On-shell Methods for ttbar + jets with Polarization

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Introduction

- Physics to be done during first run of LHC:
  - Ambitious: SUSY, Higgs
  - Guaranteed: Study the top quark
- May be possible to measure anomalous magnetic moment of top
  - Large anomalous magnetic moment = top compositeness
    - Hints of extra dimensions?

Introduction

- To study anomalous magnetic moment, consider most general QED fermion-photon vertex
- Leads to introduction of:

$$\mathcal{L}_a = \frac{a}{2m}\bar{\psi}\Sigma_{\mu\nu}F^{\mu\nu}\psi$$

  - Can add in a non-Abelian theory, too
- We will cite results in color-ordered, non-Abelian gauge theory

Introduction

- History: prospects of discovering anomalous magnetic moment at hadron colliders discussed in 1994! Rizzo, 1994
- Did everything that could be done with anomalous magnetic moment at hadron colliders at 5 points
  - Sufficient at Tevatron
  - Need more at LHC!

Introduction

- Top could be boosted at LHC and radiate many gluons
  - Necessary to consider amplitudes with many points
- *Very* inefficient to use Feynman diagrams: No. of Feynman Diagrams ~ $2^n n!$
  - Need a better way!
  - We will use BCFW recursion

Introduction

- Lore: BCFW recursion works for theories with very good behavior
  - "Need" renormalizability or even finiteness
  - e.g., $N = 4$ supersymmetric theories
- This theory is explicitly non-renormalizable so we naively don't expect BCFW to work

Outline

- Digression into spinor helicity formalism
- Introduce/remind about on-shell recursive method of Britto, Cachazo, Feng and Witten
- Recursion Relation in theory with an anomalous magnetic moment
- Introduce auxiliary theory to compute remaining amplitudes with BCFW
- Conclusions

A Brief Digression: Spinor Helicity

- We can represent any null four-vector as a rank one matrix:

$$p_\mu \sigma^\mu_{\alpha\dot{\alpha}} \equiv p_{\alpha\dot{\alpha}} = \lambda_\alpha \bar{\lambda}_{\dot{\alpha}}$$

- Can construct Lorentz invariants by contracting matrix indices:

$$\epsilon^{\alpha\beta}\lambda^i_\alpha \lambda^j_\beta = \langle ij\rangle \qquad \epsilon^{\dot{\alpha}\dot{\beta}}\bar{\lambda}^i_{\dot{\alpha}}\bar{\lambda}^j_{\dot{\beta}} = [ij]$$

- Momentum invariants:

$$s_{ij} = \langle ij\rangle[ji] = \langle iji]$$

A Brief Digression: Spinor Helicity

- Some examples:
  - Gauge boson polarization vectors:

$$\epsilon_\mu^+(p) = \frac{\langle q\gamma_\mu p]}{\sqrt{2}\langle qp\rangle} \qquad \epsilon_\mu^-(p) = -\frac{[q\gamma_\mu p\rangle}{\sqrt{2}[qp]}$$

  - Four point pure Yang-Mills amplitude:

$$\mathcal{A}(1^+,2^-,3^+,4^-) = ig^2\frac{\langle 24\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

BCFW Recursion

- BCFW is an on-shell recursive method

Britto, *et al.*, 2005

- Idea: choose two particles and shift their momenta:

$$p_i \to p_i - zq$$
$$p_j \to p_j + zq$$

- To stay on-shell: $q^2 = p_i \cdot q = p_j \cdot q = 0$

- For massless particles:

$$i\rangle \to i\rangle - z\, j\rangle, \qquad i] \to i]$$
$$j\rangle \to j\rangle, \qquad j] \to j] + z\, i]$$

BCFW Recursion

- Consider the object:

$$\oint \frac{dz}{z}\mathcal{A}(z)$$

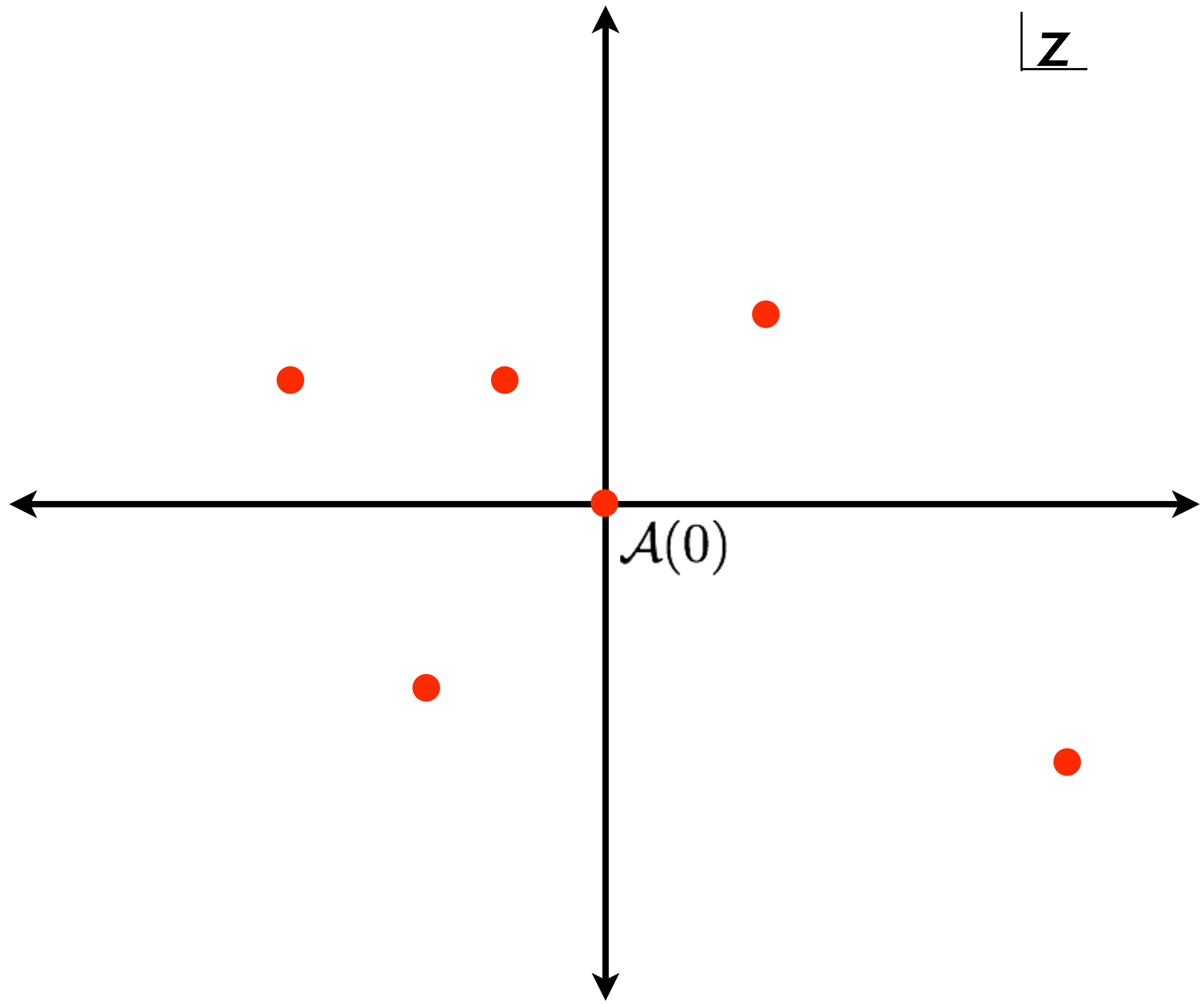
- $\mathcal{A}(0) =$ undeformed amplitude

- If $\mathcal{A}(z) \to 0$ as $z \longrightarrow \infty$:

$$\oint \frac{dz}{z}\mathcal{A}(z) = 0$$

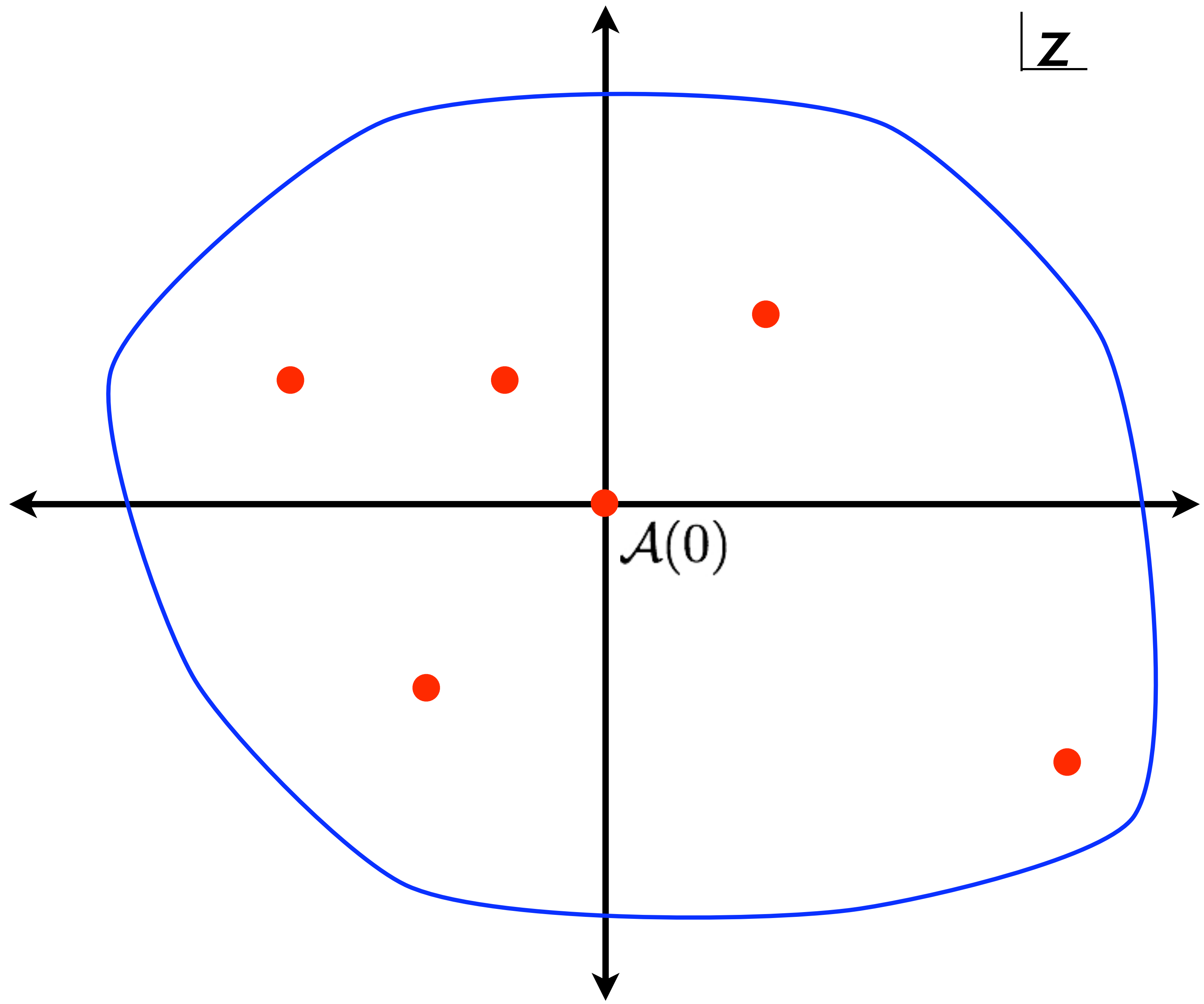
  - Can use Cauchy's Theorem!

BCFW Recursion



Poles when intermediate propagators go on-shell

BCFW Recursion

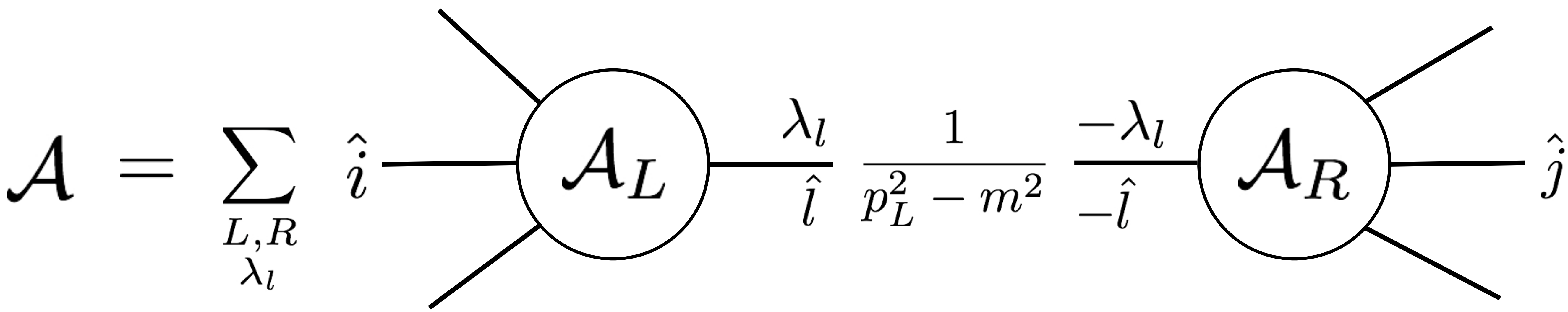


If $\mathcal{A}(z) \to 0$ as $z \to \infty$:

$$\mathcal{A}(0) = \sum \mathrm{Res}\, \mathcal{A}(z)$$

BCFW Recursion

BCFW Recursion Relation:



BCFW Recursion Review

- Requirement for BCFW recursion:

$$\mathcal{A}(z) \to 0 \text{ as } z \to \infty$$

- Increased efficiency over Feynman diagrams
- Used in QCD, gravity, supersymmetry
- Question: Are amplitudes in a theory with an anomalous magnetic moment BCFW constructible?

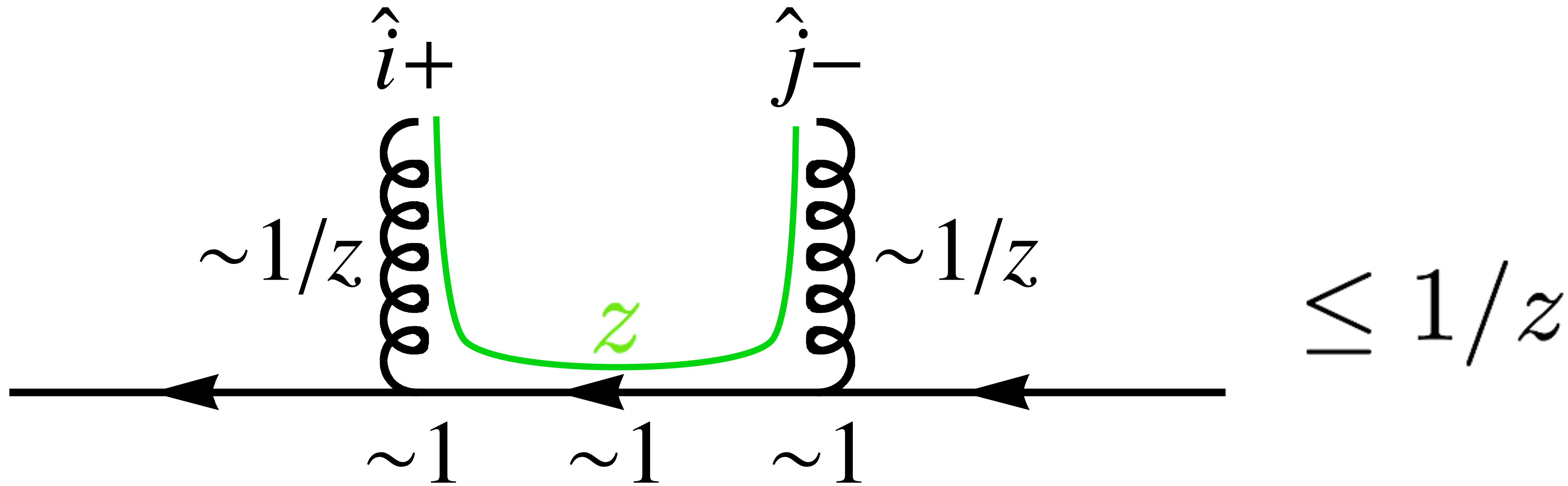
Recursion Relation for $a \neq 0$

- How do amplitudes behave when gluons are shifted?
  - Shifting massless particles maintains Lorentz invariance
- Consider four possible gluon shifts depending on helicity:

$\hat{i}$	$+$	$+$	$-$	$-$
$\hat{j}$	$+$	$-$	$+$	$-$

Recursion Relation for $a \neq 0$

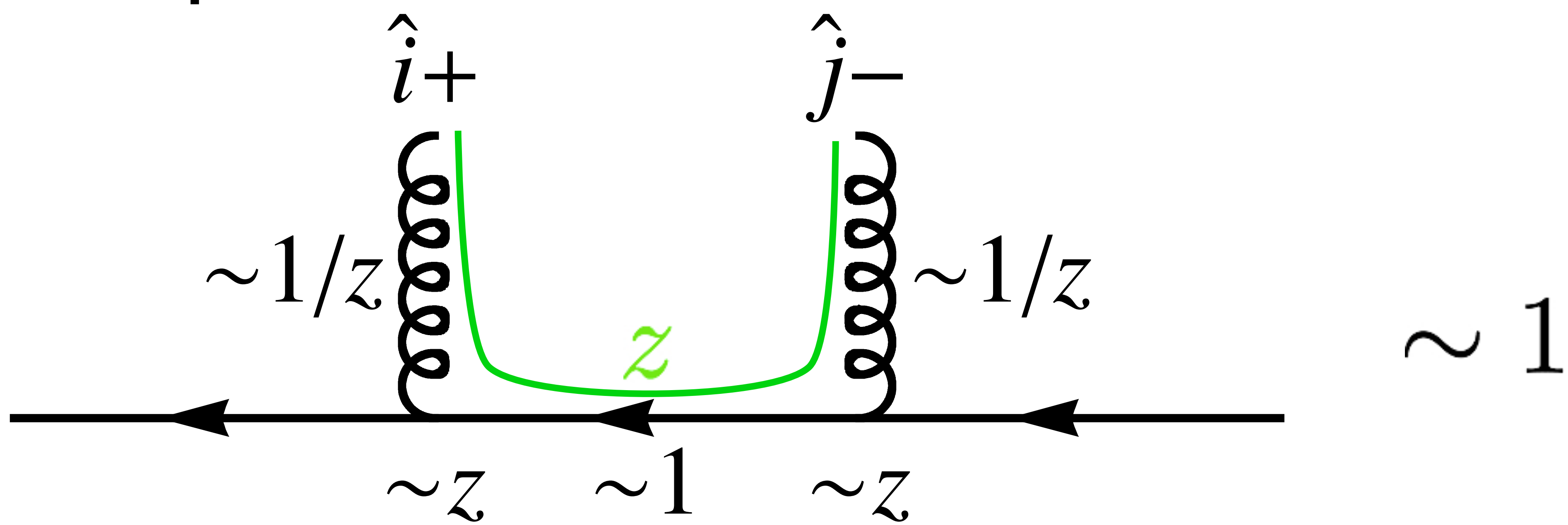
- z dependence for $a = 0$



- Fermion-gluon vertex ~ constant in z
- Fermion propagator ~ constant
- z dependence of polarizations depend on helicity

Recursion Relation for $a \neq 0$

- z dependence for $a \neq 0$



- Naively, bad behavior for large z!
- Large z dependence is actually better by considering product of fermion propagator and $a \neq 0$ vertex

Recursion Relation for *a* ≠ 0

- By considering the structure of a general Feynman diagram, one can show

$\hat{i}$	$\hat{j}$	large z
$+$	$+$	1
$+$	$-$	$1/z$
$-$	$-$	1
$-$	$+$	z^3

- Moral: Cannot use BCFW on amplitudes with all + or all - gluons!

Recursion Relation for $a \neq 0$

- Reminder: in pure $a = 0$ QCD need at least one + and one - helicity gluon
  - Maximally Helicity Violating (MHV) amplitude with minimum number of - helicity gluons
- Adding an anomalous magnetic moment changes MHV amplitude to that with no - helicity gluons!

Recursion Relation for *a* ≠ 0

- Example: Massless four point amplitudes at order a^1

$$\mathcal{A}(q^+, g_1^+, g_2^+, \bar{q}^+) = \frac{[12]^2}{\langle q\bar{q}\rangle}$$

$$\mathcal{A}(q^-, g_1^-, g_2^+, \bar{q}^-) \quad = -\frac{\langle q1\rangle^2 \langle \bar{q}1\rangle^2}{\langle q1\rangle\langle 12\rangle\langle 2\bar{q}\rangle}$$

- Shift: $1] \to 1] + z\,2] \qquad 2\rangle \to 2\rangle - z\,1\rangle$
- Can't use BCFW on first amplitude!

Anomalous Mag Moment Redux

- How can we describe the all + amplitude?
- $\Sigma_{\mu\nu}F^{\mu\nu} = \sigma_{\mu\nu}F^{\mu\nu} + \bar{\sigma}_{\mu\nu}F^{\mu\nu}$
  - First term: self-dual component, couples to left handed fermion
  - Second term: anti-self-dual, couples to right handed fermion
  - For all + helicity gluons: $\bar{\sigma}_{\mu\nu}F^{\mu\nu} = 0$
  - Can integrate out right handed component!

Anomalous Mag Moment Redux

- The result:

$$\mathcal{L} = \frac{1}{m}\bar{u}^{\dagger}\left[-D^2 - m^2 + \frac{g}{2}\sigma_{\mu\nu}F^{\mu\nu}\right]u$$

- An effective description of top's anomalous magnetic moment
- Exact description when:
  - $g = 2$
  - All gluons have + helicity!

Anomalous Mag Moment Redux

- What if $g = 2$?
  - "Square" Dirac equation:

$$(i\not{D} + m)(i\not{D} - m)\psi = (-D^2 - m^2 + \Sigma_{\mu\nu}F^{\mu\nu})\psi$$

  - Coefficient of spin term is $g/2$
  - Left and right handed components are decoupled!
  - Arises when computing functional determinants in background field gauge

Anomalous Mag Moment Redux

- Computing amplitudes in scalar theory:
  - Treat external fermions like scalars and compute amplitudes with
    $$\mathcal{L} = \frac{1}{m}\bar{u}^\dagger\left[-D^2 - m^2 + \frac{g}{2}\sigma_{\mu\nu}F^{\mu\nu}\right]u$$
  - Project onto fermion line by multiplying by $\bar{u}^\dagger$ on the left and u on the right
    - Gives exact results if g = 2
      - Even with massless fermions!

Anomalous Mag Moment Redux

- Example: 4 point amplitude in massless QCD
  - Scalar amplitude is:

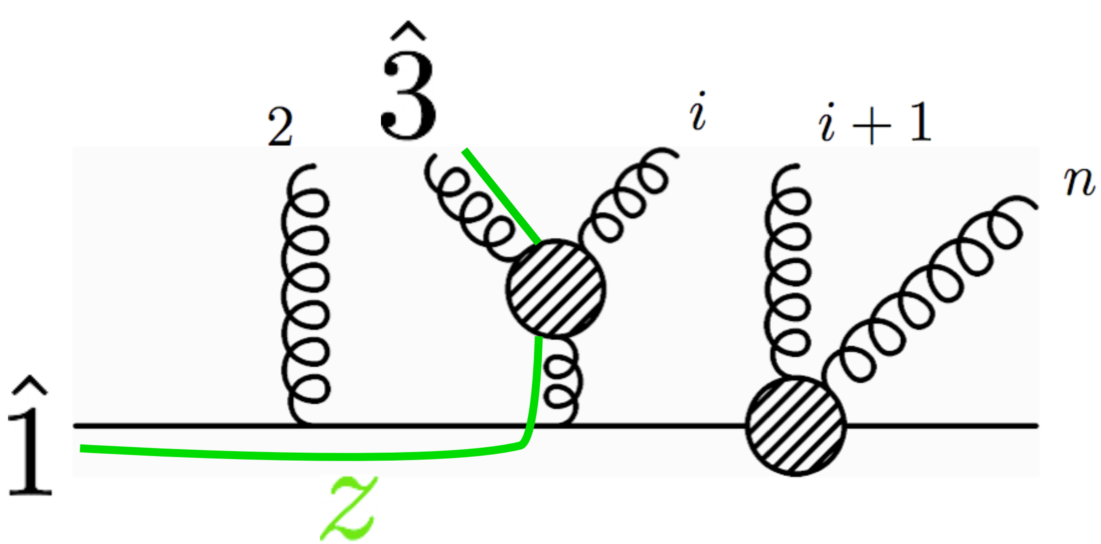
$$\mathcal{A}(1,2^+,3^-,4) = -\frac{\langle 312]^2}{\langle 212]\langle 232]} - \frac{\langle 312]}{\langle 212][23]}2][2$$

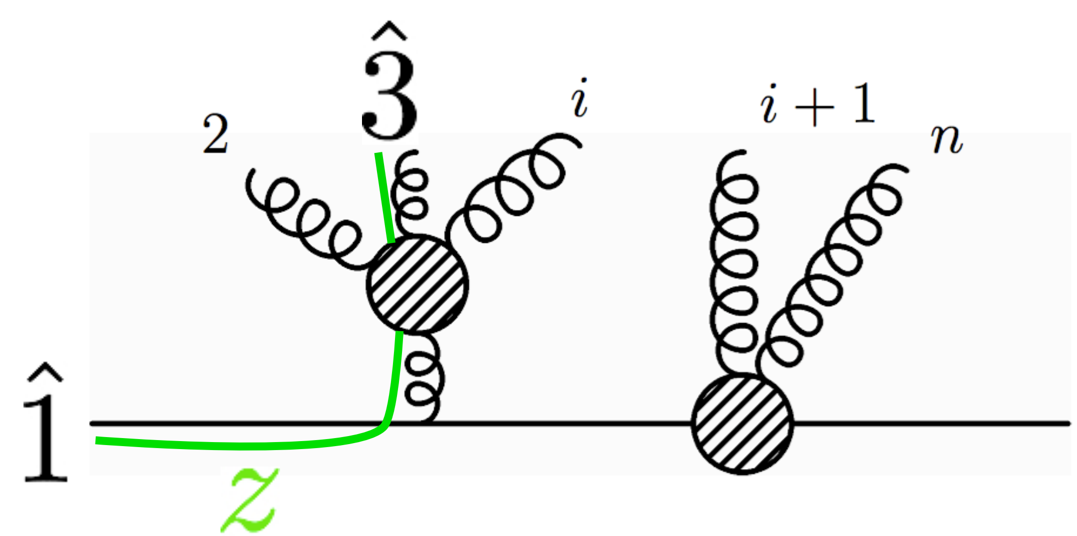
  - Multiply by $[1$ on the left and $\frac{s]}{[4s]}$ on the right
  - Result:

$$\mathcal{A}(1^+,2^+,3^-,4^-) = \frac{\langle 13\rangle\langle 34\rangle^3}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 41\rangle}$$

Anomalous Mag Moment Redux

- Can we use BCFW in this theory?
  - Surprisingly, yes!
  - Can show that shifting on "scalar" and non-adjacent gluon is OK





Anomalous Mag Moment Redux

- Review: Computing amplitudes with BCFW
- If an amplitude has at least one - and one + helicity gluon, use BCFW in fermion theory:

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi + \frac{a}{2m}\bar{\psi}\Sigma_{\mu\nu}F^{\mu\nu}\psi$$

- If amplitude has only + or - helicity gluons use BCFW in scalar theory:

$$\mathcal{L} = \frac{1}{m}\bar{\psi}\left[-D^2 - m^2 + \frac{g}{2}\Sigma_{\mu\nu}F^{\mu\nu}\right]\psi$$

Anomalous Mag Moment Redux

- Aside-Integrate out right-handed components:

$$\mathcal{L} = \bar{u}^\dagger \left[-m + \frac{a}{m}\sigma_{\mu\nu}F^{\mu\nu} - \sigma \cdot D \frac{1}{m - \frac{a}{m}\bar{\sigma}_{\mu\nu}F^{\mu\nu}} \bar{\sigma} \cdot D \right] u$$

- This is an exact description of anomalous magnetic moment

- As a phenomenological model, can introduce operators into original theory to remove terms from Taylor expansion

Conclusions

- We will learn a lot about the top quark during the first run of the LHC
- With enough tops produced, the anomalous magnetic moment could be measured
- At LHC, need to consider processes when the top radiates many gluons
- BCFW is an efficient and effective tool for computing amplitudes in this theory

Conclusions

- Ability to use BCFW in this theory is surprising
- Some effort to categorize all theories that are BCFW constructible Elvang, 2010
    - Only considered renormalizable operators
- Can more non-renormalizable theories be found that have an unexpected BCFW recursion relation?