IR-Improved DGLAP Theory and HERWIRI

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Tools for SUSY and the New Physics, Sharpening our Tools
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Outline

- **Introduction & Motivation**
- $QED \otimes QCD$ **YFS-Style Resummation** & **IR-Improved DGLAP Theory**
- **Summary and conclusion**

Why the Higher Order Radiative corrections in Colliders

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What is the current precision at LHC?

- CURRENT STATUS: UsingMC@NLO and FEWZ, HORACE, PHOTOS, etc., [a]

$$(4.1 \pm 0.3)\% = (1.51 \pm 0.75)\%(QCD) \oplus 3.79(PDF) \oplus 0.38 \pm 0.26(EW)\%$$

accuracy on single Z to leptons at LHC.

[a] Phys.Rev.D56,7284 (hep-ex/9705004)

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- Theory: uncertainitities are diff cult to estimate: non-perturbative physics, scales (μ_R, μ_F).

- Lack of precise knowledge of parton distribution functions (PDFs) (specially at "high x and low x").

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Is it possible to reduce these Uncertainties?

- the higher order radiative corrections
- The dominant source of theoretical uncertainties are from the strong interaction physics (QCD)
- Higher Order QCD corrections
- For the precision requirement of LHC, one must also check that the Higher Order EW Corrections are under appropriate control.

- This is one of the reason why we are interested QCD $\otimes$ EW corrections.

Why the Higher Order Radiative corrections in Colliders

- For the theoretical precision we need atleast $\mathcal{O}(\alpha_s^2 L^{n_1}, \alpha\alpha_s L^{n_2}, \alpha^2 L^{n_3})$.

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1				
$\alpha_i L$	α_i			
$\alpha_i^2 L^2$	$\alpha_i^2 L$	α_i^2		
$\alpha_i^3 L^3$	$\alpha_i^3 L^2$	$\alpha_i^3 L$	α_i^3	
$\alpha_i^4 L^4$	$\alpha_i^4 L^3$	$\alpha_i^4 L^2$	$\alpha_i^4 L^1$	α_i^4
$\vdots$	$\vdots$	$\vdots$		

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- One way is to treat QED and QCD simultaneously in YFS(Yennie-Frautschi-Suura)-style Resummation[a].

[a] Ann.Phys.13,379(1969)

Representative processes

$$\mathbf{P}\ \mathbf{P}\ (\bar{\mathbf{P}})\ \Rightarrow \mathbf{V}\ + \mathbf{m}(\gamma) + \mathbf{n}(\mathbf{G}) + \mathbf{X}$$

$$\Rightarrow \ell\ell' + \mathbf{m}'(\gamma) + \mathbf{n}(\mathbf{G}) + \mathbf{X}$$

where $V = W^{\pm}, Z$.

$\ell = e, \mu, \nu_e, \nu_\mu$

$\ell' = e, \mu, \nu_e, \nu_\mu$

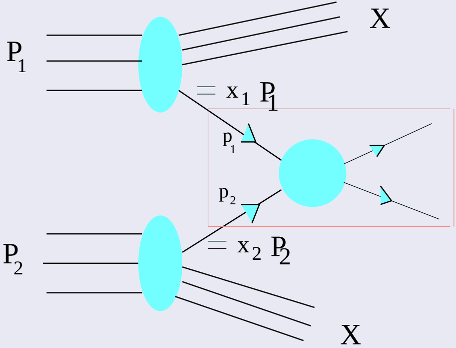
Current status of Theoretical Calculation

- NLO: Analytical (or numerical) calculations of diagrams yield parton level Monte Carlos (NLOJET++, MCFM, . . .) NLO + parton shower (MC@NLO, VINCIA)

- NNLO: selected results known (mostly inclusive kinematics)

- N^3LO: very few ...

Hadronic Cross Section



$$\sigma^{H_1H_2}(S) = \sum_{i,j=q,\bar{q},g} \int dx_1\, dx_2\, f_{i/H_1}(x_1, \mu_F^2)\, f_{j/H_2}(x_2, \mu_F^2)\, \hat{\sigma}(\hat{s}, \mu_F^2, \mu_R^2)$$

We start from the basic formula

$$d\sigma_{exp}(pp \to V + X \to \bar{\ell}\ell' + X') = \sum_{i,j} \int dx_i dx_j F_i(x_i) F_j(x_j) d\hat{\sigma}_{exp}(x_i x_j s),$$

$$\begin{aligned} d\hat{\sigma}_{\rm exp} &= \sum_n d\hat{\sigma}^n \\ &= e^{\rm SUM_{IR}(QCD)} \sum_{n=0}^{\infty} \int \prod_{j=1}^{n} \frac{d^3 k_j}{k_j} \\ & \int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_a+p_b-p_c-p_d-\sum k_j)+D_{\rm QCD}} \\ & *\tilde{\bar{\beta}}_n(k_1,\ldots,k_n)\frac{d^3p_c}{p_c^0}\frac{d^3p_d}{p_d^0} \end{aligned}$$

$$\mathrm{SUM_{IR}(QCD)} = 2\alpha_s \Re B_{QCD} + 2\alpha_s \tilde{B}_{QCD}$$

- DGLAP eqn contains all the leading logs(LLs).

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- subtract all the LLs from the YFS IR form factor.

$$\mathrm{SUM_{IR}^{nls}(QCD)} = 2\alpha_s \Re B^{nls}_{QCD} + 2\alpha_s \tilde{B}^{nls}_{QCD}$$

QED⊗QCDExponentiation

$$
\begin{aligned}
B^{nls}_{QCD} &\rightarrow B^{nls}_{QCD} + B^{nls}_{QED} \equiv B^{nls}_{QCED},\\
\tilde{B}^{nls}_{QCD} &\rightarrow \tilde{B}^{nls}_{QCD} + \tilde{B}^{nls}_{QED} \equiv \tilde{B}^{nls}_{QCED}
\end{aligned}
$$

$$
\begin{aligned}
d\hat{\sigma}_{\rm exp} &= e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}} \prod_{j_2=1}^{m} \frac{d^3 {k'}_{j_2}}{{k'}_{j_2}} \\
&\int \frac{d^4 y}{(2\pi)^4} e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum {k'}_{j_2})+D_{\rm QCED}} \\
&\tilde{\bar{\beta}}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)\frac{d^3 p_2}{p_2^0}\frac{d^3 q_2}{q_2^0},
\end{aligned}
$$

The infrared functions are now given by

$$\mathrm{SUM_{IR}(QCED)} = 2\alpha_s \Re B^{nls}_{QCED} + 2\alpha_s \tilde{B}^{nls}_{QCED}$$

- Full MC event generator realization still open.

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- HERWIRI1.0(2): MC Realization of IR-Improvement for DGLAP-CS Parton Showers. arXiv:0910.0491 [hep-ph]

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Exponentiation of QCD higher order effects: Where to apply?[a]

[a] Ann.Phys. 323 (2008) 2147; Phys. Rev.D78(2008)056001; Adv. High Energy Phys. 2008 (2008): 682312

consider

$$\frac{dq^{NS}(x,t)}{dt} = \frac{\alpha_s(t)}{2\pi}\int_x^1 \frac{dy}{y} q^{NS}(y,t) P_{qq}(x/y)$$

$$P_{qq}(z) = CF\frac{1+z^2}{1-z} \tag{1}$$

where the well-known result for the kernel Pqq(z) is, for $z < 1$ and $t = \ln(\mu/\mu_0)$

- Unintegrable singularity at z=1, usually regularized by

$$\frac{1}{(1-z)} \rightarrow \frac{1}{(1-z)_+} \tag{2}$$

such that

$$\int_0^1 dz \frac{f(z)}{(1-z)_+} = \int_0^1 dz \frac{f(z)-f(1)}{(1-z)} \tag{3}$$

- Unintegrable singularity at z=1, usually regularized by

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such that

$$\begin{aligned}\int_0^1 dz \frac{f(z)}{(1-z)_+} &= \int_0^1 dz \frac{f(z)-f(1)}{(1-z)} \\ \frac{1}{(1-z)_+} &= \frac{1}{(1-z)}\theta(1-\epsilon-z)+\ln(\epsilon)\delta(1-z) \end{aligned} \tag{5}$$

with understanding that $\epsilon \to 0$.

- Require

$$\int_0^1 dz\, P_{qq}(z) = 0 \tag{6}$$

add the virtual corrections to get

$$P_{qq}(z) = C_F\left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right) \tag{7}$$

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add the virtual corrections to get

$$P_{qq}(z) = C_F \left(\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right) \tag{9}$$

- Smooth divergent $\frac{1}{(1-z)}$ behavior as $z \rightarrow 1$has been lost. The regime $1-\epsilon < z < 1$ now has no probability. At z=1 we have a large negative contribution.

- Why set $P_{qq} = 0$ for $1 - \epsilon < z < 1$, where it actually has its largest values?

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- The experience from LEP1,2: $\frac{1}{(1-z)_+}$ should be exponentiated[a].

$$P_{AB} = \frac{1}{2} z(1-z) \overline{\sum}_{spins} \frac{|V_{A \to B+C}|^2}{p_\perp^2} \tag{10}$$

$$\Downarrow$$

$$P_{AB} = \frac{1}{2} z(1-z) \overline{\sum}_{spins} \frac{|V_{A \to B+C}|^2}{p_\perp^2} z^{\gamma_q} F_{YFS}(\gamma_q) e^{\frac{\delta_q}{2}} \tag{11}$$

where $A = q, B = G, C = q$ and $V_{A \to B+C}$ is the lowest order amplitude $q \to G(z) + q(1-z)$.

[a] see CERN YOLLOW-BOOKS,CERN-89-08

$$P_{qq}(z) = C_F F_{YFS}(\gamma_q) e^{\frac{\delta_q}{2}} \frac{1+z^2}{1-z}(1-z)^{\gamma_q} \tag{12}$$

where

$$\gamma_q = C_F \frac{\alpha_s}{\pi} t = \frac{4C_F}{\beta_0} \tag{13}$$

$$\delta_q = \frac{\gamma_q}{2} + \frac{\alpha_s C_F}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) F_{YFS}(\gamma_q) = \frac{e^{-C_E \gamma_q}}{\Gamma(1+\gamma_q)} \tag{14}$$

Note that

$$\int_{k_0} dz/z = C_0 - \ln(k_0) \qquad (15)$$

is experimentally distinguishable from

$$\int_{k_0} dz/z^{1-\gamma} = C_0' - \frac{k_0^{\gamma}}{\gamma} \qquad (16)$$

- Using the normalization condition eqn(8)

$$P_{qq}(z) = C_F\, F_{YFS}(\gamma_q)\, e^{\delta_q/2}\left[\frac{1+z^2}{(1-z)_+}(1-z)^{\gamma_q} + f_q(\gamma_q)\,\delta(1-z)\right] \tag{17}$$

where

$$f_q(\gamma_q) = \frac{2}{\gamma_q} - \frac{2}{1+\gamma_q} + \frac{1}{2+\gamma_q} \tag{18}$$

This is our IR-Improve P_{qq} DGLAP-CS Kernel.

- For $z < 1$

$$P_{Gq}(z) = P_{qq}(1-z) = C_F\, F_{YFS}(\gamma_q)\, e^{\delta_q/2}\, \frac{1+(1-z)^2}{z} z^{\gamma_q} \tag{19}$$

- Test of new theory - quark momentum sum rule:

$$\int_0^1 dz z \left(P_{Gq}(z) + P_{qq}(z)\right) = 0 \tag{20}$$

- For $P_{qG}(z)$ and $P_{GG}(z)$, we get with replacement $C_F \rightarrow C_G$ in the IR-algebra that the usual results

$$P_{GG}(z) = 2C_G\left(\frac{1-z}{z} + \frac{z}{1-z} + z(1-z)\right) \tag{21}$$

become

$$\begin{aligned} P_{GG}(z) \quad &= \quad 2C_G\, F_{YFS}(\gamma_G)\, e^{\delta_G/2} \left(\frac{1-z}{z} z^{\gamma_G} + \frac{z}{1-z}(1-z)^{\gamma_G} \right. \\ &+ \quad \frac{1}{2}\left(z^{1+\gamma_G}(1-z) + z(1-z)^{1+\gamma_G} \right) \\ &- \quad \left. f_G(\gamma_G)\delta(1-z) \right) \end{aligned} \tag{22}$$

$$P_{qG}(z) = \frac{1}{2}\left(z^2 + (1-z)^2\right) \quad (23)$$

become

$$P_{qG}(z) = F_{YFS}(\gamma_G)\, e^{\delta_G/2}\, \frac{1}{2}\left(z^2(1-z)^{\gamma_G} + (1-z)^2 z^{\gamma_G}\right) \quad (24)$$

where

$$
\begin{aligned}
\gamma_G &= C_G\frac{\alpha_s}{\pi}t = \frac{4C_G}{\beta_0} \\
\delta_G &= \frac{\gamma_G}{2} + \frac{\alpha_s C_G}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) \\
f_G(\gamma_G) &= \frac{n_f}{C_G}\frac{1}{(1+\gamma_G)(2+\gamma_G)(3+\gamma_G)} + \frac{2}{\gamma_G(1+\gamma_G)(2+\gamma_G)} \\
&+ \frac{1}{(1+\gamma_G)(2+\gamma_G)} + \frac{1}{2(3+\gamma_G)(4+\gamma_G)} \\
&+ \frac{1}{(2+\gamma_G)(3+\gamma_G)(4+\gamma_G)}
\end{aligned} \tag{25}
$$

- The gluon momentum sum rule has been used

Higher Order DGLAP-CS Kernels

- Similarly, the higher order $\mathcal{O}(\alpha_s^2), \mathcal{O}(\alpha_s^3)$ IR-Improved DGLAP-CS Kernels can be obtain from the exact DGLAP-CS Kernels using exponentiated splitting functions.

Effects on Parton Distributions

- Moments of Kernels <=> Logarithmic exponents for evolution

$$\frac{dM_n^{NS}(t)}{dt} = \frac{\alpha_s(t)}{2\pi} A_n^{NS} M_n^{NS}(t) \tag{26}$$

where

$$\begin{aligned} M_n^{NS}(t) &= \int_0^1 dz\, z^{n-1} q^{NS}(z,t) \\ A_n^{NS} &= \int_0^1 dz\, z^{n-1} P_{qq}(z) \\ &= C_F\, F_{YFS}(\gamma_q)\, e^{\delta_q/2} \Big[B(n,\gamma_q) + B(n+2,\gamma_q) - f_q(\gamma_q) \Big] \end{aligned} \tag{27}$$

Compare the Usual result

$$A_N^{NS^0} \equiv C_F \left[-\frac{1}{2} + \frac{1}{n(n+1)} - 2\sum_{j=2}^{n} \frac{1}{j} \right] \tag{28}$$

- Asymptotic behavior: IR-Improved goes to a multiple of f_q, consistent with

$$\lim_{n\to\infty} z^{n-1} = 0 \text{ for } 0 \leq z < 1; \tag{29}$$

- Usual result diverges as $-2C_F \ln(n)$.

- Different for f nite n as well: for $n = 2$ we get, for $\alpha_s = 0.118$

$$A_2^{NS} = \begin{cases} C_F(-1.33), \text{un-IR-improved} \\ C_F(-0.966), \text{IR-improved} \end{cases} \tag{30}$$

For $n = 2, Q_0 = 2$ Gev, $Q = 100$ Gev with $\Lambda_{QCD} \cong 0.2$ GeV and $n_f = 5$:
change of evovled NS moments by $\sim 5\%$
which is the expected HERA precision.[a]

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[a]see for example,T. Carli *et al.*, Proc. HERA-LHC Workshop, 2005

Threshold Corrections

- For the single Z production at LHC

$$
\begin{aligned}
r_{exp} &= \hat{\sigma}_{exp}/\hat{\sigma}_0 \\
&= \begin{cases} 1.1901 & , \text{QCED, LHC} \\ 1.1872 & , \text{QCD, LHC} \end{cases}
\end{aligned}
$$

- Only QED contribution is of the order of 0.3%
- This is stable under scale variations

MC Realization: IR-Improved Kernels in HERWIG6.5

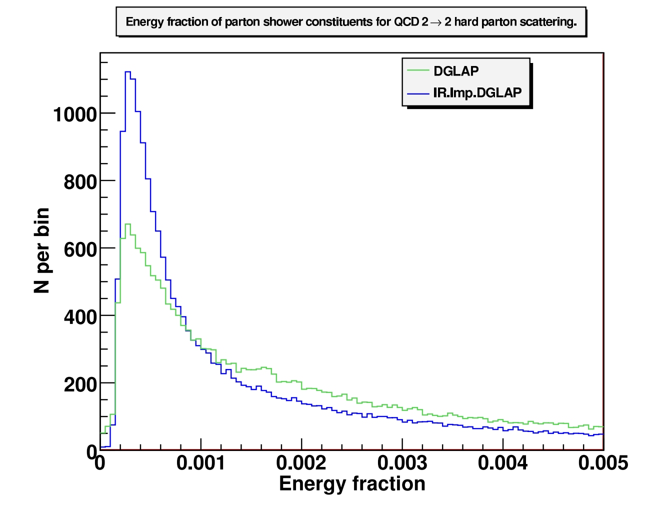
- Modify the kernels in the HWBRAN and related modules-(BW,MS)

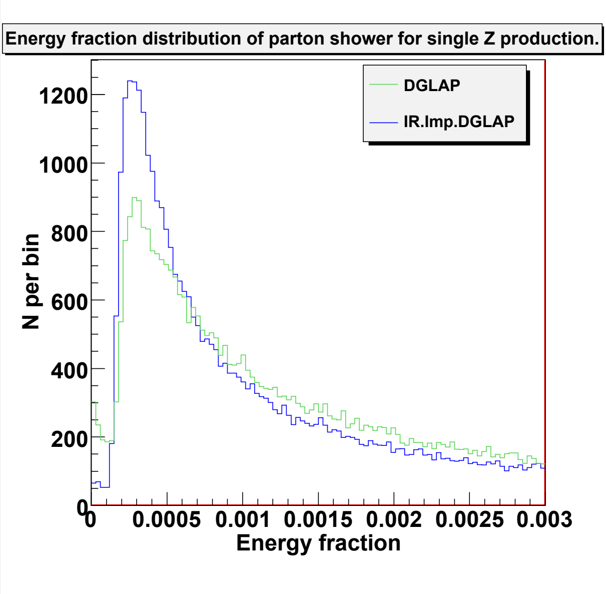
$$\text{DGLAP-CS}\, P_{AB} \Rightarrow \text{IR-I DGLAP-CS}\, P^{exp}_{AB} \tag{31}$$

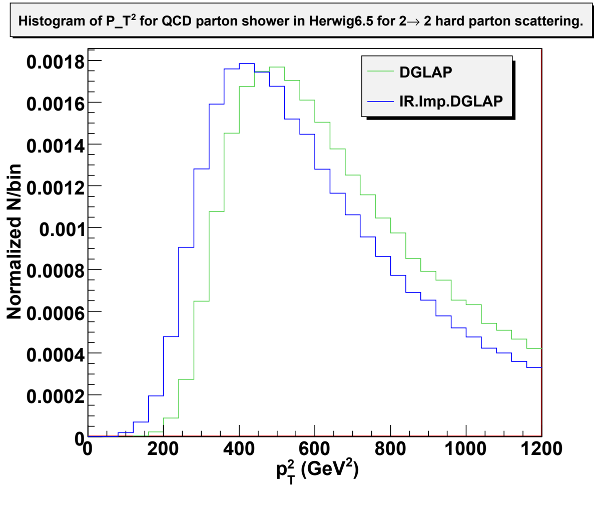
- Leave Hard Processes alone for the moment
- Issue: CTEQ and MRST are the best afte 2007, still HO EW corrections is not included.

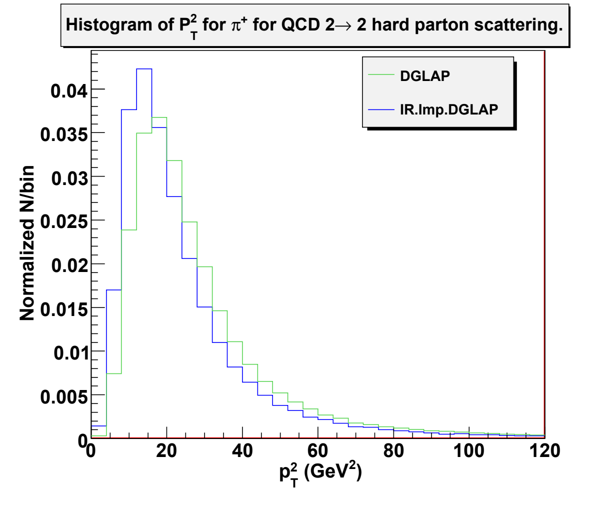
Results

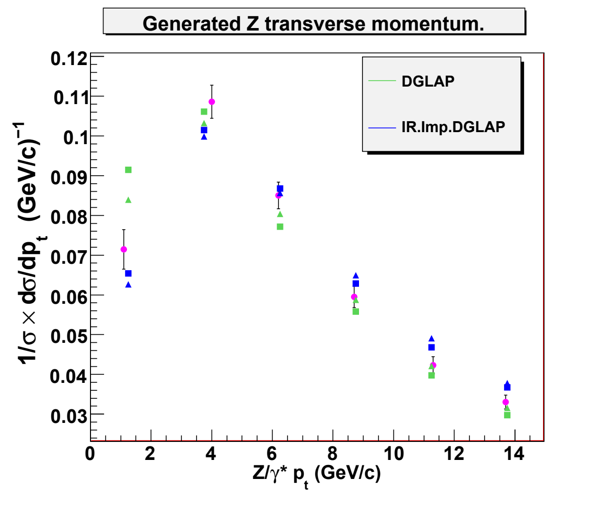
- We have taken the HERWIG6.5 program and modif ed by putting our exponentiated result which is now called HERWIRI.
- Compare the z-distributions, p_T-dist. etc. of IR-Improved and usual DGLAP-CS showers.


Energy fraction of parton shower constituents for QCD 2 → 2 hard parton scattering.
DGLAP
IR.Imp.DGLAP
N per bin
0
200
400
600
800
1000
0
0.001
0.002
0.003
0.004
0.005
Energy fraction


Energy fraction distribution of parton shower for single Z production.
DGLAP
IR.Imp.DGLAP
N per bin
0
200
400
600
800
1000
1200
0
0.0005
0.001
0.0015
0.002
0.0025
0.003
Energy fraction


Histogram of P_T^2 for QCD parton shower in Herwig6.5 for 2→ 2 hard parton scattering.
DGLAP
IR.Imp.DGLAP
Normalized N/bin
0
0.0002
0.0004
0.0006
0.0008
0.001
0.0012
0.0014
0.0016
0.0018
0
200
400
600
800
1000
1200
p_T^2 (GeV^2)


Histogram of P_T^2 for π^+ for QCD 2→ 2 hard parton scattering.
DGLAP
IR.Imp.DGLAP
Normalized N/bin
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
0
20
40
60
80
100
120
p_T^2 (GeV^2)



Generated Z transverse momentum.
DGLAP
IR.Imp.DGLAP
1/σ × dσ/dp_t (GeV/c)^-1
0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0
2
4
6
8
10
12
14
Z/γ* p_t (GeV/c)

Summary and Conclusion

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- Both QED$\otimes$*QCD* is needed for the precision luminosity determination at LHC.
- Complete O$(\alpha^2, \alpha\alpha_s, \alpha_s^2)$ MC result needed for the precision LHC(ILC!).

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- Higher order radiative corrections are necessary for the precision measurement

- Both QED⊗*QCD* is needed for the precision luminosity determination at LHC.
- Complete $O(\alpha^2, \alpha\alpha_s, \alpha_s^2)$ MC result needed for the precision LHC(ILC!).

- Theoretical uncertainties can be reduced by the radiative corrections

- Many more radiative corrections we need specially for LHC

Thank You!