Inflation From The MSSM.
N=1 Supergravity Setup

Based on works:
Phys. Rev. D100, no.9, 095027 (2019)
Outline

- **Some motivations**

- **Inflation from MSSM - model**
  sketch of construction, infl. Potential, properties etc.

- **Details of inflation** –
  spectral properties, reheating
  some implication(s)

- **Summary**
  outlook
Aim: See if MSSM can accommodate Inflation (compatible with recent data)

- With only MSSM couplings involved in the inflation process

- Predictions?

Note: In most cases inflaton is SM singlet -> unknown mass scale(s), couplings...

- In works inflation with MSSM states along $D$-flat directions (but with extra higher order terms) considered:
  
  R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, PRL 97, 191304 (2006);
  
Summary of the Results

Within the MSSM:

- Inflation is built. Inflaton -- combination of the Higgs, slepton and squark states.

- Fields along flat D-term trajectory & inflation is driven by the electron Yukawa superpotential.

- MSSM parameter $\tan \beta \approx 13$ is fixed.

Model Gives: (good agreement with data)

\[ n_s = 0.9662, \quad r = 0.00118, \quad \frac{dn_s}{d\ln k} = -5.98 \cdot 10^{-4} \]

\[ N^\text{inf}_e = 57.74, \quad \rho^{1/4}_{\text{reh}} = 2.61 \cdot 10^7 \text{GeV}, \]

\[ T_r = 1.35 \cdot 10^7 \text{GeV}. \]
Summary of the Results

➢ All parameters involved in the inflation & reheating are known -> model is very predictive.

➢ Close connection established between the particle physics model and inflationary cosmology.
Modeling inflation:

- Self consistent UV completion is important
- Symmetries may play crucial role (for potential flatness): SUSY, shift symmetries?

SUSY can guarantee Flatness & consistency

Motivations for SUSY →

- Stab. Hierarchy (Light Higgs) ↔ low SUSY scale
- MSSM → Dark Matter Candidate (LSP)
- Successful Coupling Unification -- good for GUT
The Setup: MSSM

**MSSM States:**

\[ \Phi_I = \{(q, u^c, d^c, l, e^c)_\alpha, h_u, h_d\}, \quad \alpha = 1, 2, 3 \quad \text{(Chiral superfields)} \]

\[ V_G = \{V_Y, V_{SU(2)}, V_{SU(3)}\} \quad \text{(Vector superfields)} \]

**MSSM Superpotential:**

\[ W_{\text{MSSM}} = e^c Y_E l h_d + q Y_D d^c h_d + q Y_U u^c h_u + \mu h_u h_d. \]

**Basis:**

\[ Y_E = Y_E^{\text{Diag}} = \text{Diag}(\lambda_e, \lambda_\mu, \lambda_\tau), \quad Y_D = Y_D^{\text{Diag}}, \quad Y_U = V_{CKM}^T Y_U^{\text{Diag}} \]

**Scalar Potential:**

\[ V = V_F + V_D \]
N=1 SUGRA (local SUSY)

\[ V_F = e^K \left( D_j \bar{W} K_{ji} D_I W - 3|W|^2 \right) \]

\[ D_I W = \left( \frac{\partial}{\partial \Phi_I} + \frac{\partial K}{\partial \Phi_I} \right) W \]

In our case: \( V_D = 0 \) (Flat D-terms)

Choice of the Kahler potential \( K \):

canonical form \( K \rightarrow \sum_I \Phi_I^\dagger e^{-V} \Phi_I \)

Let's Make selection:

\[ K = -\ln(1 - \sum_I \Phi_I^\dagger e^{-V} \Phi_I) \]

In small fields’ limit

\[ \Phi_I \ll 1 \quad K \rightarrow \sum_I \Phi_I^\dagger e^{-V} \Phi_I \]
Field Configuration: Along Flat D-terms

\[ V_D = \frac{g_1^2}{8} D_Y^2 + \frac{g_2^2}{2} (D_{SU(2)}^i)^2 + \frac{g_3^2}{2} (D_{SU(3)}^a)^2. \]

\[ D_Y = |h_d|^2 - |h_u|^2 - 2|\tilde{e}_\alpha|^2 + |\tilde{l}_\alpha|^2 \]
\[ -\frac{1}{3}|\tilde{q}_\alpha|^2 + \frac{4}{3}|\tilde{u}_\alpha|^2 - \frac{2}{3}|\tilde{d}_\alpha|^2, \]
\[ D_{SU(2)}^i = \frac{1}{2} \left( h_d^\dagger \tau^i h_d - h_u^\dagger \tau^i h_u + \tilde{l}_\alpha^\dagger \tau^i \tilde{l}_\alpha + \tilde{q}_\alpha^\dagger \tau^i \tilde{q}_\alpha \right) \]
\[ D_{SU(3)}^a = \frac{1}{2} \left( \tilde{q}_\alpha^\dagger \lambda^a \tilde{q}_\alpha - \tilde{u}_\alpha^\dagger \lambda^a \tilde{u}_\alpha - \tilde{d}_\alpha^\dagger \lambda^a \tilde{d}_\alpha \right). \]

There are numerous Flat D-term configurations
Consider: $e^c l q u^c$ -type flat direction

$\rightarrow$ No runaway directions / instabilities for Inflaton potential

$\langle \tilde{e}_1^c \rangle = z, \quad \langle h_d \rangle = \begin{pmatrix} z c_\theta \\ 0 \end{pmatrix}, \quad \langle \tilde{l}_2 \rangle = \begin{pmatrix} z s_\theta \\ 0 \end{pmatrix}$

$SU(2)_L$

$SU(3)_c \rightarrow$ $\quad \langle \tilde{q}_1 \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & z \end{pmatrix}$

$SU(2)_L$

$\langle \tilde{u}^c \rangle = \begin{pmatrix} 0, 0, z c_\varphi \end{pmatrix}, \quad \langle \tilde{t}^c \rangle = \begin{pmatrix} 0, 0, z s_\varphi e^{i\omega} \end{pmatrix}$

$z$-mainly inflaton d.o.f
Inflaton Potential

Only one non-vanishing F-term:

\[ F_{e-}^{*} = -\lambda_e z^2 c_\theta \quad (\cos \theta \approx 1) \]

Canonically normalized inflaton \( \phi \):

\[ z = \frac{1}{2} \tanh \left( \frac{\phi}{\sqrt{2}} \right) \]

\[ V(\phi) = V_F(\phi) \approx \frac{\lambda_e^2}{16} \tanh^4 \left( \frac{\phi}{\sqrt{2}} \right) \]
Indeed, this can be achieved:

\[ F^{(2)}_{h_u} = 0 \rightarrow V_{ud} \lambda_u c_\varphi + V_{td} e^{i\omega} \lambda_t s_\varphi = 0, \]

\[ \omega = \pi + \text{Arg} \left( \frac{V_{ud}}{V_{td}} \right), \quad \tan \varphi = \frac{\lambda_u}{\lambda_t} \left| \frac{V_{ud}}{V_{td}} \right| \simeq 3 \cdot 10^{-4} \]

\[ F_{dc} = 0 \] satisfied by adding \( W' \) [extra superpotential term(s)]

Two possible cases - (i) and (ii):

(i) \( W' = -\lambda q_1 l_2 d^c \).

\[ \langle F^*_d \rangle = z^2 (-\lambda_d c_\theta + \lambda s_\theta) = 0 \rightarrow, \quad \tan \theta = \frac{\lambda_d}{\lambda} \]
(ii) $W' = \lambda e^c_1 (q_1 l_2 u^c)(q_1 h_d d^c)$

$$\langle F^*_d \rangle = z^2 c_\theta \left(-\lambda_d + \lambda z^4 c_\varphi s_\theta\right) = 0 \rightarrow s_\theta \simeq \frac{\lambda_d}{\lambda z^4}$$

(i) - R-parity violation $\rightarrow$ Neutrino masses via loops

(ii) - Has no impact for low energy phenomenology.

For $\theta < 0.1$ $c_\theta \simeq 1$ (considered below)
Checked Inflaton Potential’s stability

(a): Potential’s dependance on $\theta$ and $\phi$. $\hat{V}_F = V_F/(85\lambda_c^2)$ and $\varphi \simeq 3 \cdot 10^{-4}$.

(b): Potential as a function of $\varphi$ and $\phi$. $\tilde{V}_F = V_F/(8\lambda_c^2)$ and $\theta \simeq 0.012$.

Plots corresponds to the case (i) and $\omega = \pi + \text{Arg} \left( \frac{V_{ud}}{V_{td}} \right)$. Arrows correspond to the inflaton’s path.

All other directions stabilized $\rightarrow$ consistent construction
**Inflation (spectral properties)**

\[ V(\phi) = V_F(\phi) \simeq \frac{\lambda_e^2}{16} \tanh^4 \left( \frac{\phi}{\sqrt{2}} \right) \]  

-- Good properties

\[ n_s = 0.9662, \quad r = 0.00118, \quad \frac{dn_s}{d\ln k} = -5.98 \cdot 10^{-4} \]

\[ N_{\text{e}}^{\text{inf}} = 57.74 \]

**Amplitude of curvature perturbation** -

\[ A_{s}^{1/2} = \frac{1}{\sqrt{12\pi}} \left| \frac{V^{3/2}}{M_{Pl}^{3} V'} \right|_{\phi_i} = 4.581 \times 10^{-5} \]

Determines \( V_i \leftrightarrow \lambda_e(M_{Pl}) = 2.435 \times 10^{-5} \)  

\[ \rightarrow \tan \beta \simeq 13.12 \]  

(MSSM parameter fixed)
Reheating

\[ E = \langle V_{inf} \rangle \rightarrow \sim T^4 \]

- Inflaton Pot.
- SM Sector: quarks, leptons, gauge bosons

\[ T_r \text{ (reheating temp.)} \]
- Via Inflaton decay

\[ T_r = \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{M_{Pl} \Gamma(\phi)} \]

\[ \varphi_{inf} \rightarrow \text{SM states} \]
Reheating: Inflaton Decay
Examining all couplings & kinematically allowed channels

**Dominant Mode:**
\[ \phi \rightarrow gg \]

\[ T_r \ (\text{reheating temp.}) \ Via \ Inflaton \ decay \]

\[
\Gamma(\phi) \simeq \Gamma(\phi \rightarrow gg) \simeq \frac{m_\phi^3 \alpha_s^2}{48 \pi^3} \left( \frac{F'}{F} + \frac{F'_g}{F_g} \right)^2
\]

\[
\frac{1}{2} F(\phi) d^T Y_D d^c, \quad F(\phi) = \tanh \frac{\phi}{\sqrt{2}} (1 - \tanh^2 \frac{\phi}{\sqrt{2}})^{1/2}
\]

From gluinos (in loop):
\[ F_g(\phi) = \sinh \frac{\phi}{\sqrt{2}} \]

\[ \rightarrow T_r \simeq 1.35 \cdot 10^7 \text{GeV} \]
Some Implications:  Neutrino masses

- If (i) $W' = -\lambda q_1 l_2 d^c$ used (for $\mathcal{V}(\phi)$ stability)
  $\rightarrow$ R-parity breaking ($L$-violation)
  $\rightarrow$ at 1-loop: $\mu_i h_u l_i$ superpotential & soft $B_i h_u l_i$ terms $\rightarrow$

  $m_{\nu\mu} \approx \frac{\lambda^2 g_2^2}{4 c_w^2} \frac{m_d^2}{\tilde{m}} \left( \frac{9}{8 \pi^2} \ln \frac{M_{Pl}}{M_Z} \right)^2$

For $\tilde{m} = 2$ TeV (SUSY scale), $\lambda \lesssim 0.1 \rightarrow m_\nu \lesssim 0.1$ eV

- Range $6 \times 10^{-4} \lesssim \lambda \lesssim 0.1 \rightarrow \cos \theta \simeq 1$ (predictive inflation)

- $W' = \lambda q_1 l_2 d^c$ $\rightarrow$ directly 1-loop $\delta m_\nu \approx \frac{3 \lambda^2}{8 \pi^2} \frac{m_d^2}{\tilde{m}} \lesssim 2 \times 10^{-3}$ eV
  (with $\lambda \lesssim 0.1$)

**Good scales for the neutrinos**, but would neutrino data (masses & mixings) accommodated? [additional $\bar{\lambda}_{ijk} e_i^c l_j l_k$, $\lambda_{ijk} q_i l_j d_k^c$ terms needed? ]

Detailed investigation needed:
  Connection between neutrino oscillations & inflation $\leftrightarrow$ Very exciting!
Summary & Outlook

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- Inflation is built. Inflaton -- combination of the Higgs, slepton and squark states.

- Fields along flat D-term trajectory & inflation is driven by the electron Yukawa superpotential.

- MSSM parameter $\tan \beta \approx 13$ is fixed.

Model Gives: (good agreement with data)

\[
\begin{align*}
    n_s &= 0.9662, \\
    r &= 0.00118, \\
    \frac{dn_s}{d \ln k} &= -5.98 \times 10^{-4} \\
    N_{e}^{\text{inf}} &= 57.74, \\
    \rho_{\text{reh}}^{1/4} &= 2.61 \times 10^7 \text{GeV}, \\
    T_r &= 1.35 \times 10^7 \text{GeV}.
\end{align*}
\]
Summary

- All parameters involved in the inflation & reheating are known -> model is very predictive.
- Close connection established between the particle physics model and inflationary cosmology.
Outlook, problems/ issues to be addressed:

1. Investigate neutrino masses / oscillations (via R-parity viol.)

2. Baryogenesis/ Leptogenesis - during inflation B & L are broken

3. GUT embedding [SU(5), SO(10)] - more predictive?

4. What symmetry may support considered Kahler potential? (a’la Kallosh, et al. 2013, 2017 ?)

5. Investigate Other issues / topics [some discussed at this conference]

....

THANK YOU
Backup Slides

Deriving Inflaton Potential

Inflation is due to the $F$-term potential:

$$V_F = e^K \left( D_J \bar{W} K^{JI} D_I W - 3|W|^2 \right), \quad (1)$$

where $D_I W = \left( \frac{\partial}{\partial \Phi_I} + \frac{\partial K}{\partial \Phi_I} \right) W$, $D_J \bar{W} = \left( \frac{\partial}{\partial \Phi^*_J} + \frac{\partial K}{\partial \Phi^*_J} \right) \bar{W}$;

$K_{IJ} = \frac{\partial^2 K}{\partial \Phi_I \partial \Phi^*_J}$. $K_{I\bar{M}} K_{\bar{M}J} = \delta^J_I$.

Considered Kähler potential is:

$$K = - \ln(1 - \sum_I \Phi^*_I e^{-V} \Phi_I), \quad (2)$$
We consider the following VEV configuration:

\[
\langle \tilde{e}_1^c \rangle = z, \quad \langle h_d \rangle = \begin{pmatrix} zc_\theta \\ 0 \end{pmatrix}, \quad \langle \tilde{l}_2 \rangle = \begin{pmatrix} zs_\theta \\ 0 \end{pmatrix}
\]

\[ SU(2)_L \]

\[
\langle \tilde{q}_1 \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & z \end{pmatrix}
\]

\[ SU(2)_L \]

\[
\langle \tilde{u}^c \rangle = \begin{pmatrix} 0 \\ 0 \\ zc_\varphi \end{pmatrix}, \quad \langle \tilde{t}^c \rangle = \begin{pmatrix} 0 \\ 0 \\ zs_\varphi e^{i\omega} \end{pmatrix}
\]

(3)
With \( \cos \theta \simeq 1 \) (fixed), the only non-zero \( F \)-term is:

\[
F_{e^-}^* = -\lambda_e z^2
\]

(4)

giving:

\[
V_F = e^\mathcal{K} \mathcal{K}^{e^-} |F_{e^-}|^2.
\]

(5)

The kinetic part, which includes \((\partial z)^2\) is

\[
\mathcal{K}_{IJ} \partial \Phi_I \partial \Phi_J^* \rightarrow (\partial V_z)^\dagger \langle \mathcal{K}(z) \rangle \partial V_z,
\]

(6)

where with (2) and (3) we have:

\[
V_z^T = (z, zc_\theta, zs_\theta, z, zc_\phi, zs_\phi e^{-i\omega}),
\]

\[\langle \mathcal{K}(z) \rangle^T = \frac{1}{1-4z^2} \mathbf{1}_{6 \times 6} + \frac{z^2}{(1-4z^2)^2} \times \]

\[
\begin{pmatrix}
1 & c_\theta & s_\theta & 1 & c_\phi & s_\phi e^{-i\omega} \\
c_\theta & c_\theta^2 & c_\theta s_\theta & c_\theta & c_\theta c_\phi & c_\theta s_\phi e^{-i\omega} \\
s_\theta & c_\theta s_\theta & s_\theta^2 & s_\theta & s_\theta c_\phi & s_\theta s_\phi e^{-i\omega} \\
1 & c_\theta & s_\theta & 1 & c_\phi & s_\phi e^{-i\omega} \\
c_\phi & c_\phi c_\phi & s_\phi c_\phi & c_\phi & c_\phi^2 & c_\phi s_\phi e^{-i\omega} \\
s_\phi e^{i\omega} & c_\phi s_\phi e^{i\omega} & s_\phi s_\phi e^{i\omega} & s_\phi e^{i\omega} & c_\phi s_\phi e^{i\omega} & s_\phi^2
\end{pmatrix}
\]

(7)
Using (7) in (6) and introducing canonically normalized real scalar $\phi$ - the inflaton - we obtain

$$K_{I\bar{J}} \partial \Phi_I \partial \Phi^*_{\bar{J}} \rightarrow 4 \frac{(\partial z)^2}{(1-4z^2)^2} \equiv \frac{1}{2} (\partial \phi)^2.$$  \hspace{1cm} (8)

$$\rightarrow z = \frac{1}{2} \tanh(\frac{\phi}{\sqrt{2}}),$$ \hspace{1cm} (9)

With these, from (5), we derive the inflaton potential $\mathcal{V}$ to have the form:

$$\mathcal{V}(\phi) = V_F(\phi) \simeq \frac{\lambda^2}{16} \tanh^4(\frac{\phi}{\sqrt{2}}).$$ \hspace{1cm} (10)
Some References

Some earlier works on inflation along $D$-flat directions by MSSM states (but with extra higher order terms):


Inflation with additional singlets by Logarithmic but slightly different Kähler potentials, investigated in works:
