R. Pittau (U. of Granada) WSA, June 29th, 2010

Why (N)NLO QCD calculations?

- Why (N)NLO QCD calculations?
- 2 Techniques (mainly NLO)

- Why (N)NLO QCD calculations?
- 2 Techniques (mainly NLO)
- 3 Tools

- Why (N)NLO QCD calculations?
- 2 Techniques (mainly NLO)
- 3 Tools
- Recent Results

Why (N)NLO QCD calculations?

• (N)NLO QCD calculations at Hadron Colliders are needed for:



 $\begin{array}{ccc} \alpha_s & m_t \\ M_W & M_H & \cdots \end{array}$

- Heavy New Physics states undergo long chain decays
- SM Processes accompanied by multi-jet activity



From Dixon's talk at HO-2010





1. Get the **best theoretical prediction** you can, whether – Basic Monte Carlo [PYTHIA, HERWIG, Sherpa, ...]

- LO QCD parton level
- LO QCD matched to parton showers [MadGraph/MadEvent, ALPGEN/PYTHIA, Sherpa, ...]
- NLO QCD at parton level
- NLO matched to parton showers [MC@NLO, POWHEG,...]
- NNLO inclusive at parton level
- NNLO with flexible cuts at parton level
- 2. Take ratios whenever possible
 - QCD effects cancel when event kinematics are similar
 - Closely related to "data driven" strategies

L. Dixon Precision QCD for the LHC

CERN HO10 23 June 2010

10

Increasing accuracy \rightarrow

W NNLO rapidity distribution at TEVATRON

Catani, Ferrera, Grazzini



• Now the normalization is trustable

Tuning LO Monte Carlos with NLO calculations

Moretti, Piccinini, R. P., Treccani using MLM matching

ALPGEN vs Tevatron W + j data



R. Pittau (U. of Granada) WSA, June 29th, 2010

Recent developments and results in theoretical perturbative QCD

The Les Houches NLO Wishlist (LHC but also Tevatron)

Priority list of processes experimentalist wish to know at NLO Z. Bern *et. al.*, arXiv:0803.0494

NLO Wishlist 2007						
• $pp \rightarrow W + j$	• $pp \rightarrow t\bar{t} + 2j$	• $pp \rightarrow V + 3j$				
• $pp \rightarrow H + 2j$	• $pp \rightarrow VVb\bar{b}$	• $pp \rightarrow t\bar{t}b\bar{b}$				
• $pp \rightarrow VVV$	• $pp \rightarrow VV + 2j$	• $pp \rightarrow b\bar{b}b\bar{b}$				

2009 update

- $pp \rightarrow t\bar{t}t\bar{t}$ $pp \rightarrow 4j$ $pp \rightarrow W+4j$ • $pp \rightarrow Z+3j$ • $pp \rightarrow Wb\bar{b}j$
- See the Les Houches 2009 Proceedings

J. R. Andersen et. al., arXiv:1003.1241 [hep-ph]

The SM and NLO multileg working group

J.R. Andersen, J. Archibald, S. Badger, R.D. Ball, G. Bevilacqua, I. Bierenbaum, T. Binoth, F. Boudjema, R. Boughezal, A. Bredenstein, R. Britto, M. Campanelli, J. Campbell, L. Carminati, G. Chachamis, V. Ciulli, G. Cullen, M. Czakon, L. Del Debbio, A. Denner, G. Dissertori, S. Dittmaier, S. Forte, R. Frederix, S. Frixione, E. Gardi, M.V. Garzelli, S. Gascon-Shotkin, T. Gehrmann, A.Gehrmann-De Ridder, W. Giele, T. Gleisberg, E.W.N. Glover, N. Greiner, A. Guffanti, J.-Ph. Guillet, A. van Hameren, G. Heinrich, S. Hoeche, M. Huber, J. Huston, M. Jaquier, S. Kallweit, S. Karg, N. Kauer, F. Krauss, J.I. Latorre, A. Lazopoulos, P. Lenzi, G. Luisoni, R. Mackeprang, L. Magnea, D. Maitre, D. Majumder, I. Malamos, F. Maltoni, K. Mazumdar, P. Nadolsky, P. Nason, C. Oleari, F. Olness, C.G. Papadopoulos, G. Passarino, E. Pilon, R. Pittau, S. Pozzorini, T. Reiter, J. Reuter, M. Rodgers, G. Rodrigo, J. Rojo, G. Sanguinetti, F.-P. Schilling, M. Schumacher, S. Schumann, R. Schwienhorst, P. Skands, H. Stenzel, F. Stoeckli, R. Thorne, M. Ubiali, P. Uwer, A. Vicini, M. Warsinsky, G. Watt, J. Weng, I. Wigmore, S. Weinzierl, J. Winter, M. Worek, G. Zanderighi

Effort Distribution at NLO



A typical 2
$$\rightarrow m$$
 process at NLO

$$\sigma^{NLO} = \int_{m} d\sigma^{B} + \int_{m} \left(d\sigma^{V} + \int_{1} d\sigma^{A} \right) + \int_{m+1} \left(d\sigma^{R} - d\sigma^{A} \right)$$

- $d\sigma^B$ is the Born cross section
- 2 $d\sigma^V$ is the Virtual correction (loop diagrams)
- **3** $d\sigma^R$ is the Real correction
- $d\sigma^A$ and $\int_1 d\sigma^A$ are unintegrated and integrated counterterms (allowing to compute the Real part in 4 dimensions)

The Virtual corrections

$$\mathcal{M}^{1-loop} = \sum_{i} d_{i} \operatorname{Box}_{i} + \sum_{i} c_{i} \operatorname{Triangle}_{i} + \sum_{i} b_{i} \operatorname{Bubble}_{i} \\ + \sum_{i} a_{i} \operatorname{Tadpole}_{i} + R + \mathcal{O}(\epsilon)$$

Scalar Loop Functions *

$$\mathsf{Tadpole}_i = \int d^n \bar{q} \frac{1}{\bar{D}_0}$$
 $\mathsf{Bubble}_i = \int d^n \bar{q} \frac{1}{\bar{D}_0 \bar{D}_1}$

$$\mathsf{Triangle}_i = \int d^n \bar{q} \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2} \quad \mathsf{Box}_i = \int d^n \bar{q} \frac{1}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3}$$

* Known analytically

$$\overline{D}_i = (\overline{q} + p_i)^2 - m_i^2$$
 and $n = 4 + \epsilon$

The OPP Method (Ossola, Papadopoulos, Pittau, 2007)

Working at the integrand level

$$\mathcal{M}^{1-loop} = \int d^n \bar{q} \left[\mathcal{A}(q) + \tilde{A}(q, \tilde{q}, \epsilon) \right]$$

$$\left(\begin{array}{c} \bar{q} = q + \tilde{q} \\ n = 4 + \epsilon \end{array}\right)$$

• For example, in the case of $pp \rightarrow t\bar{t}b\bar{b}$



The function to be sampled *numerically* to extract the coefficients

$$\begin{split} N_{i}^{(6)}(q) &= \sum_{i_{0} < i_{1} < i_{2} < i_{3}}^{5} \left[\boldsymbol{d}(i_{0}i_{1}i_{2}i_{3}) + \tilde{\boldsymbol{d}}(q;i_{0}i_{1}i_{2}i_{3}) \right] D_{i_{4}} D_{i_{5}} \\ &+ \sum_{i_{0} < i_{1} < i_{2}}^{5} \left[\boldsymbol{c}(i_{0}i_{1}i_{2}) + \tilde{\boldsymbol{c}}(q;i_{0}i_{1}i_{2}) \right] D_{i_{3}} D_{i_{4}} D_{i_{5}} \\ &+ \sum_{i_{0} < i_{1}}^{5} \left[\boldsymbol{b}(i_{0}i_{1}) + \tilde{\boldsymbol{b}}(q;i_{0}i_{1}) \right] D_{i_{2}} D_{i_{3}} D_{i_{4}} D_{i_{5}} \\ &+ \sum_{i_{0}}^{5} \left[\boldsymbol{a}(i_{0}) + \tilde{\boldsymbol{a}}(q;i_{0}) \right] D_{i_{1}} D_{i_{2}} D_{i_{3}} D_{i_{4}} D_{i_{5}} \\ &+ \tilde{\boldsymbol{P}}(q) D_{i_{0}} D_{i_{1}} D_{i_{2}} D_{i_{3}} D_{i_{4}} D_{i_{5}} \end{split}$$

Why (N)NLO QCD Techniques Tools Recent Results

Solving the OPP Equation 1

• The functional form of the *spurious* terms should be known Ossola, Papadopoulos, R. P., Nucl.Phys.B763:147-169,2007 del Aguila, R. P., JHEP 0407:017,2004

Example $(p_0 = 0)$

$$\tilde{d}(q;0123) = \tilde{d}(0123) \epsilon(qp_1p_2p_3)$$

$$\int d^n \bar{q} \frac{\tilde{d}(q;0123)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = \tilde{d}(0123) \int d^n \bar{q} \frac{\epsilon(q p_1 p_2 p_3)}{\bar{D}_0 \bar{D}_1 \bar{D}_2 \bar{D}_3} = 0$$

• The coefficients $\{d_i, c_i, b_i, a_i\}$ and $\{\tilde{d}_i, \tilde{c}_i, \tilde{b}_i, \tilde{a}_i\}$ are extracted by solving linear systems of equations

Why (N)NLO QCD Techniques Tools Recent Results

Solving the OPP Equation 2

The use of special values of q helps

$$D_0(q^{\pm}) = D_1(q^{\pm}) = D_2(q^{\pm}) = D_3(q^{\pm}) = 0$$

$$N^{(m-1)}(q^{\pm}) = \left[d(0123) + \tilde{d}(q^{\pm}; 0123) \right] \prod_{i \neq 0, 1, 2, 3}^{m-1} D_i(q^{\pm})$$

$$d(0123) = \frac{1}{2} \left[\frac{N^{(m-1)}(q^+)}{\prod_{i \neq 0, 1, 2, 3}^{m-1} D_i(q^+)} + \frac{N^{(m-1)}(q^-)}{\prod_{i \neq 0, 1, 2, 3}^{m-1} D_i(q^-)} \right]$$

. . .



•
$$N(q) = 1$$

• $D_0(q^{\pm}) = D_1(q^{\pm}) = D_2(q^{\pm}) = D_3(q^{\pm}) = 0$

$$\mathbf{d}(0123) = \frac{1}{2} \left[\frac{1}{D_4(q^+)D_5(q^+)D_6(q^+)} + \frac{1}{D_4(q^+)D_5(q^+)D_6(q^+)} \right]$$

What about $R (= R_1 + R_2)$?

The origin of R_1

$$\frac{1}{\bar{D}_i} = \frac{1}{D_i} \left(1 - \frac{\tilde{q}^2}{\bar{D}_i} \right) \; \Rightarrow \text{predicted within OPP}$$

The origin of R_2

$$R_2 = \int d^n \bar{q} \frac{\tilde{N}(q, \tilde{q}, \epsilon)}{\bar{D}_0 \cdots \bar{D}_{m-1}} \Rightarrow \text{effective tree-level Feynman Rules}^*$$

* QCD: Draggiotis, Garzelli, Papadopoulos, R. P., JHEP 0904:072,2009
 EW: Garzelli, Malamos, R. P., JHEP 1001:040,2010

Recursion Relations at 1-loop (cutting)

• OPP + hard-cut allow to use *the same tree-level Recursion Relations* for m + 2 tree-like structures



• The color can be treated *as at the tree level*



In the meanwhile · · ·

\cdots on the other side of the ocean \cdots

Cutting \cdots (Gluing \cdots)

 Double cuts ⇔ gluing 2 tree-level amplitudes (Bern, Dixon, Dunbar, Kosower 1994)



- Different double cuts are applied to disentangle 1-loop scalar functions by looking at the analytic structure of the result
- R is reconstructed by looking at collinear and infrared limits

Why (N)NLO QCD Techniques Tools Recent Results

··· and more cutting (··· more gluing)

 Quadruple cuts ⇔ gluing 4 tree-level amplitudes (Britto, Cachazo, Feng, hep-th/0412103)



- **2** q integration frozen \Rightarrow coefficient d_i of the box extracted
- 3 bubbles are connected together, the box contributions subtracted and the coefficients c_i of the triangles extracted
- 4 . . .

Generalized Unitarity (Relevant References)

- Bern, Dixon, Dunbar, Kosower (1994)
- Ossola, Papadopoulos, R. P., hep-ph/0609007
- Forde, 0704.1835
- Ellis, Giele, Kunszt, 0708.2398
- Berger et al., 0803.4180

The Real Corrections





 Feynman Diagrams avoided (Berends, Giele, Caravaglios, M. Moretti)

The Counterterms

The Catani-Seymour dipoles

- Catani, Seymour, Nucl. Phys. B485, 291 (1997)
- Catani, Dittmaier, Seymour, Trocsanyi, Nucl. Phys. B627, 189 (2002)
- Czakon, Papadopoulos, Worek, JHEP 0908 (2009) 085
 - Massless
 Massive
 Polarized

The FKS subtraction

• Frixione, Kunszt, Signer, hep-ph/9512328

The Antenna subtraction

- Kosower, Phys. Rev. D 71 (2005) 045016
- Campbell, Cullen Glover, Eur. Phys. J. C 9 (1999) 245

NLO Parton Level Tools

Analytic formulae

• MCFM [Campbell *et al.*]

Feynman Diagrams

- DKU, HAWK · · · [Bredenstein, Denner, Dittmaier, Pozzorini et al.]
- FormCalc/LoopTools/FeynCalc [Hahn et al.]
- GOLEM [Binoth et al.]
- GRACE [Belanger, Boudjema et al.]

OPP/Unitarity

- HELAC-NLO/CutTools [Papadopoulos, R. P. et al.]
- BlackHat/SHERPA [Berger et al.]
- Rocket/MCFM [Ellis et al.]
- Samurai [Mastrolia, Ossola, Reiter, Tramontano]

The Helac-NLO System

- CutTools $\{d_i, c_i, b_i, a_i\}$ and R_1
- **2** HELAC-1LOOP N(q) and R_2
- OneLOop scalar 1-loop integrals
- HELAC-DIPOLES

Real correction and CS dipoles



(figure by G. Bevilacqua)

- Ossola, Papadopoulos, R. P., JHEP 0803 (2008) 042
- van Hameren, Papadopoulos, R. P., JHEP 0909 (2009) 106
- Czakon, Papadopoulos, Worek, JHEP 0908 (2009) 085

The HELAC-NLO group *

2	*	

G. Bevilacqua A. van Hameren A. Kardos M. Worek

M. Czakon

J. Malamos C.G. Papadopoulos R. P.

M. Garzelli A. Lazopoulos

Contributors

Caffarella Draggiotis Kanaki Ossola

Tuned comparisons

Process	σ_{FD}^{LO} [fb]	$\sigma^{\rm LO}_{\rm OPP}$ [fb]	$\sigma_{\rm FD}^{\rm NLO}~{ m [fb]}$	$\sigma_{\rm OPP}^{\rm NLO}~{\rm [fb]}$		
$q\bar{q} \rightarrow t\bar{t}b\bar{b}$	85.522(26)	85.489(46)	87.698(56)	87.545(91)		
$pp \to t\bar{t}b\bar{b}$	1488.8(1.2)	1489.2(0.9)	2638(6)	2642(3)		
$pp ightarrow tar{t}bar{b} + X$ at the LHC, $\mu_F = \mu_R = m_t.$						

- Agreement between two completely different techniques
- Agreement on $pp \rightarrow ZZ + j + X$ between GOLEM and Dittmaier, Kallweit and Uwer



A proposal for
$$\Leftrightarrow$$
 can be found in

Binoth et al. arXiv:1001.1307

Tools for the Real Radiation

Automation of the subtraction methods

- Gleisberg, Krauss, 0709.2881
- Seymour, Tevlin, 0803.2231
- Hasegawa, Moch, Uwer, 0807.3701
- Frederix, Gehrmann, Greiner, 0808.2128
- Czakon, Papadopoulos, Worek, 0905.0883
- Frederix, Frixione, Maltoni, Stelzer, 0908.4272
- Frederix, Gehrmann, Greiner, 1004.2905

Adding PS consistently at NLO

- MC@NLO Frixione, Webber (2002)
- POWHEGNason(2004); Frixione, Nason, Oleari (2007)
- GenEvABauer, Tackmann, Thaler (2008)

Not yet for arbitrary complex final states

A NLO analysis of ttH production vs ttbb and ttjj backgrounds

Based on arXiv:1003.1241 [hep-ph], Phys.Rev.Lett.104:162002,2010 and JHEP 0909:109,2009

Cross sections at NLO

 $pp \rightarrow t\bar{t}b\bar{b} + X$ $\sigma^{B}_{LO} \text{ [fb]} \quad \sigma^{B}_{NLO} \text{ [fb]} \quad K\text{-factor}$ $1489.2 \pm 0.9 \quad 2642 \pm 3 \quad 1.77$ $\mu_{R} = \mu_{F} = \mu_{0} = m_{t} \text{ (CTEQ6)}$

$pp \rightarrow t\bar{t}H + X \rightarrow t\bar{t}b\bar{b} + X$ $\frac{\sigma_{LO}^{S} \text{ [fb]}}{150.375 \pm 0.077} \frac{\sigma_{NLO}^{S} \text{ [fb]}}{207.268 \pm 0.150} \frac{K\text{-factor}}{1.38}$ $\mu_{R} = \mu_{F} = \mu_{0} = m_{t} + m_{H}/2 \text{ (CTEQ6)}$

• $p_T(b) > 20 \text{ GeV}$, $\Delta R(b, \bar{b}) > 0.8$, $|\eta_b| < 2.5$

Distributions at NLO



Scale dependence of the *ttbb* Background



Scale dependence of the Signal



The effect of a jet veto on the Signal/Background ratio



• With $p_T(j) < 50$ GeV:

$$(S/B)_{LO} = 0.10 \quad (S/B)_{NLO-veto} = 0.064 (S/B)_{NLO} = 0.079$$

Scale dependence of the ttjj Background



m_{ij} distribution of the ttjj Background



Hardest jet p_T distribution of the ttjj Background



NLO QCD corrections to $pp \rightarrow e^+e^-$ at the LHC

Parameters

$$\begin{split} \sqrt{s} &= 7 \; {\rm TeV} \qquad p_T(\ell^{\pm}) > 1 \; {\rm GeV} \quad |\eta(\ell^{\pm})| < 5 \\ m_{\ell^+\ell^-} > 60 \; {\rm GeV} \quad \mu_F = \mu_R = M_Z \end{split}$$

Results cross-checked with MCFM



The $m_{\ell^+\ell^-}$ and $y_{\ell^+\ell^-}$ distributions



The $p_t(\ell^+)$ and $y(\ell^+)$ distributions



The $\Delta R_{\ell^+\ell^-}$ distribution



NLO QCD corrections to $pp \to W^+ \to e^+ \nu_e$ at the LHC

Parameters

$$\sqrt{s} = 7 \text{ TeV} \quad p_T(\ell^{\pm}) > 1 \text{ GeV} \\ |\eta(\ell^{\pm})| < 5 \quad \mu_F = \mu_R = M_W$$

Results cross-checked with MCFM



The $m_{e\nu}$ and $y_{e\nu}$ distributions



The $p_t(e^+)$ and $y(e^+)$ distributions



The $p_{t,miss}$ distribution



Single-top production at Tevatron

Schwienhorst, Frederix, Maltoni



W +3 jets at NLO

- Melnikov, Zanderighi, arXiv:0910.3671
- Ellis, Melnikov, Zanderighi, arXiv:0906.1445
- Ellis, Melnikov, Zanderighi, arXiv:0901.4101
- Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre (BlackHat)
 - arXiv:0907.1984
 - arXiv:0902.2760

$Z\,+$ 3 jets at NLO

- BlackHat
 - arXiv:0912.4927
 - arXiv:1004.1659
 - arXiv:1005.3728



R. Pittau (U. of Granada) WSA, June 29th, 2010

Recent developments and results in theoretical perturbative QCD



W +4 jets at NLO



BlackHat

W + 3j unleashed comparisons



- The use of a scale=HT reproduces the shape of the NLO calculation at LO for many relevant kinematic distributions
- The largest shape differences, of the order of 20% and 40%, are seen in the third-jet pT and HT distributions, respectively

Higgs searches at Tevatron



- Anastasiou, Melnikov, hep-ph/0207004
- Anastasiou, Melnikov, Petriello, hep-ph/0409088
- Anastasiou, Boughezal, Petriello, arXiv:0811.3458

Why (N)NLO QCD Techniques Tools Recent Results

NNLO QCD effects on $H \rightarrow WW \rightarrow \ell \nu \ell \nu$

G. Dissertori and F. Stöckli



Jet vetoing reduces the K factor

NNLO Determination of $\alpha_s(M_z)$ at LEP from event shapes

Dissertori, Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Jaquier, Luisoni, Stenzel



 $\alpha_S(M_Z)^{NLO} = 0.1200 \pm 0.0021(exp) \pm 0.0062(th)$ $\alpha_S(M_Z)^{NNLO} = 0.1153 \pm 0.0017(exp) \pm 0.0023(th)$

Understanding soft and collinear divergences at all orders

Gardi and Magnea, Becker and Neubert

$$\mathcal{M}\left(p_i/\mu, \alpha_s(\mu^2), \epsilon\right) = Z\left(p_i/\mu_f, \alpha_s(\mu_f^2), \epsilon\right) \mathcal{H}\left(p_i/\mu, \mu/\mu_f, \alpha_s(\mu^2), \epsilon\right)$$

$$Z\left(p_i/\mu, \alpha_s(\mu^2), \epsilon\right) = \exp\left\{\int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \left[\frac{1}{8} \widehat{\gamma}_K\left(\alpha_s(\lambda^2, \epsilon)\right) \sum_{i \neq j} \ln\left(\frac{2p_i \cdot p_j e^{-i\pi\phi_{ij}}}{\lambda^2}\right) T_i \cdot T_j - \frac{1}{2} \sum_{i=1}^n \gamma_{J_i}\left(\alpha_s(\lambda^2, \epsilon)\right)\right]\right\}.$$

• Very simple dipole structure

Conclusions and Outlooks

I reviewed recent developments in the field of QCD (N)NLO

calculations relevant for Hadron Collider phenomenology

- The status of multileg NLO calculations is now at the same stage of multileg tree level calculations 10 years ago
- An analysis of all of the LHC data (at least) at the NLO accuracy is possible
- In NLO public codes in preparation