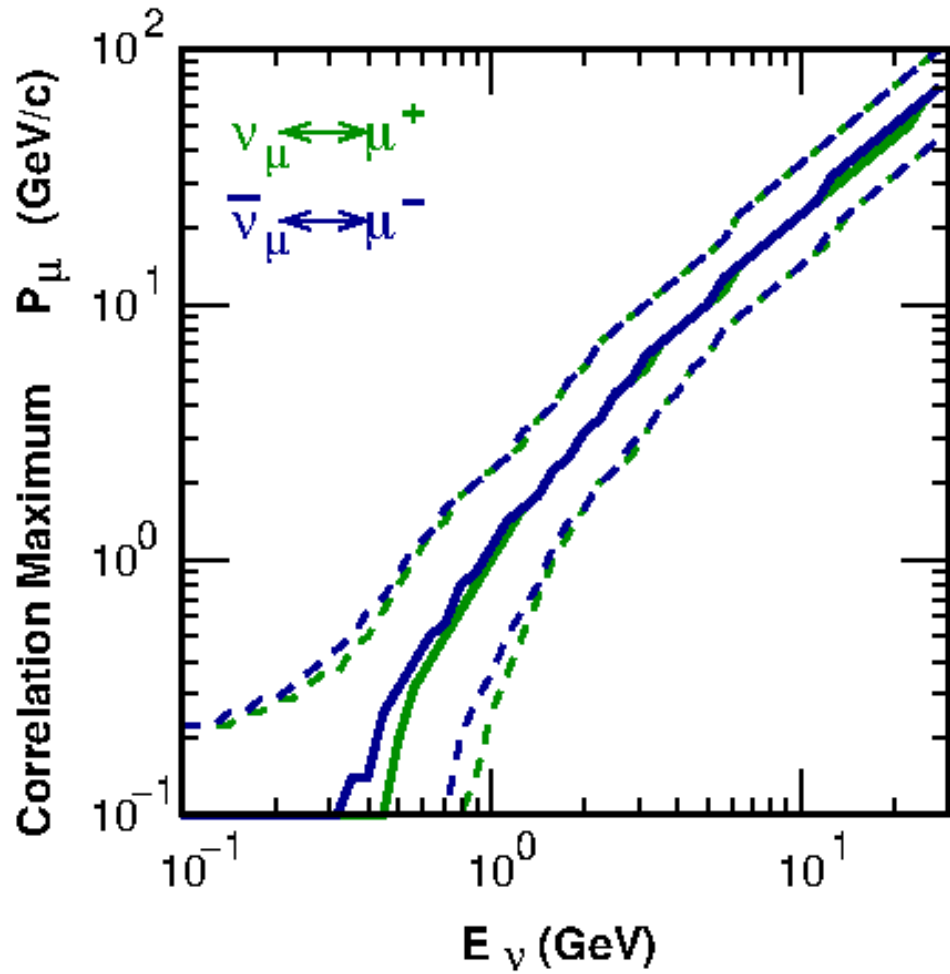
Correlation between neutrino and muon

Kamioka ν vs Kamioka μ (sea level)

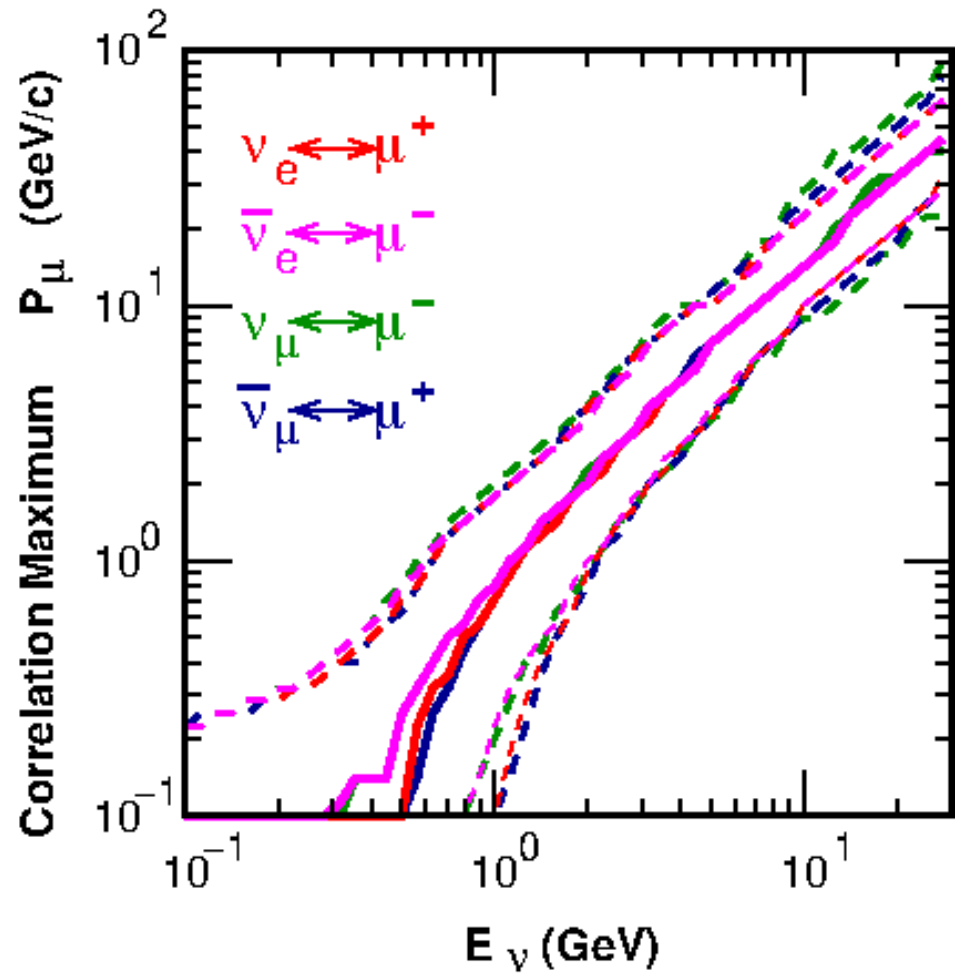


Area with the correlation > 90 % of the maximum

Direct pion decay products



Muon decay products

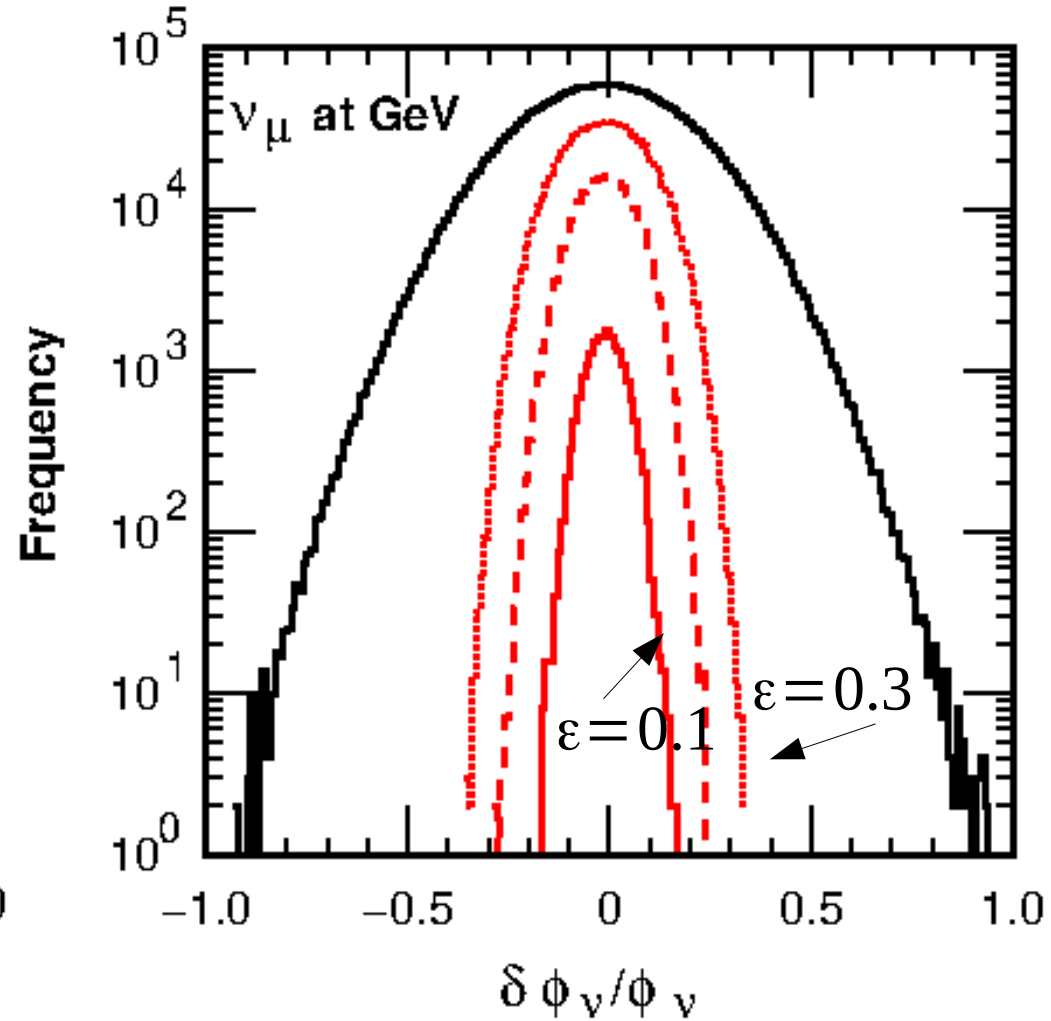
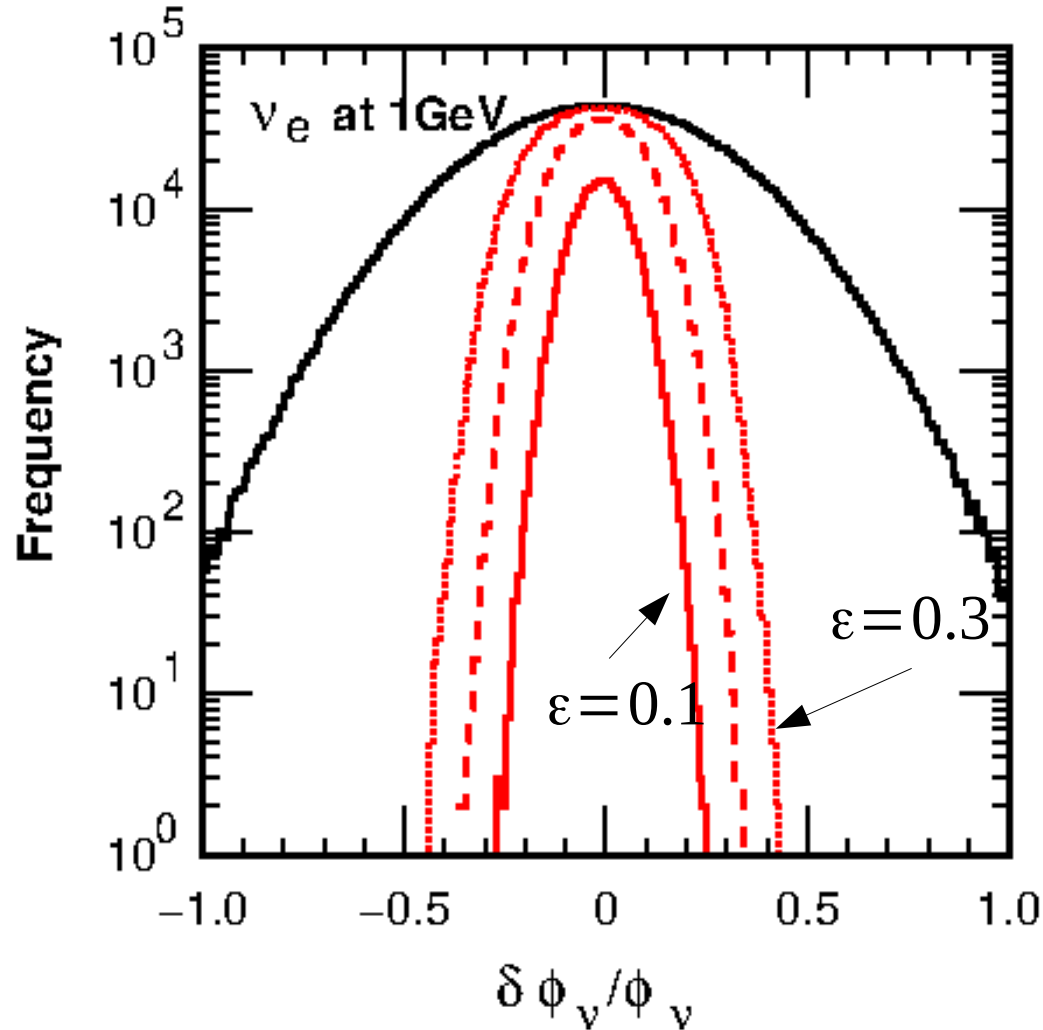


Kamioka ν vs Kamioka μ (sea level)

When the variation of muon is restricted in the correlation $> 90\%$ of maximum region as

$$\left| \frac{\Delta \Phi_\mu}{\Phi_\mu} \right| < \varepsilon ,$$

the variation of neutrino flux also shrinks as;



Each distribution for a ε is well approximated by

$$Dist\left(\varepsilon, \frac{\Delta\Phi_v}{\Phi_v}\right) = \frac{N_\varepsilon}{\sqrt{2\pi}} \exp\left(-\frac{1}{\sigma_\varepsilon} \left(\frac{\Delta\Phi_v}{\Phi_v}\right)^2\right)$$

We may define σ_{shrink} by

$$\frac{1}{\sigma_{shrink}^2} = \frac{1}{\sigma_\varepsilon^2} - \frac{1}{\sigma_\infty^2}$$

since

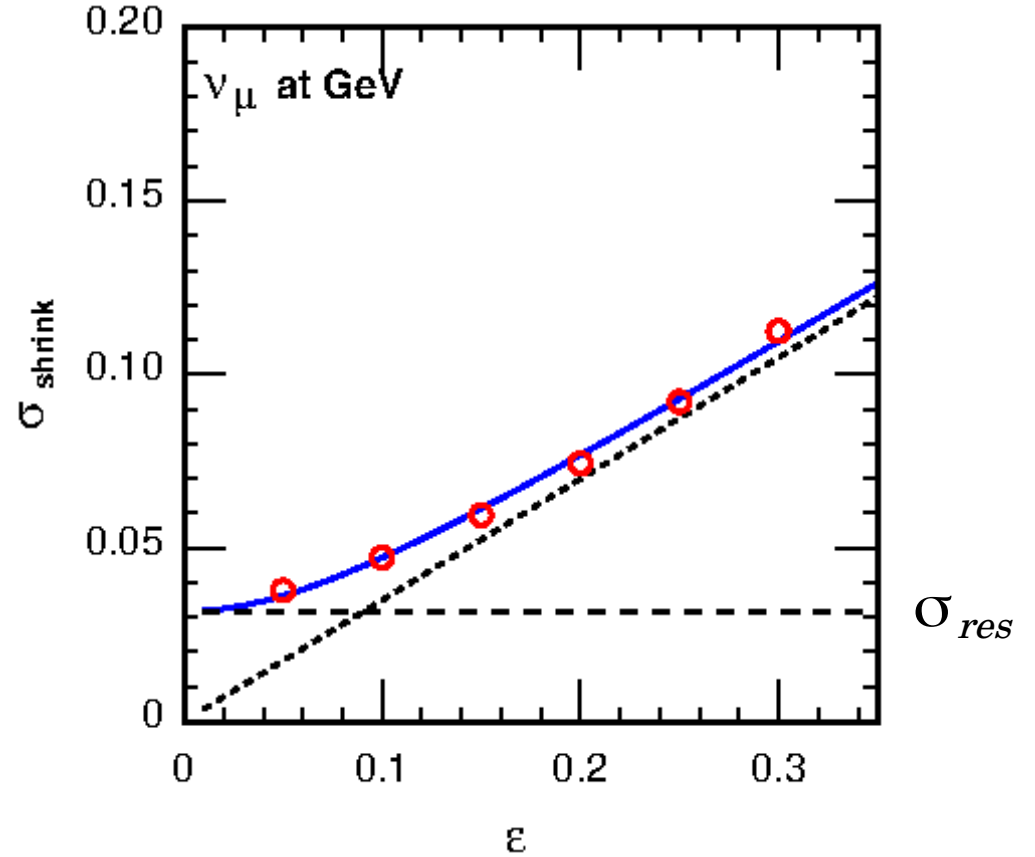
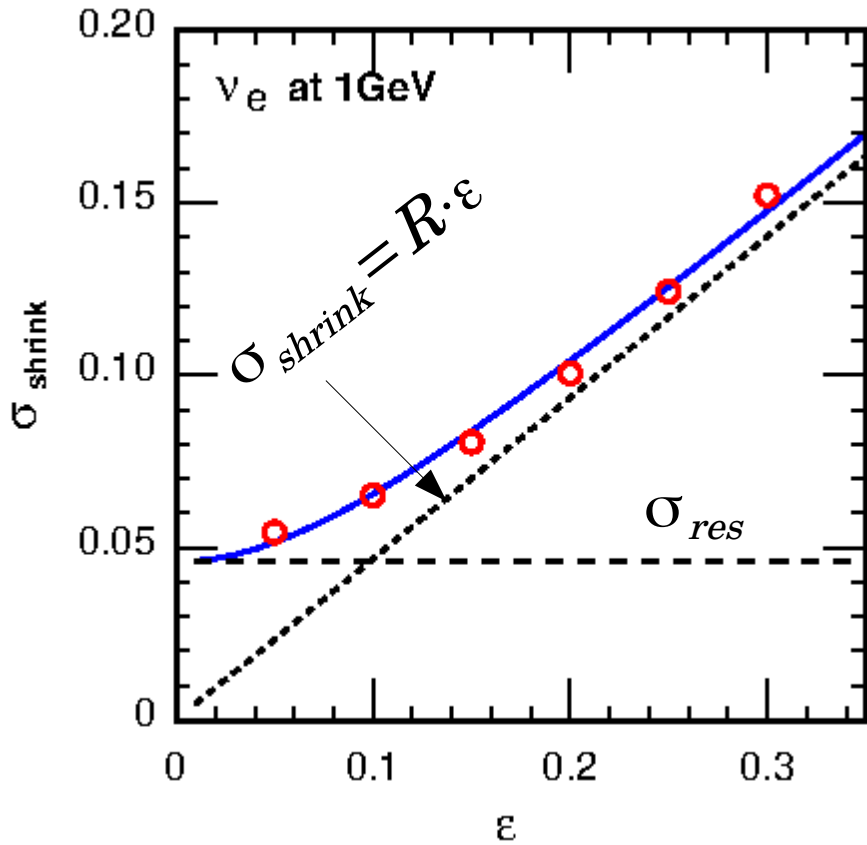
$$\frac{Dist\left(\varepsilon, \frac{\Delta\Phi_v}{\Phi_v}\right)}{Dist\left(\infty, \frac{\Delta\Phi_v}{\Phi_v}\right)} \propto \exp\left(-\frac{1}{\sigma_{shrink}} \left(\frac{\Delta\Phi_v}{\Phi_v}\right)^2\right)$$

And fit σ_{shrink} with

$$\sigma_{shrink} = \sqrt{\sigma_{res}^2 + (R \cdot \varepsilon)^2}$$

Fitting Result for 1 GeV neutrino

Kamioka ν vs Kamioka (Tsukuba) μ (sea level)



$$\sigma_{shrink} = \sqrt{\sigma_{res}^2 + (R \cdot \epsilon)^2}$$

Presently available data

Observed at

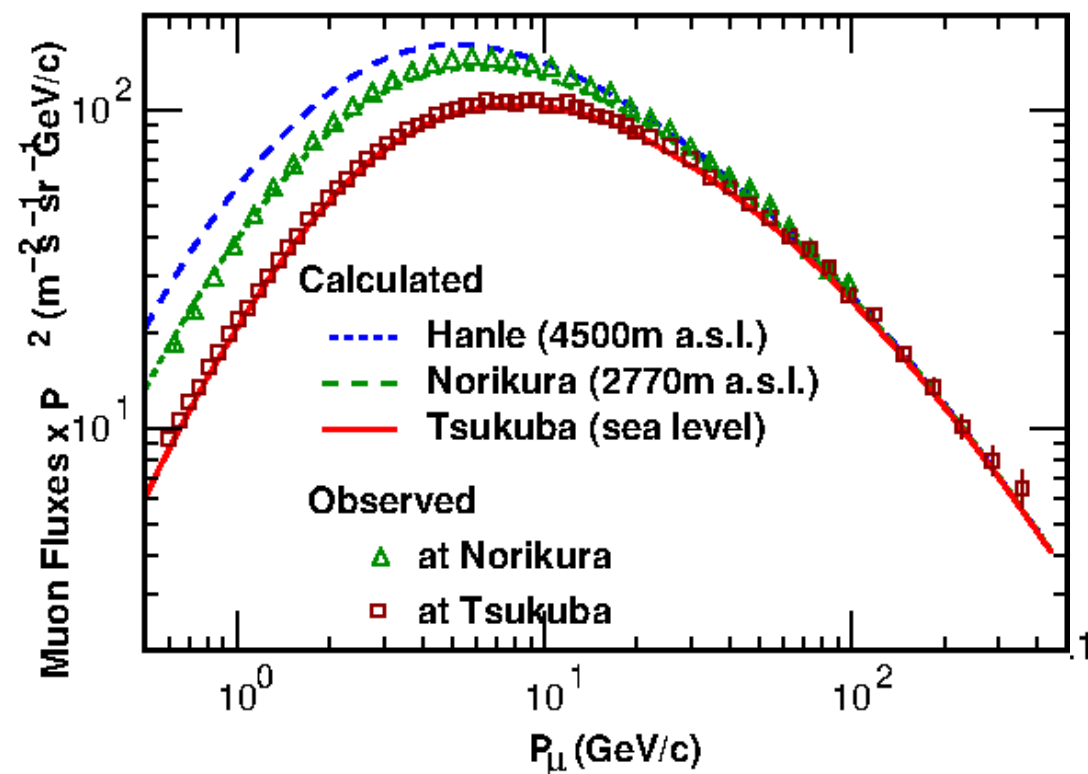
Mt. Norikura (2770m asl)

Tsukuba (KEK) (55m asl)

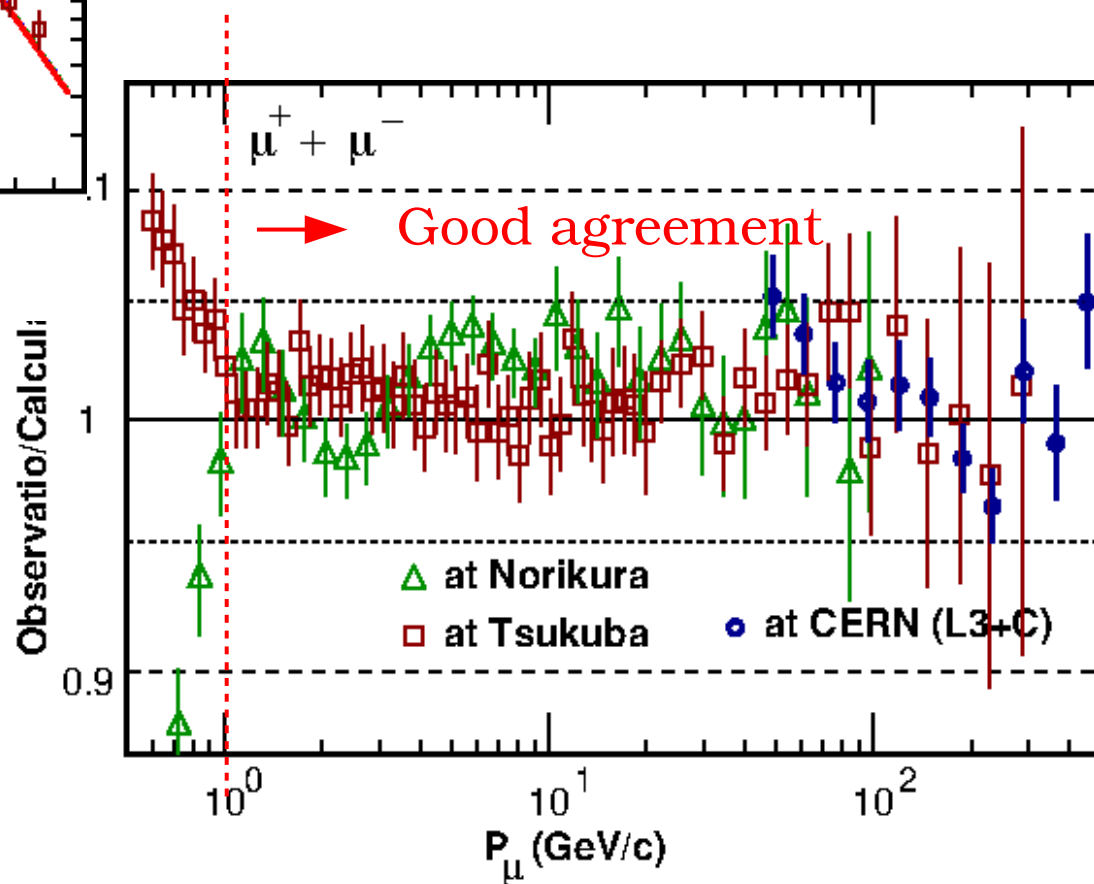


Presently available data

(Hanle is calculation only)

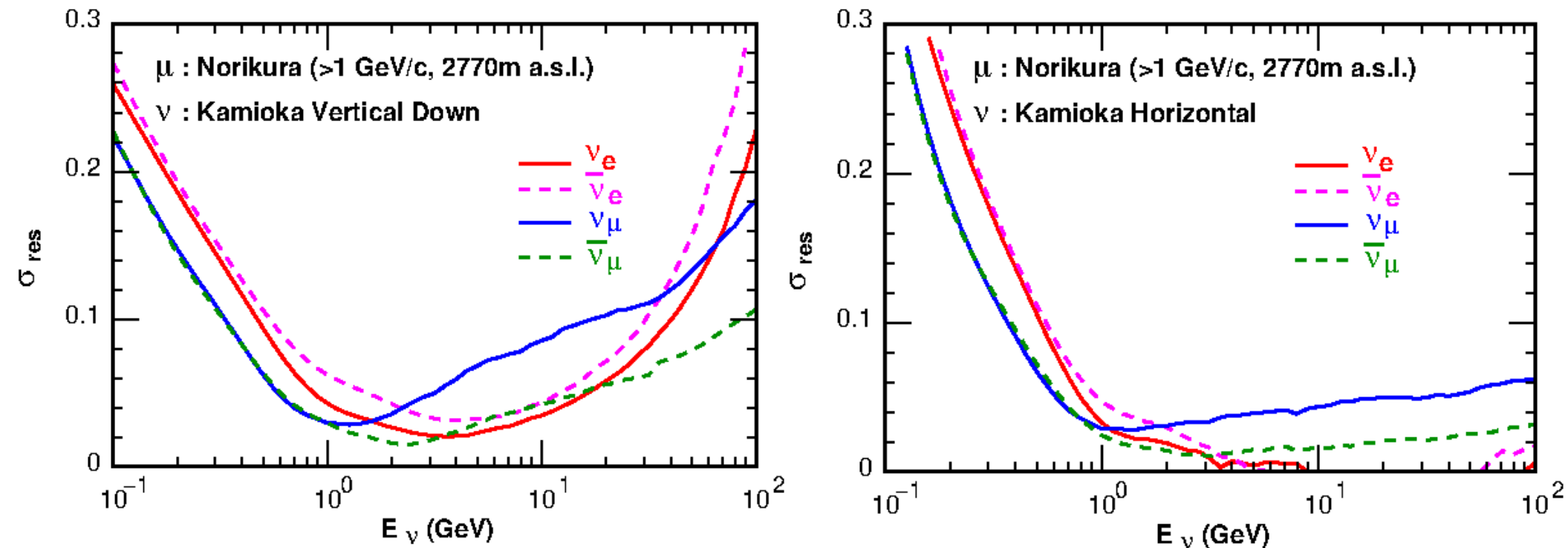


Reconstruction of flux data



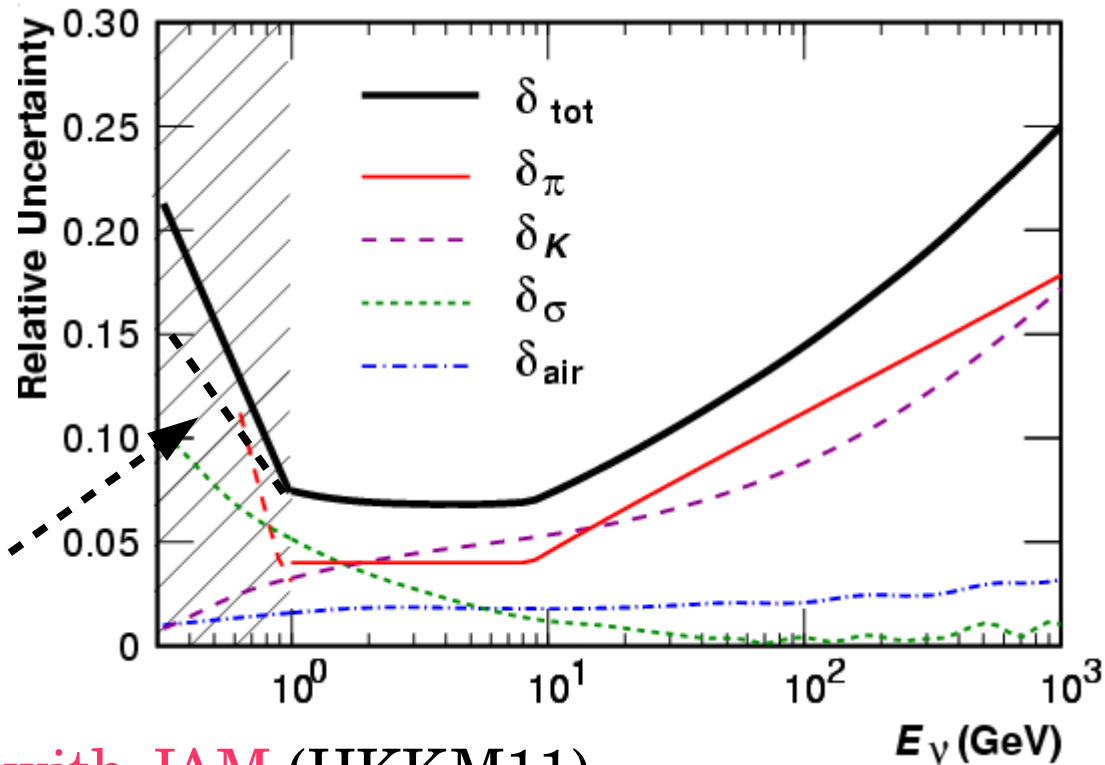
σ_{res} as the function of Neutrino Energy

Application of above study to presently available muon flux data observed at Mt. Norikura (2770m a.s.l.), limiting the momentum of available muon flux to $> 1\text{ GeV}/c$.



σ_{res} increases much quicker below 1 GeV,
but above 1 GeV is similar.

Estimated Error in Atmospheric n-flux Calculation (HKKMS07)



Possible Error with JAM (HKKM11)

-observation error + Residual of reconstruction

Kaon production uncertainty

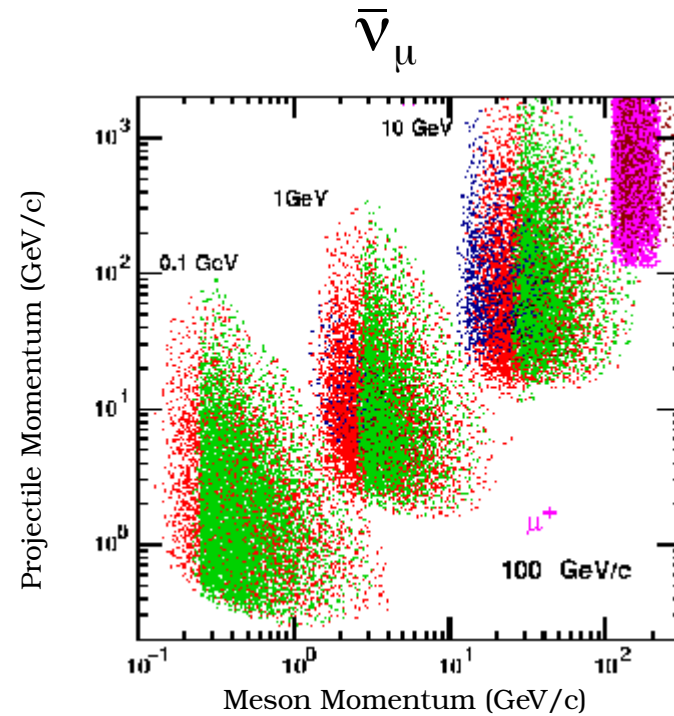
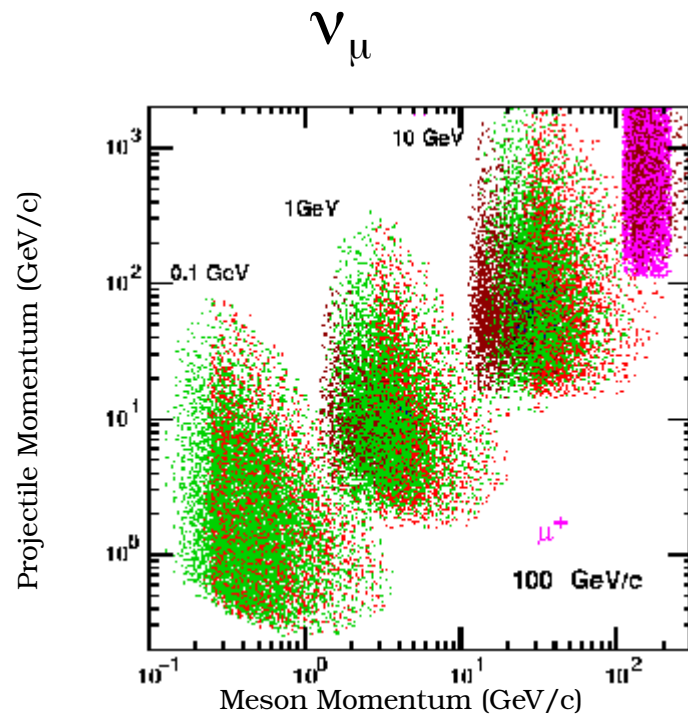
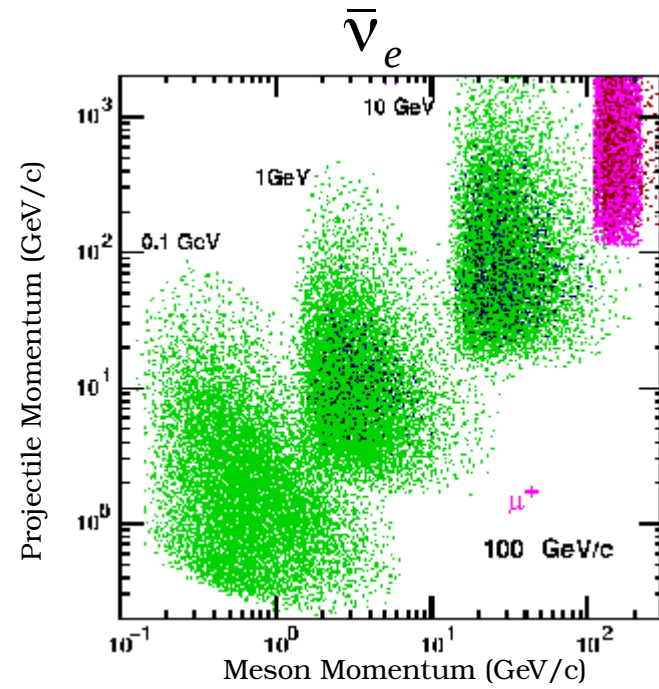
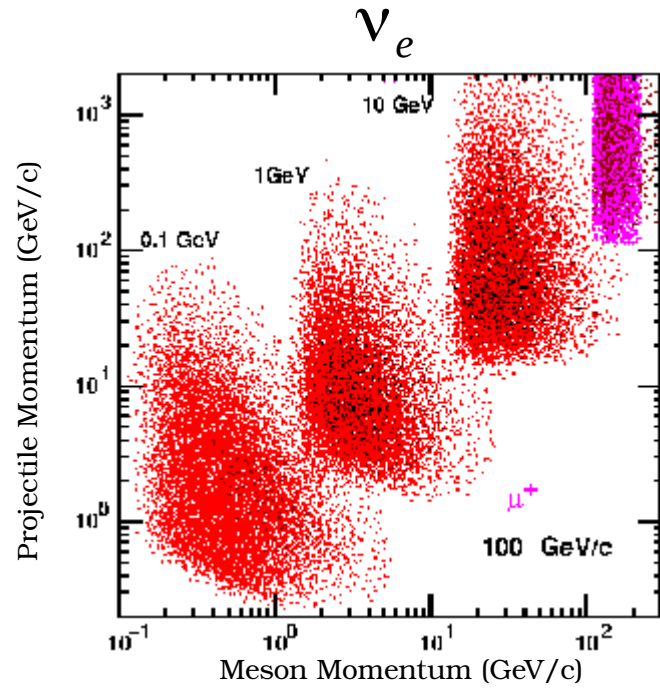
Mean free path (interaction crosssection) uncertainty

Atmosphere density profule uncertainty

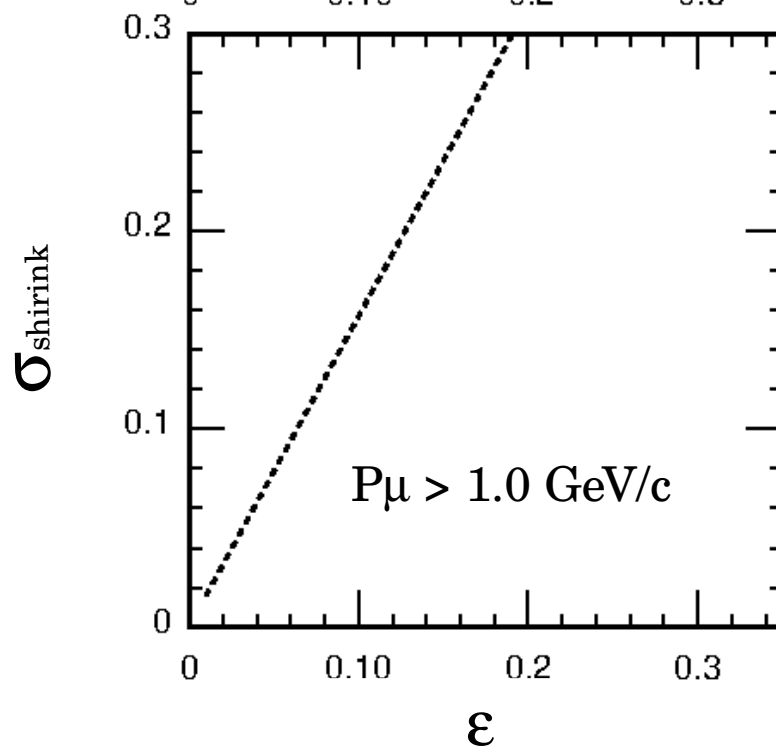
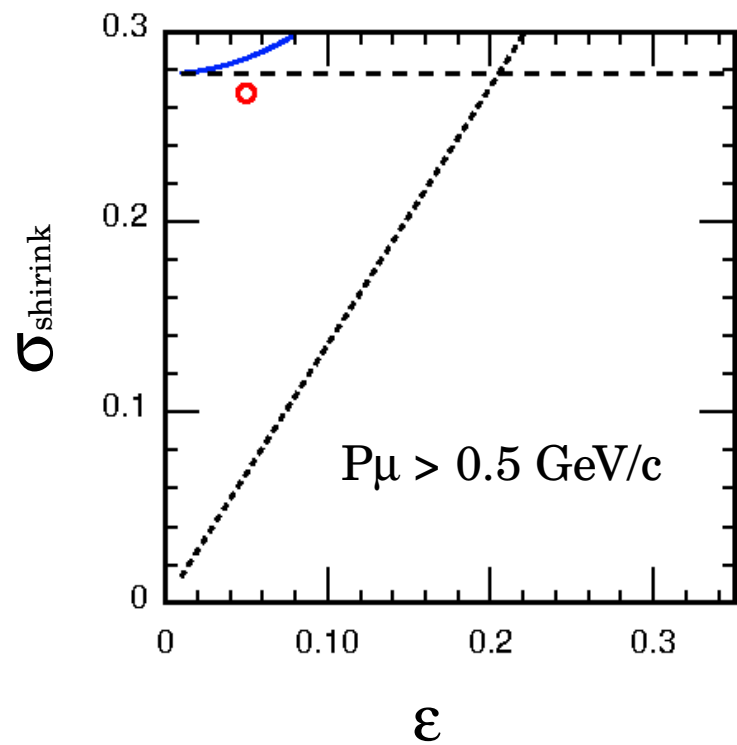
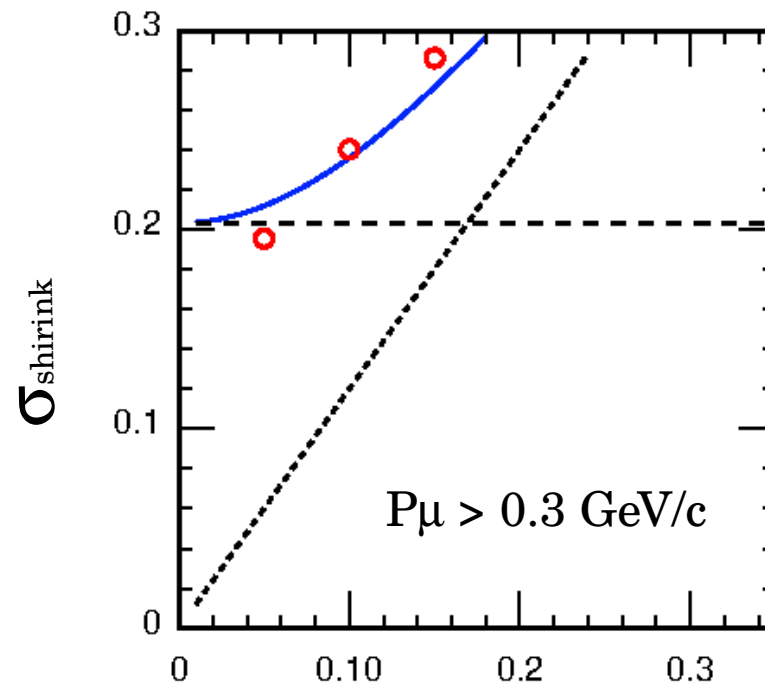
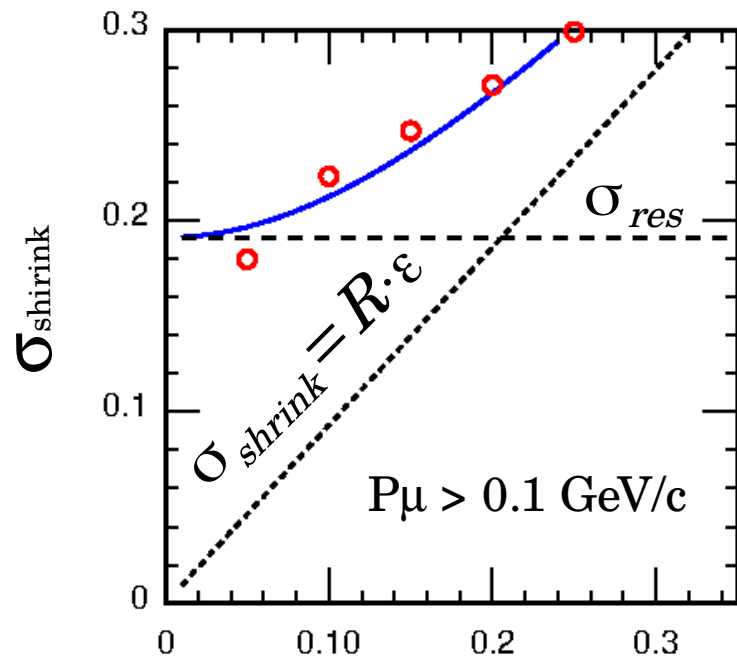
More details with muon
Observed at
Tsukuba (KEK) (55m asl)



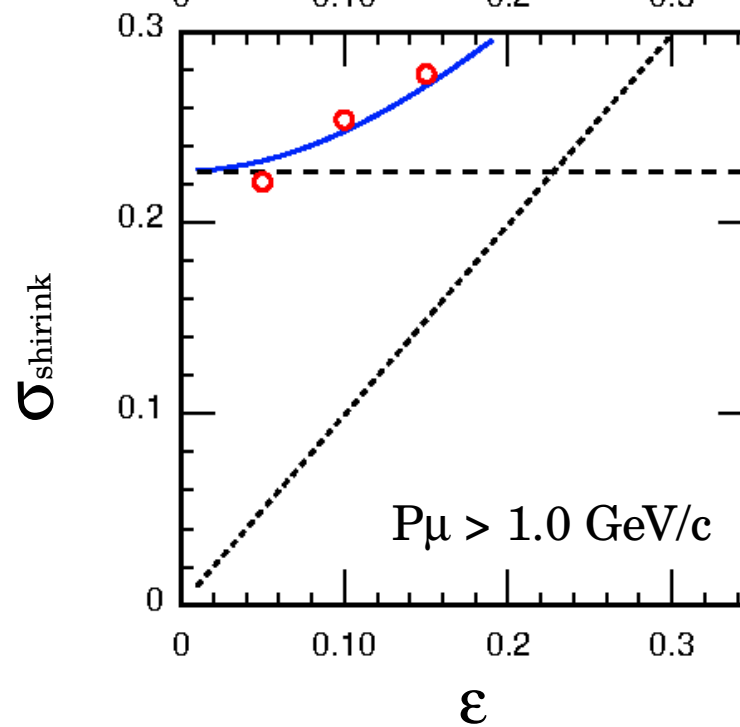
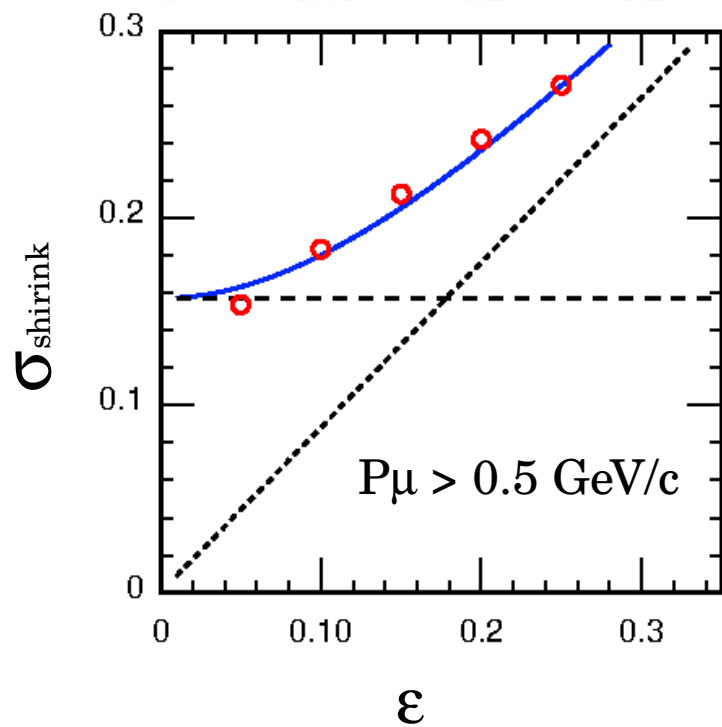
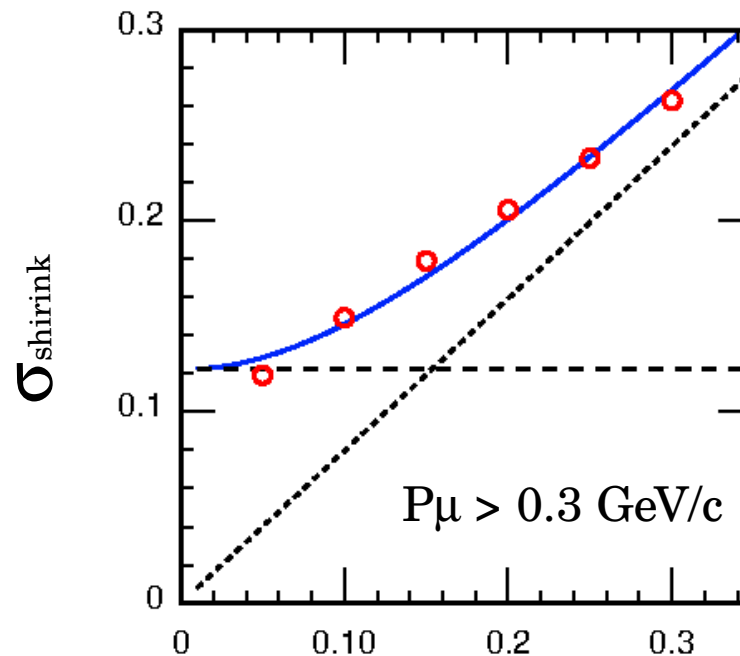
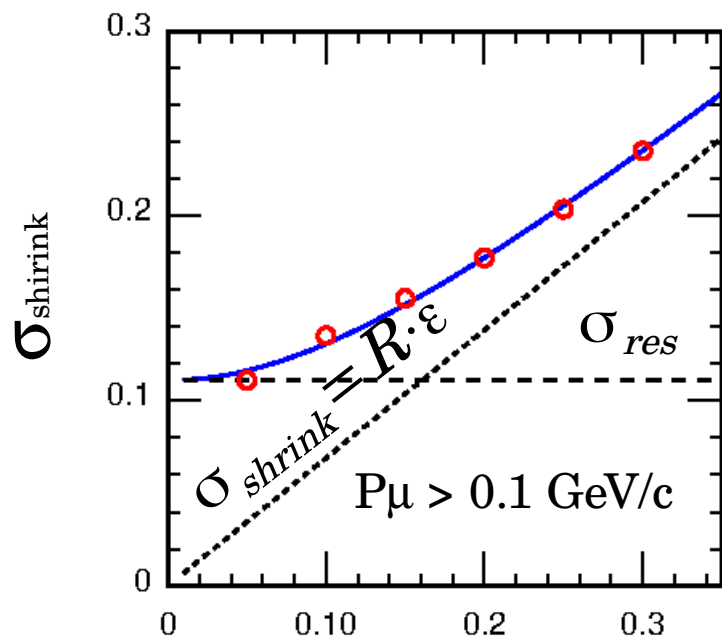
Phase Space Scan with muon at Tsukuba (Sea Level)



For $E_\nu = 0.1$ GeV with muon observation at Tsukuba



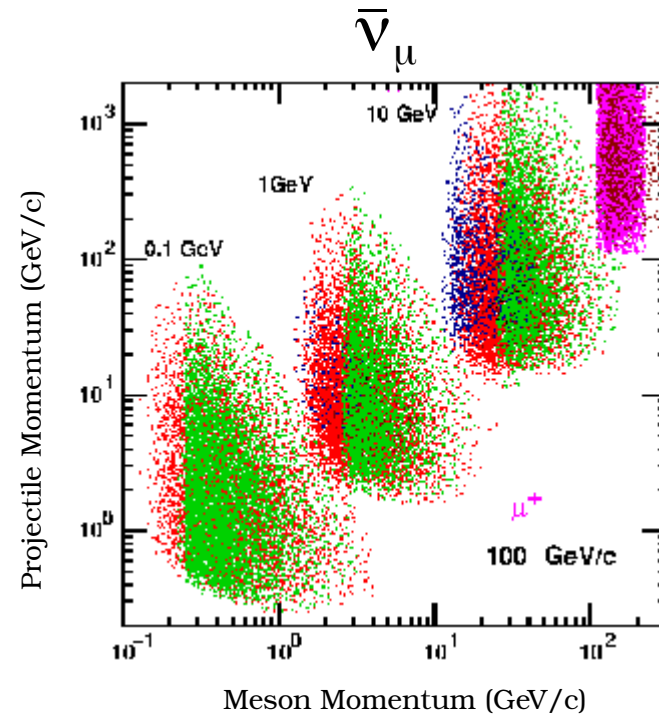
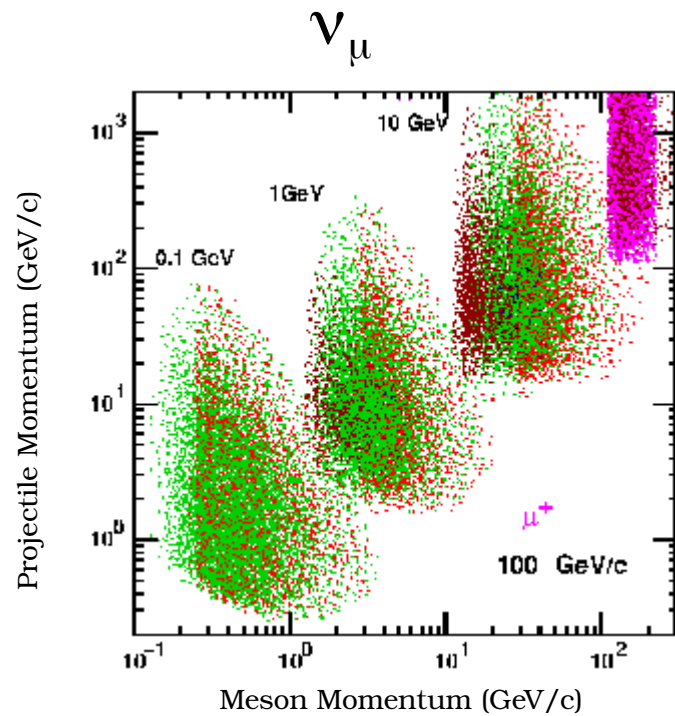
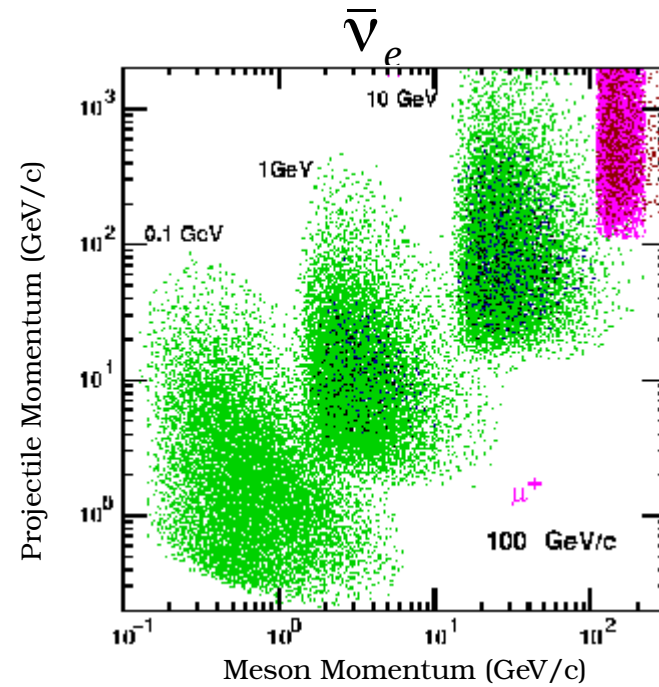
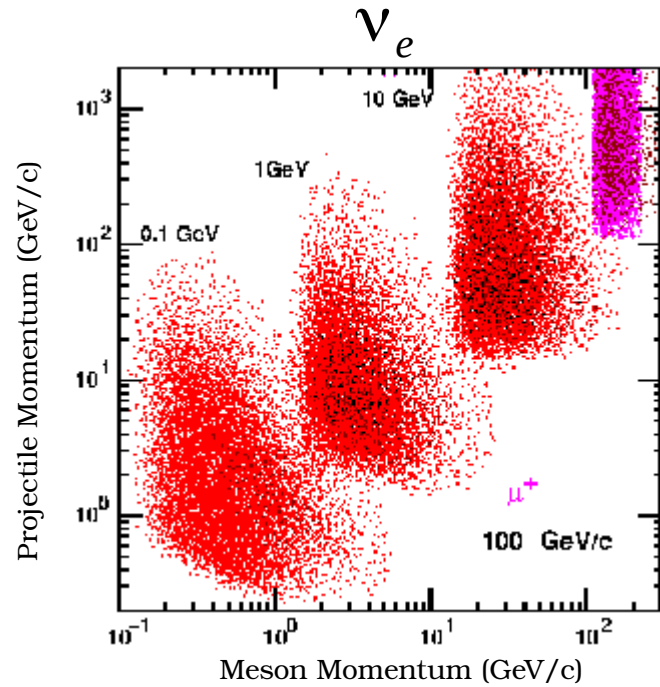
For $E\nu = 0.3$ GeV with muon observation at Tsukuba



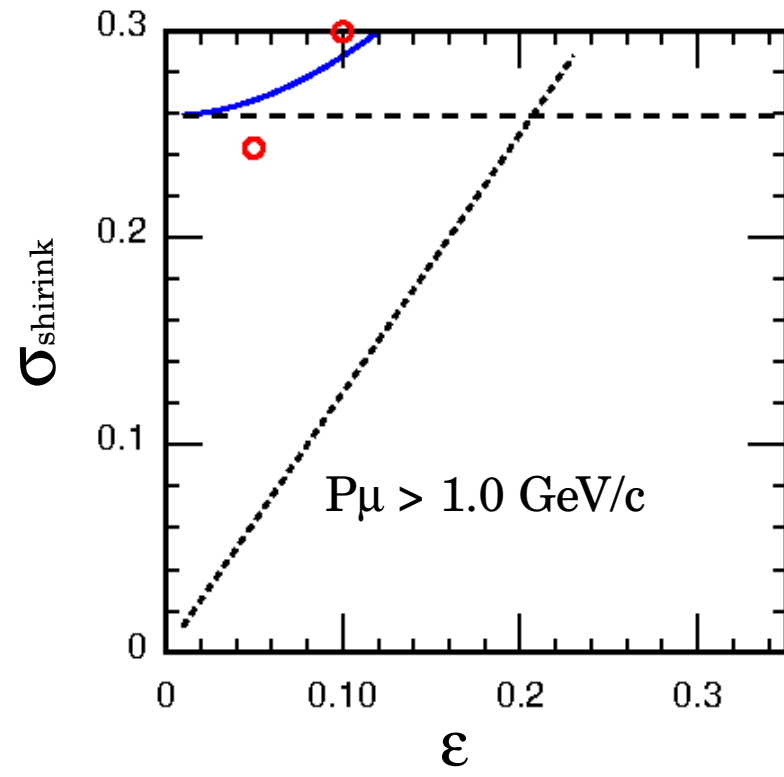
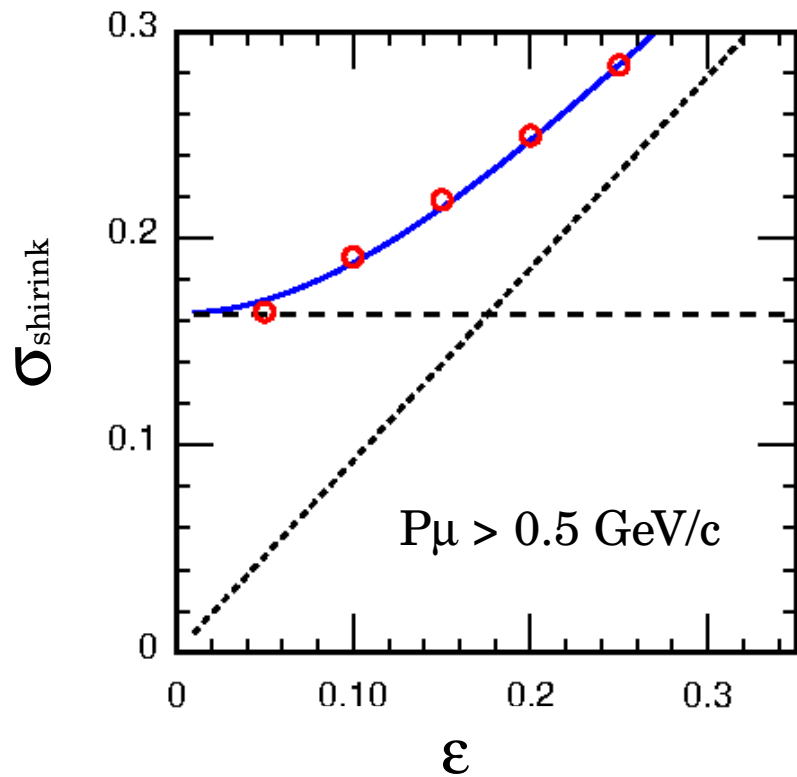
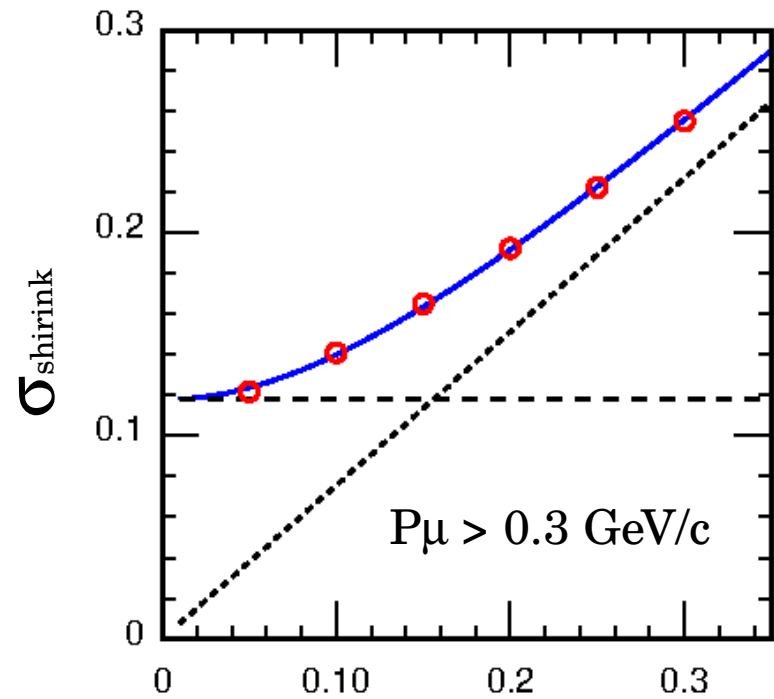
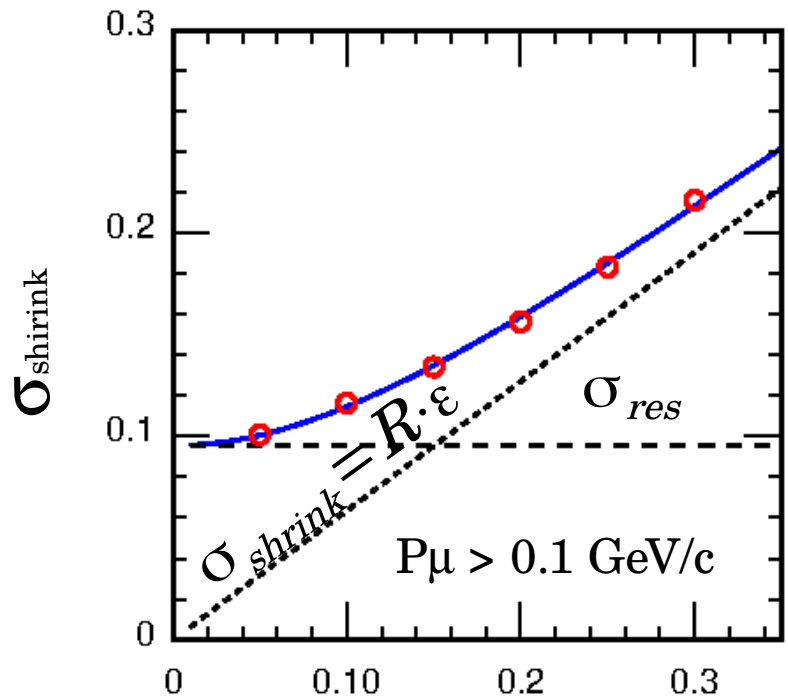
More details with muon
observed at Mt. Norikura (2770m asl)



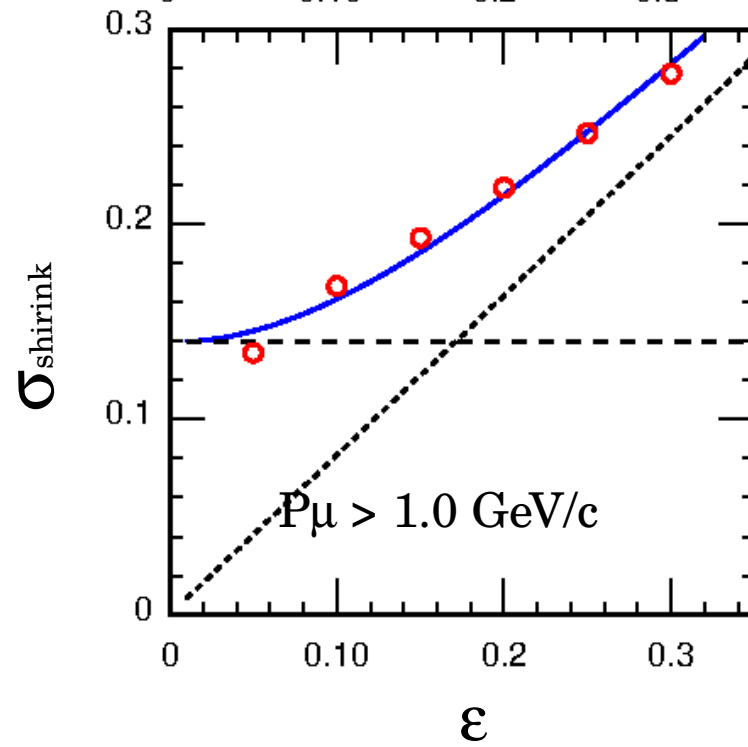
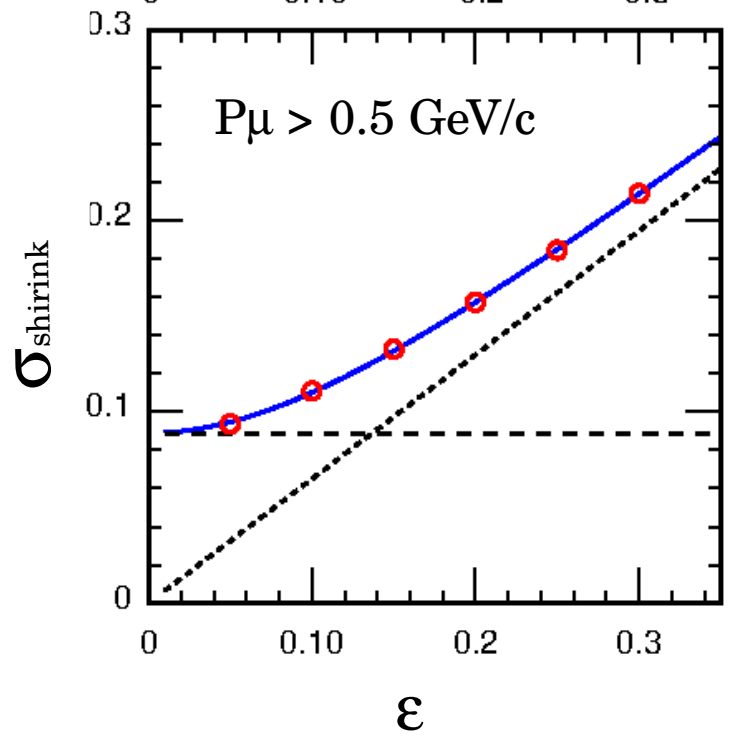
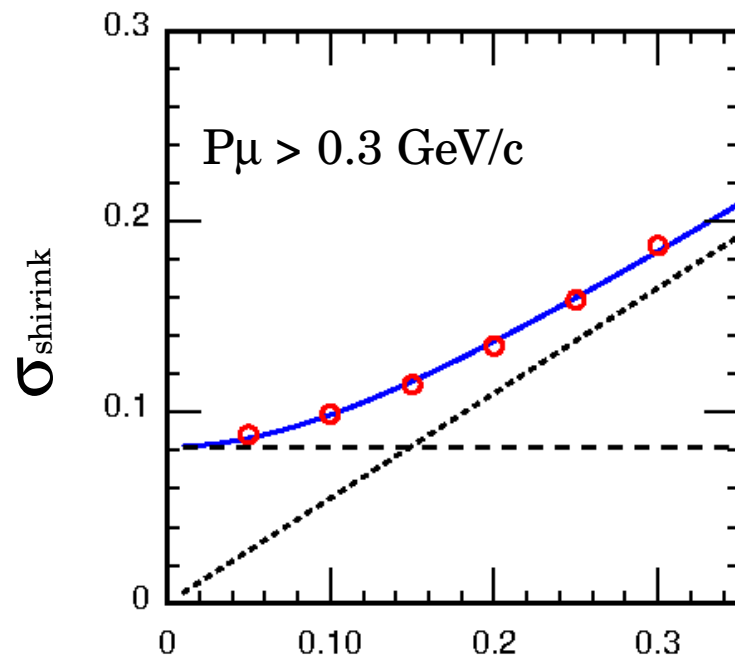
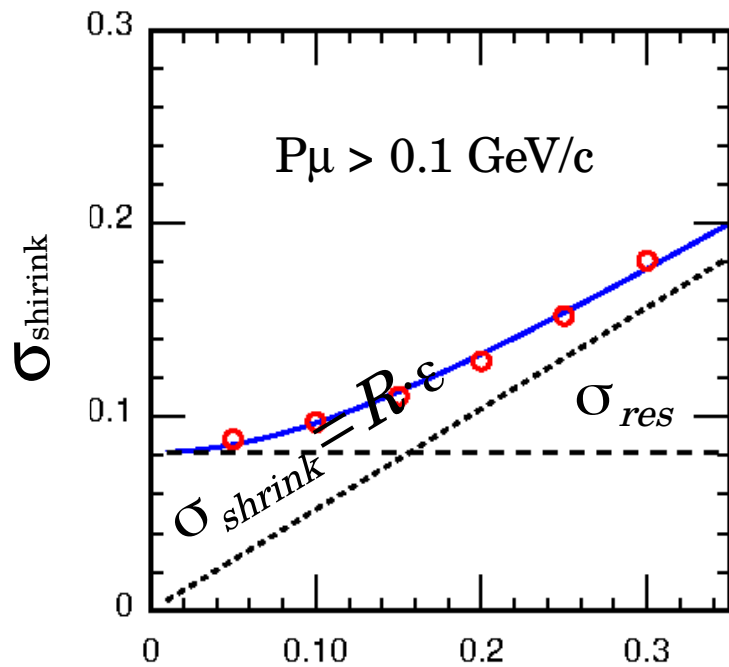
Phase Space Scan with muon at Mt.Norikura (2770m a.s.l.)



For $E_\nu = 0.1$ GeV with muon observation at Norikura

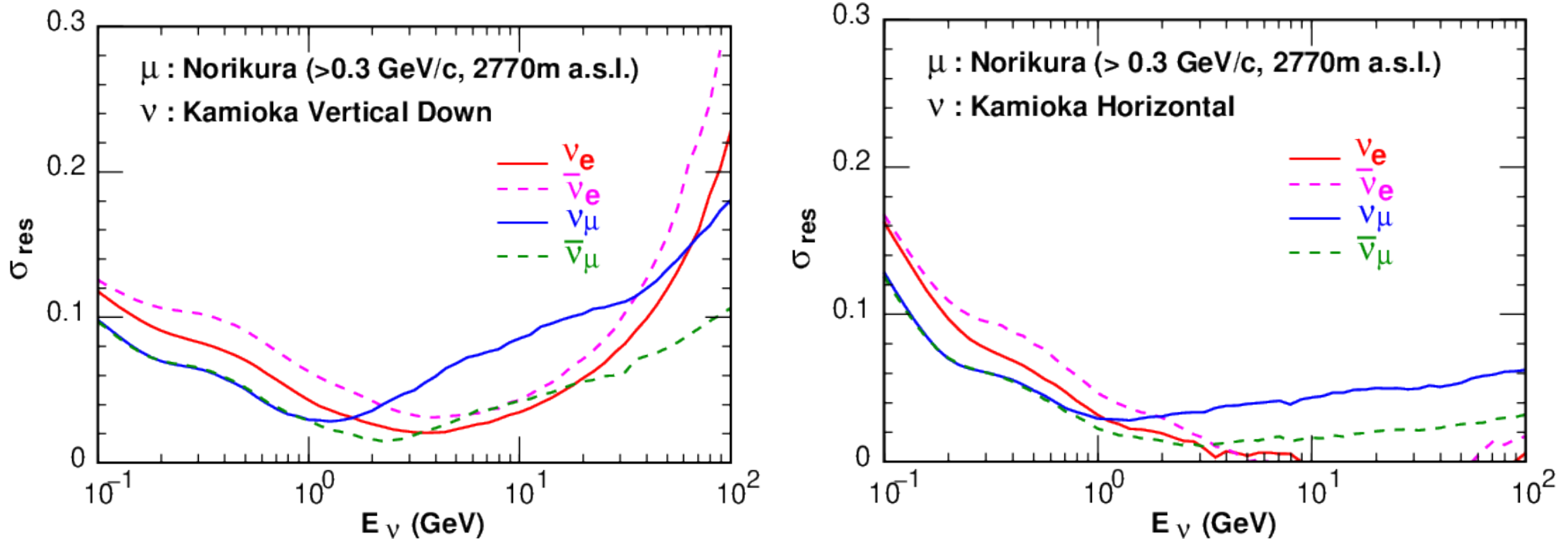


For $E_\nu = 0.3$ GeV with muon observation at Norikura



σ_{res} as the function of Neutrino Energy

If there is a accurate muon flux data observed at Mt. Norikura from 0.3 GeV/c, with the sophisticated muon detector, the precise reconstruction of the data will reduce the error of atmospheric neutrino flux prediction as,

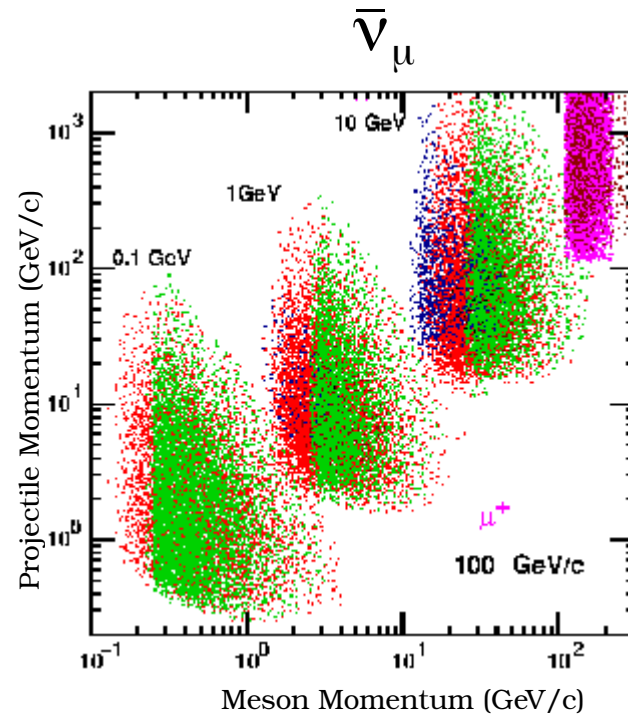
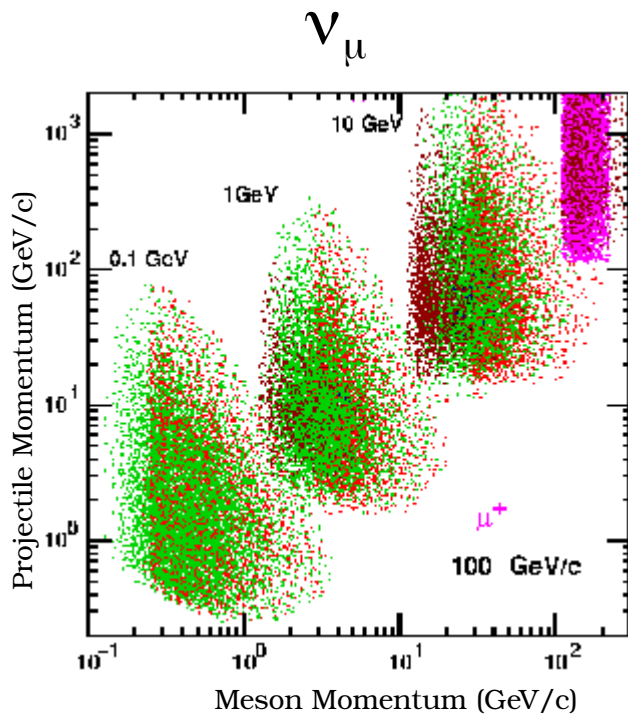
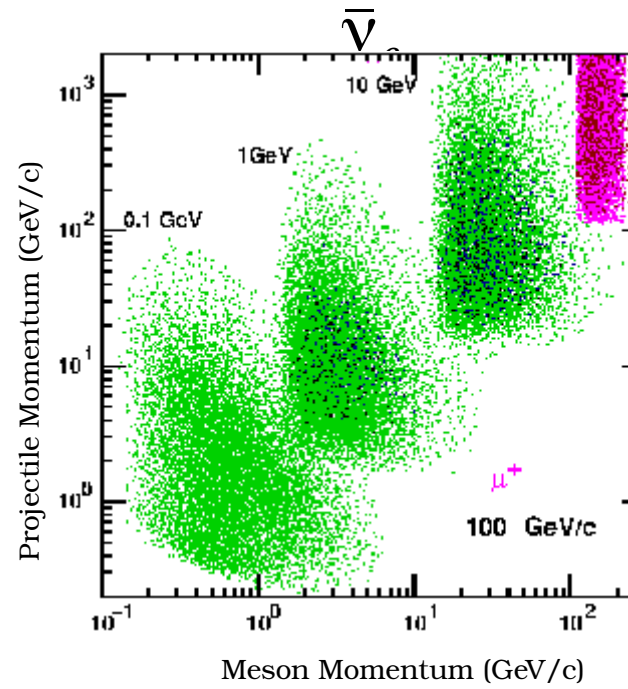
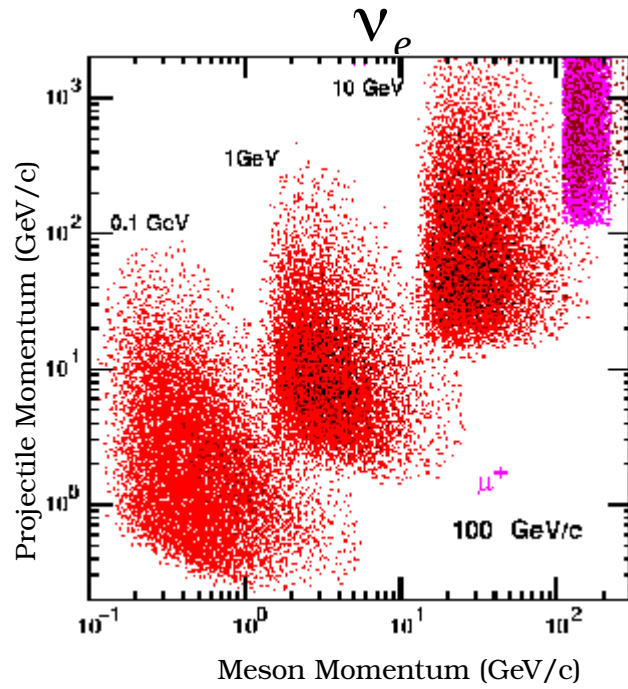


The error of the atmospheric neutrino flux prediction will be suppressed < 10 % above 0.3 GeV.

Indian Astro Observatory (Hanle, 4500m a.s.l.)

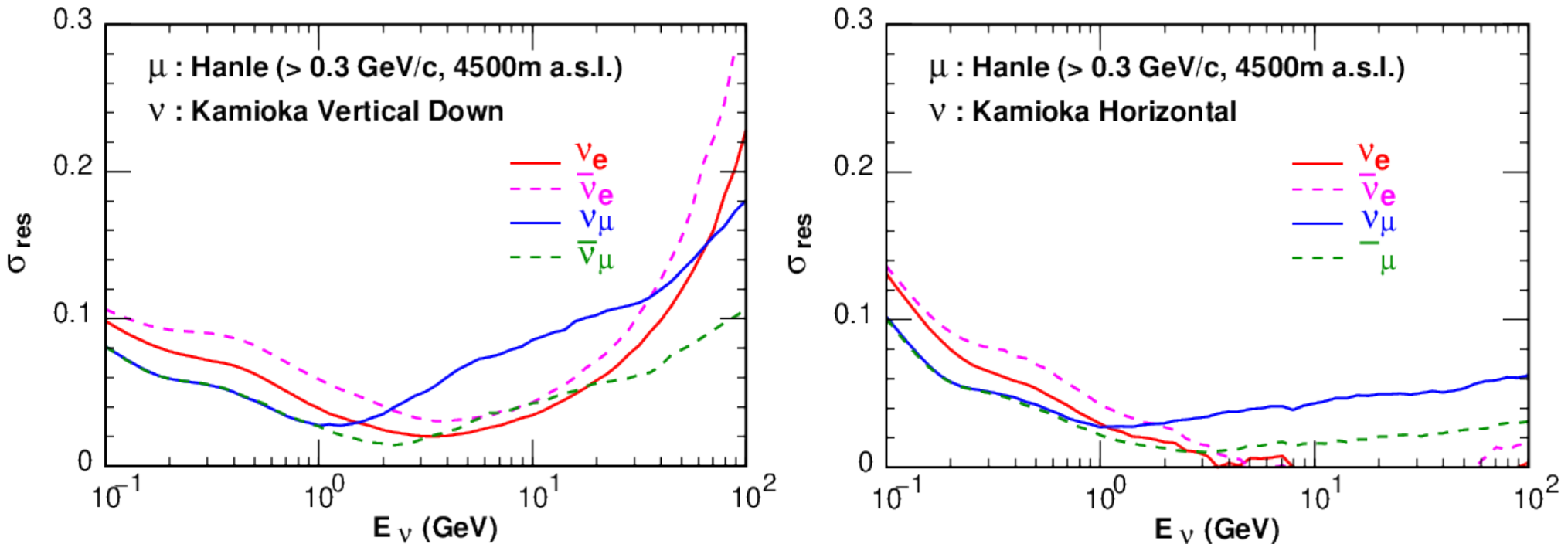


Phase Space Scan with muon at Hanle (4500m a.s.l.)



σ_{res} as the function of Neutrino Energy

If there is accurate muon flux data observed at higher altitude than Hanle (~4500m a.s.l.) from 0.3 GeV/c, the precise reconstruction of the data will reduce the error of atmospheric neutrino flux prediction as,

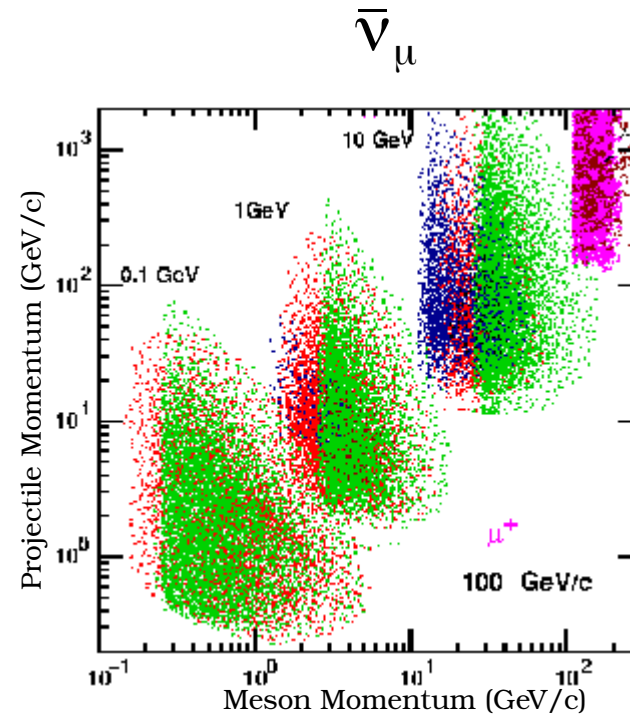
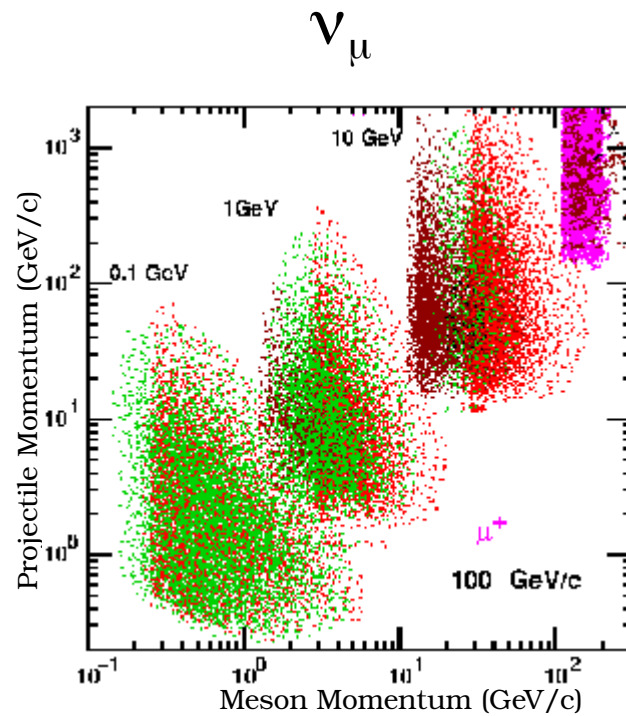
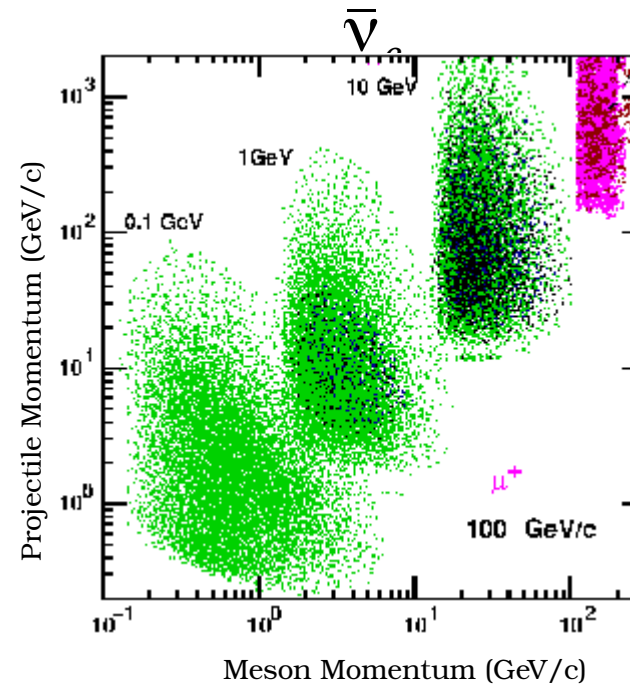
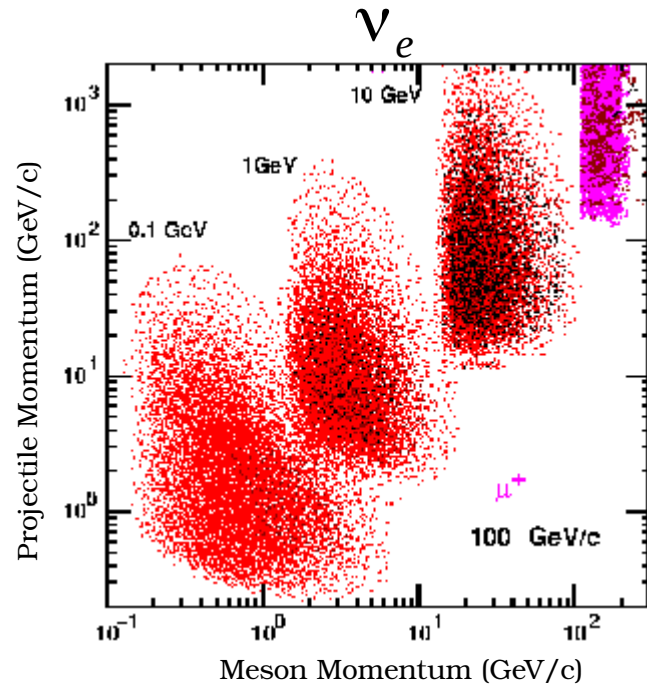


The error of the atmospheric neutrino flux prediction will be suppressed < 10 % from just above 0.1 GeV.

If higher is better, how about
Balloon altitude ?
(No photo in right altitude !)

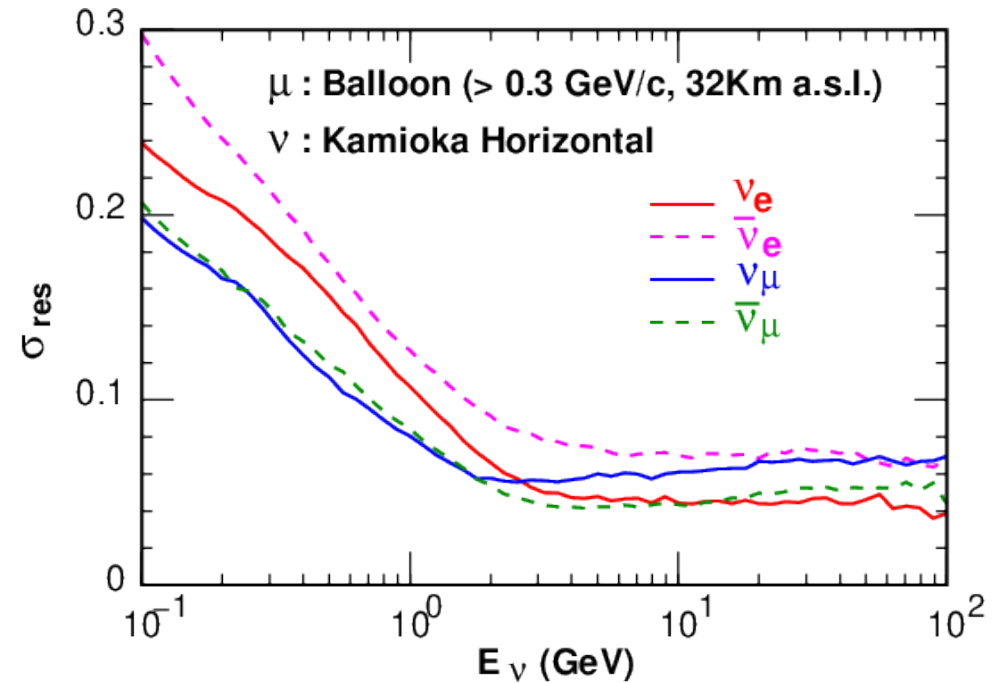
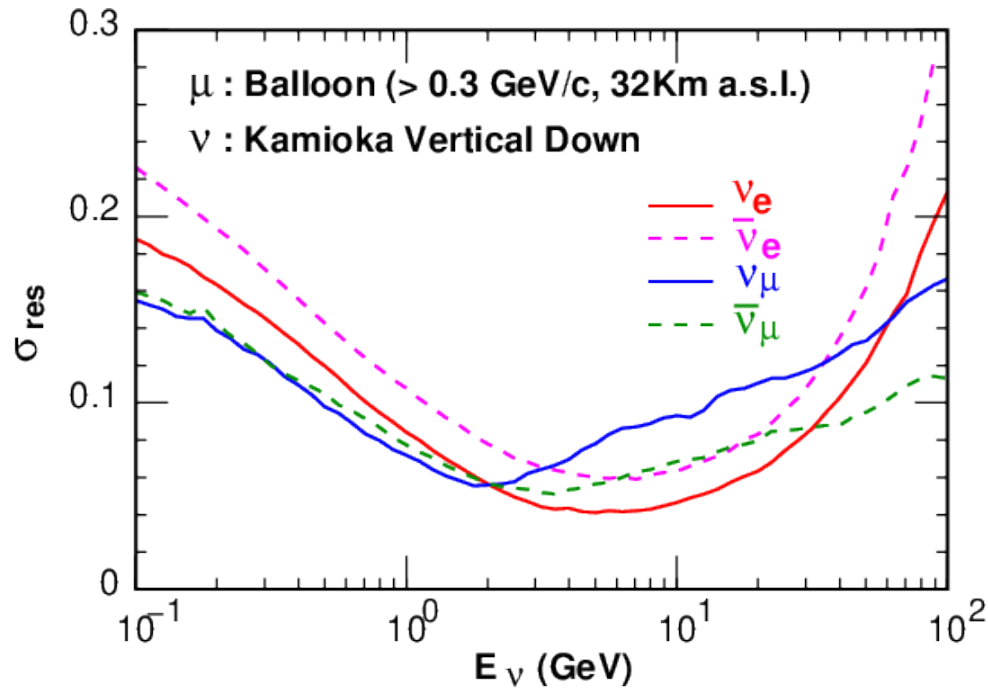


Phase Space Scan with muon at Balloon Altitude (32Km a.s.l.)



σ_{res} as the function of Neutrino Energy

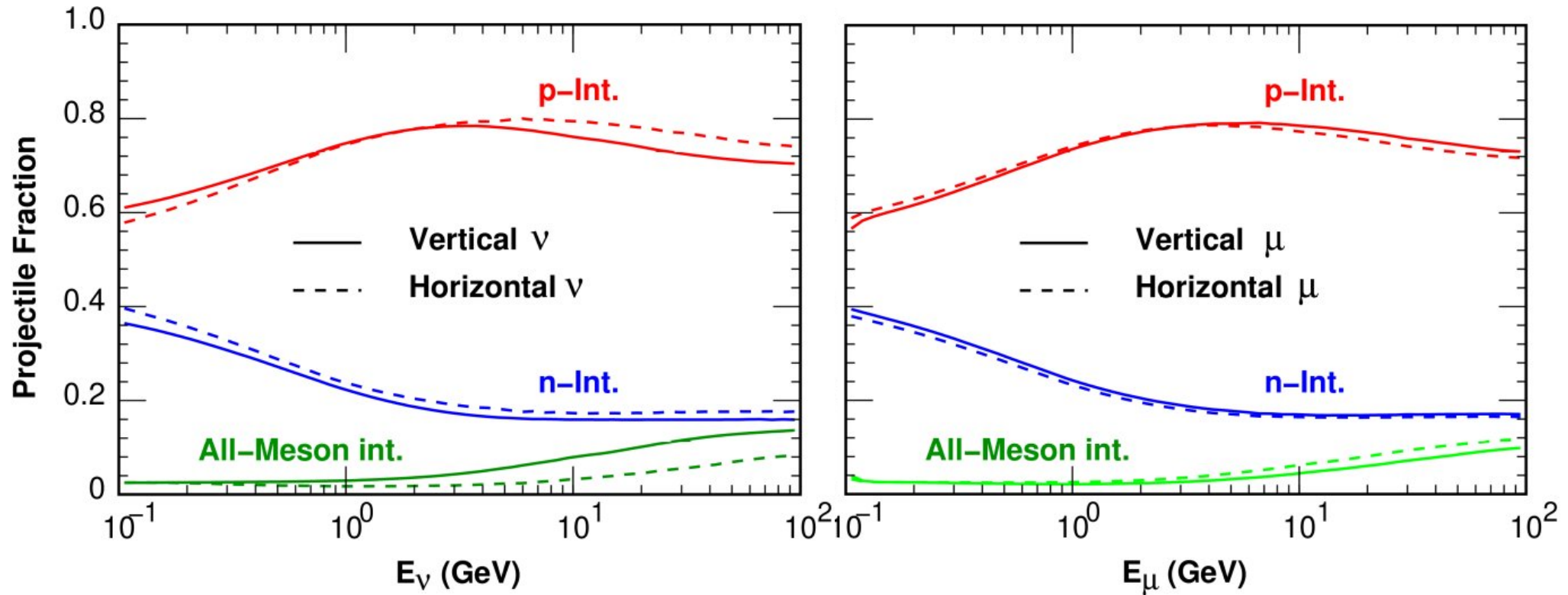
The accurate muon flux data observed at Balloon altitude (~ 32 k a.s.l.) from 0.3 GeV/c, does not help the reduction of the error of atmospheric neutrino flux prediction more than high mountain as,



The muon and neutrino phase space has a small common area.

Effect of Hadronic Interaction Variation to the Projectile particle

Study of the projectile of the last hadronic interaction

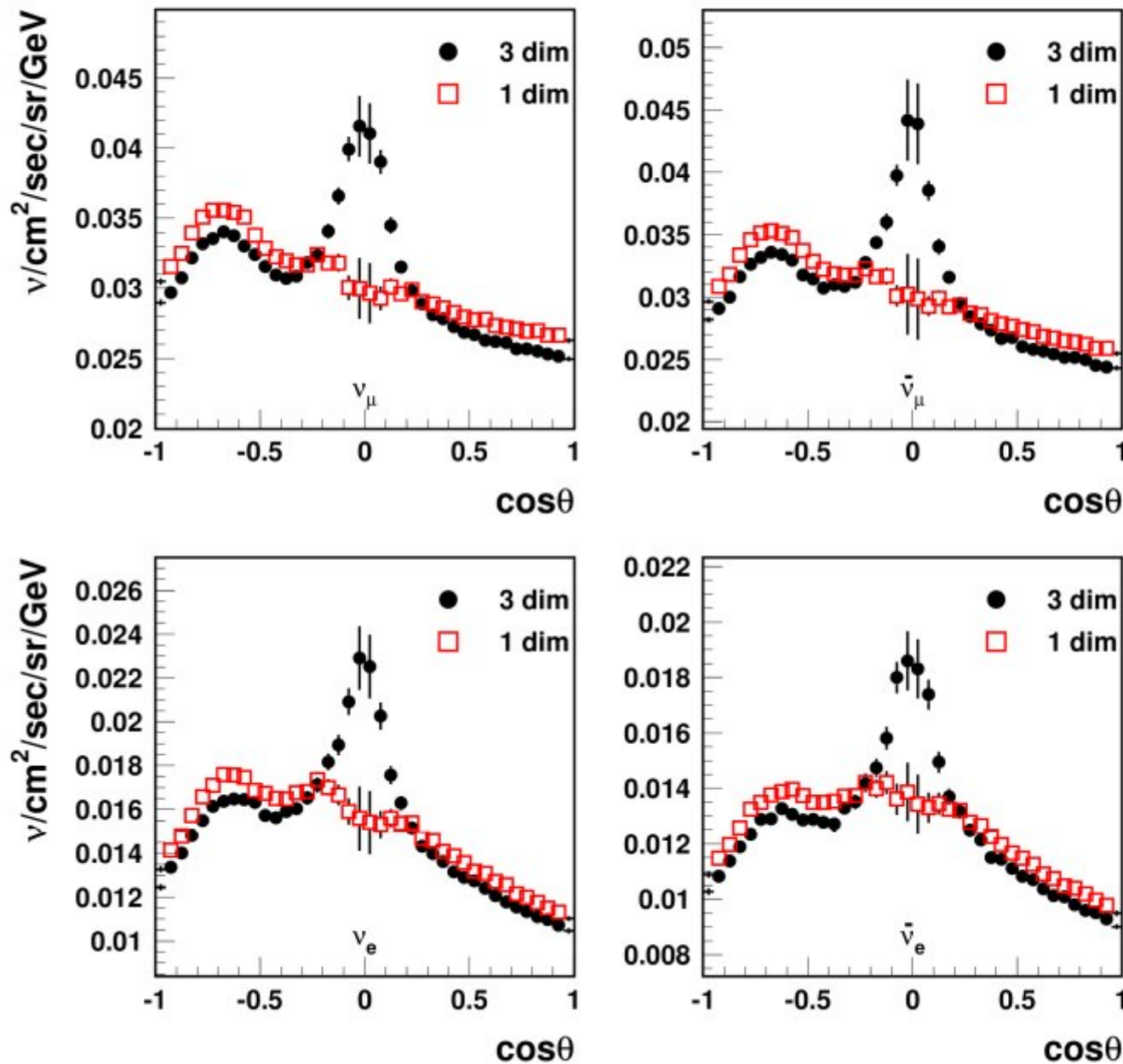


Repeated previous analyses with the varying the ratio of p-int and n-int.

The statistics has been changed, but obtained σ_{res} as the function of neutrino energy is the same.

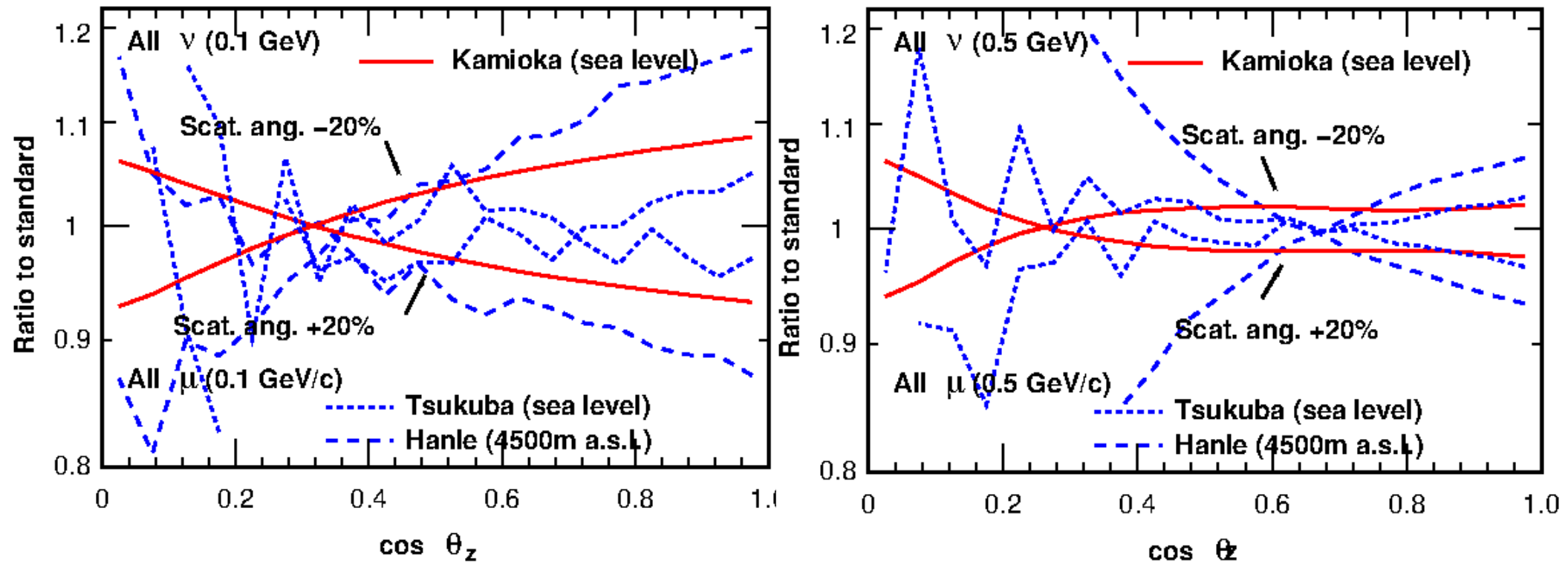
Horizontal enhancement of neutrino flux is potential source of error.

Sub-GeV flux at Kamioka



(Battistoni et al. Astropart. Phys 1999)

We have calculated the atmospheric neutrino and muon fluxes artificially changing the scattering angle by $\pm 20\%$.



Maybe if the scattering angle uncertainty is $< 10\%$, the flux error is $< 5\%$.
The zenith angle variation of muon flux at Hanle is more sensitive than the neutrino horizontal enhancement.

Summary

- The atmospheric muon and neutrino is studied in the phase space of hadronic interaction. And, the correlation is found between their response to the variation of hadronic interaction,
- We may use this correlation to reduce the prediction error of the atmospheric neutrino flux, by reconstructing the accurately measured atmospheric muon flux data.
- To reduce the atmospheric neutrino prediction error with muon flux data at low energy (< 0.3 GeV), it is preferable to measure the muon flux at high mountain (4500 ~ 5000m a.s.l) .
- Applying this study to the muon flux data exist, our former estimation of the calculation error in atmospheric neutrino flux is justified.