Automatic generation of Feynman rules with LanHEP

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Motivation

- LanHEP was developed since 1994 as a part of CompHEP project to help to create new physical models starting from the Lagrangian, the first goal was MSSM.
- A physical model in CompHEP is defined by the tables of parameters, particles and interaction vertices with implicit Lorentz structure.
- Flexible model format allows to introduce into CompHEP new gauge theories as well as various anomalous terms.
- This job is rather straightforward and can be done manually, it requires careful calculations and in the modern theories with many particles and vertices can lead to errors and misprints.
- Automatic tool is required.

A physical model in CompHEP is defined by the tables of parameters, particles and interaction vertices

Parameters

```
EE    |0.31345      |Electromagnetic coupling constant (<->1/127.9)
MW    |MZ*CW
```

Particles

```
photon       |A  |A  |2     |0     |0     |1    |G  |A
Z boson      |Z  |Z  |2     |MZ    |wZ    |1    |G  |Z
W boson      |W+ |W- |2     |MW    |wW    |1    |G  |W^+
electron     |e  |E  |1     |Me    |0     |1    |   |e
```

Vertices

```
E    |e    |A    |     |EE                          |G(m3)
E    |e    |H    |     |-EE*Me*ca/(2*MW*SW*cb)      |1
E    |e    |H3   |     |i*EE*Me*tb/(2*MW*SW)        |G5
E    |e    |Z    |     |EE/(2*S2W)                  |C2W*G(m3)*(1-G5)-2*SW^2*
E    |e    |Z.f  |     |-i*EE*Me/(2*MW*SW)          |G5
E    |e    |h    |     |EE*Me*sa/(2*MW*SW*cb)       |1
E    |ne   |H-   |     |EE*Me*Sqrt2*tb/(4*MW*SW)|(1-G5)
E    |ne   |W-   |     |-EE*Sqrt2/(4*SW)            |G(m3)*(1-G5)
E    |ne   |W-.f |     |-EE*Me*Sqrt2/(4*MW*SW)      |(1-G5)
```

LanHEP

- The LanHEP program is written on C programming language, external mathematical software is NOT required.
- LanHEP read input file which describes the physical model by the set of statements. Large projects can be split to several files.
- Conditional processing of the model file allows the user to use the same input file(s) for several species of the physical model. This feature allows, for example, to chose gauge fixing and MSSM extensions by setting some switches instead of creating several slightly different input files.
- Command-line tool: no graphical interface means easy compilation on any platform where 32-bit C compiler exists.

- The user declares physical parameters to be included in the Lagrangian. The value of a parameter can be a number or an expression:

```
parameter ee=0.31333:'elementary electric charge'.
parameter sw=0.478:'sinus of weak angle'.
parameter cw=Sqrt(1-sw**2):'cosine of weak angle'.
```

- The user declares scalar, spinor, vector, (also spin 3/2 and 2) particles . It is possible to prescribe the color structure for a particle:

```
spinor e1/E1:(electron, mass Me=0.000511).
spinor q/Q:(quark, color c3, mass Mq=10).
vector A/A:(photon, gauge).
```

- New symmetry groups are also possible. They can be defined in a way like color $SU(3)$ symmetry is defined, as well as corresponding matrices and structure constants:

```
group color:SU(3).
repres color:(c3/c3b,c8).
special lambda:(color c3, color c3b, color c8).
```

Description of the physical model for LanHEP **(cont)**

- The user can define the substitution rules, for example for covariant derivative $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$:

```
let F^mu^nu = deriv^mu*A^nu - deriv^nu*A^mu.
```

- It is possible to define multiplets, and their components:

```
let l1 = {n1,e1}, L1 = {N1,E1}.
```

- The user can write Lagrangian terms with Lorenz and multiplet indices explicitly or omit indices (all or part of them):*(QED vertex* $\bar{e}(x)\gamma^\mu A_\mu(x)e(x)$*)*

```
lterm E1^a*gamma^a^b^mu*A^mu*e1^b.
lterm E1*gamma^mu*A^mu*e1.
lterm E1*gamma*A*e1.
```

- LanHEP performs explicit summation over the indices of Lagrangian terms, if the corresponding components for multiplets and matrices are introduced.

Fields in the vertex	Variational derivative of Lagrangian by fie
$A_\mu \quad W^+{}_\nu \quad W^-{}_\rho$	$-e\big(p_2^\rho g^{\mu\nu} - p_2^\mu g^{\nu\rho} - p_3^\nu g^{\mu\rho} + p_3^\mu g^{\nu\rho} + p_1^\nu g^{\mu\rho}$
$G_{\mu p} \quad G_{\nu q} \quad G_{\rho r}$	$g_s f_{pqr}\big(p_3^\nu g^{\mu\rho} - p_3^\mu g^{\nu\rho} + p_1^\rho g^{\mu\nu} - p_1^\nu g^{\mu\rho} - p_2^\rho g$
$H \quad Z_\mu \quad Z_\nu$	$\frac{eM_W}{{c_w}^2 s_w} g^{\mu\nu}$
$\bar{c}_{ap} \quad d_{bq} \quad W^+{}_\mu$	$-\frac{1}{2}\frac{e\sqrt{2}Vcd}{s_w}\delta_{pq}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$
$\bar{\tau}_a \quad \nu^\tau{}_b \quad W^-{}_\mu$	$-\frac{1}{2}\frac{e\sqrt{2}}{s_w}\gamma^\mu_{ac}\frac{(1-\gamma^5)_{cb}}{2}$

LanHEP features

- 2-component fermion notation makes possible the introduction of supersymmetric Lagrangian in a more natural way, closer to the form used in most textbooks on the supersymmetry.
- Superpotential can be used for supersymmetric theories; this option allows to introduce easily various extensions of MSSM (R-parity violation, NMSSM, etc). Yukawa and F^*F terms are now automatically derived by the program.
- Extra dimensions: automatic generation of interaction of KK modes and 5th components of vectors.
- Generating Hermitian conjugate terms allow to simplify model description.
- Constructing the ghost Lagrangian from BRST transformation.
- Conterterms can be generated if the necassy shifts for parameters and fields are prescribed.

2-component fermion notation

The connection between two-component and four-component notations is summarized by the following relations:

$$\psi = \begin{pmatrix} \xi \\ \bar{\eta} \end{pmatrix}, \qquad \psi^c = \begin{pmatrix} \eta \\ \bar{\xi} \end{pmatrix}, \qquad \bar{\psi} = \begin{pmatrix} \eta \\ \bar{\xi} \end{pmatrix}^T, \qquad \bar{\psi}^c = \begin{pmatrix} \xi \\ \bar{\eta} \end{pmatrix}^T$$

If the user has declared a spinor particle p (with antiparticle P), the LanHEP notation for its components is:

$\xi \rightarrow$ up(p)

$\bar{\eta} \rightarrow$ down(p)

$\eta \rightarrow$ up(cc(p)) *or* up(P)

$\bar{\xi} \rightarrow$ down(cc(p)) *or* down(P)

Yukawa and $F_i^* F_i$ terms in the Lagrangian come from superpotential:

$$-\frac{1}{2}\left(\frac{\partial^2 W}{\partial A_i \partial A_j}\Psi_i \Psi_j + H.c.\right) - (\partial W/\partial A_i)(\partial W/\partial A_i)^*$$

LanHEP notation:

```
keep˙lets W.
let W=eps*(mu*H1*H2+ml1*H1*L*R+md1*H1*Q*D+mu1*H2*Q*U).
lterm - df(W,H1,H2)*fH1*fH2 - ... + AddHermConj.
lterm - df(W,H1)*df(Wc,H1c) - ....
lterm - dfdfc(W,H1) - ....
```

Extra dimensions in QED

```
let PL=(1-gamma5)/2, PR=(1+gamma5)/2.

transform A -> A + (A1*cos(1) + A2*cos(2))*Sqrt2,
          e -> e + (PL*cos(1)*e1l+PR*sin(1)*e1l +
                    PL*cos(2)*e2l+PR*sin(2)*e2l +
                    PL*sin(1)*e1r+PR*cos(1)*e1r +
                    PL*sin(2)*e2r+PR*cos(2)*e2r)*Sqrt2.

% The 5th components of the photon KK modes become goldstone bosons.

ued_5th deriv->deriv5/R, A->('A1.f'*sin(1) + 'A2.f'*sin(2))*Sqrt2.

lterm -F*F/4 where F=deriv^mu*A^nu-deriv^nu*A^mu.
lterm E*(gamma*(i*deriv - ee*A) -me)*e.
lterm  -1/2*(deriv*A)**2.
```

1-loop counterterms generation

Ultraviolet divergencies in renormalized quantum field theories are compensated by renormalization of wave functions, masses, charges, and may be some other Lagrangian paramters. Such transformation of the Lagrangian assumes changes in the Feynman rules. In particular vertices are changed due to the appearence of new terms — *counterterms*.

QED example:

```
parameter ee = 0.3133: 'Electric charge'.
vector A/A:photon.
spinor e/E:(electron, mass me=0.000511).
infinitesimal dZee, dZme, dZe1l, dZe1r, dZA.
transform ee ⟶ ee*(1+dZee), me ⟶ me+dZme,
          A ⟶ A*(1+dZA),
          e ⟶ e + (dZel/2*(1-gamma5)/2+dZer/2*(1+gamma5)/2)*e,
          E ⟶ E + (dZel/2*(1+gamma5)/2+dZer/2*(1-gamma5)/2)*E.
lterm ee*E*(i*gamma*deriv+gamma*A+me)*e.
```

LanHEP features

- Checking the correctness of the model
  - Electric charge conservation
  - Hermiticity
  - Probing kinetic and mass terms, the mass matrix is extracted
  - BRST invariance
  - Extracting classes of vertices
- Simplifying the expression for vertices
  - Orthogonal (and hermitian) matrices
  - Trigonometric expressions ($\sin\alpha \pm \beta$)
  - Lengthly expressions in the vertices can be transferred to the table of parameters.

Mass matrix

Mixing of particles [' b1', ' b2']

M11 (' B1'/' b1') = (-1/6*(3-2*SW**2)/CW**2*

= +991.671662**2

Declared mass MSb1=991.671662

M12 (' B1'/' b2') = (-1/6*(3-2*SW**2)/CW**2*MW*

= +0.000002**2

Looking for the mixing matrix...

It is recognized that these fields are rotated by matrix:

(Zd33 Zd36)

(Zd63 Zd66)

The mass matrix before introducing this rotation:

M11 = (-1/6*(3-2*SW**2)/CW**2*MW**2*c2b+1*Mb**2+1*Mq3**2)

M12 = (-1*Mb/cb*mu*sb+1/EE*MW*SW*Sqrt2*cb*ds3)

M22 = (-1/3/CW**2*MW**2*SW**2*c2b+1*Mb**2+1*Md3**2)

Some new Lagrangians implemented by LanHEP

- Complete MSSM in unitary and t'Hooft-Feynman gauges with the Higgs sector by linking with the FeynHiggs, effective potential is used to take into account radiative corrections to Higgs masses and interaction; mSUGRA and GMSB by means of SLHA interface
- MSSM extensions include:
  - MSSM with R-parity violation
  - Model with gravitino and sgoldstinos
  - NMSSM (an extension of the MSSM by a gauge singlet N with hypercharge 0)
  - MSSM with CP violation
- Complete Leptoquark model which includes Yukawa couplings for all types of LQ, gauge couplings and anomalous gauge couplings for vector LQ
- Complete two-Higgs-doublet model with conserved or broken CP invariance
- Anomalous quartic vector bosons self-couplings

More new Lagrangians implemented by LanHEP

- A new signature for color octet pseudoscalars at the LHC, in theories of extra-dim. Alfonso R. Zerwekh, Claudio O. Dib, Rogerio Rosenfeld;
- Minimal Higgsless model, Chivukula et al;
- Inert Doublet Model, Pierce and Thaler;
- Excited fermions, Boos et al;
- Technihadrons, technicolour, Zerwekh;
- Little Higgs Models, Phenomenology of littlest Higgs model with T-parity: including effects of T-odd fermions. Alexander Belyaev, Chuan-Ren Chen, Kazuhiro Tobe, C.-P. Yuan (Michigan State U.);
- Universal extra-dim, Matchev et al.

Particle table format can be tuned

New option allows to modify the format of the output particle table and to add new proprties (new columns in the table). One can add, say, PDG particle number to the table:

```
prtcformat fullname:' Full   Name ',
           name:' p ',
           aname:' ap',
           spin2,color,mass,width, aux,
           pdg:'PDG ID',
           texname:'  latex P name ',
           atexname:' latex aP name ' .
```

Then the new property value can be written in the particle declaration statement:

```
scalar h:(higgs, mass Mh, pdg 123, width wh).
```

The new property values for a set of paticles can be set (for CalcHEP):

```
prtcproperty pdg:(A=22, Z=23, 'W+'=24, G=21,  t=6,  H=35).
```

Vertices table format: explicit color structure

Color matrices and dot products can be optionally written in the Lorentz Part, e.g. QCD plus quark-photon interactions produces the following vertices file:

```
P1   |P2   |P3   |P4   |>   Factor   <|> dLagrangian/ dA(p1) dA(p2) dA(p3
G    |G    |G    |     |gg             |m2.p3*m1.m3*F(c1,c2,c3)
     |     |     |     |                -m1.p3*m2.m3*F(c1,c2,c3)
     |     |     |     |                +m3.p1*m1.m2*F(c1,c2,c3)
     |     |     |     |                -m2.p1*m1.m3*F(c1,c2,c3)
     |     |     |     |                -m3.p2*m1.m2*F(c1,c2,c3)
     |     |     |     |                +m1.p2*m2.m3*F(c1,c2,c3)
G.C  |G.c  |G    |     |-gg            |m3.p2*F(c1,c2,c3)
Q    |q    |G    |     |gg             |L(c1,c2,c3)*G(m3)
Q    |q    |A    |     |ee/3           |c1.c2*G(m3)
G    |G    |G    |G    |gg^2           |m1.m3*m2.m4*F(c1,c2,c0)*F(c3,c4,c0
     |     |     |     |                -m1.m4*m2.m3*F(c1,c2,c0)*F(c3,c4,c
     |     |     |     |                +m1.m2*m3.m4*F(c1,c3,c0)*F(c2,c4,c
     |     |     |     |                -m1.m4*m2.m3*F(c1,c3,c0)*F(c2,c4,c
     |     |     |     |                +m1.m2*m3.m4*F(c1,c4,c0)*F(c2,c3,c
     |     |     |     |                -m1.m3*m2.m4*F(c1,c4,c0)*F(c2,c3,c
```

LanHEP application to one-loop calculation: SloopS project(together with N.Baro, F. Boudjema)

- Goal — loop calculation in susy (MSSM and extensions), for Colliders and Dark Matter, On-Shell scheme
- *FeynArts* and *FormCalc* are used for matrix element calculation: *FeynArts* model format output implemented in LanHEP.
- Shifts in fields and parameters to produce counterterms by LanHEP:

```
infinitesimal dMHsq, dMZsq, dMWsq,dZAA, dZAZ, dZZA, dZZZ, dZW, dZ
infinitesimal   dEE= -(dZAA - SW/CW*dZZA)/2.
transform A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
      'W+'->'W+'*(1+dZW/2),'W-'->'W-'*(1+dZW/2), H->H*(1+dZH/2).
```

- Different normalization schemes can be used, easy to switch

- Non-linear gauge fixing

$$\begin{aligned}\mathcal{L}_{GF} = & -\frac{1}{\xi_W}|(\partial_\mu - ie\tilde{\alpha}A_\mu - igc_W\tilde{\beta}Z_\mu)W^{\mu\,+} + \xi_W\frac{g}{2}(v+\tilde{\delta}_h h + \tilde{\delta}_H H + i\tilde{\kappa}\chi_3)\chi^+|^2 \\ & -\frac{1}{2\xi_Z}(\partial.Z + \xi_Z\frac{g}{2c_W}(v+\tilde{\epsilon}_h h + \tilde{\epsilon}_H H)\chi_3)^2 - \frac{1}{2\xi_\gamma}(\partial.A)^2\end{aligned}$$

  - quite a handful of gauge parameters, but with $\xi_i = 1$, no "unphysical threshold"
  - more important: no need for higher (than the minimal set)for higher rank tensors and tedious algebraic manipulations

From the Lagrangian to the Feynman Rules

```
vector
    A/A: (photon, gauge),
    Z/Z:('Z boson', mass MZ = 91.1875, gauge),
    'W+'/'W-': ('W boson', mass MW = MZ*CW, gauge).
scalar  H/H:(Higgs, mass MH = 115).



transform A->A*(1+dZAA/2)+dZAZ*Z/2, Z->Z*(1+dZZZ/2)+dZZA*A/2,
    'W+'->'W+'*(1+dZW/2),'W-'->'W-'*(1+dZW/2).
transform H->H*(1+dZH/2), 'Z.f'->'Z.f'*(1+dZZf/2),
    'W+.f'->'W+.f'*(1+dZWf/2),'W-.f'->'W-.f'*(1+dZWf/2).

let pp = { -i*'W+.f',  (vev(2*MW/EE*SW)+H+i*'Z.f')/Sqrt2 },
PP=anti(pp).



lterm -2*lambda*(pp*anti(pp)-v**2/2)**2
     where
    lambda=(EE*MH/MW/SW)**2/16, v=2*MW*SW/EE .

let Dpp^mu^a = (deriv^mu+i*g1/2*B0^mu)*pp^a +
     i*g/2*taupm^a^b^c*WW^mu^c*pp^b.
let DPP^mu^a = (deriv^mu-i*g1/2*B0^mu)*PP^a
    -i*g/2*taupm^a^b^c*{'W-'^mu,W3^mu,'W+'^mu}^c*PP^b.
lterm DPP*Dpp.
```

Gauge fixing and BRS transformation

```
let G_Z = deriv*Z+(MW/CW+EE/SW/CW/2*nle*H)*'Z.f'.

lterm -G_A**2/2 - G_Wp*G_Wm - G_Z**2/2.

lterm -'Z.C'*brst(G_Z).
```

```
RenConst[ dMHsq ] := ReTilde[SelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZH ] := -ReTilde[DSelfEnergy[prt["H"] -> prt["H"], MH]]
RenConst[ dZZf ] := -ReTilde[DSelfEnergy[prt["Z.f"] -> prt["Z.f"],
MZ]] RenConst[ dZWf ] := -ReTilde[DSelfEnergy[prt["W+.f"] ->
prt["W+.f"], MW]]
```

Output of Feynman Rules with Counterterms !!

```
M$CouplingMatrices = {

  (*------    H  H  ------*)
   C[ S[3], S[3] ] == - I *
{
 { 0 , dZH },
 { 0 , MH^2 dZH + dMHsq }
},
  (*------    W+.f  W-.f  ------*)
   C[ S[2], -S[2] ] == - I *
{
 { 0 , dZWf },
 { 0, 0 }
}, (*------    A  Z  ------*)
   C[ V[1], V[2] ] == 1/2 I / CW^2 MW^2 *
{
 { 0, 0 },
 { 0 , dZZA },
 { 0, 0 }
},

(*------    H  H  H  ------*)
   C[ S[3], S[3], S[3] ] == -3/4 I EE / MW / SW *
{
 { 2 MH^2 , 3 MH^2 dZH -2 MH^2 / SW dSW - MH^2 / MW^2 dMWsq
},
  (*------    H  W+.f  W-.f  ------*)
   C[ S[3], S[2], -S[2] ] == -1/4 I EE / MW / SW *
{
 { 2 MH^2 , MH^2 dZH + 2 MH^2 dZWf -2 MH^2 / SW dSW - MH^2 /
},

  (*------    W-.C  A.c  W+  ------*)
   C[ -U[3], U[1], V[3] ] == - I EE *
{
 { 1 },
 { - nla }
},
```

Summary

- Effective and well-tried tool for creation of new physical model for CompHEP
  - Lagrangian can be written in form close to textbooks, in terms of initial fields before symmetry breaking
  - SUSY-friendly: 2-component fermions and superpotential notation
  - New feature: KK modes in models with extra dimensions
  - Checking the correctness of the model, simplifying the expressions for vertices
- Extentions of CompHEP tables format
- LaTeX output
- FeynArts output
- Counterterms generation for 1-loop computation
- The program is available at

  `http://theory.sinp.msu.ru/~semenov/lanhep.html`