Automatized full one-loop renormalization of the MSSM *with SloopS*

Nans BARO
Institut für Theoretische Teilchenphysik und Kosmologie, RWTH Aachen University

In collaboration with:

Fawzi BOUDJEMA, Guillaume CHALONS, Guillaume DRIEU LA ROCHELLE, Sun HAO,
(LAPTH, Annecy)

Andrei SEMENOV
(JINR, Dubna)

N. B., F. Boudjema, A. Semenov, *Phys. Rev.* **D78** (2008) 115003, 0807.4668 [hep-ph]
N. B., F. Boudjema, A. Semenov, *Phys. Lett.* **B660** (2008) 550, 0710.1821 [hep-ph]
N. B., F. Boudjema, *Phys. Rev.* **D80** (2009) 076010, 0906.1665 [hep-ph]
N. B., F. Boudjema, G. Chalons, S. Hao, *Phys. Rev.* **D81** 015005 (2010), 0910.3293 [hep-ph]

TOOLS 2010 - Winchester

OUTLINE

1. INTRODUCTION: THE NEED FOR NEW PHYSICS

2. THE CODE SLOOPS: AUTOMATIZING ONE-LOOP CALCULATIONS IN SUSY

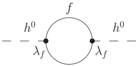
3. APPLICATIONS TO COLLIDER PHYSICS AND COSMOLOGY

OUTLINE

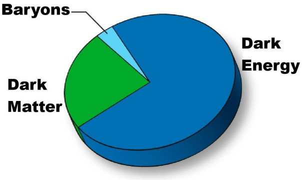
Problems: Hierarchy, Dark Matter, Unification

Standard Model

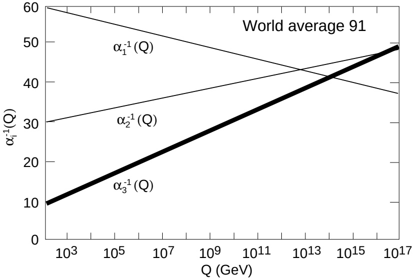
$$\Delta m_{h^0}^2$$



$$-\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{UV}^2$$

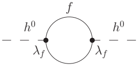


- Rotation curves
- Cosmic Microwave Background
- Galaxy clusters (Bullet cluster)
- Primordial nucleosynthetis
- ...

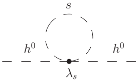


New symmetry: Fermion ↔ Boson.

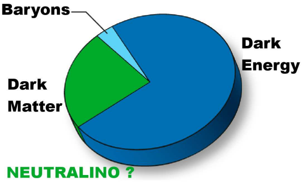
$$\Delta m^2_{h^0}$$



$$-\frac{|\lambda_f|^2}{8\pi^2}\Lambda^2_{UV}$$



$$+\frac{\lambda_S}{8\pi^2}\Lambda^2_{UV}$$



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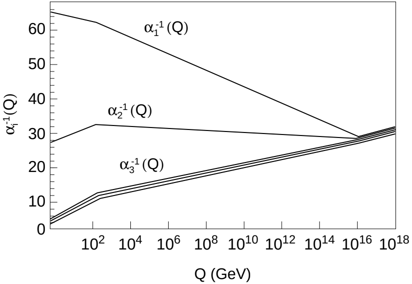


- SUSY predicts $m_f = m_b$ but... we never saw these sparticles? →SUSY must be broken! (introducing more parameters)

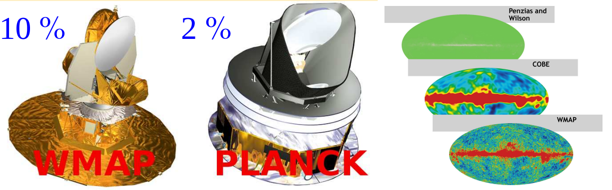
- At tree level, $m_{h^0} < m_{Z^0}$ but... we never saw the Higgs? →At one-loop, the Higgs mass receives large corrections!

⇒ Radiative corrections are important!

Relic density of Dark Matter

Precision

$0.094 < \Omega_{DM}h^2 < 0.129$

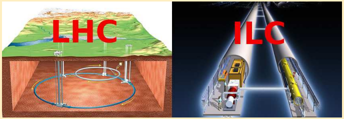


Cosmology + Particle physics

$$\Omega_{DM}h^2 \approx \frac{3 \times 10^{-27} cm^3 s^{-1}}{< \sigma(\tilde{\chi}^0\tilde{\chi}^0 \to SM)v >}$$

Precision

Need to know precisely σ
$\Rightarrow$ Computation of the relic density
$\Rightarrow$ Parameters reconstruction at the LHC/LC
$\Rightarrow$ Check the underlying cosmological scenario



SECTORS SM

Fermions	*Bosons*
u, d *Quark*	
e, ν *Lepton*	
	h^0 *Higgs*
	γ, Z^0, W^+ *EW gauge*
	g *Gluon*

A lot of parameters (124 after symmetry breaking)

A lot of interactions ($\sim$ 5000 vertices)

Calculations become extremely tedious and involved.
Even more so at one-loop...

Minimal Supersymmetric Standard Model

Sectors MSSM

Fermions	*Bosons*
u, d *Quark*	$\tilde{u}_{1,2}$, $\tilde{d}_{1,2}$ *Squark*
e, ν *Lepton*	$\tilde{e}_{1,2}$, $\tilde{\nu}$ *Slepton*
$\tilde{H}^0_{1,2}$, $\tilde{H}^+_{1,2}$	h^0, H^0, A^0, H^+ *Higgs*
$\tilde{B}$, $\tilde{W}_3$, $\tilde{W}^+$	γ, Z^0, W^+ *EW gauge*
$\tilde{g}$ *Gluino*	g *Gluon*

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u, d *Quark*	$\tilde{u}_{1,2}$, $\tilde{d}_{1,2}$ *Squark*
e, ν *Lepton*	$\tilde{e}_{1,2}$, $\tilde{\nu}$ *Slepton*
$\tilde{\chi}^0_{1,2,3,4}$ *Neutralino*	h^0, H^0, A^0, H^+ *Higgs*
$\tilde{\chi}^\pm_{1,2}$ *Chargino*	γ, Z^0, W^+ *EW gauge*
$\tilde{g}$ *Gluino*	g *Gluon*

A lot of parameters (124 after symmetry breaking)

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MINIMAL SUPERSYMMETRIC STANDARD MODEL

SECTORS MSSM

Fermions	*Bosons*
u, d *Quark*	$\tilde{u}_{1,2}$, $\tilde{d}_{1,2}$ *Squark*
e, ν *Lepton*	$\tilde{e}_{1,2}$, $\tilde{\nu}$ *Slepton*
$\tilde{\chi}^0_{1,2,3,4}$ *Neutralino*	h^0, H^0, A^0, H^+ *Higgs*
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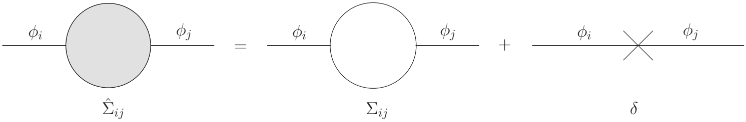
Renormalization & Definition of the parameters

Shifts

$g \longrightarrow g + \delta g$
$m_{ij}^2 \longrightarrow m_{ij}^2 + \delta m_{ij}^2$
$\phi_i \longrightarrow (\delta_{ij} + \frac{1}{2}\delta Z_{ij})\phi_j$



On-Shell renormalization scheme

- *M_i^2 is the pole of the propagator*: $\hat{\Sigma}_{ii}(M_i^2) = 0 \longrightarrow \delta M^2$
- *residue at the pole is 1*: $\hat{\Sigma}'_{ii}(M_i^2) = 0 \longrightarrow \delta Z_{ii}$
- *no transition on the external legs*: $\hat{\Sigma}_{ij}(M_i^2) = 0$ and $\hat{\Sigma}_{ji}(M_j^2) = 0 \longrightarrow \delta Z_{ij}$

Procedure and ingredients for one-loop calculations. Example: $[\tilde{\chi}^0_1 \tilde{\chi}^0_1 \to \tau^+ \tau^- (\gamma)]$

Tree Level 

[9]

Procedure and ingredients for one-loop calculations. Example: $[\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow \tau^+\tau^-(\gamma)]$

Tree Level 

[9]

Loops  $\int \frac{d^4q}{(2\pi)^4} \frac{q_\mu q_\nu \ldots q_\rho}{D_0 D_1 \ldots D_{N-1}}$

[2223,2538,855]

Procedure and ingredients for one-loop calculations. Example: $[\tilde{\chi}_1^0\tilde{\chi}_1^0 \to \tau^+\tau^-(\gamma)]$

Tree Level 

[9]

Loops 

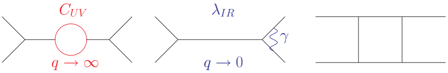
$$\int \frac{d^4q}{(2\pi)^4} \frac{q_\mu q_\nu \ldots q_\rho}{D_0 D_1 \ldots D_{N-1}} \supset C_{UV}$$

[2223,2538,855]

PROCEDURE AND INGREDIENTS FOR ONE-LOOP CALCULATIONS. EXAMPLE: $[\tilde{\chi}_1^0\tilde{\chi}_1^0 \to \tau^+\tau^-(\gamma)]$

Tree Level 

[9]

Loops 

$$\int \frac{d^4q}{(2\pi)^4} \frac{q_\mu q_\nu \ldots q_\rho}{D_0 D_1 \ldots D_{N-1}} \supset C_{UV}, \lambda_{IR}$$

[2223,2538,855]

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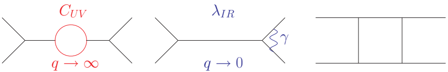
Loops 

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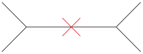
[2223,2538,855]

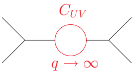
Counterterms 

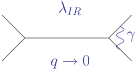
C_{UV}

[42]

Tree Level 

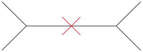
[9]

Loops 



$$\int \frac{d^4q}{(2\pi)^4}\frac{q_\mu q_\nu \ldots q_\rho}{D_0 D_1 \ldots D_{N-1}} \supset C_{UV}, \lambda_{IR}$$

[2223,2538,855]

Counterterms  C_{UV}

[42]

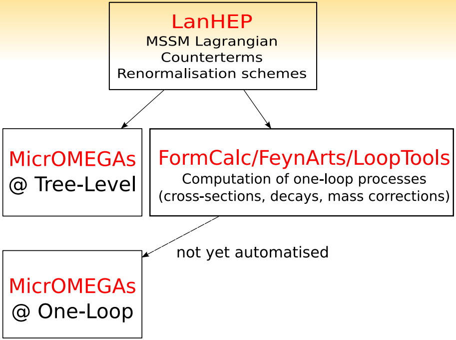
Soft$_{(E_\gamma < k_c)}$ / Hard$_{(E_\gamma > k_c)}$ 

λ_{IR}

[22]

OUTLINE

AUTOMATIC TOOL: SLOOPS



SLOOPS

A code for the calculation of loops diagrams in the MSSM with application to collider physics, astrophysics and cosmology

FEATURES OF THE CODE

- Complete and coherent renormalization of the MSSM
- Flexibility (between renormalization schemes)
- Non linear gauge fixing
- Special routine for $v = 0$ to avoid Gram det. problems

LanHEP

A.Semenov, Automatic generation of Feynman rules with LanHEP, today at 2 p.m.

Particles, lagrangian, counterterms...

```
vector A/A: (photon, gauge).

scalar h/h:('Light Higgs', mass Mh, width wh), H/H:('Heavy higgs', mass MHH, width wHh).
```

```
lterm -F**2/4 where

F=deriv^mu*B0^nu-deriv^nu*B0^mu.
```

```
lterm MG1*f_B0*f_B0/2

lterm -Mq1**2*s_q1*s_Q1 - Mq2**2*s_q2*s_Q2 - Mq3**2*s_q3*s_Q3.
```

```
transform h->h*(1+dZhlhl/2)+H*dZhlhh/2.
```

```
infinitesimal dphlhl = '-ReTilde[SelfEnergy[prt["h"]->prt["h"], Mh]]'.
```

Program similar to FeynRules C.Duhr, FeynRules Tutorial, today at 11.30 a.m.
and SARAH F.Staub, SARAH package, on Friday at 10.00 a.m.

T.Hahn, FeynArts Tutorial, today at 2.30 p.m.

Automatic generation of ~ 5000 vertices involving all the counterterms!

```
(*------ h h h ------*)
C[ S[3], S[3], S[3] ] == 3/4 I * {
{ -2 A00555 , A00519 dZhhhl -3 A00555 dZhlhl -4 A01380 dZg

+ 6 A01380 dZw3 + 2 A00555 dXwz - A01381 dXH + 2 A01382 dZb + 2 A01383 dZbw3} }
```

```
(*------ H+ H+ H- H- ------*)
C[ S[6], S[6], -S[6], -S[6] ] == -1/2 I * {

{ A03898 , 2 A03899 dZg -3 A03899 dZw3 -2 A03900 dZf1 + 2 A03901 dZf2 - A03902 dZb - A03903 dZbw3 } }
```

Usual Gauge Fixing

$$\begin{aligned}\mathcal{L}^{GF} &= -\frac{1}{\xi_W}|\partial_\mu W^{+\,\mu} \\ &+ i\xi_W \frac{g}{2} v G^+|^2 \\ &- \frac{1}{2\xi_Z}(\partial_\mu Z^{0\,\mu} + \xi_Z \frac{g}{2c_W} v G^0)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2\end{aligned}$$

$\xi = 1$ (loop library)

Non linear

Checks

- UV finite
- IR finite
- Gauge independent

Usual Gauge Fixing

$$\mathcal{L}^{GF} = -\frac{1}{\xi_W}|\partial_\mu W^{+\,\mu} + i\xi_W \frac{g}{2} v G^+|^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^{0\,\mu} + \xi_Z \frac{g}{2c_W} v G^0)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2$$

$$\frac{-i}{q^2 - m_{Z^0}^2 + i\epsilon}\left[g_{\mu\nu} + (\xi_Z - 1)\frac{q_\mu q_\nu}{q^2 - \xi_Z m_{Z^0}^2}\right]$$

$\xi = 1$ (loop library)

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- UV finite
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$$\mathcal{L}^{GF} = -\frac{1}{\xi_W}|\partial_\mu W^{+\,\mu} + i\xi_W \frac{g}{2} v G^{+}|^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^{0\,\mu} + \xi_Z \frac{g}{2c_W} v G^0)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2$$

$$\frac{-i}{q^2 - m_{Z^0}^2 + i\epsilon}\left[g_{\mu\nu} + (\xi_Z - 1)\frac{q_\mu q_\nu}{q^2 - \xi_Z m_{Z^0}^2}\right]$$

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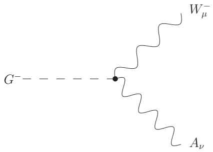
Non linear

CHECKS

- UV finite
- IR finite
- Gauge independent

Non Linear Gauge Fixing

$$\mathcal{L}^{GF} = -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z^0_\mu)W^{+\mu} + i\xi_W\frac{g}{2}(v + \tilde{\delta}h^0 + \tilde{\omega}H^0 + i\tilde{\kappa}G^0 + i\tilde{\rho}A^0)G^+|^2 - \frac{1}{2\xi_Z}(\partial_\mu Z^{0\,\mu} + \xi_Z\frac{g}{2c_W}(v + \tilde{\epsilon}h^0 + \tilde{\gamma}H^0)G^0)^2 - \frac{1}{2\xi_A}(\partial_\mu A^\mu)^2$$



$$em_W(1 + \tilde{\alpha})g_{\mu\nu}$$

$\xi = 1$ (loop library)

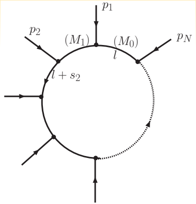
Non linear

Checks

- UV finite
- IR finite
- Gauge independent

SPECIAL ROUTINE FOR $v = 0$

Boudjema, Semenov, Temes, *Phys. Rev.* **D72** (2005) 055024, hep-ph/0507127



$$\underbrace{T^{(N)}_{\mu\nu\ldots\rho}}_{M} = \int \frac{d^4 l}{(2\pi)^4} \frac{l_\mu l_\nu \ldots l_\rho}{D_0 D_1 \ldots D_{N-1}}, \quad M \leqslant N$$

with $$D_i = \left(l + \sum_{j=1}^{i} p_j\right)^2 - M_i^2$$

(Passarino-Veltman) tensor integral reduction requires inverse Gram determinant computation:

$$\det \begin{vmatrix} p_1^2 & p_1 p_2 & p_1 p_3 \\ p_1 p_2 & p_2^2 & p_2 p_3 \\ p_1 p_3 & p_2 p_3 & p_3^2 \end{vmatrix} \propto v^2$$

$\Rightarrow$ numerical instabilities for $v \approx 0$

Segmentation of one-loop integrals at $v = 0$

$$\frac{1}{D_0 D_1 D_2 D_3} \propto \frac{1}{D_0 D_1 D_2} + \frac{A}{D_0 D_1 D_3} + \frac{B}{D_0 D_2 D_3} - \frac{1 + A + B}{D_1 D_2 D_3}$$

4-point integrals $\longrightarrow$ 3-point integrals

INPUT PARAMETERS

The MSSM contains 8×3 SUSY breaking parameters for sfermions, 3×3 fermion masses and 12 parameters for gauge couplings, scalar potential and the SUSY breaking gaugino masses:

$$\underbrace{g, g', g_s}_{\text{gauge}}, \underbrace{v_1, v_2}_{\text{v.e.v.}}, \underbrace{m_1, m_2, m_{12}}_{\text{scalar potential}}, \mu, \underbrace{M_1, M_2, M_3, M_{\tilde{Q}_L}, M_{\tilde{u}_R}, M_{\tilde{d}_R}}_{\text{SUSY breaking}}, \underbrace{A_u, A_d}_{\text{trilinear}}$$

Set of parameters directly connected to the **physical** quantities:

$$\underbrace{\alpha(0), m_W, m_Z}_{\text{EW}}, \underbrace{t_\beta = v_2/v_1, m_A, T_1, T_2}_{\textit{Higgs}}, \underbrace{m_{\chi_1^+}, m_{\chi_2^+}}_{\text{Chargino}}, \underbrace{m_{\chi_1^0}}_{\text{Neutralino}}, \underbrace{g_s, m_{\tilde{g}}}_{\text{QCD}}, \underbrace{m_{\tilde{u}_1}, m_{\tilde{d}_1}, m_{\tilde{d}_2}, \Gamma_u, \Gamma_d}_{\text{Squark}}$$

- On-Shell scheme
- Flexibility between different renormalization schemes (for example, one can also choose $m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_2^0}$, $m_{\tilde{\chi}_1^+}$ as input parameters)

HOW TO DEFINE $\tan(\beta)$?

t_β doesn't represent a physical/measurable quantity

We have many different ways/schemes to define it:

$\overline{DR}$

δt_β is a pure divergence Heinemeyer, Hollik, Weiglein, *Phys. Rept.* **425** (2006) 265, hep-ph/0412214

DCPR

δt_β is defined by the condition: $\hat{\Sigma}_{A^0 Z^0}(m^2_{A^0}) = 0$

Dabelstein, *Z. Phys.* **C67** (1995) 495, hep-ph/9409375 Chankowski, Pokorski, Rosiek, *Nucl. Phys.* **B423** (1994) 437, hep-ph/9303309

MH

δt_β is defined from the measurement of the heaviest CP-even Higgs mass m_{H^0}
(we loose a correction but the definition is physical)

$A\tau\tau$

δt_β is defined from the decay $A^0 \to \tau^+\tau^-$ (vertex $\propto m_\tau t_\beta$)

OUTLINE

First checks on the code

Tree Level calculations

Comparison with public codes: `Grace` and `CompHEP`

Cross-section [pb]	`SloopS`	`CompHEP`	`Grace`
$h^0 h^0 \to h^0 h^0$	3.932×10^{-2}	3.932×10^{-2}	3.929×10^{-2}
$W^+ W^- \to \tilde{t}_1 \bar{\tilde{t}}_1$	7.082×10^{-1}	7.082×10^{-1}	7.083×10^{-1}
$e^+ e^- \to \tilde{\tau}_1 \bar{\tilde{\tau}}_2$	2.854×10^{-3}	2.854×10^{-3}	2.854×10^{-3}
$H^+ H^- \to W^+ W^-$	6.643×10^{-1}	6.643×10^{-1}	6.644×10^{-1}
Decay [GeV]			
$A^0 \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$	1.137×10^{0}	1.137×10^{0}	1.137×10^{0}
$\tilde{\chi}_1^+ \to t \bar{\tilde{b}}_1$	5.428×10^{0}	5.428×10^{0}	5.428×10^{0}
$H^0 \to \tilde{\tau}_1 \bar{\tilde{\tau}}_1$	7.579×10^{-3}	7.579×10^{-3}	7.579×10^{-3}
$H^+ \to \tilde{\chi}_1^+ \tilde{\chi}_1^0$	1.113×10^{-1}	1.113×10^{-1}	1.113×10^{-1}

... ... # 200 processes checked

One-loop corrections that do not need renormalization

Comparisons with public codes: `PLATON` and `DarkSUSY`

Boudjema, Semenov, Temes, *Phys. Rev.* **D72** (2005) 055024, hep-ph/0507127

- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \gamma\gamma$
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to gg$
- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to Z^0\gamma$

Applications in the Higgs sector

N. B., F. Boudjema, A. Semenov, *Phys. Lett.* **B660** (2008) 550, 0710.1821 [hep-ph]

- One-loop corrections to Higgs masses H^+, h^0 Freitas, Stockinger, *Phys. Rev.* **D66** (2002) 095014, hep-ph/0205281

$t_\beta = 3$	*mhmax*	*large* μ	*nomix*
Tree Level	72.51	72.51	72.51
DCPR	134.28	97.57	112.26
MH	140.25	86.68	117.37
$A\tau\tau$	134.25	97.59	112.27
$\overline{\text{DR}}$ $\overline{\mu} = m_{A^0}$	134.87	98.10	112.86

Light Higgs mass m_{h^0}

- $A^0 \to \tau^+\tau^-$, $A^0 \to Z^0 h^0$, $H^0 \to Z^0 Z^0$, $H^0 \to \tau^+\tau^-$

$t_\beta = 3$	*mhmax*	*large* μ	*nomix*
Tree Level	9.35×10^{-3}	9.35×10^{-3}	9.35×10^{-3}
DCPR	-1.09×10^{-4}	-7.96×10^{-5}	-1.09×10^{-4}
MH	$+6.28\times10^{-3}$	-7.91×10^{-3}	$+4.47\times10^{-3}$
$A\tau\tau$	-1.45×10^{-4}	-7.09×10^{-5}	-1.01×10^{-4}
$\overline{\text{DR}}$ $\overline{\mu} = m_{A^0}$	$+5.08\times10^{-4}$	$+3.24\times10^{-4}$	$+4.17\times10^{-4}$

$H^0 \to \tau^+\tau^-$ at one-loop with no QED

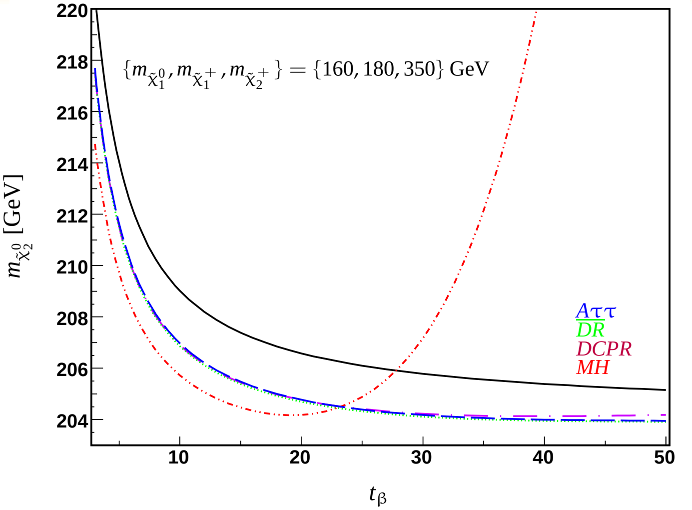
- Theoretical issue due to non-linear gauge fixing and modified Ward-Slavnov-Taylor Identity in the Higgs sector:

$$m^2_{A^0} \times A^0 \dashrightarrow \bigcirc \dashrightarrow Z^0 + m_{Z^0} \times A^0 \dashrightarrow \bigcirc \dashrightarrow G^0 = (m^2_{A^0} - m^2_{Z^0})\,\frac{ie}{s_{2W}}\,[\,\tilde{\epsilon} \times \circlearrowleft^{G^0}_{h^0} \dashrightarrow A^0 + \tilde{\gamma} \times \circlearrowleft^{G^0}_{H^0} \dashrightarrow A^0\,] \neq 0$$

Applications in the Chargino/Neutralino sector

N. B., F. Boudjema, *Phys. Rev.* **D80** (2009) 076010, 0906.1665 [hep-ph]

- One-loop corrections to neutralino masses $\tilde{\chi}^0_2$, $\tilde{\chi}^0_3$, $\tilde{\chi}^0_4$ Fritzsche, Hollik, *Eur. Phys. J.* **C24** (2002) 619, hep-ph/0203159

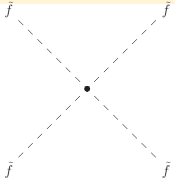


- Chargino decays at one-loop (comparison with Fujimoto *et al.*, *Phys. Rev.* **D75** (2007) 113002, hep-ph/0701200)

APPLICATIONS IN THE SFERMION SECTOR

N. B., F. Boudjema, *Phys. Rev.* **D80** (2009) 076010, 0906.1665 [hep-ph]

- 4-sfermion vertices (complicated color structure with mixing)

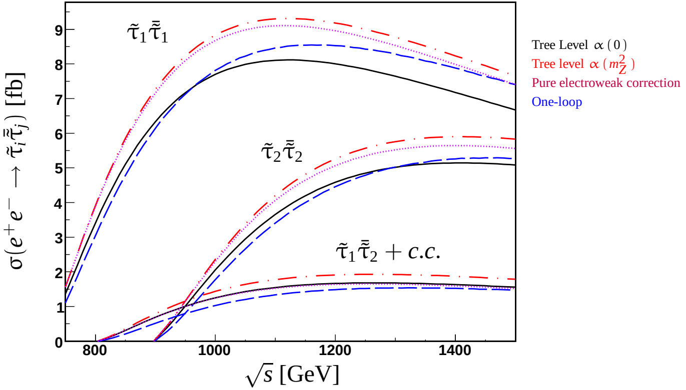


- Corrections to the sbottom and stau masses

  Hollik, Rzehak, *Eur. Phys. J.* **C32** (2003) 127, hep-ph/0305328

N. B., F. Boudjema, *Phys. Rev.* **D80** (2009) 076010, 0906.1665 [hep-ph]

- $e^+e^- \longrightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-$ Fujimoto *et al.*, *Phys. Rev.* **D75** (2007) 113002, hep-ph/0701200
- $e^+e^- \longrightarrow \tilde{\tau}_i\tilde{\tau}_j^*$ Kovarik, Weber, Eberl, Majerotto, *Phys. Rev.* **D72** (2005) 053010, hep-ph/0506021



Applications to Dark Matter

Total cross section

$\sigma v = \sum_{ij} \frac{\tilde{g}_i \tilde{g}_j}{\tilde{g}^2} (\sigma_{ij} v)$

Thermal relic

$$\Omega h^2 \simeq \frac{0.237 \times 10^{-26} cm^3/s}{x_F \int_{x_F}^{\infty} \langle \sigma v \rangle dx/x^2}$$

Effective d.o.f.

$$\tilde{g}_i = \frac{g_i}{g_{\tilde{\chi}_1^0}} (1 + \underbrace{(m_i - m_{\tilde{\chi}_1^0})/m_{\tilde{\chi}_1^0}}_{\Delta m_i})^{3/2} e^{-x \Delta m_i}$$

Examples of thermal averaging

$\langle v^2 \rangle = \frac{6}{x}$,

$\langle 1/v \rangle = \sqrt{\frac{x}{\pi}}$

Approximation

Expansion: $\sigma v = a + b v^2 \quad \Rightarrow \Omega h^2 \simeq \frac{0.237 \times 10^{-26} cm^3/s}{\sigma v (v^2 = 6/x_F \simeq 0.15)}$

Relic density calculated with the help of `MicrOmegas`

G. Bélanger, MicrOmegas Tutorial, tomorrow at 11.30 a.m.

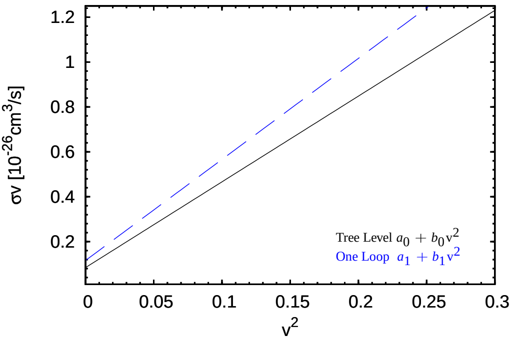
How to extract the parameters a and b?

Average

For a preliminary study, we compute the approximation:

$$< \sigma v > \simeq a + b < v^2 > \text{ with } v = \text{relative velocity} \simeq 0.1 - 0.3$$

$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \tau^+ \tau^-$



A FEW EXAMPLES

N. B., F. Boudjema, A. Semenov, *Phys. Rev.* **D78** (2008) 115003, 0807.4668 [hep-ph]

BINO-LIKE NEUTRALINO

$\tilde{\chi}_1^0\tilde{\chi}_1^0 \longrightarrow l^+l^-$

M_1	M_2	μ	M_3	$M_{\tilde{l}_{L,R}}$	A_f	M_{A^0}	t_β
90	200	-600	1000	250/110 800/800	0	500	5

COANNIHILATION WITH A STAU

$\tilde{\chi}_1^0\tilde{\tau}_1^+ \longrightarrow \tau^+\gamma$

$\tilde{\chi}_1^0\tilde{\tau}_1^+ \longrightarrow \tau^+Z^0$

$\tilde{\tau}_1^+\tilde{\tau}_1^+ \longrightarrow \tau^+\tau^+$

MIXED-LIKE NEUTRALINO

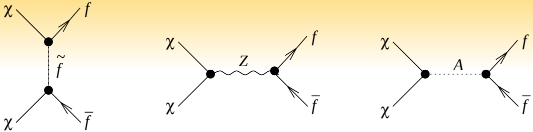
$\tilde{\chi}_1^0\tilde{\chi}_1^0 \longrightarrow W^+W^-$

$\tilde{\chi}_1^0\tilde{\chi}_1^0 \longrightarrow Z^0Z^0$

QCD CORRECTION

$\tilde{\chi}_1^0\tilde{\chi}_1^0 \longrightarrow b\bar{b}$

BINO CASE



$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to \tau^+\tau^-$ (36%)	$(\times 10^{26} cm^3/s)$ Tree	$A\tau\tau$	$\overline{DR}$	MH
a	0.081	+38%	+35%	+15%
b	3.858	+18%	+18%	+18%
Ωh^2	0.166	0.138	0.138	0.141
$\frac{\delta\Omega h^2}{\Omega h^2}$		-17%	-17%	-15%
		-2%	-2%	0%

- Helicity suppression: $\mathrm{Ampl}(v \to 0) \propto m_\tau$ thus $a \sim 0$
- $\alpha(0) \to \alpha(m_Z^2)$ implies a correction of 15%

RELIC DENSITY DOMINATED BY THE ANNIHILATION PROCESS $\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow W^+W^-$

N. B., F. Boudjema, G. Chalons, S. Hao, *Phys. Rev.* **D81** 015005 (2010), 0910.3293 [hep-ph]

- Higgsino or wino neutralino could “explain” the PAMELA/ATIC data

  Nagai, Nakayama, 0807.1634[hep-ph]

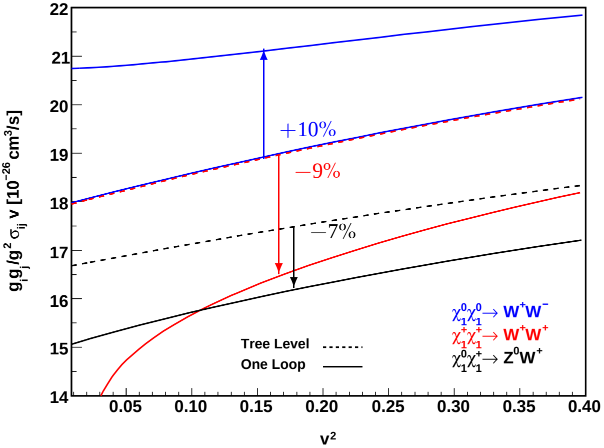
  Lattanzi, Silk, 0812.0360[hep-ph]

- Coannihilation with a chargino
- Calculating the relic density including coannihilation effects can significantly change the results
- Region of parameters difficult to probe (in mSUGRA) in colliders are regions where coannihilation comes into account for the relic density

Light Wino like

M_1	M_2	μ	M_3	$M_{\tilde{f}L,R}$	A_f	M_{A^0}	t_β
550	210	-600	1200	400/800	0	700	30

$\tilde{\chi}_1^0 = 0.0\tilde{B} - 1.0\tilde{W} - 0.2\tilde{H}_1^0 - 0.1\tilde{H}_2^0$

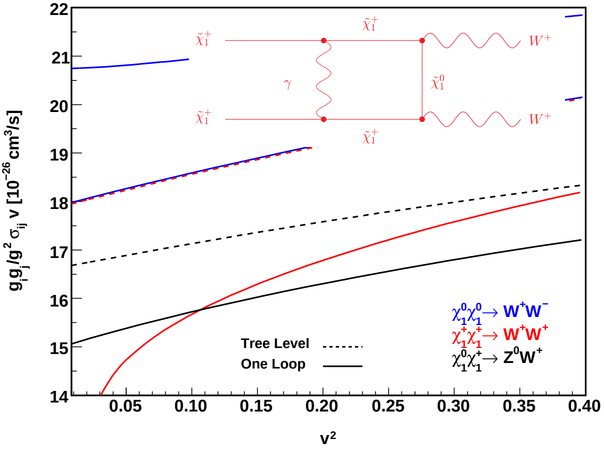


- $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ and $\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$ degenerated at Tree Level
  $\longrightarrow$ But no longer true at One Loop!
- Large corrections $\pm 10\%$
- Coulomb effect for $\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$
  $\sigma_1 v = a_1 + b_1 v^2 - \pi\alpha a_0 Q_i Q_j / v$
  $\longrightarrow \int \langle\sigma_1 v\rangle dx/x^2 = a_1 + 3/x_F b_1 \underbrace{- 2\alpha Q_i Q_j \sqrt{x_F \pi}\, a_0}_{\sim 0.12}$
- $\Omega h^2 = 0.002$ at Tree Level
  (correction at One Loop: -1.9%)
  $\longrightarrow$ Non thermal relic

Light Wino like

M_1	M_2	μ	M_3	$M_{\tilde{f}_{L,R}}$	A_f	M_{A^0}	t_β
550	210	-600	1200	400/800	0	700	30

$\tilde{\chi}_1^0 = 0.0\tilde{B} - 1.0\tilde{W} - 0.2\tilde{H}_1^0 - 0.1\tilde{H}_2^0$



- $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ and $\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$ degenerated at Tree Level
  $\longrightarrow$ But no longer true at One Loop!
- Large corrections $\pm 10\%$
- Coulomb effect for $\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$
  $\sigma_1 v = a_1 + b_1 v^2 - \pi\alpha a_0 Q_i Q_j / v$
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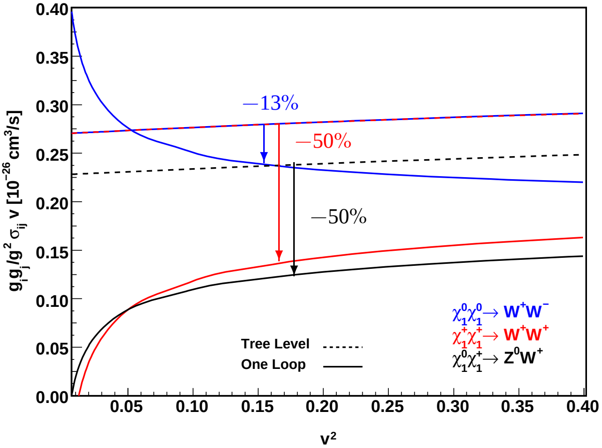
- $\Omega h^2 = 0.002$ at Tree Level
  (correction at One Loop: -1.9%)

  $\longrightarrow$ Non thermal relic

M_1	M_2	μ	M_3	$M_{\tilde{f}_{L,R}}$	A_f	M_{A^0}	t_β
3500	1800	4500	5000	5000	0	5000	15

$\tilde{\chi}_1^0 = 0.0\tilde{B} - 1.0\tilde{W} + 0.0\tilde{H}_1^0 + 0.0\tilde{H}_2^0$

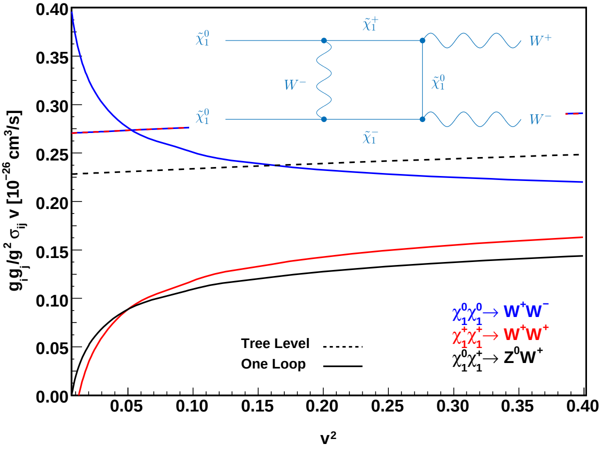


- $\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow W^+W^-$ and $\tilde{\chi}_1^+\tilde{\chi}_1^+ \rightarrow W^+W^+$ degenerated at Tree Level
  $\rightarrow$ But no longer true at One Loop!
- Very large corrections
- Sommerfeld effect for
  $\tilde{\chi}_1^0\tilde{\chi}_1^0 \rightarrow W^+W^-$
  $\tilde{\chi}_1^+\tilde{\chi}_1^+ \rightarrow W^+W^+$
  $\tilde{\chi}_1^0\tilde{\chi}_1^+ \rightarrow Z^0W^+$
  $\sigma_1 v = a_1 + b_1 v^2 + c_1/(d_1 + v)$
- $\Omega h^2 = 0.100$ at Tree Level
  (correction at One Loop: $+9.3\%$)

HEAVY WINO LIKE

M_1	M_2	μ	M_3	$M_{\tilde{f}_{L,R}}$	A_f	M_{A^0}	t_β
3500	1800	4500	5000	5000	0	5000	15

$\tilde{\chi}_1^0 = 0.0\tilde{B} - 1.0\tilde{W} + 0.0\tilde{H}_1^0 + 0.0\tilde{H}_2^0$



- $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$ and $\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$ degenerated at Tree Level
  $\to$ But no longer true at One Loop!
- Very large corrections
- Sommerfeld effect for
  $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to W^+W^-$
  $\tilde{\chi}_1^+\tilde{\chi}_1^+ \to W^+W^+$
  $\tilde{\chi}_1^0\tilde{\chi}_1^+ \to Z^0W^+$
  $\sigma_1 v = a_1 + b_1 v^2 + c_1/(d_1 + v)$
- $\Omega h^2 = 0.100$ at Tree Level
  (correction at One Loop: $+9.3\%$)

Conclusion

- Complete EW renormalization of the MSSM and modularity with different schemes
- One-loop corrections to masses, decays, cross-sections at the colliders
- Importance of radiative corrections in the relic density calculation
  - Corrections seem to be small for the bino case either in the bulk through coannihilation (after reabsorbing $\alpha(0) \longrightarrow \alpha(m_Z^2)$) but still needed
  - Large corrections for the mixed case
- Coannihilation with a stau
- Coannihilation with a chargino
- Coulomb-Sommerfeld enhancement
- First steps done for the connection with `micrOMEGAs`
- New renormalisation schemes (chargino/neutralino sector)