CHARYBDIS2: Modelling higher dimensional black hole events

Marco Sampaio

sampaio@hep.phy.cam.ac.uk

CFP, Physics Department, University of Porto

June 30, 2010

In collaboration with James A. Frost, Jonathan R. Gaunt, Marc Casals, Sam R. Dolan, M. Andrew Parker and Bryan R. Webber

References: JHEP10(2009)014 [arXiv:0904.0979], http://projects.hepforge.org/charybdis2/

Acknowledgements



Cambridge SUSY working group

FCT - SFRH/BD/23052/2005



Gravity



Gravity



Gravity



Gravity



Particle Physics



Gravity



Particle Physics



Gravity



Particle Physics



Gravity



Particle Physics



How to explain this hierarchy of scales?

Aim of this talk

The aim of this talk

To introduce the Physics of Black Hole production and decay in theories with extra dimensions.

Describe the incorporation of the theory into a Monte Carlo program CHARYBDIS2.

Present some phenomenological features of the results and how they affect observables at the LHC.

Outline



Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes
- 2 Modelling BH events CHARYBDIS2
 - The production
 - The decay
 - CHARYBDIS2 & other generators
- Phenomenology using CHARYBDIS2
 - Classical signatures
 - The effects of rotation



Outline



Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes
- 2 Modelling BH events CHARYBDIS2
 - The production
 - The decay
 - CHARYBDIS2 & other generators
- 3 Phenomenology using CHARYBDIS2
 - Classical signatures
 - The effects of rotation

4 Conclusions and Outlook

- 1 Higgs particle (s = 0),
- 3 families of leptons and 3 of quarks (s = 1/2),
- 1 non-abelian SU(3)_C gluon field, 3 massive vector bosons, 1 neutral U(1) Maxwell field (s = 1).

"Low" energy degrees of freedom (after symmetry breaking):

$$\mathcal{L}_{SM} = \frac{1}{2} \partial^{\mu} h \partial_{\mu} h - \frac{m_h^2}{2} h^2$$

• 1 Higgs particle (s = 0),

- 3 families of leptons and 3 of quarks (s = 1/2),
- 1 non-abelian SU(3)_C gluon field, 3 massive vector bosons, 1 neutral U(1) Maxwell field (s = 1).

$$\begin{split} \mathcal{L}_{SM} &= \frac{1}{2} \partial^{\mu} h \partial_{\mu} h - \frac{m_{h}^{2}}{2} h^{2} + \bar{e}^{a} \left(i \not{\partial} - m_{e_{a}} \right) e^{a} + \bar{\nu}^{a} i \not{\partial} \nu^{a} + \bar{u}^{a} \left(i \not{\partial} - m_{u_{a}} \right) u^{a} + \\ &+ \bar{d}^{a} \left(i \not{\partial} - m_{d_{a}} \right) d^{a} \end{split}$$

- 1 Higgs particle (s = 0),
- 3 families of leptons and 3 of quarks (s = 1/2),
- 1 non-abelian SU(3)_C gluon field, 3 massive vector bosons, 1 neutral U(1) Maxwell field (s = 1).

$$\begin{split} \mathcal{L}_{SM} &= \frac{1}{2} \partial^{\mu} h \partial_{\mu} h - \frac{m_{h}^{2}}{2} h^{2} + \bar{e}^{a} \left(i \not{\partial} - m_{e_{a}} \right) e^{a} + \bar{\nu}^{a} i \not{\partial} \nu^{a} + \bar{u}^{a} \left(i \not{\partial} - m_{u_{a}} \right) u^{a} + \\ &+ \bar{d}^{a} \left(i \not{\partial} - m_{d_{a}} \right) d^{a} - \frac{1}{4} \mathbf{G}_{\mu\nu} \cdot \mathbf{G}^{\mu\nu} - \frac{1}{2} W^{\dagger}_{\mu\nu} W^{\mu\nu} + m_{W}^{2} W^{\dagger}_{\mu} W^{\mu} + \\ &- \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{m_{Z}^{2}}{2} Z_{\mu} Z^{\mu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} \end{split}$$

- 1 Higgs particle (s = 0),
- 3 families of leptons and 3 of quarks (s = 1/2),
- 1 non-abelian SU(3)_C gluon field, 3 massive vector bosons, 1 neutral U(1) Maxwell field (s = 1).

$$\begin{split} \mathcal{L}_{SM} &= \frac{1}{2} \partial^{\mu} h \partial_{\mu} h - \frac{m_{h}^{2}}{2} h^{2} + \bar{e}^{a} \left(i \not{\partial} - m_{e_{a}} \right) e^{a} + \bar{\nu}^{a} i \not{\partial} \nu^{a} + \bar{u}^{a} \left(i \not{\partial} - m_{u_{a}} \right) u^{a} + \\ &+ \bar{d}^{a} \left(i \not{\partial} - m_{d_{a}} \right) d^{a} - \frac{1}{4} \mathbf{G}_{\mu\nu} \cdot \mathbf{G}^{\mu\nu} - \frac{1}{2} W^{\dagger}_{\mu\nu} W^{\mu\nu} + m_{W}^{2} W^{\dagger}_{\mu} W^{\mu} + \\ &- \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} + \frac{m_{Z}^{2}}{2} Z_{\mu} Z^{\mu} - \frac{1}{4} A_{\mu\nu} A^{\mu\nu} + Interactions \end{split}$$

- 1 Higgs particle (s = 0),
- 3 families of leptons and 3 of quarks (s = 1/2),
- 1 non-abelian SU(3)_C gluon field, 3 massive vector bosons, 1 neutral U(1) Maxwell field (s = 1).

The Standard Model – Interactions









The hierarchy problem: SM vs Gravity

The action for gravity coupled to matter is

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left[rac{M_4^2}{2} R + \mathcal{L}_{SM}
ight]$$

The hierarchy problem: SM vs Gravity

The action for gravity coupled to matter is

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left[rac{M_4^2}{2} R + \mathcal{L}_{SM}
ight]$$

Linear perturbations $g_{\mu\nu} = \eta_{\mu\nu} + \frac{E}{M_4} h_{\mu\nu}$ (units $x \to x/(E^{-1})$)

$$\mathcal{S} = \int \left[\mathcal{L}_{h_{\mu\nu}, \textit{kinetic}} + \mathcal{L}_{SM} + \frac{E}{2M_4} T_{\mu\nu} h^{\mu\nu} + \ldots \right], \frac{1 \text{ TeV}}{M_4} \sim \sqrt{\alpha}_{\rm G} \sim 10^{-16}$$

The hierarchy problem: SM vs Gravity

The action for gravity coupled to matter is

$$\mathcal{S} = \int d^4x \sqrt{|g|} \left[rac{M_4^2}{2} R + \mathcal{L}_{SM}
ight]$$

Linear perturbations $g_{\mu\nu} = \eta_{\mu\nu} + \frac{E}{M_4} h_{\mu\nu}$ (units $x \to x/(E^{-1})$)

$$\mathcal{S} = \int \left[\mathcal{L}_{h_{\mu\nu}, \text{kinetic}} + \mathcal{L}_{SM} + \frac{E}{2M_4} T_{\mu\nu} h^{\mu\nu} + \ldots \right], \frac{1 \text{ TeV}}{M_4} \sim \sqrt{\alpha}_{G} \sim 10^{-16}$$

Operator type	Couplings	at $E \sim 1 { m TeV}$
$\mathcal{T}_{lphaeta} \mathbf{h}^{lphaeta}$	E/M_4	10 ⁻¹⁶
SM Interactions	$\sim m{e}, m{g}_{QCD}, rac{m_H}{v}, rac{v}{E}, rac{m_f}{v}$	$O(10^{-6}) - O(1)$

$M_4 \sim 10^{16} M_{\text{EW}}$

Hierarchy due to taking the scale for new physics from gravity (mesoscopic) rather than the electroweak scale (microscopic). **The ADD solution:** Assume M_{EW} is more fundamental.

N. Arkani-Hamed et al. hep-th/9803315 (ADD)

Assume our space time is 4+n dimensional

$$\textbf{S}_{\textbf{G}} \sim \int \mathrm{d}^{4+n} \textbf{x} \, \textbf{M}_{(4+n)}^{2+n} \, \sqrt{-\textbf{g}} \, \mathcal{R}^{(4+n)}$$

 $\bullet~$ Take $M_{EW} \sim$ 1 ${\rm TeV} \rightarrow M_{4+n}$ as the fundamental scale

At large distances M²

 ${f S}_{f G}\sim\int {
m d}^4{f x}\,{f M}^{2+n\,f \dagger}_{(4+n)}{f R}^n\,\sqrt{-g}\,{\cal R}^{(4)}{\Rightarrow}\,4{
m D}$ gravity diluted

 $M_4 \sim 10^{16} M_{EW}$

Hierarchy due to taking the scale for new physics from gravity (mesoscopic) rather than the electroweak scale (microscopic).

The ADD solution: Assume M_{EW} is more fundamental.

N. Arkani-Hamed et al. hep-th/9803315 (ADD)

Assume our space time is 4+n dimensional

SM effective theory on a thin brane

$$\textbf{S}_{\textbf{G}} \sim \int \mathrm{d}^{4+n} \textbf{x} \, \textbf{M}_{(4+n)}^{2+n} \, \sqrt{-\textbf{g}} \, \mathcal{R}^{(4+n)}$$



- $\bullet~$ Take $M_{EW} \sim$ 1 ${\rm TeV} \rightarrow M_{4+n}$ as the fundamental scale
- At large distances M_4^2 $S_G \sim \int d^4 x M_{(4+n)}^{2+n \frac{1}{4}} R^n \sqrt{-g} \mathcal{R}^{(4)} \Rightarrow 4D$ gravity diluted

 $M_4 \sim 10^{16} M_{EW}$

Hierarchy due to taking the scale for new physics from gravity (mesoscopic) rather than the electroweak scale (microscopic).

The ADD solution: Assume M_{EW} is more fundamental.

N. Arkani-Hamed et al. hep-th/9803315 (ADD)

Assume our space time is 4+n dimensional SM effective

SM effective theory on a thin brane

$$\textbf{S}_{\textbf{G}} \sim \int \mathrm{d}^{\textbf{4}+\textbf{n}} \textbf{x} \, \textbf{M}_{(\textbf{4}+\textbf{n})}^{\textbf{2}+\textbf{n}} \, \sqrt{-\textbf{g}} \, \mathcal{R}^{(\textbf{4}+\textbf{n})}$$



- $\bullet~$ Take $M_{EW} \sim 1~{\rm TeV} \rightarrow M_{4+n}$ as the fundamental scale
- At large distances M_4^2 $S_G \sim \int d^4 x M_{(4+n)}^{2+n i} R^n \sqrt{-g} \mathcal{R}^{(4)} \Rightarrow 4D$ gravity diluted

 $M_4 \sim 10^{16} M_{EW}$

Hierarchy due to taking the scale for new physics from gravity (mesoscopic) rather than the electroweak scale (microscopic).

The ADD solution: Assume M_{EW} is more fundamental.

N. Arkani-Hamed et al. hep-th/9803315 (ADD)

Assume our space time is 4+n dimensional SM ef

SM effective theory on a thin brane

$$\textbf{S}_{\textbf{G}} \sim \int \mathrm{d}^{\textbf{4}+\textbf{n}} \textbf{x} \, \textbf{M}_{(\textbf{4}+\textbf{n})}^{\textbf{2}+\textbf{n}} \, \sqrt{-\textbf{g}} \, \mathcal{R}^{(\textbf{4}+\textbf{n})}$$

Our 4D spacetime brane

Extra dimensions

 $\bullet~$ Take $M_{EW} \sim 1~{\rm TeV} \rightarrow M_{4+n}$ as the fundamental scale

RÎ

• At large distances M_4^2 $S_G \sim \int d^4 x M_{(4+n)}^{2+n \dagger} R^n \sqrt{-g} \mathcal{R}^{(4)} \Rightarrow 4D$ gravity diluted

So how does gravity look like in ADD?

$$F_{r \ll R} \sim \frac{1}{M_{(4+n)}^{2+n} r^{2+n}}, \quad F_{r \gg R} \sim \frac{1}{M_{(4+n)}^{2+n} R^n r^2} \left(1 + 2n e^{-\frac{r}{R}} + \ldots\right)$$

- Predicts deviations from Newtonian gravity as we approach short distances.
- Contains KK gravitons from the 4D point of view.
- **Gravity is higher dimensional** at very short distances.

This can be used to put <u>bounds on *R* as a function of *n*.</u>

So how does gravity look like in ADD?

$$F_{r\ll R} \sim \frac{1}{M_{(4+n)}^{2+n}r^{2+n}}, \quad F_{r\gg R} \sim \frac{1}{M_{(4+n)}^{2+n}R^{n}r^{2}}\left(1+2ne^{-\frac{r}{R}}+\ldots\right)$$

- Predicts deviations from Newtonian gravity as we approach short distances.
- Contains KK gravitons from the 4D point of view.
- **Gravity is higher dimensional** at very short distances.

This can be used to put <u>bounds on *R* as a function of *n*.</u>

So how does gravity look like in ADD?

$$F_{r\ll R} \sim \frac{1}{M_{(4+n)}^{2+n}r^{2+n}}, \quad F_{r\gg R} \sim \frac{1}{M_{(4+n)}^{2+n}R^{n}r^{2}}\left(1+2ne^{-\frac{r}{R}}+\ldots\right)$$

- Predicts deviations from Newtonian gravity as we approach short distances.
- Contains KK gravitons from the 4D point of view.
- **Gravity is higher dimensional** at very short distances.

This can be used to put <u>bounds on *R* as a function of *n*.</u>

So how does gravity look like in ADD?

$$F_{r\ll R} \sim rac{1}{M^{2+n}_{(4+n)}r^{2+n}}, \quad F_{r\gg R} \sim rac{1}{M^{2+n}_{(4+n)}R^nr^2} \left(1+2ne^{-rac{r}{R}}+\ldots\right)$$

- Predicts deviations from Newtonian gravity as we approach short distances.
- Ocntains KK gravitons from the 4D point of view.
- **Gravity is higher dimensional** at very short distances.

This can be used to put <u>bounds on *R* as a function of *n*.</u>

So how does gravity look like in ADD?

$$F_{r\ll R} \sim rac{1}{M^{2+n}_{(4+n)}r^{2+n}}, \quad F_{r\gg R} \sim rac{1}{M^{2+n}_{(4+n)}R^nr^2} \left(1+2ne^{-rac{r}{R}}+\ldots\right)$$

- Predicts deviations from Newtonian gravity as we approach short distances.
- Contains KK gravitons from the 4D point of view.
- **Gravity is higher dimensional** at very short distances.

This can be used to put bounds on *R* as a function of *n*.

$$\Rightarrow$$
 Translates as a bound on M_{4+n} .

Bounds on extra dimensions

$M_4^2 = \frac{R^n M_{(4+n)}^{2+n}}{$	R in μm (n = 2)	$M_{4+n} \sim 1 \text{TeV OK}$
Deviations from r ⁻² in torsion-balance	\lesssim 55	<i>n</i> > 1
KK graviton produc- tion @ colliders	\lesssim 800	n > 2
KK graviton produc- tion in Supernovae	$\lesssim 5.1 imes 10^{-4}$	n > 3
KK gravitons early Universe production	\lesssim 2.2 $ imes$ 10 ⁻⁵	n > 3

Bounds on extra dimensions

$M_4^2 = \frac{R^n M_{(4+n)}^{2+n}}{$	R in µm (n = 2)	$M_{4+n} \sim 1 \text{TeV OK}$
Deviations from r ⁻² in torsion-balance	\lesssim 55	<i>n</i> > 1
KK graviton produc- tion @ colliders	\lesssim 800	n > 2
KK graviton produc- tion in Supernovae	$\lesssim 5.1 imes 10^{-4}$	n > 3
KK gravitons early Universe production	\lesssim 2.2 $ imes$ 10 ⁻⁵	n > 3

 $\bullet\,$ SM on a 4D brane of thickness L $\lesssim (1{\rm TeV})^{-1} \sim 10^{-13} \mu m$

To avoid bounds from Electroweak precision and fast proton decay. Quarks and leptons may have to be on sub-branes for $L \lesssim (1 \text{TeV})^{-1}$.

• All SM particles propagating on a single brane.

Good approximation if process occurs at large scales compared to *L*.

Outline



Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes
- 2 Modelling BH events CHARYBDIS2
 - The production
 - The decay
 - CHARYBDIS2 & other generators
- 3 Phenomenology using CHARYBDIS2
 - Classical signatures
 - The effects of rotation

4 Conclusions and Outlook
At short distances gravity is higher dimensional

$$\Rightarrow \sqrt{\alpha_G} \sim \frac{E}{M_4} \rightarrow \frac{E}{M_{4+n}} \sim \frac{E}{1\text{TeV}}$$

So gravity becomes the strongest force above 1 TeV! \Rightarrow Small impact parameter, high energy collision \rightarrow BHs!

At short distances gravity is higher dimensional

$$\Rightarrow \sqrt{\alpha_G} \sim \frac{E}{M_4} \rightarrow \frac{E}{M_{4+n}} \sim \frac{E}{1\text{TeV}}$$

So gravity becomes the strongest force above 1 TeV! \Rightarrow Small impact parameter, high energy collision \rightarrow BHs!

At short distances gravity is higher dimensional

$$\Rightarrow \sqrt{\alpha_{G}} \sim \frac{E}{M_{4}} \rightarrow \frac{E}{M_{4+n}} \sim \frac{E}{1\text{TeV}}$$

So gravity becomes the strongest force above 1 TeV! \Rightarrow Small impact parameter, high energy collision \rightarrow BHs!



At short distances gravity is higher dimensional

$$\Rightarrow \sqrt{\alpha_{G}} \sim \frac{E}{M_{4}} \rightarrow \frac{E}{M_{4+n}} \sim \frac{E}{1\text{TeV}}$$

So gravity becomes the strongest force above 1 TeV! \Rightarrow Small impact parameter, high energy collision \rightarrow BHs!



S. Dimopoulos and G. Landsberg, hep-ph/0106295



Outline



Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes

2 Modelling BH events – CHARYBDIS2

- The production
- The decay
- CHARYBDIS2 & other generators
- 3 Phenomenology using CHARYBDIS2
 - Classical signatures
 - The effects of rotation

4 Conclusions and Outlook

Modelling production - The ideal solution



Ideally:

• Set up spatial metric for two highly boosted particles,



- Set up spatial metric for two highly boosted particles,
- Include the spin and charge,



- Set up spatial metric for two highly boosted particles,
- Include the spin and charge,
- Evolve this system using Einstein's equations,



- Set up spatial metric for two highly boosted particles,
- Include the spin and charge,
- Evolve this system using Einstein's equations,
- Obtain final Black Hole + radiation



- Set up spatial metric for two highly boosted particles,
- Include the spin and charge,
- Evolve this system using Einstein's equations,
- Obtain final Black Hole + radiation \rightarrow 4D so far.

U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738 b = 0

- M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 $b \neq 0$
- U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, T. Hinderer, N. Yunes arXiv:0907.1252 $b \neq 0$
- M. Choptuik, F. Pretorius, arXiv:0908.1780 b = 0 (solitons)

Zilhao, Witek, Sperhake, Cardoso, Gualtieri, Herdeiro, Nerozzi arXiv:1001.2302 4 + n



- Set up spatial metric for two highly boosted particles,
- Include the spin and charge,
- Evolve this system using Einstein's equations,
- Obtain final Black Hole + radiation \rightarrow 4D so far.

U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, J. Gonzalez, arXiv:0806.1738 b = 0

- M. Shibata, H. Okawa, T. Yamamoto, arXiv:0810.4735 $b \neq 0$
- U. Sperhake, V. Cardoso, F. Pretorius, E. Berti, T. Hinderer, N. Yunes arXiv:0907.1252 $b \neq 0$
- M. Choptuik, F. Pretorius, arXiv:0908.1780 b = 0 (solitons)

Zilhao, Witek, Sperhake, Cardoso, Gualtieri, Herdeiro, Nerozzi arXiv:1001.2302 4 + n

H. Yoshino and V. S. Rychkov hep-th/0503171

H. Yoshino and V. S. Rychkov hep-th/0503171

• Lower bounds on $b_{max}^{(n)} \Rightarrow$ Enhanced cross section $F_n \sigma_{disk}$

$$\sigma_{PP \to BH} = \sum_{i,j}^{\text{partons}} \int_{\tau_m}^1 d\tau \int_{\tau}^1 \frac{dx}{x} \mathbf{f}_i(\mathbf{x}) \mathbf{f}_j\left(\frac{\tau}{\mathbf{x}}\right) \mathbf{F}_{\mathbf{n}} \sigma_{\mathsf{disk}}(\tau \mathbf{s})$$

 $\Rightarrow \sigma_{PP \rightarrow BH} = 70$, 160, 280 pb $\Rightarrow 1 s^{-1}$ @ design at 7 TeV $\Rightarrow 10^{-4}$ suppression.

H. Yoshino and V. S. Rychkov hep-th/0503171

• Lower bounds on $b_{max}^{(n)} \Rightarrow$ Enhanced cross section $F_n \sigma_{disk}$

$$\sigma_{PP \to BH} = \sum_{i,j}^{\text{partons}} \int_{\tau_m}^1 d\tau \int_{\tau}^1 \frac{dx}{x} \mathbf{f}_i(\mathbf{x}) \mathbf{f}_j\left(\frac{\tau}{\mathbf{x}}\right) \mathbf{F}_{\mathbf{n}} \sigma_{\mathsf{disk}}(\tau \mathbf{s})$$

⇒ $\sigma_{\mathbf{PP}\to\mathbf{BH}} = \mathbf{70}$, **160**, **280** pb ⇒ 1 s^{-1} @ design at 7 TeV ⇒ 10⁻⁴ suppression.

H. Yoshino and V. S. Rychkov hep-th/0503171

• Lower bounds on $b_{max}^{(n)} \Rightarrow$ Enhanced cross section $F_n \sigma_{disk}$

$$\sigma_{PP \to BH} = \sum_{i,j}^{\text{partons}} \int_{\tau_m}^1 d\tau \int_{\tau}^1 \frac{dx}{x} \mathbf{f}_i(\mathbf{x}) \mathbf{f}_j\left(\frac{\tau}{\mathbf{x}}\right) \mathbf{F}_{\mathbf{n}} \sigma_{\mathsf{disk}}(\tau \mathbf{s})$$

 $\Rightarrow \sigma_{\mathbf{PP} \to \mathbf{BH}} = \mathbf{70}, \ \mathbf{160}, \ \mathbf{280} \ \mathrm{pb} \Rightarrow \mathbf{1s^{-1}} \ @ \text{ design}$ at 7 TeV $\Rightarrow \mathbf{10^{-4}}$ suppression.

H. Yoshino and V. S. Rychkov hep-th/0503171

• Lower bounds on $b_{max}^{(n)} \Rightarrow$ Enhanced cross section $F_n \sigma_{disk}$

$$\sigma_{PP \to BH} = \sum_{i,j}^{\text{partons}} \int_{\tau_m}^1 d\tau \int_{\tau}^1 \frac{dx}{x} \mathbf{f}_i(\mathbf{x}) \mathbf{f}_j\left(\frac{\tau}{\mathbf{x}}\right) \mathbf{F}_{\mathbf{n}} \sigma_{\mathsf{disk}}(\tau \mathbf{s})$$

 $\Rightarrow \sigma_{\mathbf{PP} \rightarrow \mathbf{BH}} = \mathbf{70}, \ \mathbf{160}, \ \mathbf{280} \ \mathrm{pb} \Rightarrow \mathbf{1s^{-1}} @$ design at 7 TeV $\Rightarrow \mathbf{10^{-4}}$ suppression.

Bounds on M and J lost into gravitational radiation.

H. Yoshino and V. S. Rychkov hep-th/0503171

• Lower bounds on $b_{max}^{(n)} \Rightarrow$ Enhanced cross section $F_n \sigma_{disk}$

$$\sigma_{PP \to BH} = \sum_{i,j}^{\text{partons}} \int_{\tau_m}^1 d\tau \int_{\tau}^1 \frac{dx}{x} \mathbf{f}_i(\mathbf{x}) \mathbf{f}_j\left(\frac{\tau}{\mathbf{x}}\right) \mathbf{F}_{\mathbf{n}} \sigma_{\mathsf{disk}}(\tau \mathbf{s})$$

 $\Rightarrow \sigma_{\mathbf{PP} \rightarrow \mathbf{BH}} = \mathbf{70}, \ \mathbf{160}, \ \mathbf{280} \ \mathrm{pb} \Rightarrow \mathbf{1s^{-1}} @ \text{ design}$ at 7 TeV $\Rightarrow \mathbf{10^{-4}}$ suppression.

Bounds on M and J lost into gravitational radiation.



H. Yoshino and V. S. Rychkov hep-th/0503171

• Lower bounds on $b_{max}^{(n)} \Rightarrow$ Enhanced cross section $F_n \sigma_{disk}$

$$\sigma_{PP \to BH} = \sum_{i,j}^{\text{partons}} \int_{\tau_m}^1 d\tau \int_{\tau}^1 \frac{dx}{x} \mathbf{f}_i(\mathbf{x}) \mathbf{f}_j\left(\frac{\tau}{\mathbf{x}}\right) \mathbf{F}_{\mathbf{n}} \sigma_{\mathsf{disk}}(\tau \mathbf{s})$$

 $\Rightarrow \sigma_{\mathbf{PP} \rightarrow \mathbf{BH}} = \mathbf{70}, \ \mathbf{160}, \ \mathbf{280} \ \mathrm{pb} \Rightarrow \mathbf{1s^{-1}} @ \text{ design}$ at 7 TeV $\Rightarrow \mathbf{10^{-4}}$ suppression.

Bounds on M and J lost into gravitational radiation.











Outline



Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes

2 Modelling BH events – CHARYBDIS2

- The production
- The decay
- CHARYBDIS2 & other generators
- 3 Phenomenology using CHARYBDIS2
 - Classical signatures
 - The effects of rotation

4 Conclusions and Outlook

• After formation, classically nothing else happens! (if BH relatively slowly rotating, otherwise instabilities).

$$ds^{2} = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^{2} + \frac{2a\mu\sin^{2}\theta}{\Sigma r^{n-1}} dt d\phi - \frac{\Sigma}{\Delta} dr^{2} - \sum d\theta^{2} - \left(r^{2} + a^{2} + \frac{a^{2}\mu\sin^{2}\theta}{\Sigma r^{n-1}}\right) \sin^{2}\theta d\phi^{2} - r^{2}\cos^{2}\theta d\Omega_{n}^{2},$$

- 1974, Hawking's quantum instability \Rightarrow BH decays $10^{-26}s$.
- Gravity couples Universally

<i>s</i> = 0	Higgs + W_L/Z_L	4
<i>s</i> = 1/2		
<i>s</i> = 1		

$$S_{brane} = \int d^4x \sqrt{|g|} \left(-f^4 + \mathcal{L}_0 + \mathcal{L}_{\frac{1}{2}} + \mathcal{L}_1 \right)$$

• After formation, classically nothing else happens! (if BH relatively slowly rotating, otherwise instabilities).

$$ds^{2} = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^{2} + \frac{2a\mu\sin^{2}\theta}{\Sigma r^{n-1}} dt d\phi - \frac{\Sigma}{\Delta} dr^{2} - \sum d\theta^{2} - \left(r^{2} + a^{2} + \frac{a^{2}\mu\sin^{2}\theta}{\Sigma r^{n-1}}\right) \sin^{2}\theta d\phi^{2} - r^{2}\cos^{2}\theta d\Omega_{n}^{2},$$

- 1974, Hawking's quantum instability \Rightarrow BH decays $10^{-26}s$.
- Gravity couples Universally

s = 0	Higgs + W_L/Z_L	4
<i>s</i> = 1/2		
<i>s</i> = 1		

$$S_{brane} = \int d^4x \sqrt{|g|} \left(-f^4 + \mathcal{L}_0 + \mathcal{L}_{\frac{1}{2}} + \mathcal{L}_1 \right)$$

• After formation, classically nothing else happens! (if BH relatively slowly rotating, otherwise instabilities).

$$ds^{2} = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^{2} + \frac{2a\mu \sin^{2}\theta}{\Sigma r^{n-1}} dt d\phi - \frac{\Sigma}{\Delta} dr^{2} - \sum d\theta^{2} - \left(r^{2} + a^{2} + \frac{a^{2}\mu \sin^{2}\theta}{\Sigma r^{n-1}}\right) \sin^{2}\theta d\phi^{2} - r^{2} \cos^{2}\theta d\Omega_{n}^{2},$$

- 1974, Hawking's quantum instability \Rightarrow BH decays $10^{-26}s$.
- Gravity couples Universally

s = 0	Higgs + W_L/Z_L	4
<i>s</i> = 1/2		
<i>s</i> = 1		

$$S_{brane} = \int d^4x \sqrt{|g|} \left(-f^4 + \mathcal{L}_0 + \mathcal{L}_{\frac{1}{2}} + \mathcal{L}_1 \right)$$

• After formation, classically nothing else happens! (if BH relatively slowly rotating, otherwise instabilities).

$$ds^{2} = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^{2} + \frac{2a\mu \sin^{2}\theta}{\Sigma r^{n-1}} dt d\phi - \frac{\Sigma}{\Delta} dr^{2} - \sum d\theta^{2} - \left(r^{2} + a^{2} + \frac{a^{2}\mu \sin^{2}\theta}{\Sigma r^{n-1}}\right) \sin^{2}\theta d\phi^{2} - r^{2} \cos^{2}\theta d\Omega_{n}^{2},$$

• 1974, Hawking's quantum instability \Rightarrow BH decays $10^{-26}s$.

• Gravity couples Universally

s = 0	Higgs + W_L/Z_L	4
<i>s</i> = 1/2		
<i>s</i> = 1		

$$S_{brane} = \int d^4x \sqrt{|g|} \left(-f^4 + \mathcal{L}_0 + \mathcal{L}_{\frac{1}{2}} + \mathcal{L}_1 \right)$$

• After formation, classically nothing else happens! (if BH relatively slowly rotating, otherwise instabilities).

$$ds^{2} = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^{2} + \frac{2a\mu \sin^{2}\theta}{\Sigma r^{n-1}} dt d\phi - \frac{\Sigma}{\Delta} dr^{2} - \sum d\theta^{2} - \left(r^{2} + a^{2} + \frac{a^{2}\mu \sin^{2}\theta}{\Sigma r^{n-1}}\right) \sin^{2}\theta d\phi^{2} - r^{2} \cos^{2}\theta d\Omega_{n}^{2},$$

- 1974, Hawking's quantum instability \Rightarrow BH decays $10^{-26}s$.
- Gravity couples Universally

<i>s</i> = 0	Higgs + W_L/Z_L	4
<i>s</i> = 1/2	Quarks + Leptons	90
<i>s</i> = 1	$G + \gamma + W_T/Z_T$	24

$$S_{brane} = \int d^4x \sqrt{|g|} \left(-f^4 + \mathcal{L}_0 + \mathcal{L}_{\frac{1}{2}} + \mathcal{L}_1 \right)$$

 After formation, classically nothing else happens! (if BH relatively slowly rotating, otherwise instabilities).

$$ds^{2} = \left(1 - \frac{\mu}{\Sigma r^{n-1}}\right) dt^{2} + \frac{2a\mu\sin^{2}\theta}{\Sigma r^{n-1}} dt d\phi - \frac{\Sigma}{\Delta} dr^{2} - \sum d\theta^{2} - \left(r^{2} + a^{2} + \frac{a^{2}\mu\sin^{2}\theta}{\Sigma r^{n-1}}\right) \sin^{2}\theta d\phi^{2} - r^{2}\cos^{2}\theta d\Omega_{n}^{2},$$

- 1974, Hawking's quantum instability \Rightarrow BH decays $10^{-26}s$.
- Gravity couples Universally

<i>s</i> = 0	Higgs + W_L/Z_L	4
<i>s</i> = 1/2	Quarks + Leptons	90
<i>s</i> = 1	$\mathbf{G} + \gamma + W_T / Z_T$	24

$$S_{brane} = \int d^4x \sqrt{|g|} \left(-f^4 + \mathcal{L}_0 + \mathcal{L}_{\frac{1}{2}} + \mathcal{L}_1 \right)$$

- The decay can be described through Hawking radiation
- The **time between emission** of one particle is **large** (true for large BH mass)
- Brane emission is dominant (large number of SM degrees of freedom)
- **Backreaction** of the metric between emissions is **small** (true for large BH mass)

• The decay can be described through Hawking radiation

- The time between emission of one particle is large (true for large BH mass)
- Brane emission is dominant (large number of SM degrees of freedom)
- **Backreaction** of the metric between emissions is **small** (true for large BH mass)

- The decay can be described through Hawking radiation
- The time between emission of one particle is large (true for large BH mass)
- Brane emission is dominant (large number of SM degrees of freedom)
- **Backreaction** of the metric between emissions is **small** (true for large BH mass)

- The decay can be described through Hawking radiation
- The time between emission of one particle is large (true for large BH mass)
- Brane emission is dominant (large number of SM degrees of freedom)
- Backreaction of the metric between emissions is small (true for large BH mass)
Some of the underlying assumptions!

- The decay can be described through Hawking radiation
- The time between emission of one particle is large (true for large BH mass)
- Brane emission is dominant (large number of SM degrees of freedom)
- Backreaction of the metric between emissions is small (true for large BH mass)

$$\frac{\mathsf{d}\mathsf{E}_{h}}{\mathsf{d}\mathsf{t}\mathsf{d}\omega\mathsf{d}\Omega} = \sum_{m,j} \frac{\omega}{2\pi} \frac{\mathbb{T}_{k}^{(n)}(\omega, a_{*})}{\exp(\frac{\omega - m\Omega_{H}}{T_{H}}) \pm 1} \left|{}_{h}S_{j}^{m}(\Omega, \omega a_{*})\right|^{2}$$

• Dependence on *a**

$${f T}_{f H}=rac{(n\!+\!1)\!+\!(n\!-\!1)a_{*}^{2}}{4\pi(1\!+\!a_{*}^{2})r_{H}},\,\Omega_{H}\!=rac{1}{r_{H}}rac{a_{*}}{1\!+\!a_{*}^{2}}$$

- Spheroidal angular functions
- Harder spectrum
- 2 $m = j \ (m > 0)$ dominant \Rightarrow Spin-down
- Similar for Scalars and Vectors

$$\frac{\mathsf{d}\mathsf{E}_{h}}{\mathsf{d}\mathsf{t}\mathsf{d}\omega\mathsf{d}\Omega} = \sum_{m,j} \frac{\omega}{2\pi} \frac{\mathbb{T}_{k}^{(n)}(\omega, \boldsymbol{a}_{*})}{\exp(\frac{\omega - m\Omega_{H}}{T_{H}}) \pm 1} \left| {}_{h}S_{j}^{m}(\Omega, \omega \boldsymbol{a}_{*}) \right|^{2}$$

Dependence on a_{*}

 $\mathbf{T}_{\mathbf{H}} = \frac{(\mathbf{n}+1)+(\mathbf{n}-1)\mathbf{a}_{*}^{2}}{4\pi(1+\mathbf{a}_{*}^{2})\mathbf{r}_{\mathbf{H}}}, \ \Omega_{H} = \frac{1}{\mathbf{r}_{\mathbf{H}}}\frac{\mathbf{a}_{*}}{1+\mathbf{a}_{*}^{2}}$ • Spheroidal angular functions

Harder spectrum

- (a) $m = j \ (m > 0)$ dominant \Rightarrow Spin-down
- Similar for Scalars and Vectors

$$\frac{\mathsf{d}\mathsf{E}_{h}}{\mathsf{d}\mathsf{t}\mathsf{d}\omega\mathsf{d}\Omega} = \sum_{m,j} \frac{\omega}{2\pi} \frac{\mathbb{T}_{k}^{(n)}(\omega, a_{*})}{\exp(\frac{\omega - m\Omega_{H}}{T_{H}}) \pm 1} \left|{}_{h}S_{j}^{m}(\Omega, \omega a_{*})\right|^{2}$$

• Dependence on a_{*}

 $\textbf{T}_{\textbf{H}} = \frac{(n+1) + (n-1)a_{*}^{2}}{4\pi(1+a_{*}^{2})\textbf{r}_{\textbf{H}}}, \, \Omega_{\textbf{H}} = \frac{1}{\textbf{r}_{\textbf{H}}} \frac{a_{*}}{1+a_{*}^{2}}$

- Spheroidal angular functions
- Harder spectrum
- 2 $m = j \ (m > 0)$ dominant \Rightarrow Spin-down
- Similar for Scalars and Vectors

$$\frac{\mathsf{d}\mathsf{E}_{h}}{\mathsf{d}\mathsf{t}\mathsf{d}\omega\mathsf{d}\Omega} = \sum_{m,j} \frac{\omega}{2\pi} \frac{\mathbb{T}_{k}^{(n)}(\omega, a_{*})}{\exp(\frac{\omega - m\Omega_{H}}{T_{H}}) \pm 1} \left|{}_{h}S_{j}^{m}(\Omega, \omega a_{*})\right|^{2}$$

Dependence on a_{*}

$$\mathbf{T}_{\mathbf{H}} = rac{(\mathbf{n+1})+(\mathbf{n-1})\mathbf{a}_{*}^{2}}{4\pi(\mathbf{1}+\mathbf{a}_{*}^{2})\mathbf{r}_{\mathbf{H}}}, \, \Omega_{H} = rac{1}{\mathbf{r}_{\mathbf{H}}}rac{\mathbf{a}_{*}}{\mathbf{1}+\mathbf{a}_{*}^{2}}$$

- Spheroidal angular functions
- Harder spectrum
- 2 $m = j \ (m > 0)$ dominant \Rightarrow Spin-down
- Similar for Scalars and Vectors





- Dependence on a_{*}
- $\textbf{T}_{\textbf{H}} = \tfrac{(\textbf{n+1}) + (\textbf{n-1})\textbf{a}_{*}^{2}}{4\pi(\textbf{1} + \textbf{a}_{*}^{2})\textbf{r}_{\textbf{H}}}, \, \Omega_{\textbf{H}} = \tfrac{\textbf{1}}{\textbf{r}_{\textbf{H}}} \tfrac{\textbf{a}_{*}}{\textbf{1} + \textbf{a}_{*}^{2}}$
- Spheroidal angular functions
- Harder spectrum
- $m = j \ (m > 0) \ dominant$ $\Rightarrow Spin-down$
- Similar for Scalars and Vectors



Hawking radiation – Angular spectrum

- High rotation makes angular distributions equatorial.
- Output: Book and the second second
 - Each peak comes from different polarisation contributions.
 - Study of asymmetries in vector boson decays.
 - Similar effect for fermions.



M. Casals, S. R. Dolan, P. Kanti and E. Winstanley, JHEP 0703 (2007) 019 [hep-th/0608193]

Hawking radiation – Angular spectrum

- High rotation makes angular distributions equatorial.
 - However note lower energy vectors with axial peaks!:
 - Each peak comes from different polarisation contributions.
 - Study of asymmetries in vector boson decays.
 - Similar effect for fermions.



M. Casals, S. R. Dolan, P. Kanti and E. Winstanley, JHEP 0703 (2007) 019 [hep-th/0608193]

Hawking radiation – Angular spectrum

- High rotation makes angular distributions equatorial.
- Output: Book and the second second
 - Each peak comes from different polarisation contributions.
 - Study of asymmetries in vector boson decays.
 - Similar effect for fermions.



M. Casals, S. R. Dolan, P. Kanti and E. Winstanley, JHEP 0703 (2007) 019 [hep-th/0608193]



Non-rotating

• Mass drops linearly.

•
$$T \sim r_S^{-1} \rightarrow$$
 speed up last $\sim 15\% t$.



Non-rotating

- Mass drops linearly.
- $T \sim r_S^{-1} \rightarrow$ speed up last $\sim 15\% t$.

Rotating

- Initial spin down $\sim 15\% t$. J drops by $\sim 80\%$. M drops by $\sim 30\%$
- Followed by a Schwarzchild phase.
- Note: BH with large *M* and *J*. More semi-classical, Spins down efficiently Maybe not the case for most BHs @ LHC



Non-rotating

- Mass drops linearly.
- $T \sim r_S^{-1} \rightarrow$ speed up last $\sim 15\% t$.

Rotating

- Initial spin down $\sim 15\% t$. J drops by $\sim 80\%$. M drops by $\sim 30\%$
- Followed by a Schwarzchild phase.
- Note: BH with large *M* and *J*.
 More semi-classical, Spins down efficiently
 Maybe not the case for most BHs @ LHC

















Outline



Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes

2 Modelling BH events – CHARYBDIS2

- The production
- The decay
- CHARYBDIS2 & other generators
- 3 Phenomenology using CHARYBDIS2
 - Classical signatures
 - The effects of rotation

4 Conclusions and Outlook

BH event generators

J = 0 generators

- TRUENOIR: Fixed T, no T⁽ⁿ⁾_k.
 S. Dimopoulos et al. hep-ph/0106295
- CHARYBDIS1: Variable T, no T⁽ⁿ⁾
 C. M. Harris et al. hep-ph/0307305
- CATFISH: Energy loss, variable T, no $\mathbb{T}_k^{(n)}$. Cavaglia et al. hep-ph/0609001

$J \neq 0$ generators

- BlackMax: Energy loss, variable T, split branes, $\mathbb{T}_k^{(n)}$. Dai et al. arXiv:0711.3012
- CHARYBDIS2: Energy loss model, polarisation, variable T, remnant options, $\mathbb{T}_{k}^{(n)}$.

J. A. Frost, J. R. Gaunt, MS, M. Casals, S. R. Dolan, M. A. Parker and

B. R. Webber, arXiV:0904.0979 http://projects.hepforge.org/charybdis2/

BH event generators

J = 0 generators

- TRUENOIR: Fixed T, no T⁽ⁿ⁾_k.
 S. Dimopoulos et al. hep-ph/0106295
- CHARYBDIS1: Variable T, no T⁽ⁿ⁾
 C. M. Harris et al. hep-ph/0307305
- CATFISH: Energy loss, variable T, no $\mathbb{T}_{k}^{(n)}$. Cavaglia et al. hep-ph/0609001

$J \neq 0$ generators

- BlackMax: Energy loss, variable T, split branes, $\mathbb{T}_k^{(n)}$. Dai et al. arXiv:0711.3012
- CHARYBDIS2: Energy loss model, polarisation, variable T, remnant options, $\mathbb{T}_{k}^{(n)}$.

J. A. Frost, J. R. Gaunt, MS, M. Casals, S. R. Dolan, M. A. Parker and

B. R. Webber, arXiV:0904.0979 http://projects.hepforge.org/charybdis2/

BH event generators

J = 0 generators

- TRUENOIR: Fixed T, no T⁽ⁿ⁾_k.
 S. Dimopoulos et al. hep-ph/0106295
- CHARYBDIS1: Variable T, no T⁽ⁿ⁾
 C. M. Harris et al. hep-ph/0307305
- CATFISH: Energy loss, variable T, no $\mathbb{T}_k^{(n)}$. Cavaglia et al. hep-ph/0609001

$J \neq 0$ generators

- BlackMax: Energy loss, variable T, split branes, $\mathbb{T}_{k}^{(n)}$. Dai et al. arXiv:0711.3012
- CHARYBDIS2: Energy loss model, polarisation, variable T, remnant options, $\mathbb{T}_{k}^{(n)}$.

J. A. Frost, J. R. Gaunt, MS, M. Casals, S. R. Dolan, M. A. Parker and

B. R. Webber, arXiV:0904.0979 http://projects.hepforge.org/charybdis2/

Outline



Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes
- 2 Modelling BH events CHARYBDIS2
 - The production
 - The decay
 - CHARYBDIS2 & other generators
- Phenomenology using CHARYBDIS2
 Classical signatures
 - The effects of rotation

4 Conclusions and Outlook







- High multiplicity events with large number or jets. In the SM, SUSY and other BSM models this is usually suppressed. Even more if also leptons are present.
- Very High *P_T* tails.
- Allow for large boost particles.
- For high multiplicity events, virtually any combination of particles in the final state.

- High multiplicity events with large number or jets. In the SM, SUSY and other BSM models this is usually suppressed. Even more if also leptons are present.
- Very High *P_T* tails.
- Allow for large boost particles.
- For high multiplicity events, virtually any combination of particles in the final state.

- High multiplicity events with large number or jets. In the SM, SUSY and other BSM models this is usually suppressed. Even more if also leptons are present.
- Very High P_T tails.
- Allow for large boost particles.
- For high multiplicity events, virtually any combination of particles in the final state.

- High multiplicity events with large number or jets. In the SM, SUSY and other BSM models this is usually suppressed. Even more if also leptons are present.
- Very High P_T tails.
- Allow for large boost particles.
- For high multiplicity events, virtually any combination of particles in the final state.

- High multiplicity events with large number or jets. In the SM, SUSY and other BSM models this is usually suppressed. Even more if also leptons are present.
- Very High P_T tails.
- Allow for large boost particles.
- For high multiplicity events, virtually any combination of particles in the final state.

Outline



Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes
- 2 Modelling BH events CHARYBDIS2
 - The production
 - The decay
 - CHARYBDIS2 & other generators
- Operation State State
 - Classical signatures
 - The effects of rotation

Conclusions and Outlook
Cross section



Rotation effects – Final state particles

Rotation effects – Final state particles



Rotation effects - Final state particles



Rotation effects – Final state particles



Massless approximation in the generator! \rightarrow **But** $m_H, m_W, m_Z, m_t \sim 0.1$ **TeV**

Gauge charges of the particles not included in the Hawking radiation calculations.

MOPS, JHEP 10 (2009) 008 [arXiv:0907.5107] MOPS, JHEP 02 (2010) 042 [arXiv:0911.0688]

Not clear whether **graviton emission** will be enhanced with rotation and compete with brane emission.

→ **need full numerical analysis** of gravitons.

Massless approximation in the generator! \rightarrow **But** m_H , m_W , m_Z , $m_t \sim 0.1$ **TeV**

Gauge charges of the particles not included in the **Hawking** radiation calculations.

MOPS, JHEP 10 (2009) 008 [arXiv:0907.5107] MOPS, JHEP 02 (2010) 042 [arXiv:0911.0688]

Not clear whether **graviton emission** will be enhanced with rotation and compete with brane emission.

Massless approximation in the generator!

 \rightarrow But $m_H, m_W, m_Z, m_t \sim 0.1$ TeV

Gauge charges of the particles not included in the Hawking radiation calculations.

MOPS, JHEP 10 (2009) 008 [arXiv:0907.5107] MOPS, JHEP 02 (2010) 042 [arXiv:0911.0688]

Not clear whether **graviton emission** will be enhanced with rotation and compete with brane emission.

Massless approximation in the generator!

 \rightarrow But $m_H, m_W, m_Z, m_t \sim 0.1$ TeV

Gauge charges of the particles not included in the Hawking radiation calculations.

MOPS, JHEP 10 (2009) 008 [arXiv:0907.5107] MOPS, JHEP 02 (2010) 042 [arXiv:0911.0688]

Not clear whether **graviton emission** will be enhanced with rotation and compete with brane emission.

Massless approximation in the generator!

 \rightarrow But $m_H, m_W, m_Z, m_t \sim 0.1$ TeV

Gauge charges of the particles not included in the Hawking radiation calculations.

MOPS, JHEP 10 (2009) 008 [arXiv:0907.5107] MOPS, JHEP 02 (2010) 042 [arXiv:0911.0688]

Not clear whether **graviton emission** will be enhanced with rotation and compete with brane emission.

Introduction

- The hierarchy problem Extra dimensions
- Strong gravity & Black Holes
- 2 Modelling BH events CHARYBDIS2
 - The production
 - The decay
 - CHARYBDIS2 & other generators
- 3 Phenomenology using CHARYBDIS2
 - Classical signatures
 - The effects of rotation



- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy → classical signatures roughly remain.
 - Potential of rotation: angular correlations (to be explored).
- Future work will involve detailed studies of:
 - asymmetries and angular correlations,
 - refinement of the modelling of production and evaporation to include mass and charge effects, etc...
 - Studies of exclusion limits for the various LHC runs.

- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - 2 The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy → classical signatures roughly remain.
 - 2 Potential of rotation: angular correlations (to be explored).
- Future work will involve detailed studies of:
 - asymmetries and angular correlations,
 - refinement of the modelling of production and evaporation to include mass and charge effects, etc...
 - Studies of exclusion limits for the various LHC runs.

- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy → classical signatures roughly remain.
 - Potential of rotation: angular correlations (to be explored).

- asymmetries and angular correlations,
- refinement of the modelling of production and evaporation to include mass and charge effects, etc...
- Studies of exclusion limits for the various LHC runs.

- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy \rightarrow classical signatures roughly remain.
 - Potential of rotation: angular correlations (to be explored).

- asymmetries and angular correlations,
- refinement of the modelling of production and evaporation to include mass and charge effects, etc...
- Studies of exclusion limits for the various LHC runs.

- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy \rightarrow classical signatures roughly remain.
 - Potential of rotation: angular correlations (to be explored).

- asymmetries and angular correlations,
- refinement of the modelling of production and evaporation to include mass and charge effects, etc...
- Studies of exclusion limits for the various LHC runs.

- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy \rightarrow classical signatures roughly remain.
 - Potential of rotation: angular correlations (to be explored).
- Future work will involve detailed studies of:
 - asymmetries and angular correlations,
 - refinement of the modelling of production and evaporation to include mass and charge effects, etc...
 - Studies of exclusion limits for the various LHC runs.

- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy \rightarrow classical signatures roughly remain.
 - Potential of rotation: angular correlations (to be explored).

- asymmetries and angular correlations,
- refinement of the modelling of production and evaporation to include mass and charge effects, etc...
- Studies of exclusion limits for the various LHC runs.

- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy \rightarrow classical signatures roughly remain.
 - Potential of rotation: angular correlations (to be explored).

- asymmetries and angular correlations,
- refinement of the modelling of production and evaporation to include mass and charge effects, etc...
- Studies of exclusion limits for the various LHC runs.

- We have described the Physics of production and decay of BHs in theories with extra dimensions, in CHARYBDIS2.
- We have looked into some interesting effects/models:
 - M and J distributions at formation;
 - The effect of rotation on energy and angular distributions;
- Phenomenologically:
 - Large cross sections, large multiplicities, large P_T and missing energy \rightarrow classical signatures roughly remain.
 - Potential of rotation: angular correlations (to be explored).

Future work will involve detailed studies of:

- asymmetries and angular correlations,
- refinement of the modelling of production and evaporation to include mass and charge effects, etc...
- Studies of exclusion limits for the various LHC runs.

Thanks for your attention! Questions?

BACKUP

More effects of rotation – Species

Enhancement of Vector emission



Look at radiative corrections to Higgs mass:





Look at radiative corrections to Higgs mass:





Higgs mass runs from high scale:

$$\delta m_h^2 = \left(|\lambda_f|^2 - \frac{1}{2}\lambda \right) \frac{\Lambda_{\rm cutoff}^2}{8\pi^2} + \dots$$

Look at radiative corrections to Higgs mass:





Higgs mass runs from high scale:

$$\delta m_h^2 = \left(|\lambda_f|^2 - \frac{1}{2}\lambda \right) \frac{\Lambda_{\text{cutoff}}^2}{8\pi^2} + \dots$$

If $\Lambda_{\rm cutoff} \sim M_4 \sim 10^{16} \text{ TeV} \Rightarrow$ fine tuning of $\sim 10^{-16}$

• Arrange cancellation of quadratic divergences.

 \Rightarrow New particles: SUSY, Little Higgs, etc...

Arrange cancellation of quadratic divergences.

 \Rightarrow New particles: SUSY, Little Higgs, etc...

Change the running to exponential.

 \Rightarrow Strong dynamics: the Higgs is a pion field of a new strongly coupled sector.

Arrange cancellation of quadratic divergences.

 \Rightarrow New particles: SUSY, Little Higgs, etc...

Change the running to exponential.

 \Rightarrow Strong dynamics: the Higgs is a pion field of a new strongly coupled sector.

Assume the fundamental Planck scale is 1 TeV.

 \Rightarrow Extra dimensions.

Arrange cancellation of quadratic divergences.

 \Rightarrow New particles: SUSY, Little Higgs, etc...

Change the running to exponential.

 \Rightarrow Strong dynamics: the Higgs is a pion field of a new strongly coupled sector.

Solution 3 Assume the fundamental Planck scale is 1 TeV.

 \Rightarrow Extra dimensions.



Bounds on extra dimensions

$\mathbf{M}_{\mathbf{PI}}^{2} = \mathbf{R}^{\mathbf{n}} \mathbf{M}_{(4+\mathbf{n})}^{2+\mathbf{n}}$	R in μm (n = 2)	$M_{4+n} \sim 1 \text{TeV OK}$
Deviations from r ⁻² in torsion-balance	\lesssim 55	<i>n</i> > 1
KK graviton produc- tion @ colliders	\lesssim 800	n > 2
KK graviton produc- tion in Supernovae	$\lesssim 5.1 imes 10^{-4}$	n > 3
KK gravitons early Universe production	\lesssim 2.2 $ imes$ 10 ⁻⁵	n > 3



For the **classical approximation** for production to be valid we need the wavelength of each colliding particle to be small compared to the interaction length.

For the **classical approximation** for production to be valid we need the wavelength of each colliding particle to be small compared to the interaction length.

$$\Delta x \sim rac{1}{p} \ll r_S$$

For the **classical approximation** for production to be valid we need the wavelength of each colliding particle to be small compared to the interaction length.

$$\Delta x \sim \frac{1}{p} \ll r_S$$

But:

• $p \text{ large} \Rightarrow \Delta x \text{ small}$
Classical approximation

For the **classical approximation** for production to be valid we need the wavelength of each colliding particle to be small compared to the interaction length.

$$\Delta x \sim \frac{1}{p} \ll r_S$$

But:

- $p \text{ large} \Rightarrow \Delta x \text{ small}$
- $p \text{ large} \Rightarrow \sqrt{s} \equiv E_{CM} \text{ large} \Rightarrow r_S \text{ large}$

Classical approximation

For the **classical approximation** for production to be valid we need the wavelength of each colliding particle to be small compared to the interaction length.

$$\Delta x \sim \frac{1}{p} \ll r_S$$

But:

- $p \text{ large} \Rightarrow \Delta x \text{ small}$
- $p \text{ large} \Rightarrow \sqrt{s} \equiv E_{CM} \text{ large} \Rightarrow r_S \text{ large}$

The condition is satisfied when $\sqrt{s} \gg M_{4+n}$ (trans-Planckian).

Classical approximation

For the **classical approximation** for production to be valid we need the wavelength of each colliding particle to be small compared to the interaction length.

$$\Delta x \sim \frac{1}{p} \ll r_S$$

But:

• $p \text{ large} \Rightarrow \Delta x \text{ small}$

• $p \text{ large} \Rightarrow \sqrt{s} \equiv E_{CM} \text{ large} \Rightarrow r_S \text{ large}$

The condition is satisfied when $\sqrt{s} \gg M_{4+n}$ (trans-Planckian).

Also quantum gravity approximations indicate small corrections:

- T. Banks and W. Fischler, hep-th/9906038
- S. N. Solodukhin, hep-ph/0201248
- S. D. H. Hsu, hep-ph/0203154

• During formation we should have an asymmetric BH with electric and gravitational multipole moments.

 \rightarrow Distorted geometry.

- The time for loss of multipoles is **r**_S (natural units).
- We will look next into the Hawking decay and realise that the typical timescale there is

$$\Delta t \sim r_{S} \left(\frac{M_{BH}}{M_{4+n}} \right)^{\frac{n+2}{n+1}} \gg r_{S} \; . \label{eq:delta_state}$$

We assume a **quick loss of asymmetries** ⇒ BH settles down to a stationary axisymmetric solu

 During formation we should have an asymmetric BH with electric and gravitational multipole moments.

\rightarrow Distorted geometry.



- The time for loss of multipoles is **r**_S (natural units).
- We will look next into the Hawking decay and realise that the typical timescale there is

$$\Delta t \sim r_{S} \left(\frac{M_{BH}}{M_{4+n}} \right)^{\frac{n+2}{n+1}} \gg r_{S} \; . \label{eq:delta_state}$$

We assume a **quick loss of asymmetries** ⇒ BH settles down to a stationary axisymmetric s

• During formation we should have an asymmetric BH with electric and gravitational multipole moments.

 \rightarrow Distorted geometry.



- The time for loss of multipoles is r_s (natural units).
- We will look next into the Hawking decay and realise that the typical timescale there is

$$\Delta t \sim r_{S} \left(\frac{M_{BH}}{M_{4+n}} \right)^{\frac{n+2}{n+1}} \gg r_{S} \; . \label{eq:delta_state}$$

We assume a **quick loss of asymmetries**

 \Rightarrow BH settles down to a stationary axisymmetric solution.

• During formation we should have an asymmetric BH with electric and gravitational multipole moments.

 \rightarrow Distorted geometry.



- The time for loss of multipoles is r_s (natural units).
- We will look next into the Hawking decay and realise that the typical timescale there is

$$\Delta t \sim r_{S} \left(\frac{M_{BH}}{M_{4+n}} \right)^{\frac{n+2}{n+1}} \gg r_{S} \; . \label{eq:delta_states}$$

We assume a **quick loss of asymmetries** ⇒ BH settles down to a stationary axisymmetric solutio

• During formation we should have an asymmetric BH with electric and gravitational multipole moments.

 \rightarrow Distorted geometry.



- The time for loss of multipoles is r_S (natural units).
- We will look next into the Hawking decay and realise that the typical timescale there is

$$\Delta t \sim r_{S} \left(\frac{M_{BH}}{M_{4+n}} \right)^{\frac{n+2}{n+1}} \gg r_{S} \; . \label{eq:delta_state}$$

We assume a quick loss of asymmetries

 \Rightarrow BH settles down to a stationary axisymmetric solution.

More effects of rotation – BH parameters

More effects of rotation – BH parameters



More effects of rotation – BH parameters





Angular correlations - In progress ...

- Extract angular correlations by forming correlators of the type $x_{i,j} = \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{|\mathbf{p}_i||\mathbf{p}_i|}$ in the frame of the initial BH.
- Why?

Angular correlations – In progress ...

- Extract angular correlations by forming correlators of the type $x_{i,j} = \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{\|\mathbf{p}_i\|\|\mathbf{p}_j\|}$ in the frame of the initial BH.
- Why?

Angular correlations – In progress ...

- Extract angular correlations by forming correlators of the type $x_{i,j} = \frac{\mathbf{P}_i \cdot \mathbf{P}_j}{|\mathbf{P}_i||\mathbf{P}_i|}$ in the frame of the initial BH.
- Why?

Hawking radiation – Angular spectrum

- High rotation makes angular distributions equatorial.
- Output: Book and the second second
 - Each peak comes from different polarisation contributions.
 - Study of asymmetries in vector boson decays.
 - Similar effect for fermions.



M. Casals, S. R. Dolan, P. Kanti and E. Winstanley, JHEP 0703 (2007) 019 [hep-th/0608193]

Angular correlations – In progress ...

• Extract angular correlations by forming correlators of the type $x_{i,j} = \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{\|\mathbf{p}_i\|\|\mathbf{p}_j\|}$ in the frame of the initial BH.

• Why?



- BH settles down to (4+n)D Myers-Perry rotating BH.
- Mass M and angular momentum J as seen from infinity.

- Typical size/curvature of the horizon is r_H.
- Spheroidal horizon with oblateness a.

$$\frac{x^2 + y^2}{1 + {\mathbf{a}_*}^2} + z^2 = {\mathbf{r}_{\mathsf{H}}}^2 \,.$$

- BH settles down to (4+n)D Myers-Perry rotating BH.
- Mass M and angular momentum J as seen from infinity.

- Typical size/curvature of the horizon is r_H.
- Spheroidal horizon with oblateness a.

$$\frac{x^2 + y^2}{1 + {\mathbf{a}_*}^2} + z^2 = {\mathbf{r}_{\mathsf{H}}}^2 \,.$$

- BH settles down to (4+n)D Myers-Perry rotating BH.
- Mass M and angular momentum J as seen from infinity.

Geometrical properties

- Typical size/curvature of the horizon is r_H.
- Spheroidal horizon with oblateness a*

$$\frac{x^2 + y^2}{1 + {\mathbf{a}_*}^2} + z^2 = {\mathbf{r}_{\mathsf{H}}}^2 \,.$$

- BH settles down to (4+n)D Myers-Perry rotating BH.
- Mass M and angular momentum J as seen from infinity.

Geometrical properties

- Typical size/curvature of the horizon is r_H.
- Spheroidal horizon with oblateness a.

$$\frac{x^2 + y^2}{1 + {\bf a_*}^2} + z^2 = {\bf r_H}^2 \,.$$

- BH settles down to (4+n)D Myers-Perry rotating BH.
- Mass M and angular momentum J as seen from infinity.

Geometrical properties

- Typical size/curvature of the horizon is r_H.
- Spheroidal horizon with oblateness **a***

$$\frac{x^2 + y^2}{1 + {\bf a}_*^2} + z^2 = {\bf r}_{\rm H}^2 \,.$$



• When $M_{BH} < M_{4+n}$, we reach the **quantum gravity** regime which is not known.

 \rightarrow A model must be provided.

• To make robust predictions we must try to minimise the effect of this final stage.

 \rightarrow Cut on events with M_{BH} well above $M_{4+n} \sim 1 \,\mathrm{TeV}$.

 A remnant fixed N-body phase space decay is performed in CHARYBDIS1 if M_{BH} < M_{4+n}.
 In addition If KINCUT=.TRUE. this occurs earlier, if a kinematically disallowed energy is selected.

When M_{BH} < M_{4+n}, we reach the quantum gravity regime which is not known.

\rightarrow A model must be provided.

• To make robust predictions we must try to minimise the effect of this final stage.

 \rightarrow Cut on events with M_{BH} well above $M_{4+n} \sim 1 \,\mathrm{TeV}$.

 A remnant fixed N-body phase space decay is performed in CHARYBDIS1 if M_{BH} < M_{4+n}.
 In addition If KINCUT=.TRUE. this occurs earlier, if a kinematically disallowed energy is selected.

When M_{BH} < M_{4+n}, we reach the quantum gravity regime which is not known.

 \rightarrow A model must be provided.

• To make robust predictions we must try to minimise the effect of this final stage.

 \rightarrow Cut on events with M_{BH} well above $M_{4+n} \sim 1 \text{TeV}$.

 A remnant fixed N-body phase space decay is performed in CHARYBDIS1 if M_{BH} < M_{4+n}.
 In addition If KINCUT=. TRUE. this occurs earlier, if a

kinematically disallowed energy is selected.

When M_{BH} < M_{4+n}, we reach the quantum gravity regime which is not known.

 \rightarrow A model must be provided.

• To make robust predictions we must try to minimise the effect of this final stage.

 \rightarrow Cut on events with M_{BH} well above $M_{4+n} \sim 1 \text{TeV}$.

• A remnant fixed N-body phase space decay is performed in CHARYBDIS1 if $M_{BH} < M_{4+n}$.

In addition If KINCUT=.TRUE. this occurs earlier, if a kinematically disallowed energy is selected.

When M_{BH} < M_{4+n}, we reach the quantum gravity regime which is not known.

 \rightarrow A model must be provided.

• To make robust predictions we must try to minimise the effect of this final stage.

 \rightarrow Cut on events with M_{BH} well above $M_{4+n} \sim 1 \text{TeV}$.

 A remnant fixed N-body phase space decay is performed in CHARYBDIS1 if M_{BH} < M_{4+n}.
 In addition If KINCUT=.TRUE. this occurs earlier, if a kinematically disallowed energy is selected.

 When M_{BH} < M_{4+n}, we reach the quantum gravity regime which is not known.

 \rightarrow A model must be provided.

• To make robust predictions we must try to minimise the effect of this final stage.

 \rightarrow Cut on events with M_{BH} well above $M_{4+n} \sim 1 \text{TeV}$.

 A remnant fixed N-body phase space decay is performed in CHARYBDIS1 if M_{BH} < M_{4+n}.
 In addition If KINCUT=.TRUE. this occurs earlier, if a kinematically disallowed energy is selected.

New remnant models 1

Termination criteria:

- KINCUT: as before.
- INBODYAVERAGE: Go to remnant if,

$$\langle N \rangle \simeq Mr_H \frac{\sum_i g_i \left(\frac{1}{r_H} \frac{dN}{dt}\right)_i}{\sum_j g_j \left(\frac{dE}{dt}\right)_j} < NBODY - 1 = 1 \text{ or } 2, \dots$$

Remnant models

- Phase space constrained model (next slide).
- RMBOIL: Remnant evaporates at T_H = THWMAX. Motivated by string balls.

S. Dimopoulos et al. hep-ph/0108060

Stable remnant Q = 0, ±1.
 Motivated by modified uncertainty principle.
 B. Koch et al. hep-ph/0507138

New remnant models 1

Termination criteria:

- KINCUT: as before.
- NBODYAVERAGE: Go to remnant if,

$$\langle N \rangle \simeq Mr_H \frac{\sum_i g_i \left(\frac{1}{r_H} \frac{dN}{dt}\right)_i}{\sum_j g_j \left(\frac{dE}{dt}\right)_j} < NBODY - 1 = 1 \text{ or } 2, \dots$$

Remnant models

- Phase space constrained model (next slide).
- RMBOIL: Remnant evaporates at T_H = THWMAX. Motivated by string balls.

S. Dimopoulos et al. hep-ph/0108060

Stable remnant Q = 0, ±1.
 Motivated by modified uncertainty principle.
 B. Koch et al. hep-ph/0507138

New remnant models 1

Termination criteria:

- KINCUT: as before.
- NBODYAVERAGE: Go to remnant if,

$$\langle N \rangle \simeq Mr_H \frac{\sum_i g_i \left(\frac{1}{r_H} \frac{dN}{dt}\right)_i}{\sum_j g_j \left(\frac{dE}{dt}\right)_j} < NBODY - 1 = 1 \text{ or } 2, \dots$$

Remnant models

- Phase space constrained model (next slide).
- **RMBOIL:** Remnant evaporates at T_H = THWMAX. Motivated by string balls.

S. Dimopoulos et al. hep-ph/0108060

Stable remnant Q = 0, ±1.
 Motivated by modified uncertainty principle.
 B. Koch et al. hep-ph/0507138

NBODYVAR = .TRUE.: Choose multiplicity n + 1 using

$$P_{\delta t}(n) = e^{-\langle N \rangle} \frac{\langle N \rangle^n}{n!}$$

Motivated by sudden final burst approximation. $\langle N \rangle$ physically motivated.

NBODYPHASE = .FALSE.: Use constrained phase space

$$\mathrm{d}m{P}\propto\delta^{(4)}\left(\sum_{i}m{p}_{i}-m{P}_{BH}
ight)\prod_{i}
ho_{i}\left(E_{i},\Omega_{i}
ight)\mathrm{d}^{3}m{p}_{i}$$

with

$$ho_{l}\left(E_{l},\Omega_{l}
ight)=rac{\mathbb{T}_{k}^{\left(n
ight)}\left(E_{l}r_{H},a_{*}
ight)}{\exp(ilde{E}_{l}/T_{H})\pm1}\left|S_{k}(\Omega_{l})
ight|^{2}$$

NBODYVAR = .TRUE.: Choose multiplicity n + 1 using

$$P_{\delta t}(n) = e^{-\langle N \rangle} \frac{\langle N \rangle^n}{n!}$$

Motivated by sudden final burst approximation. $\langle \textit{N} \rangle$ physically motivated.

2 NBODYPHASE = .FALSE.: Use constrained phase space $dP \propto \delta^{(4)} \left(\sum_{i} p_{i} - P_{BH} \right) \prod_{i} \rho_{i} \left(E_{i}, \Omega_{i} \right) d^{3} \mathbf{p}_{i}$ with $\rho_{i} \left(E_{i}, \Omega_{i} \right) = \frac{T_{k}^{(0)} (E_{i} f_{H}, a_{*})}{\exp(E_{i} / T_{H}) \pm 1} |S_{k}(\Omega_{i})|^{2}$

1 NBODYVAR = .**TRUE**.: Choose multiplicity n + 1 using

$$P_{\delta t}(n) = e^{-\langle N \rangle} \frac{\langle N \rangle^n}{n!}$$

Motivated by sudden final burst approximation. $\langle \textit{N} \rangle$ physically motivated.

NBODYPHASE = .FALSE.: Use constrained phase space

$$\mathrm{d}\boldsymbol{P}\propto\delta^{(4)}\left(\sum_{i}\boldsymbol{p}_{i}-\boldsymbol{P}_{BH}\right)\prod_{i}\rho_{i}\left(\boldsymbol{E}_{i},\Omega_{i}\right)\mathrm{d}^{3}\mathbf{p}_{i}$$

with

$$\rho_i(\boldsymbol{E}_i, \boldsymbol{\Omega}_i) = \frac{\mathbb{T}_k^{(n)}(\boldsymbol{E}_i \boldsymbol{r}_H, \boldsymbol{a}_*)}{\exp(\tilde{\boldsymbol{E}}_i/T_H) \pm 1} |\boldsymbol{S}_k(\boldsymbol{\Omega}_i)|^2$$

1 NBODYVAR = .**TRUE**.: Choose multiplicity n + 1 using

$$P_{\delta t}(n) = e^{-\langle N \rangle} \frac{\langle N \rangle^n}{n!}$$

Motivated by sudden final burst approximation. $\langle N \rangle$ physically motivated.

NBODYPHASE = .FALSE.: Use constrained phase space

 $\mathrm{d} \boldsymbol{P} \propto \delta^{(4)} \left(\sum_{i} \boldsymbol{p}_{i} - \boldsymbol{P}_{BH}\right) \prod_{i} \rho_{i} \left(\boldsymbol{E}_{i}, \Omega_{i}\right) \mathrm{d}^{3} \mathbf{p}_{i}$

with

$$\rho_i(E_i,\Omega_i) = \frac{\mathbb{T}_k^{(n)}(E_i r_H,a_*)}{\exp(\tilde{E}_i/T_H) \pm 1} |S_k(\Omega_i)|^2$$