CP violation in sub-GeV atmospheric neutrinos.

JHEP 10 (2020) 120 e-Print: 2005.07719 [hep-ph] + some additional plots

Ara N. Ioannisian

YerPhI and ITPM, Armenia

2nd Workshop for Atmospheric Neutrino Production in the MeV to PeV range
12 January, 2021

sub-GeV neutrinos ($E_\nu < 1$ GeV)

There are 2 kind/class of oscillations in matter
Δm^2_{21} slow oscillations (due to solar $\Delta m^2_\odot$ and neutrino potential in matter)
$\Delta m^2_{31}, \Delta m^2_{32}$ fast oscillations ($\Delta m^2_{atm} \simeq \Delta m^2_{31}$)
$\Delta m^2_{21} \ll |\Delta m^2_{32}|$
averaging over fast oscillation

- averaging over fast oscillation / CP violation effects remains
- analytic averaging over fast oscillation / $3\nu \to 2\nu$ oscillation problem
- discuss our findings, uncover physics

For any neutrino trajectory the oscillation S-matrix can be presented

$$S_{\alpha\beta} = U_m \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi_{21}} & 0 \\ 0 & 0 & e^{-i\phi_{31}} \end{pmatrix} U_m^\dagger$$

$(\alpha, \beta = \nu_e, \nu_\mu, \nu_\tau)$

$$U_m = O_{23}^m \, U_\delta^m \, O_{13}^m \, O_{12}^m$$

$$O_{12}^m = \begin{pmatrix} \cos\theta_{12}^m & \sin\theta_{12}^m & 0 \\ -\sin\theta_{12}^m & \cos\theta_{12}^m & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad O_{13}^m = \begin{pmatrix} \cos\theta_{13}^m & 0 & \sin\theta_{13}^m \\ 0 & 1 & 0 \\ -\sin\theta_{13}^m & 0 & \cos\theta_{13}^m \end{pmatrix}$$

$$O_{23}^m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23}^m & \sin\theta_{23}^m \\ 0 & -\sin\theta_{23}^m & \cos\theta_{23}^m \end{pmatrix} \qquad U_\delta^m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta^m} \end{pmatrix}$$

It is shown (Al and Pokorski) in matter $\delta^m \simeq \delta$ and $\theta_{23}^m \simeq \theta_{23}$

$$U_m \simeq O_{23} \, U_\delta \, O_{13}^m \, O_{12}^m$$

Lets average over fast oscillations

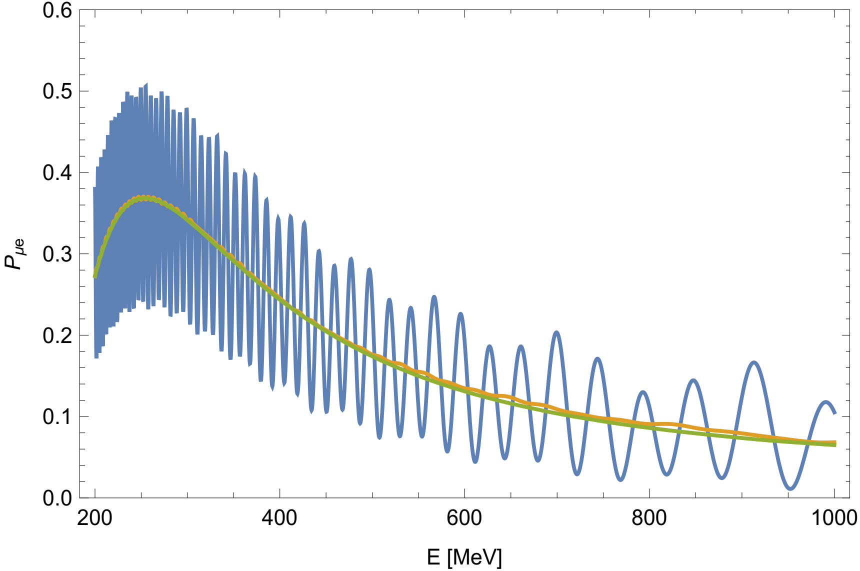
$$P_{\nu_\mu \to \nu_e} = \sin^2 2\theta_{13}^m s_{23}^2 \left[c_{12}^{m2} \sin^2 \frac{\phi_{31}}{2} + s_{12}^{m2} \sin^2 \frac{\phi_{32}}{2} \right]$$
$$+\frac{1}{2} c_{13}^m \sin 2\theta_{13}^m \sin 2\theta_{12}^m \sin 2\theta_{23} \cos\delta \; \sin \frac{\phi_{21}}{2} \sin \frac{\phi_{31}+\phi_{32}}{2}$$
$$-c_{13}^m \sin 2\theta_{13}^m \sin 2\theta_{12}^m \sin 2\theta_{23} \sin\delta \; \sin \frac{\phi_{21}}{2} \sin \frac{\phi_{31}}{2} \sin \frac{\phi_{32}}{2}$$
$$+\left[c_{13}^{m2} \sin^2 2\theta_{12}^m (c_{23}^2 - s_{23}^2 s_{13}^{m2}) + \frac{1}{4} c_{13}^m \sin 2\theta_{13}^m \sin 4\theta_{12}^m \sin 2\theta_{23} \cos\delta \right] \sin^2 \frac{\phi_{21}}{2}$$

$$\phi_{ij} = \frac{\Delta m_{ij}^2}{2E} L \quad i,j = 1,2,3 \quad .$$

$$\sin \frac{\phi_{21}}{2} \sin \frac{\phi_{31}}{2} \sin \frac{\phi_{32}}{2} = \sin \frac{\phi_{21}}{2} \sin \frac{\phi_{31}}{2} \sin \frac{\phi_{31}-\phi_{21}}{2} =$$
$$= \sin \frac{\phi_{21}}{2} \sin^2 \frac{\phi_{31}}{2} \cos \frac{\phi_{21}}{2} - \frac{1}{2} \sin^2 \frac{\phi_{21}}{2} \sin \phi_{31} \approx \frac{1}{4} \sin \phi_{21}$$

$$P_{\nu_\mu \to \nu_e} = \sin^2 2\theta_{13}^m s_{23}^2 \frac{1}{2}$$
$$+0$$
$$-c_{13}^m \sin 2\theta_{13}^m \sin 2\theta_{12}^m \sin 2\theta_{23} \sin\delta \; \frac{1}{4} \sin \phi_{21}$$
$$+\left[c_{13}^{m2} \sin^2 2\theta_{12}^m (c_{23}^2 - s_{23}^2 s_{13}^{m2}) + \frac{1}{4} c_{13}^m \sin 2\theta_{13}^m \sin 4\theta_{12}^m \sin 2\theta_{23} \cos\delta \right] \sin^2 \frac{\phi_{21}}{2}$$

After averaging over fast oscillations the CP violation (odd term) does not vanish!!!



$\delta_{cp} = \pi/2$, azimuthal angle is $\pi/10$

electron neutrino flux is, approximately, half of the muon neutrino flux

$$j_{\nu_e}(E_\nu, \Theta_{azimut}, \ldots) \simeq \frac{1}{2} j_{\nu_\mu}(E_\nu, \Theta_{azimut}, \ldots)$$

$$\begin{aligned} P_{\nu_e \to \nu_e} &= 1 - \sin^2 2\theta_{13}^m (\cos^2\theta_{12}^m \sin^2\frac{\phi_{31}}{2} + \sin^2\theta_{12}^m \sin^2\frac{\phi_{32}}{2}) - \cos^4\theta_{13}^m \sin^2 2\theta_{12}^m \sin^2\frac{\phi_{21}}{2} \\ &\simeq 1 - \frac{1}{2}\sin^2 2\theta_{13}^m - \cos^4\theta_{13}^m \sin^2 2\theta_{12}^m \sin^2\frac{\phi_{21}}{2} \end{aligned}$$

$$\begin{aligned} P_{\nu_e \to \nu_e} + 2P_{\nu_\mu \to \nu_e} = \; & 1 + \sin^2 2\theta_{13}^m (s_{23}^2 - \frac{1}{2}) \\ & - c_{13}^m \sin 2\theta_{13}^m \sin 2\theta_{12}^m \sin 2\theta_{23} \sin\delta \; \frac{1}{2} \sin\phi_{21} \\ & + c_{13}^{m2} \sin^2 2\theta_{12}^m (2c_{23}^2 - c_{13}^2) \sin^2\frac{\phi_{21}}{2} \\ & + 2\left[-c_{13}^{m2} \sin^2 2\theta_{12}^m s_{23}^2 s_{13}^{m2} + \frac{1}{4} c_{13}^m \sin 2\theta_{13}^m \sin 4\theta_{12}^m \sin 2\theta_{23} \cos\delta \right] \sin^2\frac{\phi_{21}}{2} \end{aligned}$$

"Three Neutrino Oscillations in Matter"
Ara Ioannisian , Stefan Pokorski .
Phys.Lett. B782 (2018) 641-645

For the mixing angles θ_{13}^m and θ_{12}^m in matter

$$\sin 2\theta_{13}^m = \frac{\sin 2\theta_{13}}{\sqrt{(\cos 2\theta_{13} - \epsilon_a)^2 + \sin^2 2\theta_{13}}}, \qquad \cos 2\theta_{13}^m = \frac{\cos 2\theta_{13} - \epsilon_a}{\sqrt{(\cos 2\theta_{13} - \epsilon_a)^2 + \sin^2 2\theta_{13}}},$$

$$\sin 2\theta_{12}^m = \frac{\cos \theta_{13}' \sin 2\theta_{12}}{\sqrt{(\cos 2\theta_{12} - \epsilon_\odot)^2 + \cos^2 \theta_{13}' \sin^2 2\theta_{12}}}, \quad \cos 2\theta_{12}^m = \frac{\cos 2\theta_{12} - \epsilon_\odot}{\sqrt{(\cos 2\theta_{12} - \epsilon_\odot)^2 + \cos^2 \theta_{13}' \sin^2 2\theta_{12}}}$$

where

$$\theta_{13}' = \theta_{13}^m - \theta_{13}, \quad \epsilon_a = \frac{2EV}{\Delta m_{ee}^2}, \quad \epsilon_\odot = \frac{2EV}{\Delta m_\odot^2}(\cos^2 \theta_{13}^m + \frac{\sin^2 \theta_{13}'}{\epsilon_a}), \quad \Delta m_{ee}^2 = \Delta m_a^2 - s_{12}^2 \Delta m_\odot^2$$

Mass square differences in matter

$$\frac{\Delta m_{21}^2}{2E} = \frac{\Delta m_\odot^2}{2E} \sqrt{(\cos 2\theta_{12} - \epsilon_\odot)^2 + \cos^2 \theta_{13}' \sin^2 2\theta_{12}},$$

$$\frac{\Delta m_{31}^2}{2E} = \frac{3}{4} \frac{\Delta m_{ee}^2}{2E} \sqrt{(\cos 2\theta_{13} - \epsilon_a)^2 + \sin^2 2\theta_{13}} + \frac{1}{4}[\frac{\Delta m_{ee}^2}{2E} + V] + \frac{1}{4E}(\Delta m_{21}^2 - \Delta m_\odot^2 \cos 2\theta_{12})$$

In the case of non uniform matter density profile (up to an unimportant overall phase denoted as $e^{i\xi}$) the oscillation amplitude, S, can be expressed as

$$S = e^{i\xi}\, U_a\, T\, \Pi_i \left(O^m_{i13}\; O^m_{i12} \mathcal{E}_i\; O^{mT}_{i12}\; O^{mT}_{i13} \right)\; U_a^\dagger$$

$$U_a = O_{23}\, U_\delta$$

$$\mathcal{E}_i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\frac{\Delta m^2_{21i}}{2E}\Delta x_i} & 0 \\ 0 & 0 & e^{-i\frac{\Delta m^2_{31i}}{2E}\Delta x_i} \end{pmatrix}$$

$$\ldots \mathcal{E}_i\; O^{mT}_{i12}\; O^{mT}_{i13}\; O^m_{(i+1)13}\; O^m_{(i+1)12}\; \mathcal{E}_{i+1} \ldots$$

with the most inner multiplication matrix depending on the differences of the θ^m_{13} mixing angle between the neighbouring layers:

$$O^{mT}_{i13}\; O^m_{(i+1)13} = \begin{pmatrix} \cos(\theta^m_{i13} - \theta^m_{(i+1)13}) & 0 & \sin(\theta^m_{i13} - \theta^m_{(i+1)13}) \\ 0 & 1 & 0 \\ -\sin(\theta^m_{i13} - \theta^m_{(i+1)13}) & 0 & \cos(\theta^m_{i13} - \theta^m_{(i+1)13}) \end{pmatrix}$$

for the neutrino energies below $E < \mathcal{O}(1)$ GeV and typical values of the Earth density, θ^m_{i13} angle in matter vary only very slightly. The differences $\theta^m_{i13} - \theta^m_{(i+1)13}$ between the layers are typically of the order of 0.01 radian, even less for the lower neutrino energies. Therefore, to a good approximation products of $O^{mT}_{i13}\; O^m_{(i+1)13}$ can be replaced by the unit matrices. In contrast, the dependence of the θ^m_{i12} on the matter density is stronger for this energy range. Then, neglecting the overall phase,

$$O^m_{13-first}\; T\, \Pi_i \left(O^m_{i12} \mathcal{E}_i\; O^{mT}_{i12} \right) O^{mT}_{13-last}$$

$O^m_{13-first} = O^m_{13-first} \simeq O_{13}$

$$T\,\Pi_i\left(O^m_{i12}\mathcal{E}_i\,O^{mT}_{i12}\right)=\begin{pmatrix}X_{11}&X_{12}&0\\X_{12}&X_{22}&0\\0&0&0\end{pmatrix}+e^{-i\Phi_{33}}\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}$$

$$\Phi_{33}=\sum_i\frac{\Delta m^2_{31i}}{2E}\Delta x_i=\int\frac{\Delta m^2_{31}(x)}{2E}dx$$

$$\begin{pmatrix}X_{11}&X_{12}\\X_{12}&X_{22}\end{pmatrix}=T\,\Pi_i\begin{pmatrix}\cos\theta^i_{12}&\sin\theta^i_{12}\\-\sin\theta^i_{12}&\cos\theta^i_{12}\end{pmatrix}\begin{pmatrix}1&0\\0&e^{-i\frac{\Delta m^2_{21i}}{2E}\Delta x_i}\end{pmatrix}\begin{pmatrix}\cos\theta^i_{12}&-\sin\theta^i_{12}\\\sin\theta^i_{12}&\cos\theta^i_{12}\end{pmatrix}$$

X is symmetric matrix

$$S\approx U_0\begin{pmatrix}X_{11}&X_{12}&0\\X_{12}&X_{22}&0\\0&0&0\end{pmatrix}U_0^\dagger+e^{-i\Phi_{33}}\,U_0\begin{pmatrix}0&0&0\\0&0&0\\0&0&1\end{pmatrix}U_0^\dagger$$

$$\equiv A+e^{-i\Phi_{33}}\,B$$

Oscillation probabilities are

$$P_{\alpha\beta}=|S_{\beta\alpha}|^2=|A_{\beta\alpha}|^2+2\mathrm{Re}\left[A^*_{\beta\alpha}B_{\beta\alpha}\,e^{-i\Phi_{33}}\right]+|B_{\beta\alpha}|^2$$

$$=|A_{\beta\alpha}|^2+|B_{\beta\alpha}|^2$$

$$|B_{\nu_\mu\to\nu_e}|^2=|B_{\nu_e\to\nu_\mu}|^2=\frac{1}{2}\sin^2 2\theta_{13}\sin^2\theta_{23}\simeq 0.02$$

$$|B_{\nu_e\to\nu_e}|^2=\sin^4\theta_{13}$$

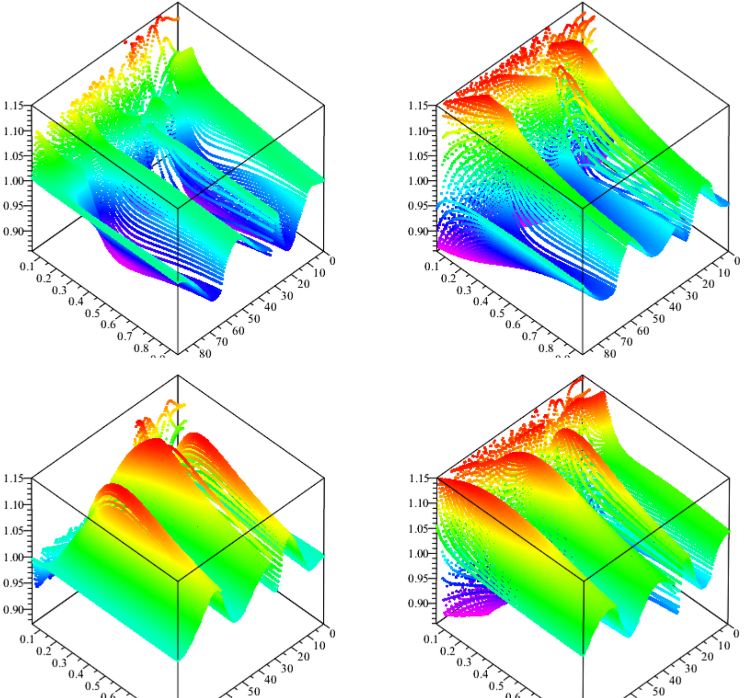
$$|B_{\nu_\mu\to\nu_\mu}|^2=\cos^4\theta_{13}\sin^4\theta_{23}$$

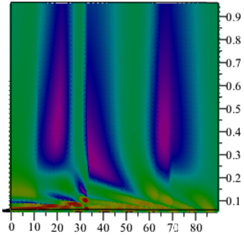
The averaging over fast oscillations is (avoiding fast oscillations and 3-dim.) just computation of 2-dimensional matrix X.

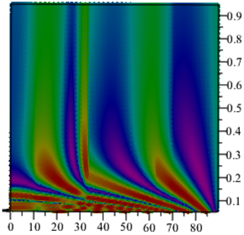
Numerical plots based on our $3\nu \to 2\nu$

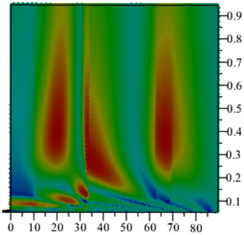
$$
\begin{aligned}
\delta_{cp} = & \quad 0, \qquad \pi/2, \\
& \quad \pi/2, \qquad \pi \\
E_\nu = & \quad 0.05 GeV \ldots 0.95 GeV \\
\Theta = & \quad 0^o \ldots 90^o \\
& \quad \Theta = 0^o \ \textit{core crossing trajectory} \\
& \quad \Theta = 90^o \ \textit{horizontal trajectory} \\
Z \equiv & \quad P_{\nu_e \to \nu_e} + 2 P_{\nu_\mu \to \nu_e}
\end{aligned}
$$

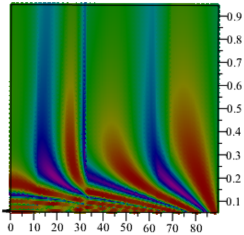
Neutrinos

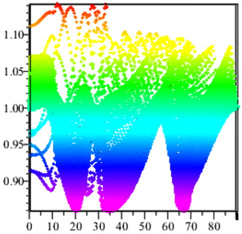

1.15
1.10
1.05
1.00
0.95
0.90
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
10
20
30
40
50
60
70
80

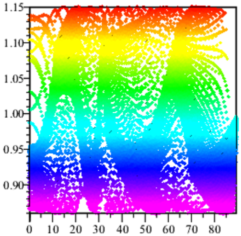

0 10 20 30 40 50 60 70 80
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

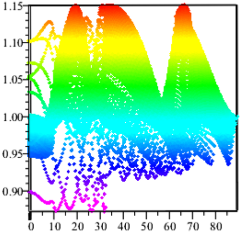

0 10 20 30 40 50 60 70 80
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

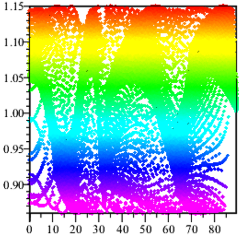

0 10 20 30 40 50 60 70 80
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9


0 10 20 30 40 50 60 70 80
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9

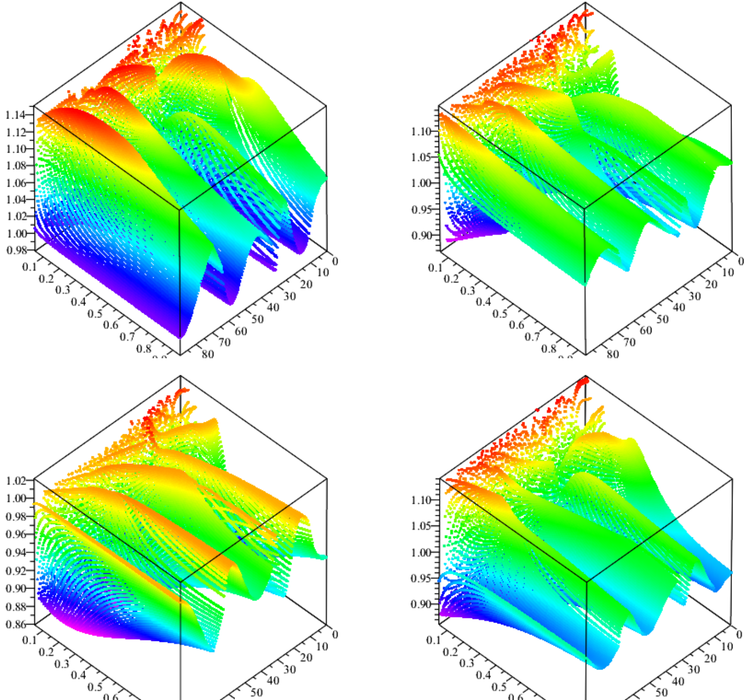

1.10
1.05
1.00
0.95
0.90
0 10 20 30 40 50 60 70 80


1.15
1.10
1.05
1.00
0.95
0.90
0 10 20 30 40 50 60 70 80


1.15
1.10
1.05
1.00
0.95
0.90
0 10 20 30 40 50 60 70 80


1.15
1.10
1.05
1.00
0.95
0.90
0 10 20 30 40 50 60 70 80

Anti-Neutrinos



Summary

averaging over fast oscillation (energy bins)

- CP violation effects remains
- $3\nu \to 2\nu$ oscillation problem
- energy and angular oscillation features uncovered
- calculations are very fast (and avoid mistakes during numerical calculations)

THANK YOU