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#### **1** Why (N)NLO QCD calculations?

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- **1** Why (N)NLO QCD calculations?
- **2** Techniques (mainly NLO)

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- **4** Recent Results

### Why (N)NLO QCD calculations?

(N)NLO QCD calculations at Hadron Colliders are needed for:



<span id="page-5-0"></span> $\alpha_s$   $m_t$  $M_W$   $M_H$   $\cdots$ 

- Heavy New Physics states undergo long chain decays
- **SM Processes accompanied by multi-jet activity**



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### From Dixon's talk at HO-2010





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### W NNLO rapidity distribution at TEVATRON

#### Catani, Ferrera, Grazzini



• Now the normalization is trustable

Moretti, Piccinini, R. P., Treccani using MLM matching

ALPGEN vs Tevatron  $W + i$  data



### The Les Houches NLO Wishlist (LHC but also Tevatron)

Priority list of processes experimentalist wish to know at NLO Z. Bern et. al., arXiv:0803.0494



#### 2009 update

- $pp \rightarrow t\bar{t}t\bar{t}$   $pp \rightarrow 4j$   $pp \rightarrow W + 4j$ •  $pp \rightarrow Z + 3j$  •  $pp \rightarrow W b \bar{b} i$
- See the Les Houches 2009 Proceedings

J. R. Andersen et. al., arXiv:1003.1241 [hep-ph]

### The SM and NLO multileg working group

J.R. Andersen, J. Archibald, S. Badger, R.D. Ball, G. Bevilacqua, I. Bierenbaum, T. Binoth, F. Boudjema, R. Boughezal, A. Bredenstein, R. Britto, M. Campanelli, J. Campbell, L. Carminati, G. Chachamis, V. Ciulli, G. Cullen, M. Czakon, L. Del Debbio, A. Denner, G. Dissertori, S. Dittmaier, S. Forte, R. Frederix, S. Frixione, E. Gardi, M.V. Garzelli, S. Gascon-Shotkin, T. Gehrmann, A.Gehrmann-De Ridder, W. Giele, T. Gleisberg, E.W.N. Glover, N. Greiner, A. Guffanti, J.-Ph. Guillet, A. van Hameren, G. Heinrich, S. Hoeche, M. Huber, J. Huston, M. Jaquier, S. Kallweit, S. Karg, N. Kauer, F. Krauss, J.I. Latorre, A. Lazopoulos, P. Lenzi, G. Luisoni, R. Mackeprang, L. Magnea, D. Maitre, D. Majumder, I. Malamos, F. Maltoni, K. Mazumdar, P. Nadolsky, P. Nason, C. Oleari, F. Olness, C.G. Papadopoulos, G. Passarino, E. Pilon, R. Pittau, S. Pozzorini, T. Reiter, J. Reuter, M. Rodgers, G. Rodrigo, J. Rojo, G. Sanguinetti, F.-P. Schilling, M. Schumacher, S. Schumann, R. Schwienhorst, P. Skands, H. Stenzel, F. Stoeckli, R. Thorne, M. Ubiali, P. Uwer, A. Vicini, M. Warsinsky, G. Watt, J. Weng, I. Wigmore, S. Weinzierl, J. Winter, M. Worek, G. Zanderighi

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### Effort Distribution at NLO



A typical 
$$
2 \to m
$$
 process at NLO  
\n
$$
\sigma^{NLO} = \int_m d\sigma^B + \int_m \left( d\sigma^V + \int_1 d\sigma^A \right) + \int_{m+1} \left( d\sigma^R - d\sigma^A \right)
$$

- $\mathbf{\Omega}$   $d\sigma^B$  is the Born cross section
- $\bm{2}$   $d\sigma^V$  is the Virtual correction (loop diagrams)
- $\mathbf{3}$   $d\sigma^R$  is the Real correction
- <span id="page-12-0"></span> $\bullet$   $d\sigma^{A}$  and  $\int_{1}d\sigma^{A}$  are *unintegrated* and *integrated* counterterms (allowing to compute the Real part in 4 dimensions)

### The Virtual corrections

$$
\mathcal{M}^{1-loop} = \sum_{i} d_i \text{ Box}_i + \sum_{i} c_i \text{ Triangle}_i + \sum_{i} b_i \text{ Bubble}_i + \sum_{i} a_i \text{ Tadpole}_i + R + \mathcal{O}(\epsilon)
$$

Scalar Loop Functions ∗

$$
\text{Tadpole}_i = \int d^n \bar{q} \frac{1}{\bar{D}_0} \qquad \qquad \text{Bubble}_i = \int d^n \bar{q} \frac{1}{\bar{D}_0 \bar{D}_1}
$$

Triangle<sub>i</sub> = 
$$
\int d^n \bar{q} \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2}
$$
 Box<sub>i</sub> =  $\int d^n \bar{q} \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3}$ 

∗ Known analytically

$$
\bar{D}_i = (\bar{q} + p_i)^2 - m_i^2 \quad \text{and} \quad n = 4 + \epsilon
$$

#### The OPP Method (Ossola, Papadopoulos, Pittau, 2007)

Working at the *integrand* level

$$
\mathcal{M}^{1-loop}=\int d^n\bar{q}\;\left[\mathcal{A}(q)+\tilde{A}(q,\tilde{q},\epsilon)\right]
$$

$$
\left(\begin{array}{c} \bar{q}=q+\tilde{q}\\ n=4+\epsilon\end{array}\right)
$$

• For example, in the case of  $pp \rightarrow t\bar{t}b\bar{b}$ 



The function to be sampled numerically to extract the coefficients

$$
N_i^{(6)}(q) = \sum_{i_0 < i_1 < i_2 < i_3}^{5} \left[ d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] D_{i_4} D_{i_5} + \sum_{i_0 < i_1 < i_2}^{5} \left[ c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] D_{i_3} D_{i_4} D_{i_5} + \sum_{i_0 < i_1}^{5} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] D_{i_2} D_{i_3} D_{i_4} D_{i_5} + \sum_{i_0}^{5} \left[ a(i_0) + \tilde{a}(q; i_0) \right] D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5} + \tilde{P}(q) D_{i_0} D_{i_1} D_{i_2} D_{i_3} D_{i_4} D_{i_5}
$$

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### Solving the OPP Equation 1

• The functional form of the *spurious* terms should be known Ossola, Papadopoulos, R. P., Nucl.Phys.B763:147-169,2007 del Aguila, R. P., JHEP 0407:017,2004

Example 
$$
(p_0 = 0)
$$
  
\n
$$
\tilde{d}(q; 0123) = \tilde{d}(0123) \epsilon(qp_1p_2p_3)
$$
\n
$$
\int d^n \bar{q} \frac{\tilde{d}(q; 0123)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = \tilde{d}(0123) \int d^n \bar{q} \frac{\epsilon(qp_1p_2p_3)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = 0
$$

The coefficients  $\{d_i, c_i, b_i, a_i\}$  and  $\{\tilde{d}_i, \tilde{c}_i, \tilde{b}_i, \tilde{a}_i\}$  are extracted by solving linear systems of equations

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### Solving the OPP Equation 2

The use of special values of  $q$  helps

$$
D_0(q^{\pm}) = D_1(q^{\pm}) = D_2(q^{\pm}) = D_3(q^{\pm}) = 0
$$

$$
N^{(m-1)}(q^{\pm}) = \left[d(0123) + \tilde{d}(q^{\pm}; 0123)\right] \prod_{i \neq 0,1,2,3}^{m-1} D_i(q^{\pm})
$$

$$
d(0123) = \frac{1}{2} \left[ \frac{N^{(m-1)}(q^+)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(q^+)} + \frac{N^{(m-1)}(q^-)}{\prod_{i \neq 0,1,2,3}^{m-1} D_i(q^-)} \right]
$$

· · ·



• 
$$
N(q) = 1
$$
  
\n•  $D_0(q^{\pm}) = D_1(q^{\pm}) = D_2(q^{\pm}) = D_3(q^{\pm}) = 0$   
\n $d(0123) = \frac{1}{2} \left[ \frac{1}{D_4(q^+)D_5(q^+)D_6(q^+)} + \frac{1}{D_4(q^+)D_5(q^+)D_6(q^+)} \right]$ 

### What about  $R (= R_1 + R_2)?$

#### The origin of  $R_1$

$$
\frac{1}{\bar{D}_i} = \frac{1}{D_i} \left( 1 - \frac{\tilde{q}^2}{\bar{D}_i} \right) \ \Rightarrow \text{predicted within OPP}
$$

#### The origin of  $R_2$

$$
R_2 = \int d^n \bar{q} \frac{\tilde{N}(q, \tilde{q}, \epsilon)}{\bar{D}_0 \cdots \bar{D}_{m-1}} \Rightarrow \text{effective tree-level Feynman Rules*}
$$

∗ QCD: Draggiotis, Garzelli, Papadopoulos, R. P., JHEP 0904:072,2009 EW: Garzelli, Malamos, R. P., JHEP 1001:040,2010

### Recursion Relations at 1-loop (cutting)

 $\bullet$  OPP + hard-cut allow to use the same tree-level Recursion Relations for  $m + 2$  tree-like structures



• The color can be treated as at the tree level



# In the meanwhile  $\cdots$

## $\cdots$  on the other side of the ocean $\cdots$

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### Cutting  $\cdots$  (Gluing  $\cdots$ )

• Double cuts  $\Leftrightarrow$  gluing 2 tree-level amplitudes (Bern, Dixon, Dunbar, Kosower 1994)



• Different double cuts are applied to disentangle 1-loop scalar functions by looking at the analytic structure of the result

• R is reconstructed by looking at collinear and infrared limits

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### $\cdots$  and more cutting  $(\cdots$  more gluing)

 $\bigcirc$  Quadruple cuts  $\Leftrightarrow$  gluing 4 tree-level amplitudes (Britto, Cachazo, Feng, hep-th/0412103)



- **2** q integration frozen  $\Rightarrow$  coefficient  $d_i$  of the box extracted
- **3** 3 bubbles are connected together, the box contributions subtracted and the *coefficients*  $c_i$  of the triangles extracted
- <sup>4</sup> · · ·

### Generalized Unitarity (Relevant References)

- Bern, Dixon, Dunbar, Kosower (1994)
- Ossola, Papadopoulos, R. P., hep-ph/0609007
- Forde, 0704.1835
- Ellis, Giele, Kunszt, 0708.2398
- Berger et al., 0803.4180

### The Real Corrections





**•** Feynman Diagrams avoided (Berends, Giele, Caravaglios, M. Moretti)

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### The Counterterms

#### The Catani-Seymour dipoles

- Catani, Seymour, Nucl. Phys. B485, 291 (1997)
- Catani, Dittmaier, Seymour, Trocsanyi, Nucl. Phys. B627, 189 (2002)
- Czakon, Papadopoulos, Worek, JHEP 0908 (2009) 085
	- Massless Massive Polarized

#### The FKS subtraction

• Frixione, Kunszt, Signer, hep-ph/9512328

#### The Antenna subtraction

- Kosower, Phys. Rev. D 71 (2005) 045016
- Campbell, Cullen Glover, Eur. Phys. J. C 9 (1999) 245

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### NLO Parton Level Tools

#### Analytic formulae

**• MCFM** [Campbell et al.]

#### Feynman Diagrams

- O DKU, HAWK · · · [Bredenstein, Denner, Dittmaier, Pozzorini et al.]
- FormCalc/LoopTools/FeynCalc [Hahn et al.]
- **GOLEM** [Binoth et al.]
- **GRACE** [Belanger, Boudjema et al.]

#### <span id="page-27-0"></span>OPP/Unitarity

- **HELAC-NLO/CutTools** [Papadopoulos, R. P. et al.]
- **BlackHat/SHERPA** [Berger et al.]
- Rocket/MCFM [Ellis et al.]
- **Samurai** [Mastrolia, Ossola, Reiter, Tramontano]

### The Helac-NLO System

- $\Omega$  CutTools  $\{d_i, c_i, b_i, a_i\}$  and  $\mathsf{R}_1$
- <sup>2</sup> HELAC-1LOOP  $N(q)$  and R<sub>2</sub>
- **3** OneLOop scalar 1-loop integrals
- **4** HELAC-DIPOLES

Real correction and CS dipoles



(figure by G. Bevilacqua)

- Ossola, Papadopoulos, R. P., JHEP 0803 (2008) 042
- van Hameren, Papadopoulos, R. P., JHEP 0909 (2009) 106
- Czakon, Papadopoulos, Worek, JHEP 0908 (2009) 085

## The HELAC-NLO group  $*$



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J. Malamos C.G. Papadopoulos R. P.

### **Contributors**

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### Tuned comparisons



- Agreement between two completely different techniques
- Agreement on  $pp \rightarrow ZZ + i + X$  between GOLEM and Dittmaier, Kallweit and Uwer

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A proposal for 
$$
\leftrightarrow
$$
 can be found in

Binoth et al. arXiv:1001.1307

### Tools for the Real Radiation

#### Automation of the subtraction methods

- Gleisberg, Krauss, 0709.2881
- Seymour, Tevlin, 0803.2231
- Hasegawa, Moch, Uwer, 0807.3701
- Frederix, Gehrmann, Greiner, 0808.2128
- Czakon, Papadopoulos, Worek, 0905.0883
- Frederix, Frixione, Maltoni, Stelzer, 0908.4272
- Frederix, Gehrmann, Greiner, 1004.2905

#### Adding PS consistently at NLO

- MC@NLO Frixione, Webber (2002)
- POWHEGNason(2004); Frixione, Nason, Oleari (2007)
- GenEvABauer, Tackmann, Thaler (2008)

#### Not yet for arbitrary complex final states

## A NLO analysis of  $ttH$  production vs  $ttbb$  and  $ttjj$ backgrounds

#### <span id="page-33-0"></span>Based on arXiv:1003.1241 [hep-ph], Phys.Rev.Lett.104:162002,2010 and JHEP 0909:109,2009

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### Cross sections at NLO

 $pp \rightarrow t\bar{t}bb + X$ 

$$
\begin{array}{c|c|c}\n\sigma_{LO}^B \text{ [fb]} & \sigma_{NLO}^B \text{ [fb]} & K\text{-factor} \\
\hline\n1489.2 \pm 0.9 & 2642 \pm 3 & 1.77\n\end{array}
$$

 $\mu_R = \mu_F = \mu_0 = m_t$  (CTEQ6)

#### $pp \rightarrow t\bar{t}H + X \rightarrow t\bar{t}b\bar{b} + X$

$$
\begin{array}{c|c|c|c}\n\sigma_{LO}^{S} \text{ [fb]} & \sigma_{NLO}^{S} \text{ [fb]} & K\text{-factor} \\
\hline\n150.375 \pm 0.077 & 207.268 \pm 0.150 & 1.38\n\end{array}
$$

 $\mu_R = \mu_F = \mu_0 = m_t + m_H/2$  (CTEQ6)

 $p_T (b) > 20 \text{ GeV}, \ \Delta R (b, \bar{b}) > 0.8, \ |\eta_b| < 2.5$ 

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### Distributions at NLO



### Scale dependence of the ttbb Background



### Scale dependence of the Signal



### The effect of a jet veto on the Signal/Background ratio



• With  $p_T(i) < 50$  GeV:

$$
(S/B)_{LO}
$$
 = 0.10  $(S/B)_{NLO-veto}$  = 0.064  
\n $(S/B)_{NLO}$  = 0.079

### Scale dependence of the  $ttjj$  Background



### $m_{ij}$  distribution of the  $ttjj$  Background



### Hardest jet  $p_T$  distribution of the  $ttjj$  Background



## NLO QCD corrections to  $pp \rightarrow e^+e^-$  at the LHC

### **Parameters**

$$
\sqrt{s} = 7 \text{ TeV} \qquad p_T(\ell^{\pm}) > 1 \text{ GeV} \quad |\eta(\ell^{\pm})| < 5
$$
  

$$
m_{\ell^+\ell^-} > 60 \text{ GeV} \quad \mu_F = \mu_R = M_Z
$$

### Results cross-checked with MCFM







### The  $p_t(\ell^+)$  and  $y(\ell^+)$  distributions



#### The  $\Delta R_{\ell^+\ell^-}$  distribution



## NLO QCD corrections to  $pp \rightarrow W^+ \rightarrow e^+ \nu_e$  at the LHC

### **Parameters**

$$
\begin{aligned} \sqrt{s} &= 7 \text{ TeV} & p_T(\ell^{\pm}) > 1 \text{ GeV} \\ |\eta(\ell^{\pm})| < 5 & \mu_F &= \mu_R = M_W \end{aligned}
$$

### Results cross-checked with MCFM





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### Single-top production at Tevatron

Schwienhorst, Frederix, Maltoni



#### $W$  +3 jets at NLO

- Melnikov, Zanderighi, arXiv:0910.3671
- Ellis, Melnikov, Zanderighi, arXiv:0906.1445
- Ellis, Melnikov, Zanderighi, arXiv:0901.4101
- Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre (BlackHat)
	- arXiv:0907.1984
	- arXiv:0902.2760

#### $Z + 3$  jets at NLO

- BlackHat
	- a  $arXiv:0912.4927$
	- arXiv:1004.1659
	- arXiv:1005.3728



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### $W$  +4 jets at NLO



#### BlackHat

### $W + 3i$  unleashed comparisons



- The use of a scale=HT reproduces the shape of the NLO calculation at LO for many relevant kinematic distributions
- The largest shape differences, of the order of 20% and 40%. are seen in the third-jet pT and HT distributions, respectively

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### Higgs searches at Tevatron



- Anastasiou, Melnikov, hep-ph/0207004  $\bullet$
- Anastasiou, Melnikov, Petriello, hep-ph/0409088  $\bullet$
- Anastasiou, Boughezal, Petriello, arXiv:0811.3458  $\bullet$

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### NNLO QCD effects on  $H \to WW \to \ell \nu \ell \nu$

G. Dissertori and F. Stöckli



• Jet vetoing reduces the K factor

### NNLO Determination of  $\alpha_s(M_z)$  at LEP from event shapes

Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Jaquier, Luisoni, Stenzel



 $\alpha_S(M_Z)^{NLO}$  = 0.1200  $\pm$  0.0021( $exp) \pm 0.0062(th)$  $\alpha_S(M_Z)^{NNLO}$  = 0.1153 ± 0.0017( $exp$ ) ± 0.0023(th)

### Understanding soft and collinear divergences at all orders

Gardi and Magnea, Becker and Neubert

$$
\mathcal{M}\left(p_i/\mu,\alpha_s(\mu^2),\epsilon\right) = Z\left(p_i/\mu_f,\alpha_s(\mu_f^2),\epsilon\right) \; \mathcal{H}\left(p_i/\mu,\mu/\mu_f,\alpha_s(\mu^2),\epsilon\right)
$$

$$
Z(p_i/\mu, \alpha_s(\mu^2), \epsilon) = \exp\left\{ \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \left[ \frac{1}{8} \hat{\gamma}_K(\alpha_s(\lambda^2, \epsilon)) \sum_{i \neq j} \ln\left(\frac{2p_i \cdot p_j e^{-i\pi \phi_{ij}}}{\lambda^2}\right) T_i \cdot T_j - \frac{1}{2} \sum_{i=1}^n \gamma_{J_i}(\alpha_s(\lambda^2, \epsilon)) \right] \right\}.
$$

#### Very simple dipole structure

### Conclusions and Outlooks

### **1** I reviewed recent developments in the field of QCD (N)NLO

calculations relevant for Hadron Collider phenomenology

- **2** The status of multileg NLO calculations is now at the same stage of multileg tree level calculations 10 years ago
- <sup>3</sup> An analysis of *all of the LHC data* (at least) at the NLO accuracy is possible
- <span id="page-58-0"></span><sup>4</sup> NLO public codes in preparation