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Global Portraits of Nonminimal Inflation

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2104.10183, PRD...

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2104.14258, Universe 7 (2021) 6, 179

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Overview

Inflation

- must be an attractor, but not a fixed point,
- rather realized by a heteroclinic orbit in phase space
  from a saddle or nonhyperbolic point to an attractor point
  c.f. Alho, Uggla 1406.0438, ...

In this work we introduce a physically motivated set of variables and

- draw global phase portraits in the Jordan frame which
- clearly distinguish different asymptotic regimes,
- reveal the inflationary attractor orbit,
- show the correct slow roll approximation in the Jordan frame,
- visualize the range of good initial conditions (> 50 efolds).

First paper

- A scalar field nonminimally coupled to curvature (Jordan frame),

$$S = \frac{1}{2} \int d^4x \sqrt{-g} \, \{F(\phi)R - \partial_\mu \phi \, \partial^\mu \phi - 2V(\phi)\} \, , \qquad (1)$$

- in flat FLRW cosmology,
- and use dynamical variables (and evolution in e-folds $N = \ln a$)
  Dutta, LJ, Khyllep, Tõkke 2007.06601

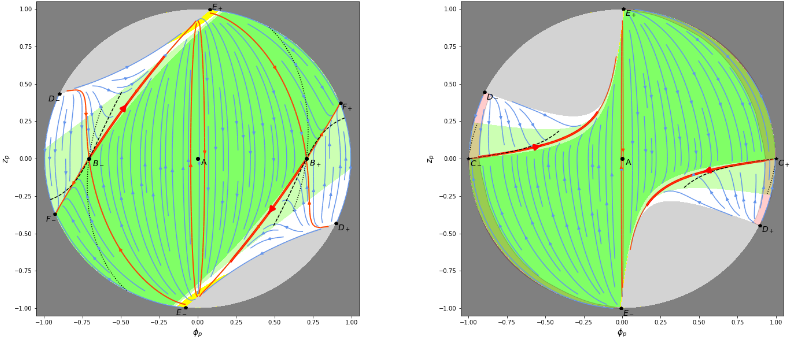
$$\phi \, , \qquad z = \frac{\dot{\phi}}{H} = \frac{d\phi}{dN} \qquad (2)$$

- study global portraits in Poincaré compactification

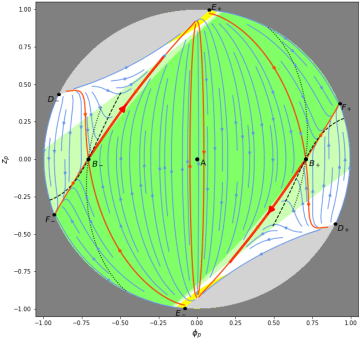
$$\phi_p = \frac{\phi}{\sqrt{1 + \phi^2 + z^2}} \, , \qquad z_p = \frac{z}{\sqrt{1 + \phi^2 + z^2}} \, . \qquad (3)$$

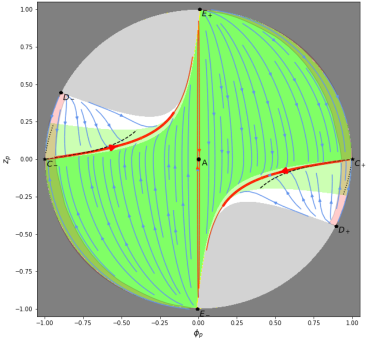
What we get

For example: nonminimal coupling $F = 1 + \xi\phi^2$,
quadratic $V = \frac{m^2}{2}\phi^2$ (left), quartic $V = \frac{\lambda}{4}\phi^4$ (right) potential.

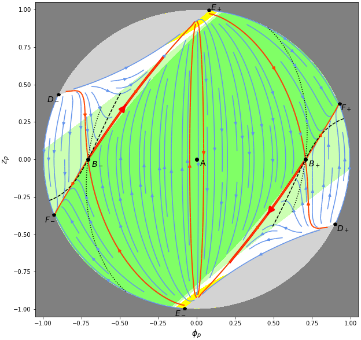


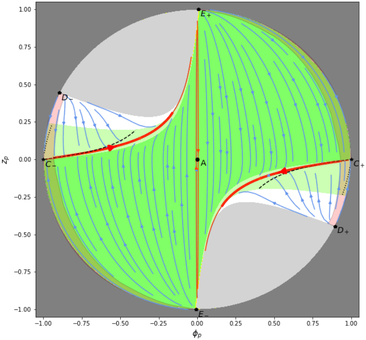
Green – superacceleration, light green – acceleration, white – deceleleration, yellow – superstiff expansion, grey – unphysical region.





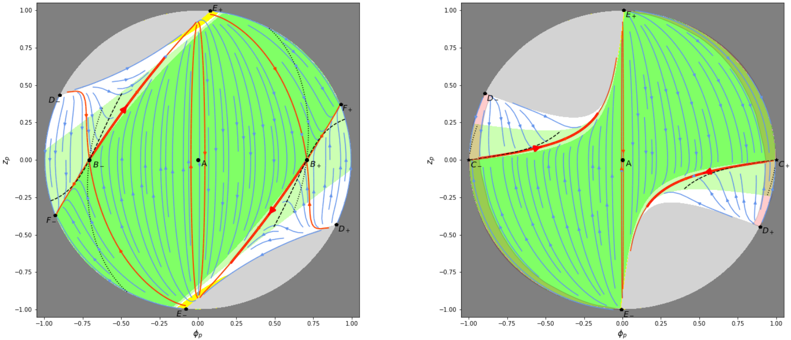
1. Variables (ϕ, z) distinguish different asymptotic regimes:
   fixed points C, F - asymptotic de Sitter, D, E - kinetic dominated,
   otherwise mapped to the same point in $(\phi, \dot{\phi})$.





2. Clearly identify inflation as a heteroclinic orbit $B \to A$ or $C \to A$, not realized before in easy to interpret variables, cf. Alho, Uggla 1406.0438, ...

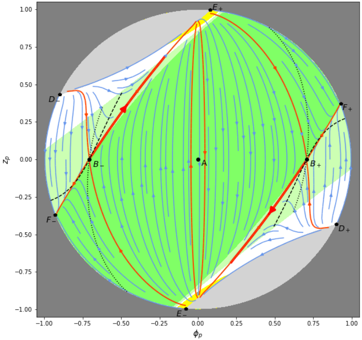
Why is it interesting : slow roll approximation

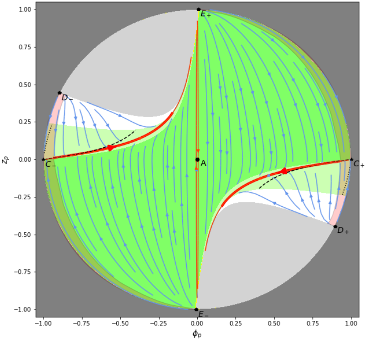


3. Leading inflationary orbit is approximated
   - not so well by "generalised slow roll" in the Jordan frame (dotted),
   + but much better by our mechanical analogue [slide 11] (dashed line) which matches Einstein frame slow roll translated into Jordan frame

     c.f. Akın, Arapoglu, Yükselci 2007.10850

Why is it interesting : initial conditions





4. Can see the range of "good" initial conditions (trajectories) that lead to at least 50 e-folds of accelerated expansion (pink shadow).
   - For increasing ξ the range shrinks for quadratic, but enlarges for quartic potentials.

Second paper: nonminimal teleparallel inflation

- Take a scalar field nonminimally coupled to torsion in the teleparallel framework (review Bahamonde et al. 2106.13793)

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\{F(\phi)T + \partial_\mu\phi\,\partial^\mu\phi - 2V(\phi)\} \qquad (4)$$

and do the same analysis.

- Incidentally, the background flat FLRW cosmology is the same for a scalar field nonminimally coupled to nonmetricity in the symmetric teleparallel framework

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,\{-F(\phi)Q + \partial_\mu\phi\,\partial^\mu\phi - 2V(\phi)\}\ . \qquad (5)$$

LJ, Rünkla, Saal, Vilson 1802.00492

Generic method to find fixed points in the Jordan frame

- Substitute in $\dot{H}$ to write the scalar field equation as

$$\ddot{\phi} = \frac{1}{m_{\text{eff}}}(-V_{\text{eff},\phi}) - 3\dot{\phi}H - \dot{\phi}^2(\text{extra friction})\,. \qquad (6)$$

- Fixed points

	Effective mass m_{eff}	Effective potential V_{eff}	Fixed point condition	Stability condition condition
Minimally coupled	1	V	$\frac{V_{,\phi}}{V} = 0$	$\frac{V_{,\phi\phi}}{V} > 0$
Scalar-curvature	$\frac{2F+3F^2_{,\phi}}{F^3}$	$\frac{V}{F^2}$	$\frac{1}{m_{\text{eff}}}\frac{V_{\text{eff},\phi}}{V} = 0$	$\frac{1}{m_{\text{eff}}}\frac{V_{\text{eff},\phi\phi}}{V} > 0$
Scalar-torsion	F	FV	$\frac{1}{m_{\text{eff}}}\frac{V_{\text{eff},\phi}}{V} = 0$	$\frac{1}{m_{\text{eff}}}\frac{V_{\text{eff},\phi\phi}}{V} > 0$

Previous discussions on the effective potential Chiba, Yamaguchi 0807.4965; Skugoreva, Toporensky, Vernov 1404.6226; Skugoreva,Toporensky 1605.01989;
LJ et al. 1612.06863; Dutta, LJ, Khyllep, Tõkke 2007.06601

Generic method to find slow roll in the Jordan frame

- Substitute in $\dot{H}$ to write the scalar field equation as

$$\ddot{\phi} = \frac{1}{m_{\rm eff}}(-V_{{\rm eff},\phi}) - 3\dot{\phi}H - \dot{\phi}^2(\text{extra friction})\,. \tag{7}$$

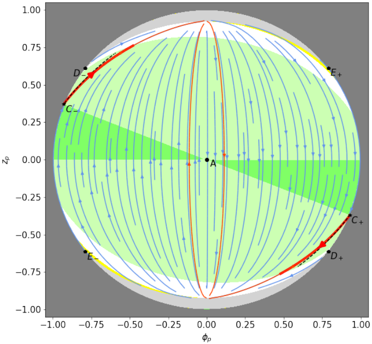
- Slow roll condition

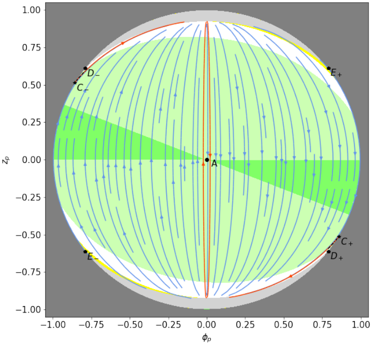
$$3FH^2 \simeq V\,, \qquad 3\dot{\phi}H = \frac{1}{m_{\rm eff}}(-V_{{\rm eff},\phi}) \tag{8}$$

  - The same scheme works for nonminimal scalar-curvature and scalar-torsion models.

What we get (nonminimal teleparallel models)

For example: nonminimal coupling $F = 1 + \xi\phi^2$,
quadratic $V = \frac{m^2}{2}\phi^2$ (left), quartic $V = \frac{\lambda}{4}\phi^4$ (right) potential.





Inflation possible for very small ξ only. C.f. Gonzalez-Espinoza et al. 1904.08068; Raatikainen, Räsanen, 1910.03488.
Perhaps should try some other form of nonminimal coupling $F(\phi)$?

Conclusions

- Inflation is realized by a heteroclinic orbit in the phase space (from a saddle or nonhyperbolic point to an attractor point).
- For nonminimal scalar in Jordan frame useful variables are $(\phi, \frac{\dot{\phi}}{H})$ to draw global phase portraits which
    - clearly distinguish different asymptotic regimes,
    - reveal the inflationary attractor orbit,
    - assess the correctness of slow roll approximation,
    - visualize the range of good initial conditions ($>$ 50 efolds).
- Can use the mechanical analogy with effective potential and effective mass to derive fixed points and slow roll conditions in the Jordan frame.
- Teleparallel models with quadratic nonminimal coupling do not enjoy inflation.

For more, see Järv, Toporensky 2104.10183; Järv, Lember 2104.14258.