

Genetic Algorithms: New **tools** to Explore **SUSY** Parameter Spaces

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Yashar Akrami

In collaboration with:

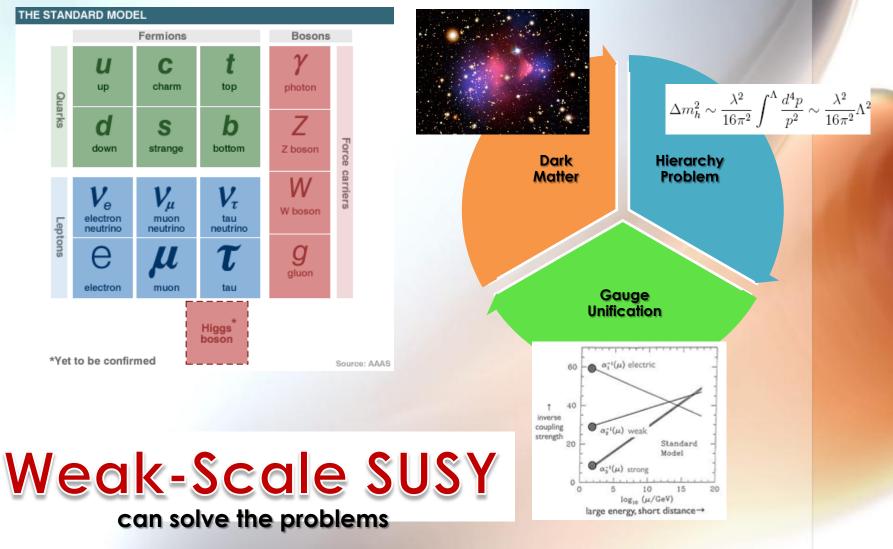
Pat Scott, Joakim Edsjö, Jan Conrad and Lars Bergström JHEP 04, 057 (2010) [arXiv:0910.3950]



Oskar Klein Center for Cosmoparticle Physics (OKC)



Demands for New Physics at TeV Scales



Field Content:

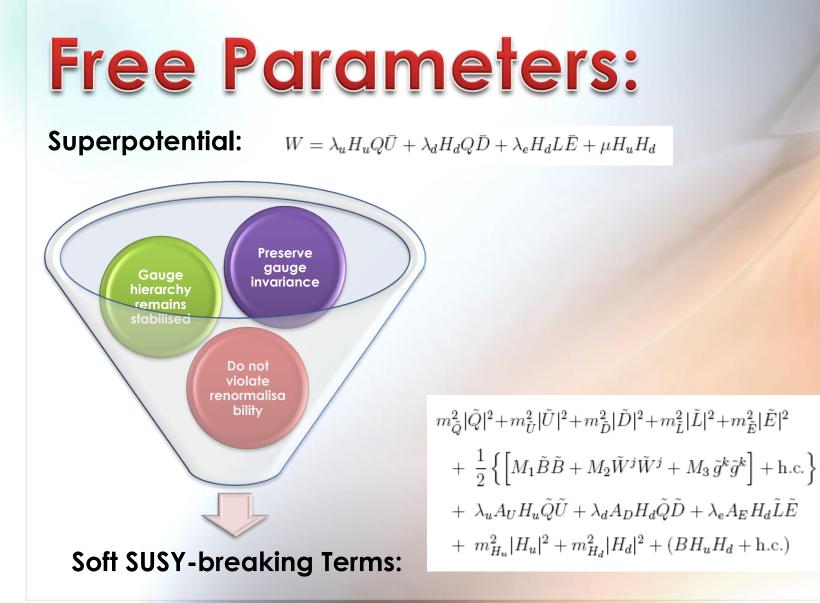
$$\begin{split} \tilde{e}_{L,R}, \tilde{\mu}_{L,R}, \tilde{\tau}_{L,R} & \text{[Sleptons (Spin 0)]} \\ \tilde{\nu}_{e}, \tilde{\nu}_{\mu}, \tilde{\nu}_{\tau} \\ \tilde{u}_{L,R}, \tilde{c}_{L,R}, \tilde{t}_{L,R} & \text{[Squarks (Spin 0)]} \\ \tilde{d}_{L,R}, \tilde{s}_{L,R}, \tilde{b}_{L,R} \\ \tilde{B}, \tilde{W}^{0}, \tilde{H}_{u}^{0}, \tilde{H}_{d}^{0} & \text{[Neutralinos (Spin 1/2)]} \\ \tilde{W}^{+}, \tilde{H}_{u}^{+} & \text{[Charginos (Spin 1/2)]} \\ \tilde{W}^{-}, \tilde{H}_{d}^{-} \end{split}$$

 \tilde{g} [Gluinos (Spin 1/2)]



After EW Symmetry Breaking:

 h, H, A, H^{\pm} [Higgs Bosons (Spin 0)]



4

Free Parameters:

Superpotential:

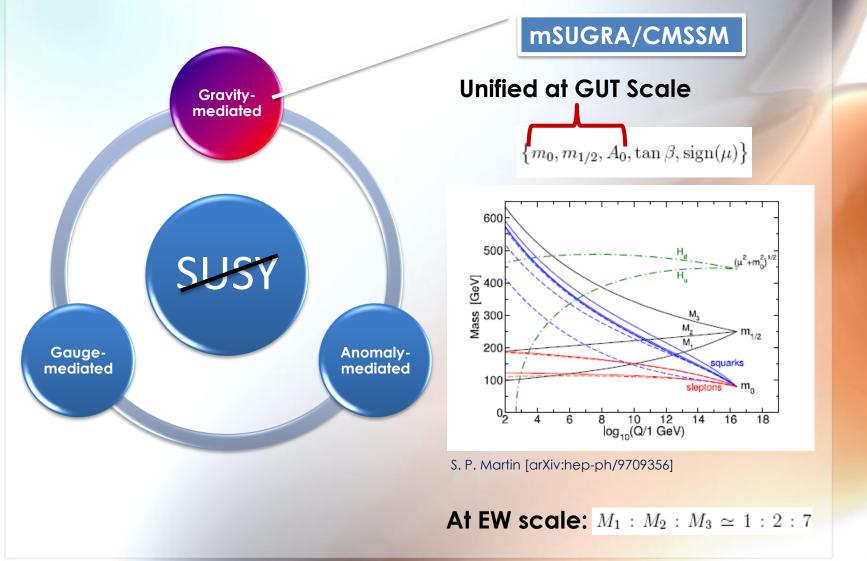
 $W = \lambda_u H_u Q \bar{U} + \lambda_d H_d Q \bar{D} + \lambda_e H_d L \bar{E} + \mu H_u H_d$

Soft SUSY-breaking terms:

$$\begin{split} m_{\tilde{Q}}^{2} |\tilde{Q}|^{2} + m_{\tilde{U}}^{2} |\tilde{U}|^{2} + m_{\tilde{D}}^{2} |\tilde{D}|^{2} + m_{\tilde{L}}^{2} |\tilde{L}|^{2} + m_{\tilde{E}}^{2} |\tilde{E}|^{2} \\ &+ \frac{1}{2} \left\{ \left[M_{1} \tilde{B} \tilde{B} + M_{2} \tilde{W}^{j} \tilde{W}^{j} + M_{3} \tilde{g}^{k} \tilde{g}^{k} \right] + \text{h.c.} \right\} \\ &+ \lambda_{u} A_{U} H_{u} \tilde{Q} \tilde{U} + \lambda_{d} A_{D} H_{d} \tilde{Q} \tilde{D} + \lambda_{e} A_{E} H_{d} \tilde{L} \tilde{E} \\ &+ m_{H_{u}}^{2} |H_{u}|^{2} + m_{H_{d}}^{2} |H_{d}|^{2} + (B H_{u} H_{d} + \text{h.c.}) \end{split}$$

105 new parameters

Constraints on FCNCs and CP-violation can help



Scanning Supersymmetric Parameter Spaces

Goal: given a particular version of supersymmetry, determine which parameter combinations fit all experiments, and how well

SUSY Global Fit

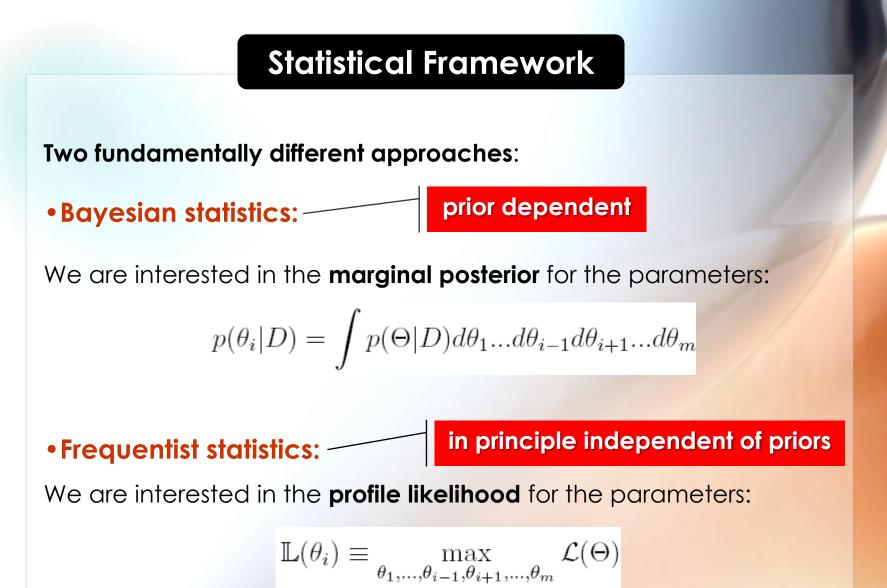
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Issue 1: Combining fits to different experiments **Easy** – composite likelihood $(L_1 \times L_2 \equiv \chi_1^2 + \chi_2^2)$

- dark matter relic density from WMAP
- precision electroweak tests at LEP
- LEP limits on sparticle masses
- B-factory data (rare decays, $b \rightarrow s\gamma$)
- muon anomalous magnetic moment

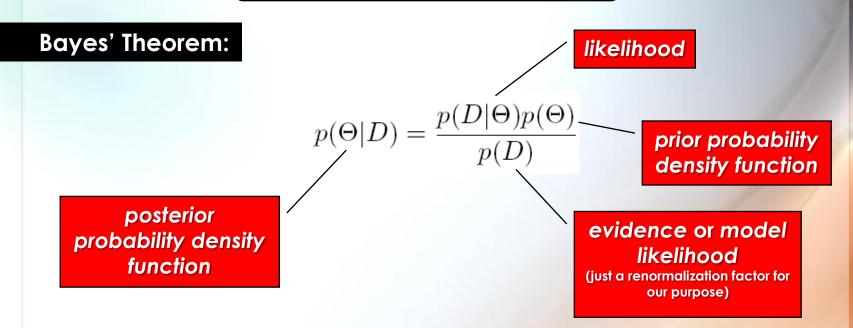
Issue 2: Finding the points with the best likelihoods **Tough** – grid scans, MCMCs, nested sampling or genetic algorithms

Public codes: SuperBayeS, SFitter, Fittino



Thus in the profile likelihood one maximizes the value of the likelihood along the hidden dimensions, rather than integrating it out as in the marginal posterior.

Statistical Framework

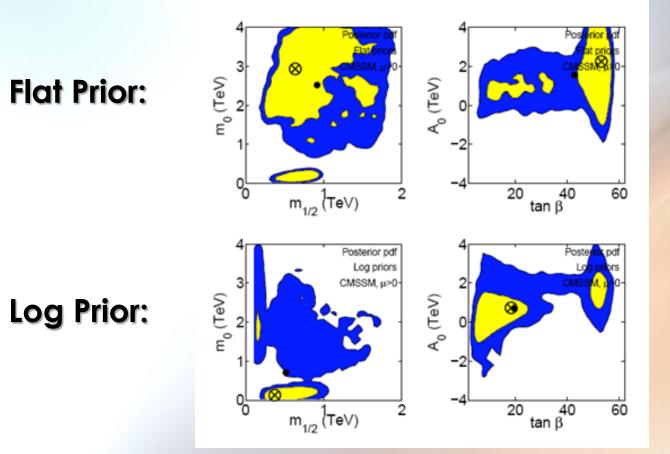


• One practically interesting consequence of Bayesian inference is that it gives a powerful way of estimating **how robust a fit is**, i.e., if the **posterior** is strongly **dependent** on **different priors**, this actually means that **the data** are **not sufficient or accurate enough** to constrain the model parameters.

• If a fit is robust, the Bayesian and frequentist methods should result in similar confidence regions of the parameter space. This is **NOT** the case for **SUSY models**.

Statistical Framework

Some CMSSM Scans with SuperBayeS:



R. Trotta, F. Feroz, M.P. Hobson, L. Roszkowski and R. Ruiz de Austri, The impact of priors and observables on parameter inferences in the Constrained MSSM, JHEP 12 (2008) 024 [arXiv:0809.3792]



• Developed by Roberto Ruiz de Austri, Roberto Trotta, Farhan Feroz, Leszek Roszkowski, and Mike Hobson.

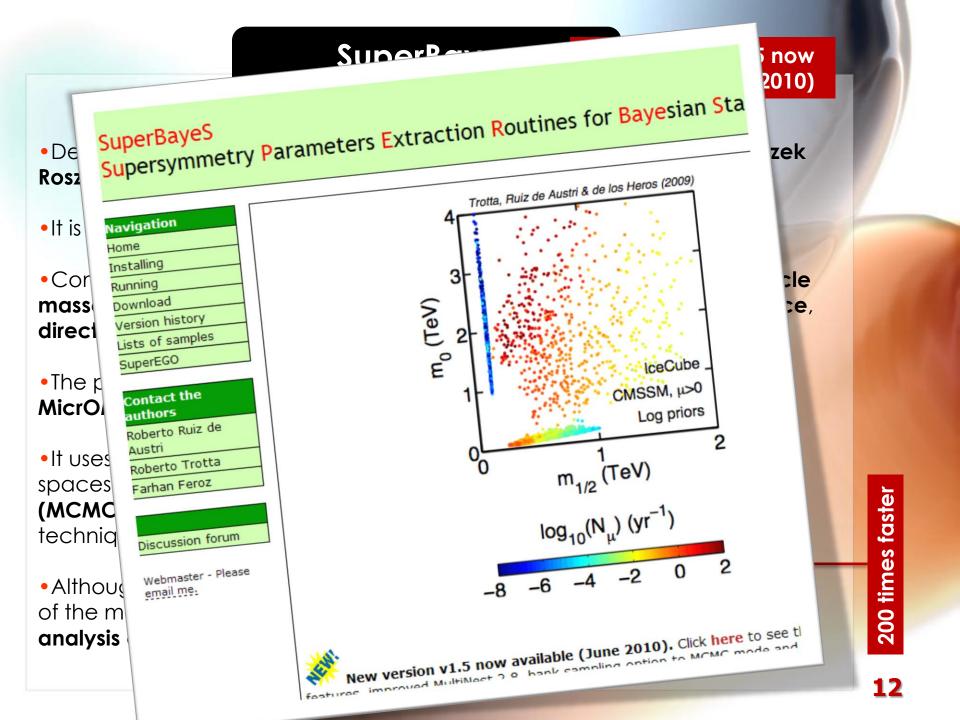
• It is a package for **fast** and **efficient** sampling of the CMSSM.

• Compares SUSY predictions with observable quantities, including **sparticle masses**, **collider observables**, **B-factory data**, **dark matter relic abundance**, **direct detection cross sections**, **indirect detection quantities** etc.

• The package combines **SoftSusy**, **DarkSusy**, **FeynHiggs**, **Bdecay** and **MicrOMEGAs**.

 It uses Bayesian techniques to explore multidimensional SUSY parameter spaces. Scanning can be performed using Markov Chain Monte Carlo (MCMC) technology or more efficiently by employing the new scanning technique called nested sampling (MultiNest algorithm).

• Although these methods have been used for the **profile likelihood** analysis of the model, they are essentially **optimized for the marginal posterior analysis** of the model.





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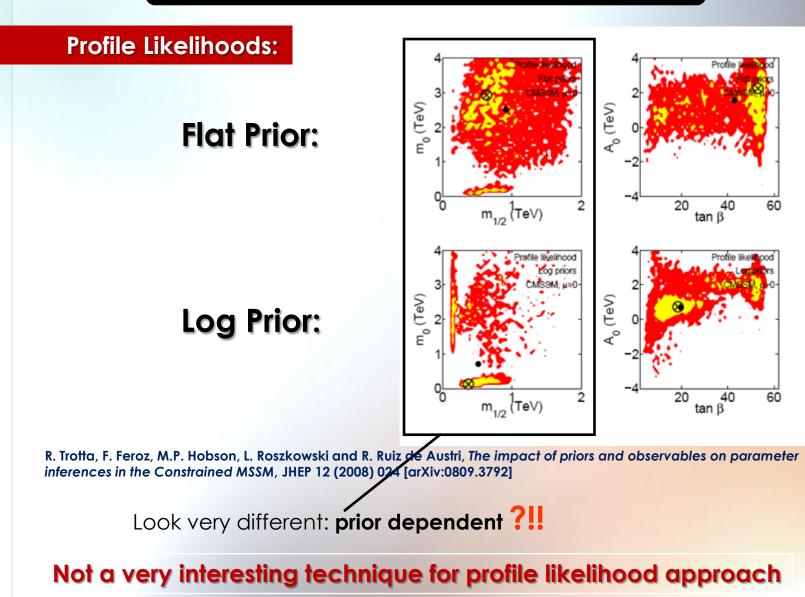
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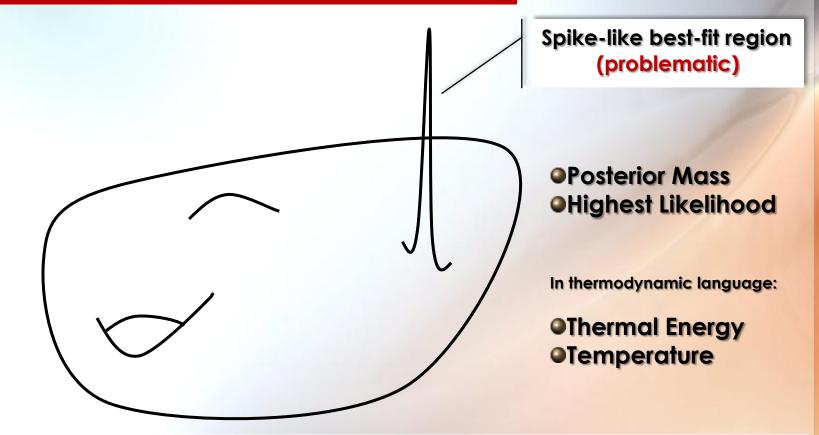




According to the SB people, MCMC scans give similar results up to some statistical noise

Complex Parameter Spaces





In order to make a profile likelihood analysis of a model correctly, it is extremely important to know, with enough accuracy, the highest value of the likelihood function in the parameter space of the model. Otherwise, the calculated confidence regions might be very far from the real ones.

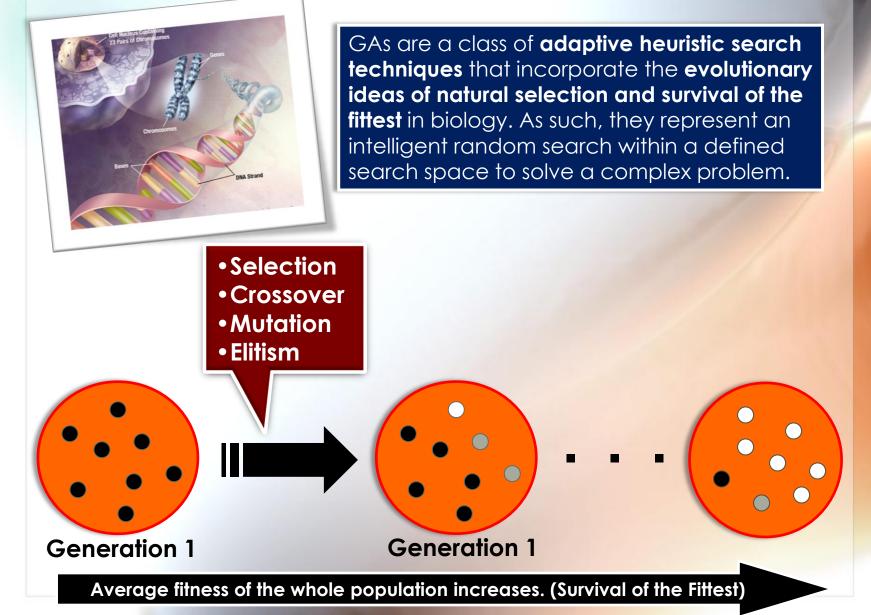
Genetic Algorithms (GAs)

GAs can be helpful, because:

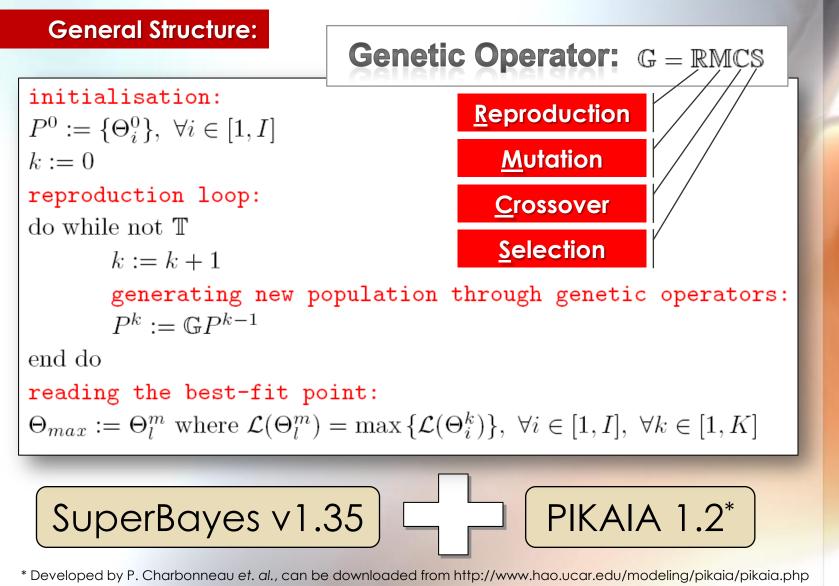
•The actual use of these algorithms is to **maximize/minimize** a specific function; this is exactly what we need in the case of a profile likelihood scan.

•GAs are usually considered as **powerful methods** in probing global extrema when the parameter space is **very large**, **complex** or **poorly understood**; these are precisely what we have in the case of the supersymmetric models including the CMSSM.

Genetic Algorithms (GAs)



Genetic Algorithms (GAs)



Model + Nuisance Parameters

• **CMSSM:** GUT-scale parameterisation

 m_0 : scalar mass parameter $m_{1/2}$: gaugino mass parameter $tan\beta$: ratio of Higgs VEVs A_0 : trilinear coupling $sgn \mu$: Higgs mass parameter (+ve in our scans)

Just a testbed – techniques are applicable to any MSSM parameterisation

• SM nuisances: reflecting our imperfect knowledge of the values of relevant SM parameters

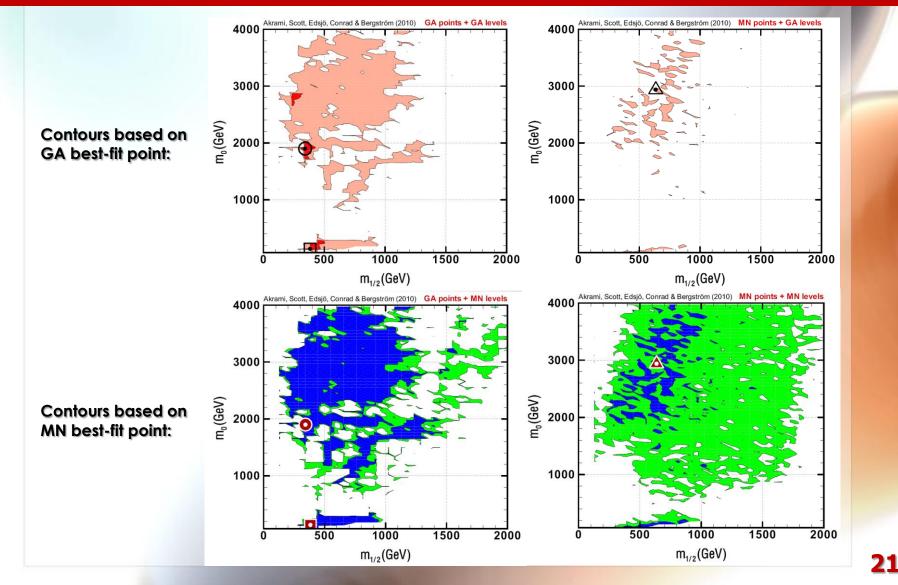
m_t: pole top quark mass a_{em}: EM coupling constant m_b: bottom quark mass
a_s: strong coupling constant

Data and Other Constraints Included

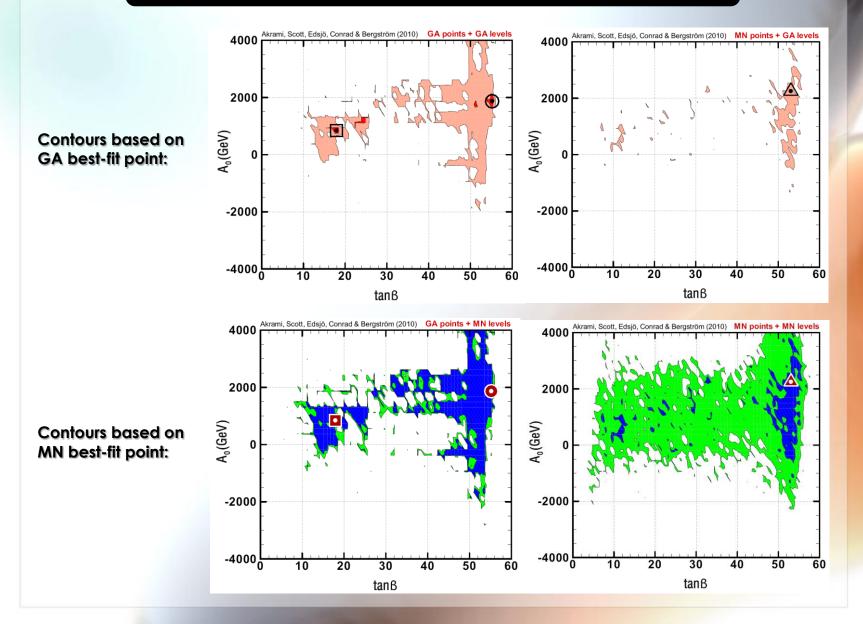
Observable	Mean value	experiment	ties viations) theoretical	
$\begin{array}{c} \hline m_t \\ m_b(m_b)^{\overline{MS}} \\ \alpha_s(m_Z)^{\overline{MS}} \\ 1/\alpha_{em}(m_Z)^{\overline{MS}} \\ \hline m_W \\ \sin^2 \theta_{eff} \\ \delta a_\mu^{SUSY} \times 10^{10} \\ BR(\overline{B} \to X_s \gamma) \times \overline{\Delta} \\ \Delta M_{B_s} \\ BR(\overline{B}_u \to \tau \nu) \times \\ \Omega_{\chi} h^2 \\ \hline \end{array}$ $\begin{array}{c} \hline BR(\overline{B}_s \to \mu^+ \mu^- m_h \\ \zeta_h^2 \\ m_{\tilde{\chi}_1^1} \\ m_{\tilde{\kappa}_R} \\ m_{\tilde{\chi}_1^1} \\ m_{\tilde{e}_R} \\ m_{\tilde{\mu}_R} \\ m_{\tilde{\tau}_1} \\ m_{\tilde{\nu}} \\ m_{\tilde{t}_1} \\ m_{\tilde{\nu}_1} \\ m_{\tilde{t}_1} \\ m_{\tilde{t}_$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c} 0.07 \text{ GeV} \\ 0.002 \\ 0.03 \\ \hline \\ 25 \text{ MeV} \\ 16 \times 10^{-5} \\ 8.8 \\ 0.26 \\ 0.12 \text{ ps}^{-1} \\ 0.49 \\ 0.0062 \\ \hline \\ \text{CL} \\ \hline \\ \text{Higgs} \\ \hline \\ \text{GeV} \\ \text{W} \\ \text{V} \\ \text{V} \\ \text{V} \\ \text{V} \\ \text{V} \\ \text{W} \\ \text{W} \\ \end{array}$	$\begin{array}{c} - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - \\ - $	<section-header></section-header>

2D Profile Likelihoods in $m_0-m_{1/2}$ Plane

GAs find better fits than nested sampling ($X^2 = 9.35$ vs. $X^2 = 13.51$).

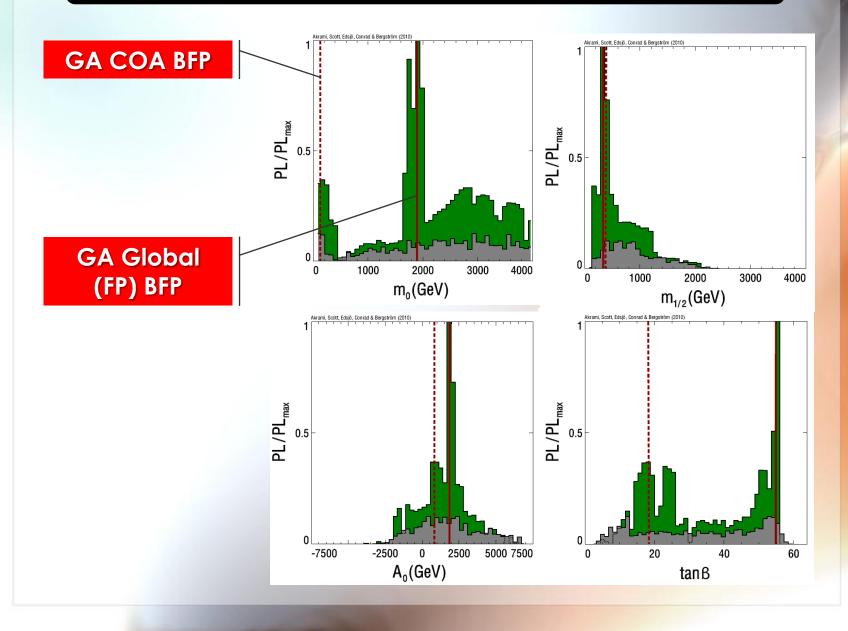


2D Profile Likelihoods in A₀-tanβ Plane



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1D Profile Likelihoods for CMSSM Parameters



23

Best-fit Parameter Values

) parameter	'S
	model (+nuisance) parameter	rs Pregion) GA COA BFP
	model (+nuisance) parent GA global BFP (located in FF	133.9 Gev
	GA giobal	$_{383.1}~{\rm GeV}$
	1900.5 GeV	840.6 GeV
m_0	342.8 GeV	17.9
$m_{1/2}$	1873.9 GeV	17.9 173.3 GeV
A_0	55.0	173.5 GC
an eta	172.9 GeV	$4.20 { m GeV}$
m.	4.19 GeV	0.1183
MS		127.955
	0.1172	
$\frac{m_b(m_b)}{\alpha_s(m_Z)^{\overline{MS}}}$ $\frac{1}{\alpha_{\rm em}(m_Z)^{\overline{MS}}}$	127.955	A PEP
$1/\alpha_{\rm em}(m_Z)$	observables	FP region) GA COA BFP
	GA global BFP (located in	FP region) 80.371 GeV
	80.366 GeV	0.23153
	80.300 Ge 1	14.5
mw	0.23156	2.97
$\frac{\sin^2 \theta_{\text{eff}}}{\delta a_{\mu}^{\text{SUSY}} \times 10^{10}}$	5.9	19.0 ps^{-1}
$\frac{\delta a_{\mu}^{SOUT} \times 10}{BR(\overline{B} \to X_s \gamma)}$	$\times 10^4$ 3.58	1.46
$BR(B \rightarrow \Lambda_s))$	$\times 10^{-10}$ 17.37 ps ⁻¹	0.10985
		0.10900
$\frac{\Delta M_{B_s}}{BR(\overline{B}_u \to \tau \nu)}$	0.10949	3.87×10^{-8}
0 6		
$\frac{\Omega_{\chi}n}{BR(\overline{B}_s \to \mu^+\mu)}$	-) 4.01	the second se
Dire		

Best-fit Parameter Values

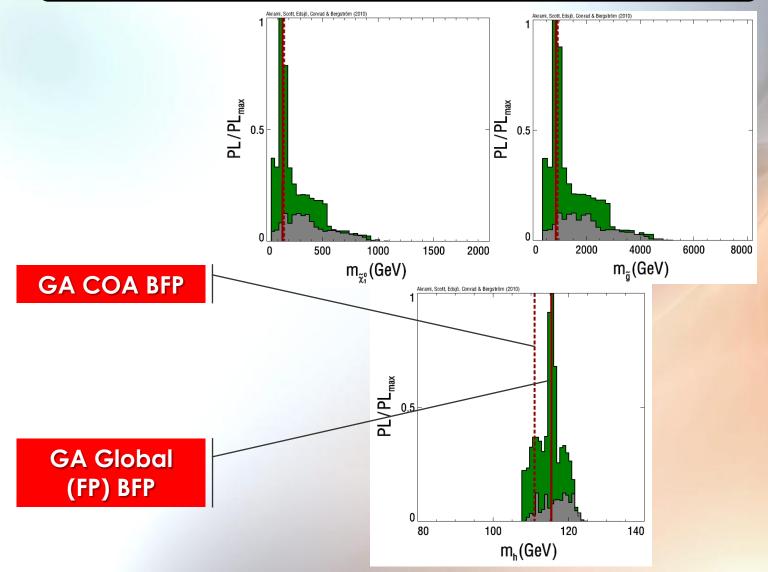
			fractional contributi	to the total χ^2	in %)
		2 (fractional contributi	MN global BFP	MN global BFP
		Post	GA COA BFP	MN globar Dr	WITH 108 F
		GA global BFP	GA COM	with flat priors	0.81 (6.78%)
	observable	located in FP region	(0.10%)	0.48(3.56%)	0.69 (5.83%)
	Observe	located in 1	0.35(3.10%)	1.48(10.92%)	0.09(0.034%) 0.0040(0.034%)
1	nuisance parameters	0.12 (1.27%)	0.83 (7.29%)	0.07 (0.49%)	0.0040 (0.0018%)
	nuisance pare	1.21 (12.95%)	$\sim 10^{-4} (0.001\%)$	9.21 (68.20%)	2.40(20.18%)
	mW	0.024 (0.26%)	2.86 (25.21%)	0.10 (0.74%)	3.83(32.20%)
	$\sin^2 \theta_{\text{eff}}$	7 09 (75.79%)	3.03 (26.76%)	0.10(0.1470)	0.29(2.41%)
	δa_{μ}^{SUSY}	0.010 (0.11%)	0.26(2.31%)	0.09(0.66%)	0.043 (0.36%)
	$BR(\overline{B} \to X_s \gamma)$	0.028 (0.30%)	0.26(2.044%) 0.050(0.44%)	1.91 (14.14%)	0.13 (1.07%)
	ΔM_{Bs}	$\sim 10^{-5} (10^{-4}\%)$	0.050 (0.4170)	0.03~(0.2%)	0.00 (0.00%)
	$\frac{\Delta MB_s}{BR(\overline{B}_u \to \tau \nu)}$	$\sim 10^{-10}$ (10)	$\sim 10^{-5} (10^{-4}\%)$	0.00(0.00%)	3.70(31.13%)
	BR(Du	0.0011 (0.012%)	0.00 (0.00%)	0.15(1.09%)	0.00 (0.00%)
	$\Omega_{\chi}h^2$	0.016 (0.17%)	3.96 (34.88%)	0.00(0.00%)	20 (100 %)
	$\frac{M_{\chi}n}{BR(\overline{B}_s \to \mu^+ \mu^-)}$	0.85 (9.14%)	0.00(0.00%)	= = 1 (100 %	0.00 (811) %) 11.90 (100 %)
	m_h	0.00(0.00%)	11.34 (100 %) 13.31 (12)	
	sparticles	$9.35\ (100\ \%)$	11.0 - (
	all				

Mass Spectrum at Best-fit Points

			11		GA global BFP	GA COA BFP
	G	A global Dri	GA COA BFP	(GeV)	located in FP region 1994	798.1 832.4
(Ge	V) loca	ated in FP region 1908	294.1 202	$m_{\tilde{d}_R}$ $m_{\tilde{s}_L}$	2000 1994	798.1 765
mě mě	R	1903 1907	294.1 201.9	$m_{\tilde{s}_R}$ $m_{\tilde{b}_1}$	1354 1492	793.4 152.6
	μ <u>L</u> μ̃R l ī 1	1901 1100	160.1 289.2	$m_{\tilde{b}_2}$ $m_{\tilde{\chi}_1^0}$	140.4 269.9	285.4 451.1
n	$n_{\tilde{\tau}_2}$ $n_{\tilde{\nu}_e}$	1560 1906	283.3 283.3	$\begin{array}{c} m_{\tilde{\chi}^0_2} \\ m_{\tilde{\chi}^0_3} \end{array}$	519.7 529.7	469.6 286.9
1	$m \tilde{\nu}_{\mu}$ $m \tilde{\nu}_{\tau}$	1905 1560	272.5 826.1	$m_{\tilde{\chi}_4^0}$ $m_{\tilde{\chi}_1^\pm}$	270.4 530.3	468.5
	m _{ũL} m _{ũR}	1998 1996	805.4 826.1	$m_{\tilde{\chi}_2^{\pm}}$ m_h	115.55	111.11 504.24 504.04
	$m_{\tilde{c}L}$	1998 1996	805.4 672.8		4 179.83 A 201.14	504.04 510.67 898.8
	$m_{\tilde{e}_R}$ $m_{\tilde{t}_1}$ $m_{\tilde{t}_2}$	1194 1364	803 832.4	m _H m	811.1	050.0
	m ₃ .	2001				

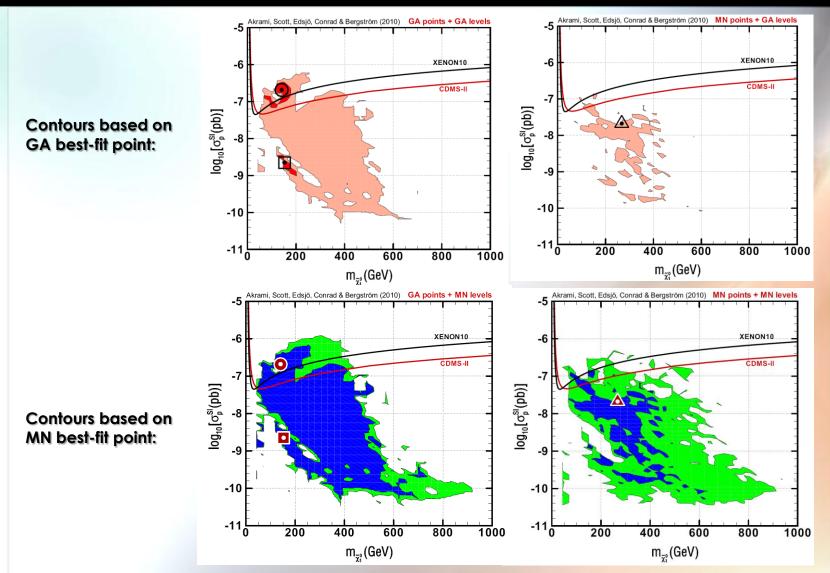
 $m_{\tilde{d}L}$

1D Profile Likelihoods for Some Observables



• The LHC is in principle able to investigate a large fraction of the high-likelihood points in the CMSSM parameter space if it explores sparticle masses up to around 3 TeV.

2D Profile Likelihoods in DM Direct Detection Plane (SI)



Best-fit point is actually ruled out by direct detection (under standard halo assumptions).

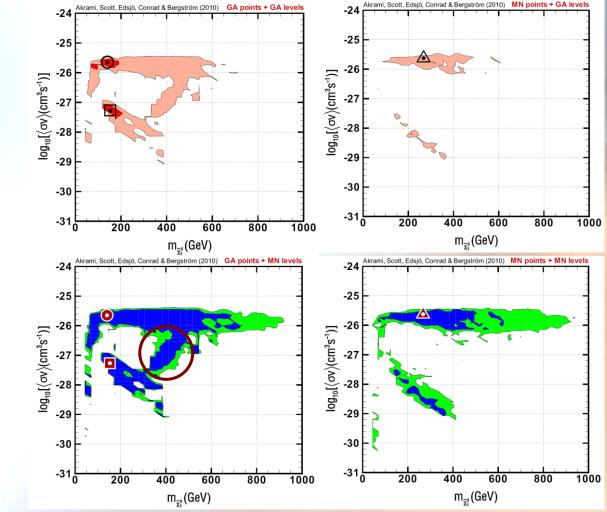
Secondary maximum still OK.

2D Profile Likelihoods in DM Indirect Detection Plane

Contours based on GA best-fit point:

Contours based on

MN best-fit point:



• Global best-fit point should be probed soon by Fermi (See e.g. P. Scott, J. Conrad, J. Edsjö, L. Bergström, C. Farnier & YA. Direct Constraints on Minimal Supersymmetry from Fermi-LAT Observations of the Dwarf Galaxy Segue 1, JCAP 01, 031 (2010) [arXiv:0909.3300])

• The GA turns up a 'new' region at moderate $\langle \sigma v \rangle$, around 400 GeV. This region is a high-m₀ stau coannihilation region, apparently missed in other scans.

Summary and Conclusions

- 1. Constraining the parameter space of the MSSM using existing data is under no circumstances an easy or straightforward task. Even in the case of the CMSSM, a highly simplified and economical version of the model, the present data are not sufficient to constrain the parameters in a way completely independent of computational and statistical techniques.
- 1. Many recent activities in this field have used scanning methods optimised for calculating the Bayesian evidence and posterior PDF. Highly successful in revealing the complex structure of SUSY models, demonstrating that some patience will be required before we can place any strong constraints on their parameters.
- 2. Bayesian scanning methods have also been employed for frequentist analyses of the problem, particularly in the framework of the profile likelihood. These methods are not optimised for such frequentist analyses, so care should be taken in applying them to such tasks.
- 3. We have employed a completely new scanning algorithm, based on GAs. They seem to be a powerful tool for frequentist approaches to the problem of scanning the CMSSM parameter space. We compared the outcomes of GA scans directly with those of the state-of-the-art Bayesian algorithm MultiNest, in the framework of the CMSSM.
- 4. We found many new high-likelihood CMSSM points, which have a strong impact on the final statistical conclusions of the study. These not only influence considerably the inferred high-likelihood regions and confidence levels on the parameter values, but also indicate that the applicability of the conventional Bayesian scanning techniques is highly questionable in a frequentist context.

Summary and Conclusions

- 5. Although our initial motivation in using GAs was to gain a correct estimate of the likelihood at the global best-fit point, which is crucial in a profile likelihood analysis, we also realised that they can find many new and interesting points in almost all the relevant regions of parameter SPACE. These points strongly affect the inferred confidence regions around the best-t point. Even though we cannot be confident of exactly how completely our algorithm is really mapping these high-likelihood regions, it has certainly covered large parts of them better than any previous algorithm.
- 6. By improving the different ingredients of GAs, such as the crossover and mutation schemes, this ability might even be enhanced further. We largely employed the standard, simplest versions of the genetic operators in our analysis, as well as very typical genetic parameters. These turned out to work sufficiently well for our purposes. Although we believe that tuning the algorithm might produce even more interesting results, it is good news that satisfactory results can be produced even with a very generic version. This likely means that one can apply the method to more complicated SUSY models without extensive ne-tuning.
- 7. We have also compared our algorithm with MultiNest in terms of speed and convergence, and argued that GAs are no worse than MultiNest in this respect. GAs have a large potential for parallelisation, reducing considerably the time required for a typical run. This property, as well as the fact that the computational eort scales linearly (i.e. as kN for an N-dimensional parameter space), also makes GAs an excellent method for the frequentist exploration of higher-dimensional SUSY parameter spaces.

Summary and Conclusions

8. The focus point region is favoured in our analysis over the co-annihilation region, in contrast to findings from some other MCMC studies, where the opposite is claimed. We also found a rather large part of the stau co-annihilation region, consistent with all experimental data, located at high m₀. That is, at least in our particular setup, high masses, corresponding either to the FP or the COA regions, are by no means disfavoured by current data (except perhaps direct detection of dark matter). The discrepancy might originate in the different scanning algorithms employed, or in the different physics and likelihood calculations performed in each analysis. We have however shown, by comparing our results with others produced using exactly the same setup except for the scanning algorithm, that one should not be at all confident that all the relevant points for a frequentist analysis can be found by scanning techniques optimised for Bayesian statistics, such as nested sampling and MCMCs.

The **bottom line** of our work is that:

We once again see that even the CMSSM, despite its simplicity, possesses a highly complex and poorly-understood structure, with many small, finetuned regions. This makes investigation of the model parameter space very difficult and still very challenging for modern statistical scanning techniques. Although the method proposed in this paper seems to outperform the usual Bayesian techniques in a frequentist analysis, it is important to remember that it may by no means be the final word in this direction. Dependence of the results on the chosen statistical framework, measure and method calls for caution in drawing strong conclusions based on such scans. The situation will of course improve significantly with additional constraints provided by forthcoming data.

OKC PROSPECTS WORKSHOP

(http://agenda.albanova.se/conferenceDisplay.py?confld=1983)



THANK VOU for your attention

GA global BF1 σ_p^{SI} 2.057 × 10 ⁻⁷ pb 2.5 σ_p^{SI} 2.435 × 10 ⁻⁶ pb 4.5	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	$\frac{1}{3 \text{ A BFP in COA region}}{5.385 \times 10^{-28} \text{ cm}^3 \text{s}^{-1}}$

