Semileptonic *B* decays and related studies at Belle

Youngjoon Kwon
Yonsei University
for the Belle collaboration

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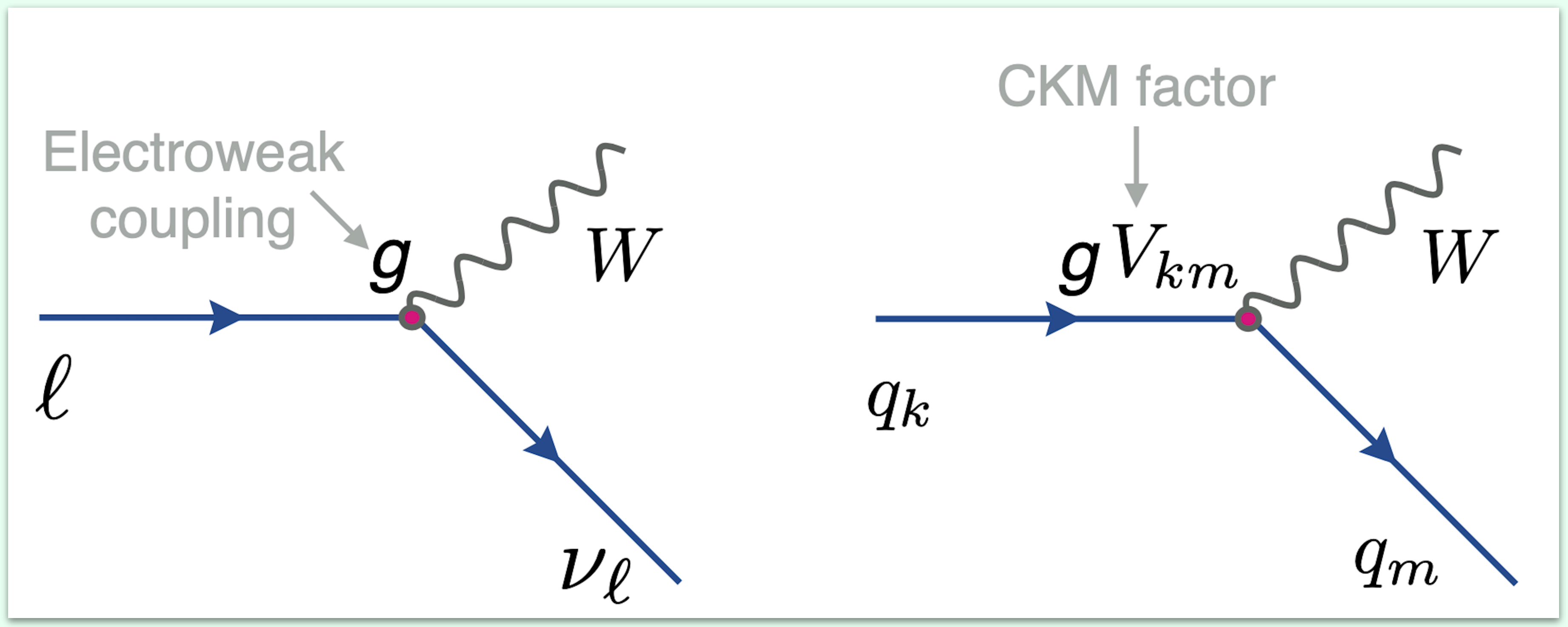
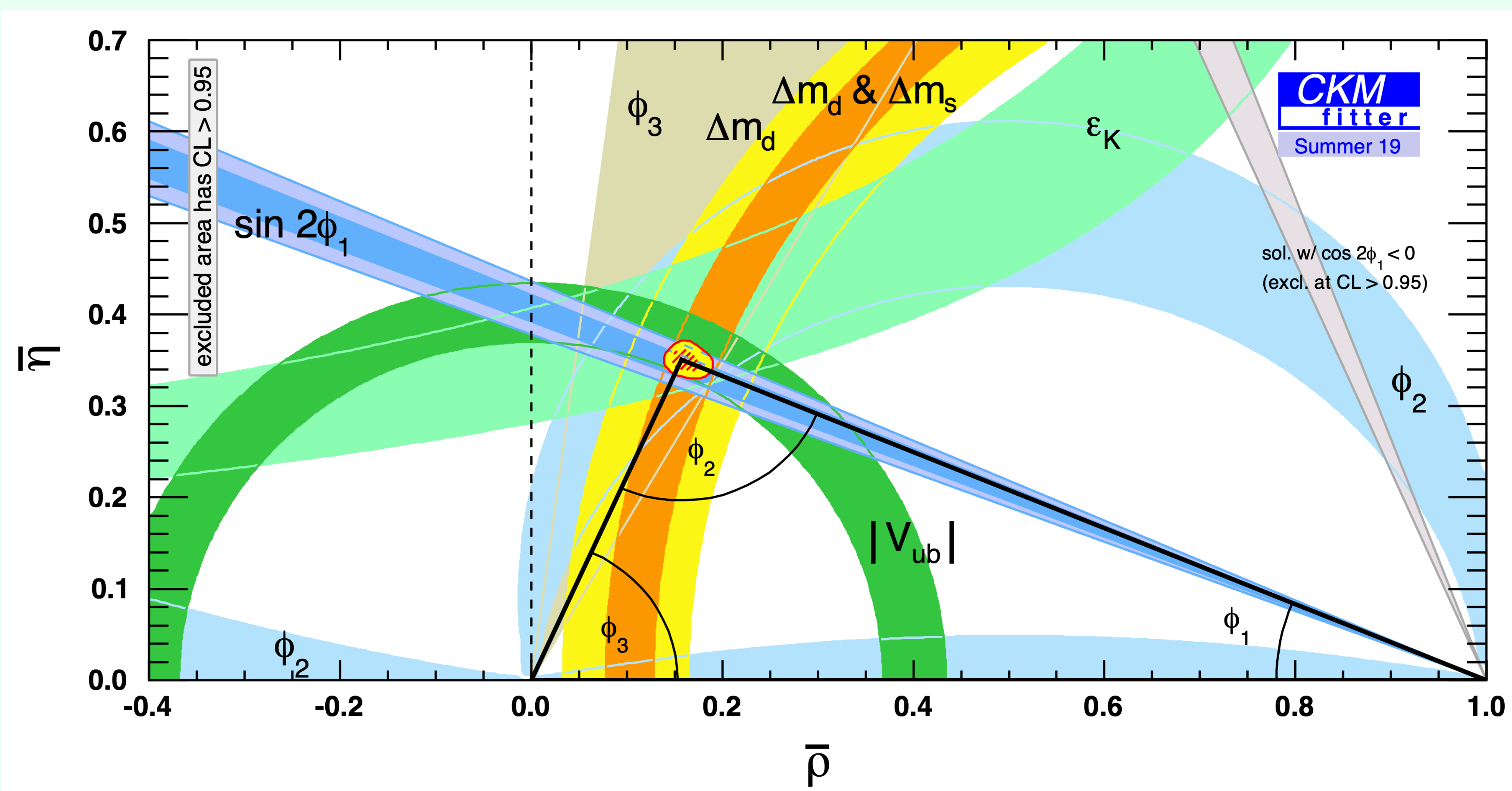
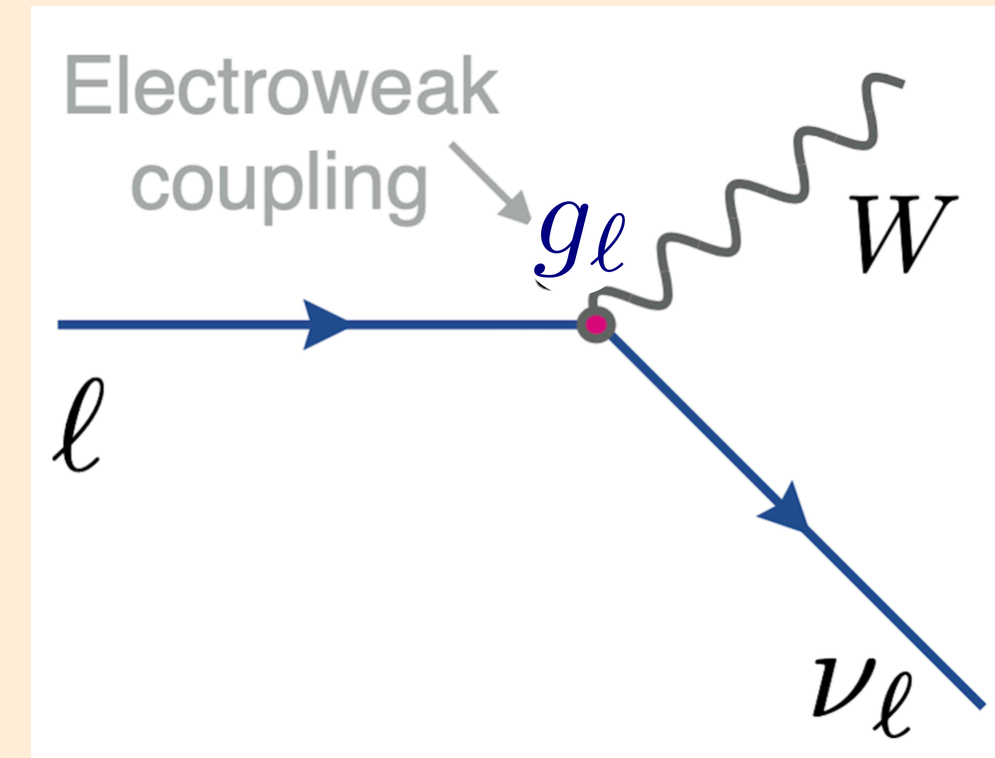
1-page summary of semileptonic decays

Precision measurements of CKM UT





Test of lepton universality in $R(D^{(*)})$



$$g_\ell \ (\ell = e, \ \mu, \ \tau)$$

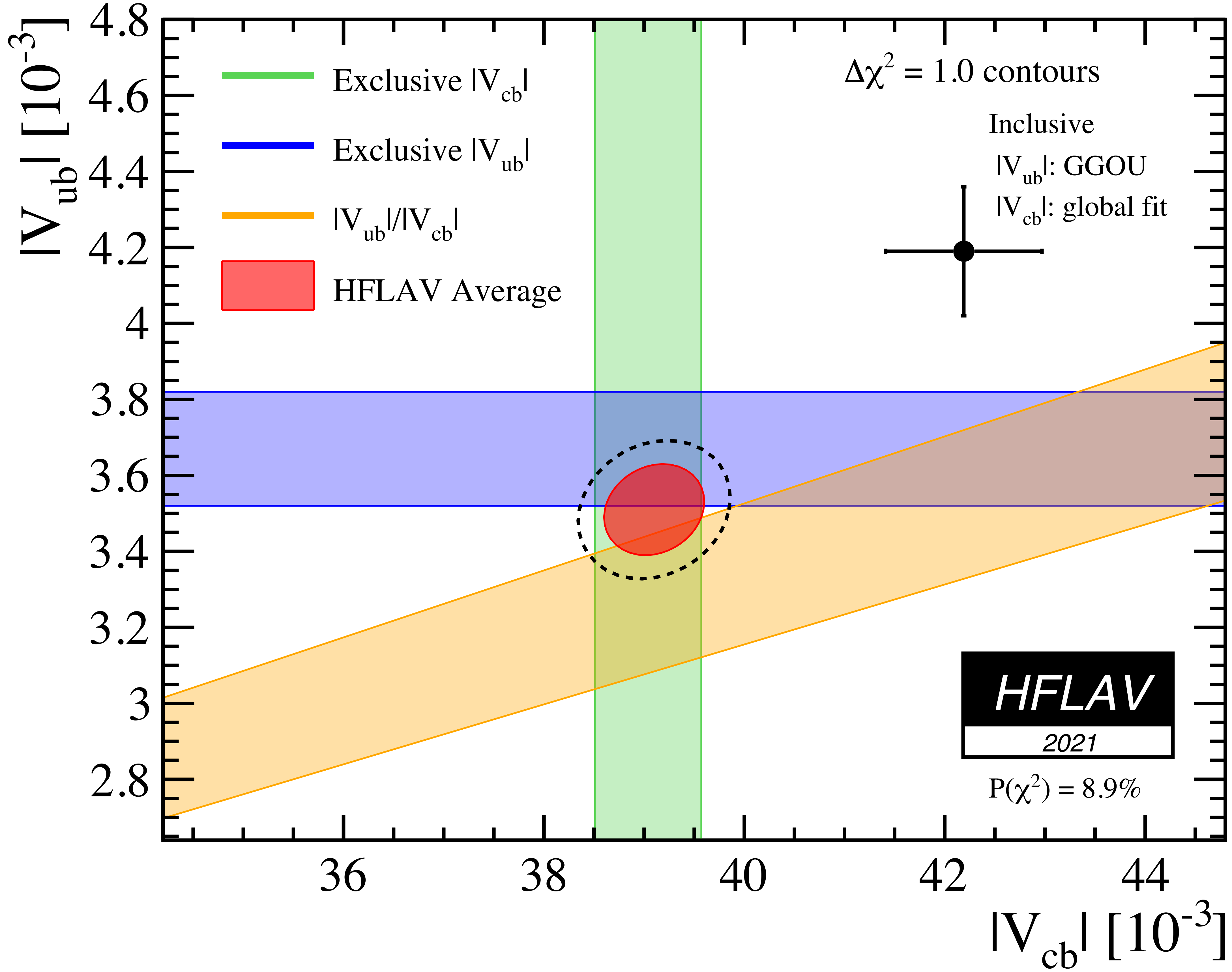
Is $g_\tau = g_\mu$, and/or g_e?

$$R(D^{(*)}) \equiv \frac{\mathscr{B}(B \to D^{(*)}\tau^+\nu)}{\mathscr{B}(B \to D^{(*)}\ell^+\nu)}$$

- the most recent Belle result on $R(D^{(*)})$ PRL 124, 161803 (2020)
- for a recent comprehensive review, arXiv:2101.08326 (accepted to RMP)

Inclusive vs. Exclusive Tension

in the measurements of $|V_{cb}|$, $|V_{ub}|$ between inclusive and exclusive approaches



$$|V_{ub}|_{\rm incl.} = (4.19 \pm 0.12^{+0.11}_{-0.12}) \times 10^{-3}$$

$$|V_{ub}|_{\rm excl.} = (3.51 \pm 0.12) \times 10^{-3}$$

$\sim 3\sigma$ tension for each ($|V_{cb}|$, $|V_{ub}|$)

$$|V_{cb}|_{\rm excl.} = (39.10 \pm 0.50) \times 10^{-3}$$

$$|V_{cb}|_{\rm incl.} = (42.19 \pm 0.78) \times 10^{-3}$$

Menu

- *starter*
  - ✓ $B^+ \to \pi^+\pi^-\ell^+\nu$ — PRD 103, 112001 (2021)
  - ✓ $B^+ \to \eta^{(\prime)}\ell^+\nu$ — arXiv:2104.13354 (submitted to PRD)
- *primo*
  - ✓ q^2 moments of $B \to X_c\ell^+\nu$ — ***preliminary***
- *secondi*
  - ✓ $B \to X_u\ell^+\nu$ inclusive for $|V_{ub}|$ — PRD 104, 012008 (2021)
  - ✓ $B \to X_u\ell^+\nu$ differential distributions — arXiv:2107.13855 (submitted to PRL)

 *dessert*

$B^+ \to \pi^+\pi^-\ell^+\nu_\ell$

Motivation

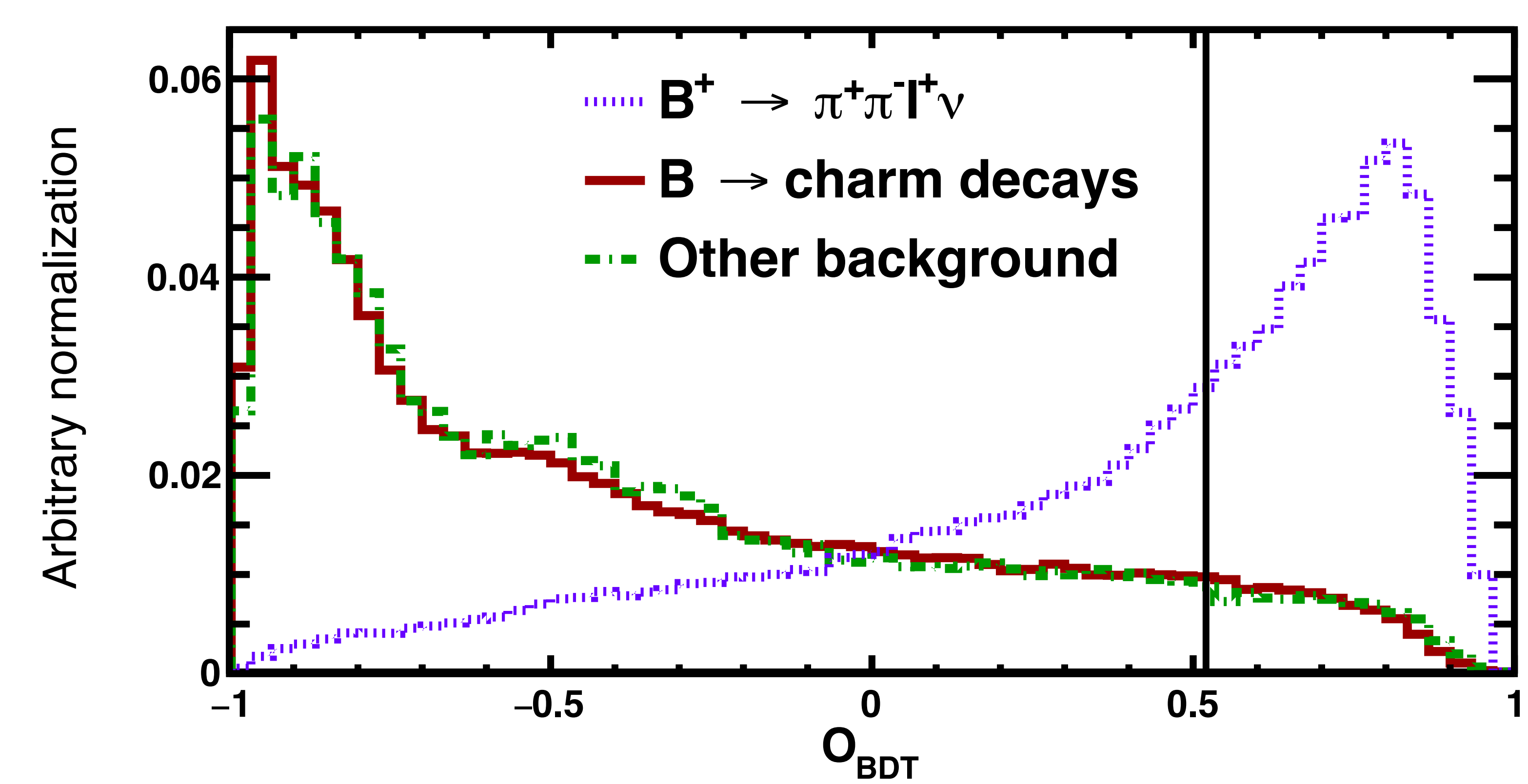
- measured exclusive $B \to X_u\ell^+\nu_\ell$ modes are far from saturating the whole $b \to u\ell^+\nu_\ell$ processes,
- thus causing large modeling (systematic) uncertainties

Interests in $B^+ \to \pi^+\pi^-\ell^+\nu_\ell$

- a potential clue to understanding the existing tension, (*inclusive-vs.-exclusive*)
- can be a probe for the internal structure of light mesons
- can improve the $B \to \pi\pi$ form-factor calculation
- $\exists$ measurements of $B \to \rho\ell^+\nu_\ell$ only

main feature

- hadronic B tagging ($B^+_{\rm tag}$) using NN
- suppress (*dominant*) $B \to X_c\ell^+\nu_\ell$ bkgd. using BDT

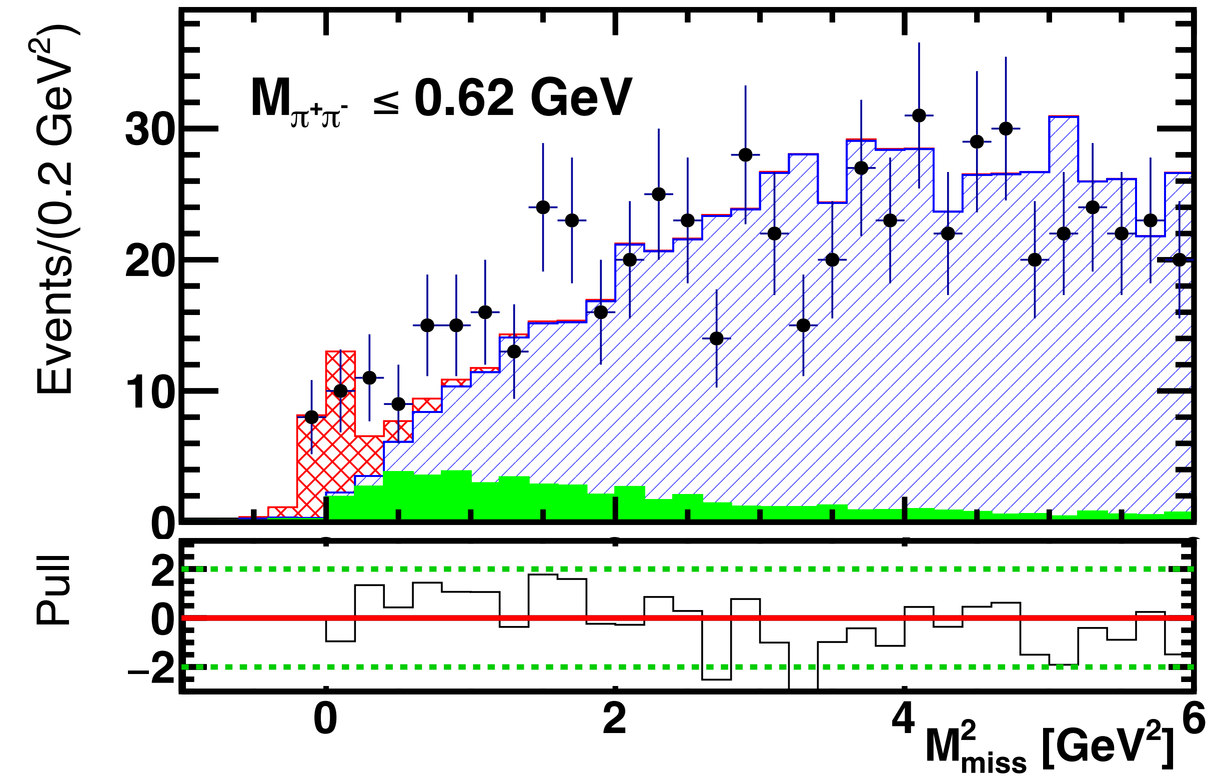


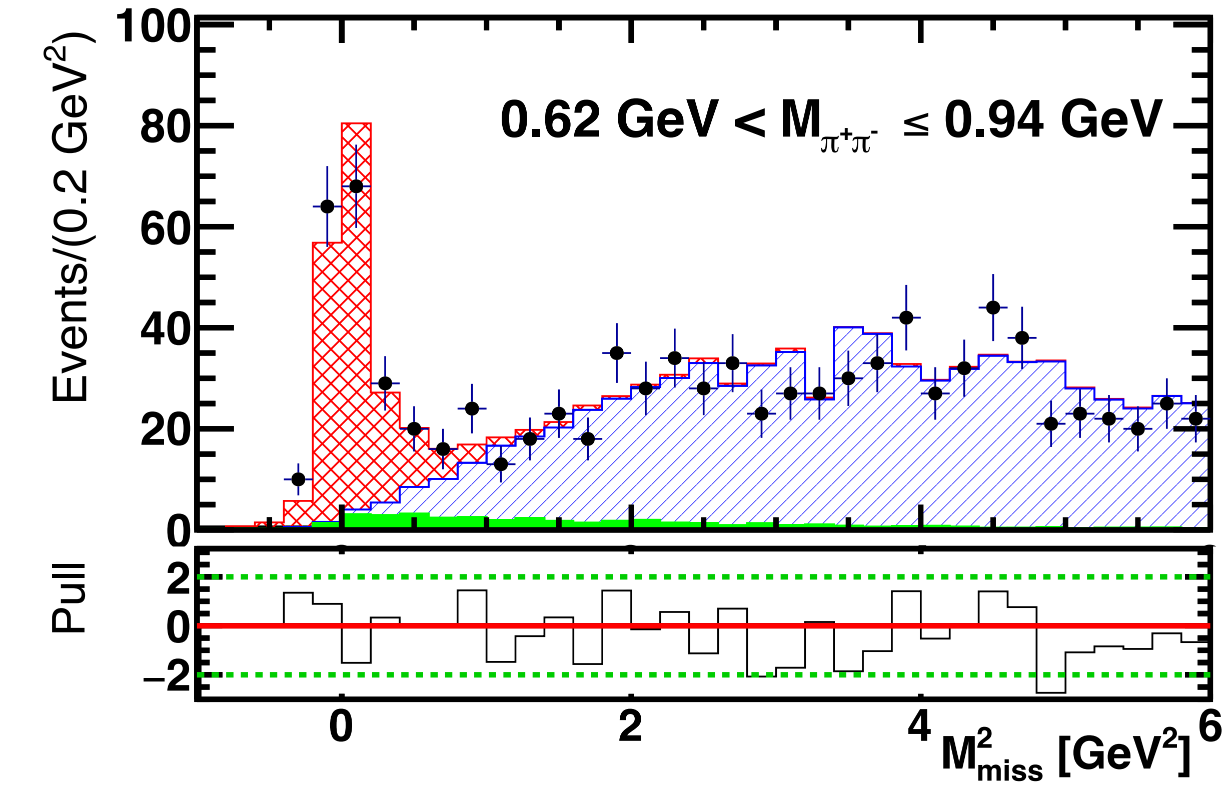
$B^+ \to \pi^+\pi^-\ell^+\nu_\ell$

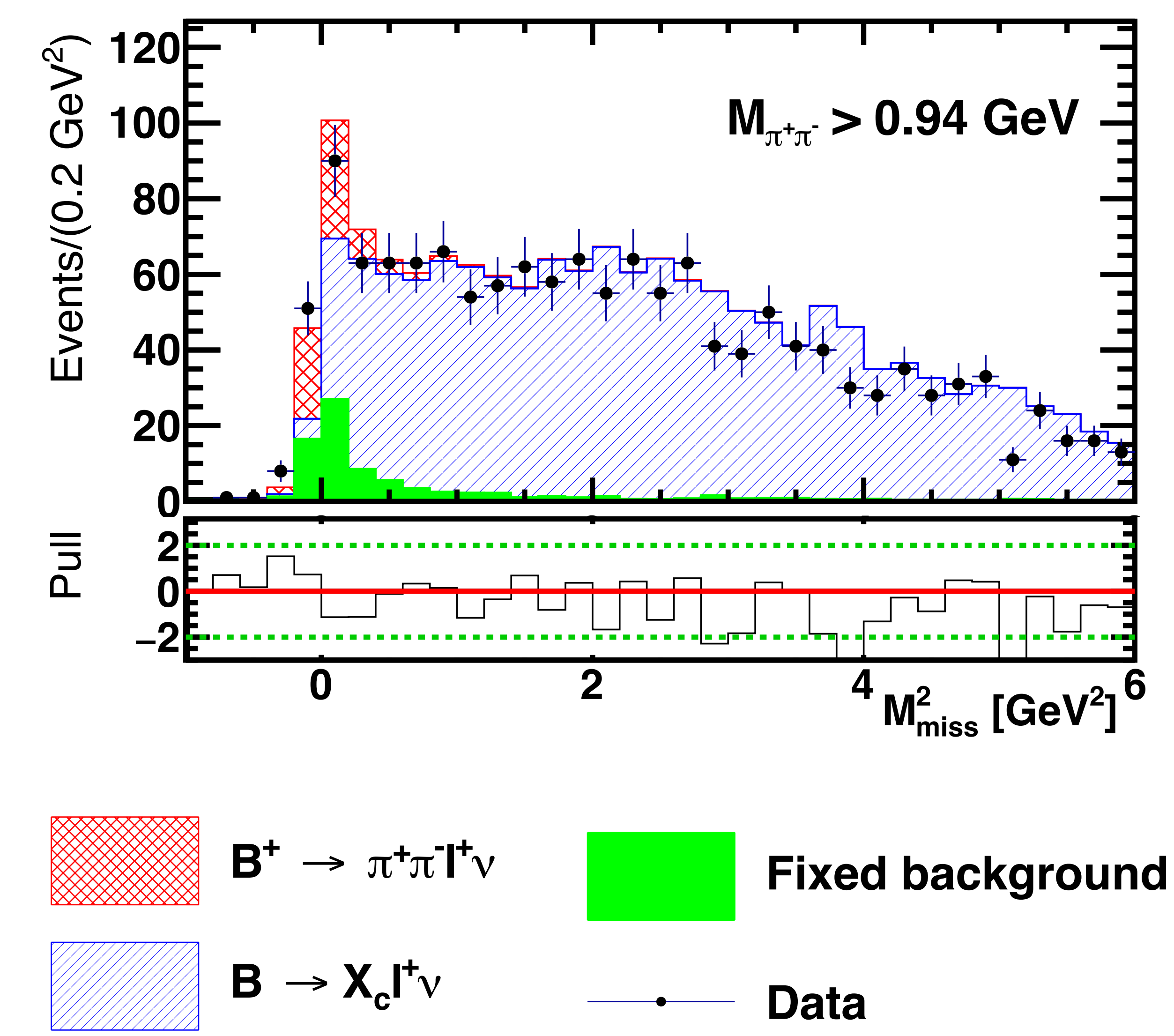
Signal extraction

- by binned fit to $M^2_{\rm miss}$
- for signal, $B \to X_c\ell^+\nu_\ell$, and other fixed bkgd.
- in bins of $M_{\pi\pi}$ or q^2, or in 2D ($M_{\pi\pi}$, q^2)

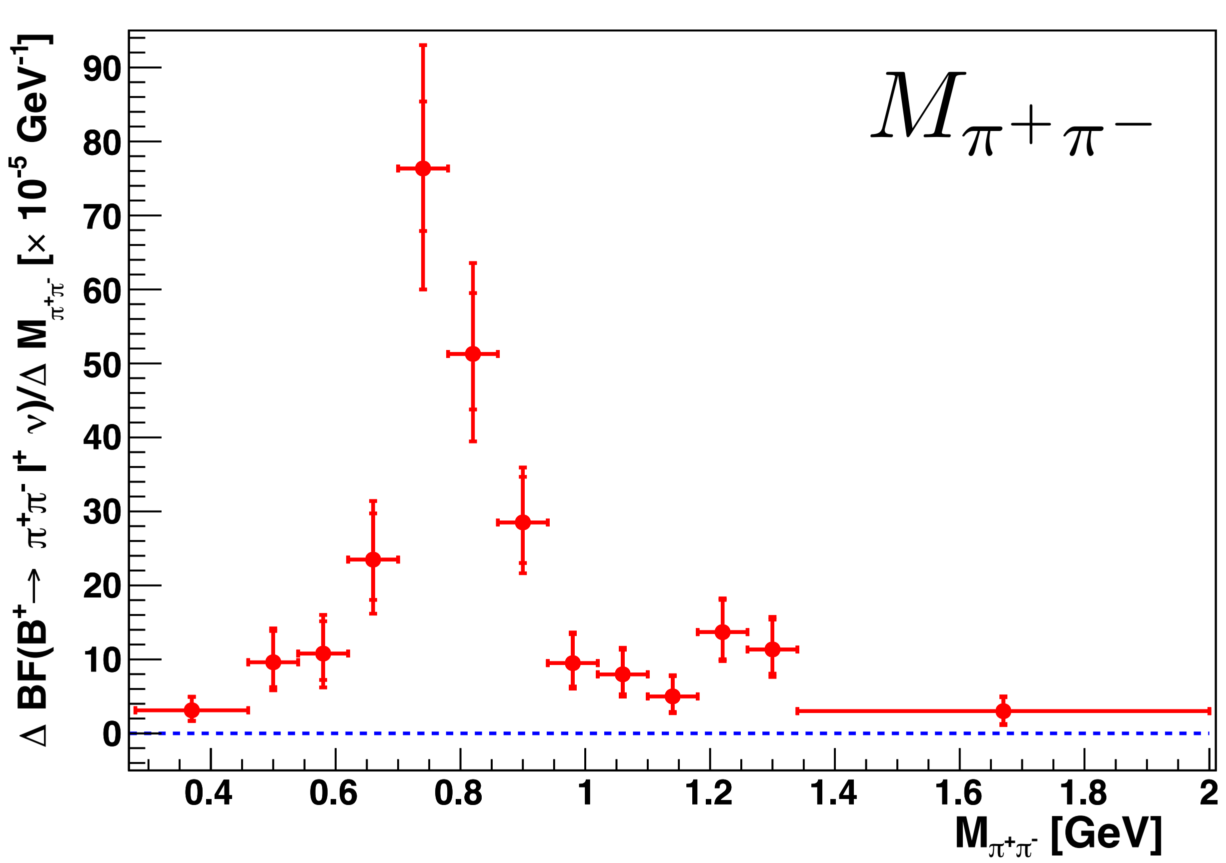
Binned fit to $M^2_{\rm miss}$ for signal extraction

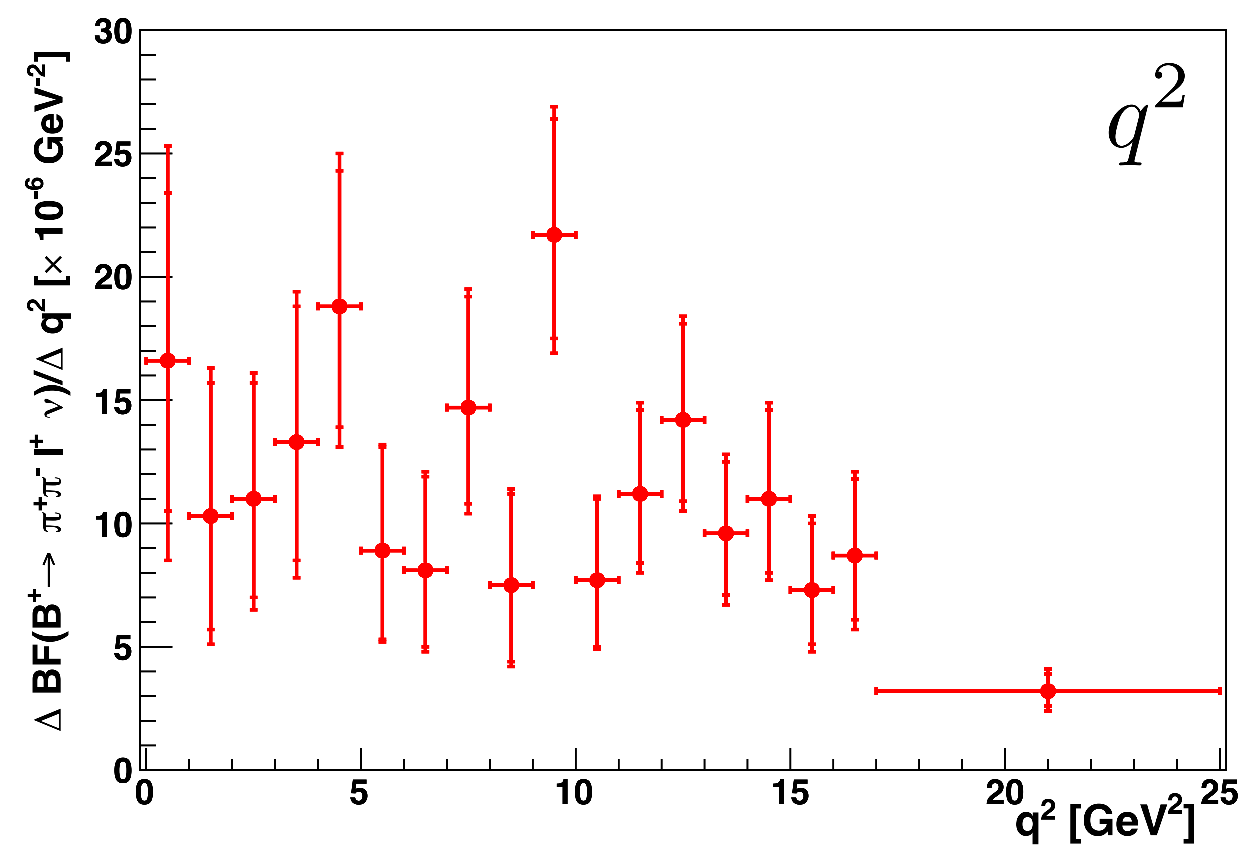






Background-subtracted distributions





$B^+ \to \pi^+\pi^-\ell^+\nu_\ell$

- 3 fit results and branching fractions

$$\mathscr{B}(B^+ \to \pi^+\pi^-\ell^+\nu_\ell)\ [\mathrm{1D}\ (M_{\pi\pi})] = (22.3^{+2.0}_{-1.8} \pm 3.9) \times 10^{-5}$$

$$\boxed{\mathscr{B}(B^+ \to \pi^+\pi^-\ell^+\nu_\ell)\ [\mathrm{1D}\ (q^2)] = (22.7^{+1.9}_{-1.6} \pm 3.4) \times 10^{-5}}$$ $\Leftarrow$ *Belle's final result*

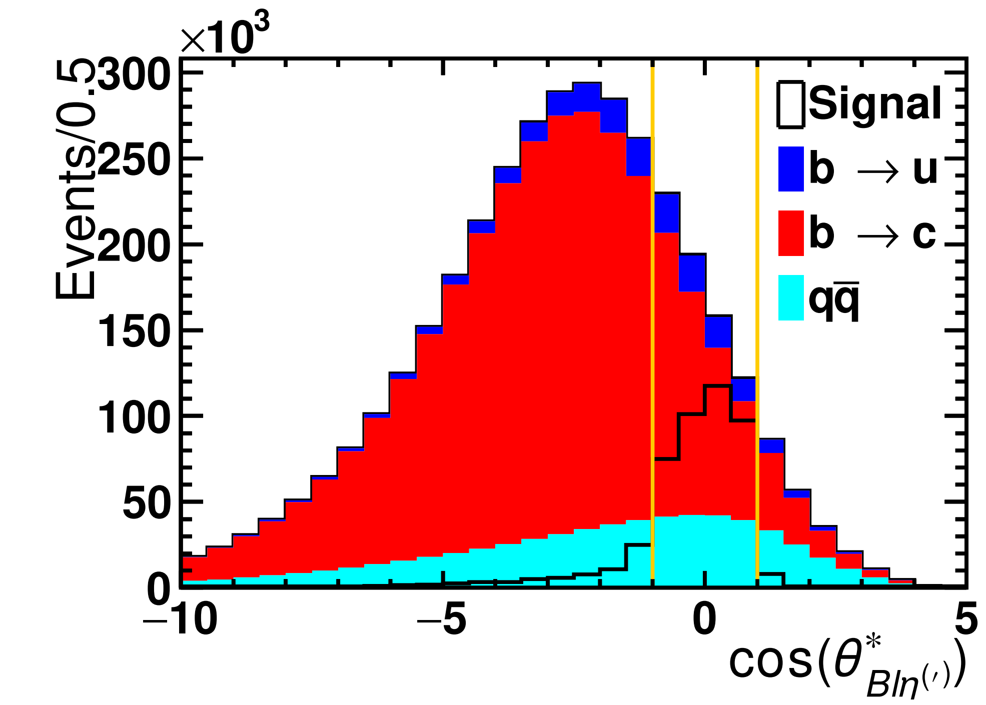
$$\mathscr{B}(B^+ \to \pi^+\pi^-\ell^+\nu_\ell)\ [\mathrm{2D}] = (23.0^{+1.9}_{-1.7} \pm 3.0) \times 10^{-5}$$

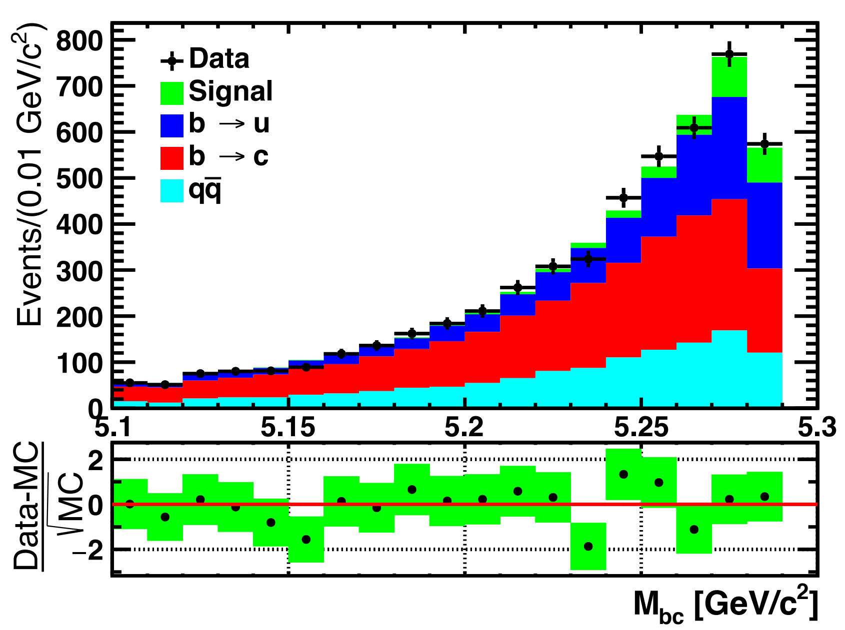
$B^+ \to \eta^{(')}\ell^+\nu_\ell$

Features

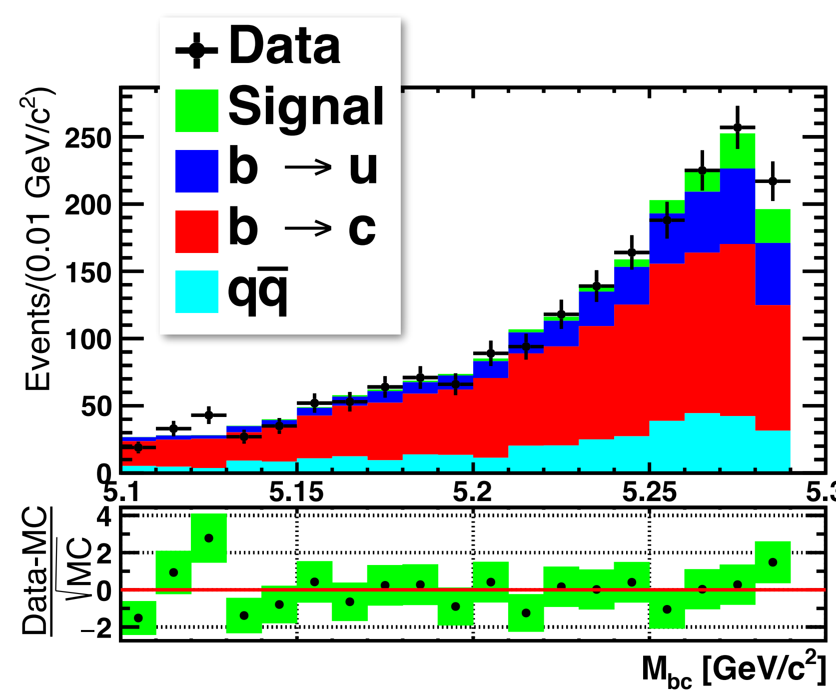
- untagged ("loose ν reconstruction")
- background suppression by $\cos\theta^*_{BX}$ $(X = \eta^{(')}\ell^+)$ and two BDTs (against $B\overline{B}$, continuum)
- binned ML fit in $(M_{\rm bc}, \Delta E)$ covering entire q^2 range

$$\cos(\theta^\star_{B\ell\eta^{(\prime)}}) = \frac{2E^\star_B E^\star_{\ell\eta^{(\prime)}} - m_B^2c^4 - m^2_{\ell\eta^{(\prime)}}c^4}{2|\vec{p}^\star_B||\vec{p}^\star_{\ell\eta^{(\prime)}}|c^2}$$

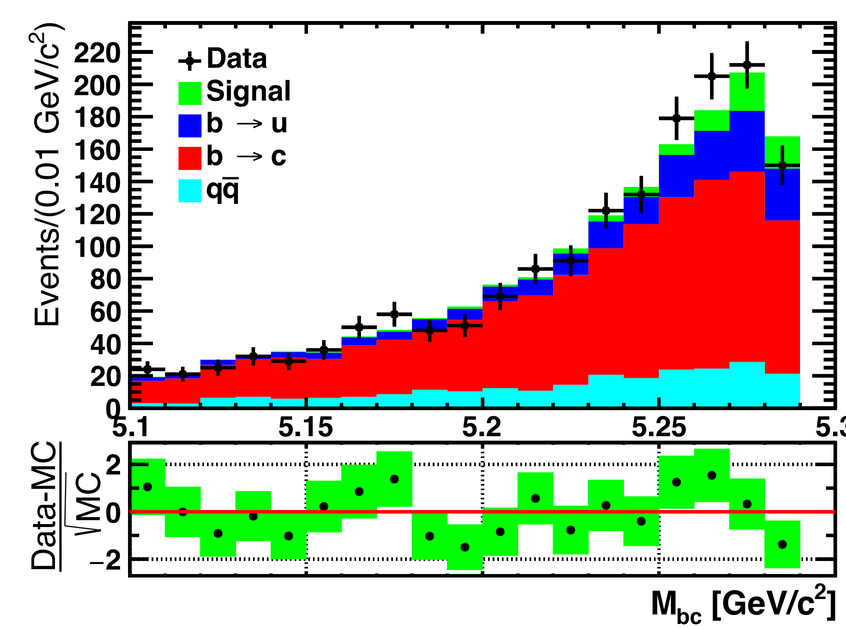




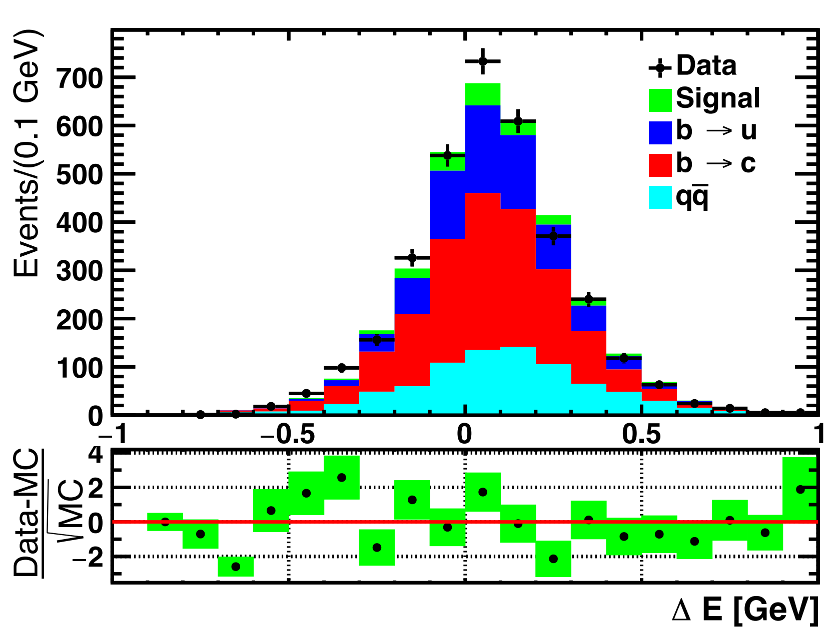
(a) $M_{\rm bc}(\eta\to\gamma\gamma)$



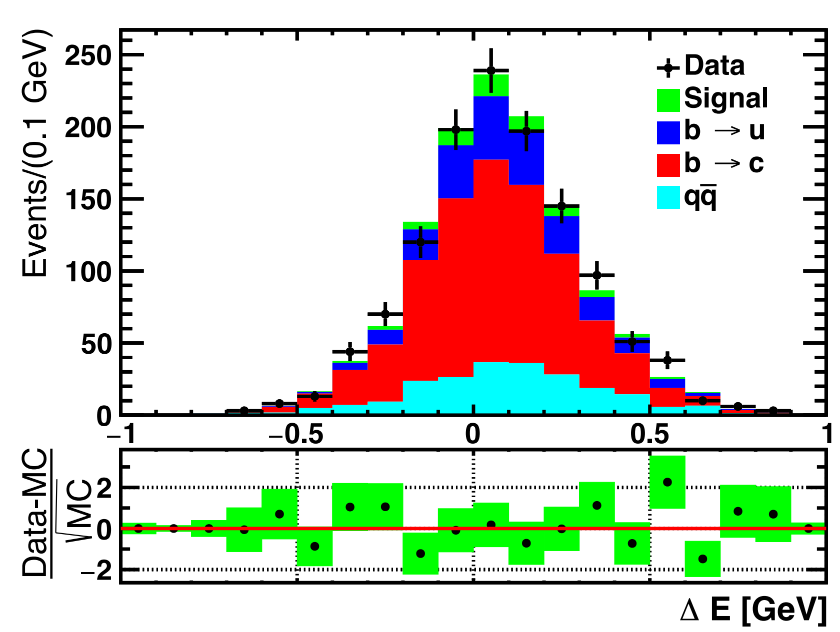
(b) $M_{\rm bc}(\eta\to\pi^+\pi^-\pi^0)$



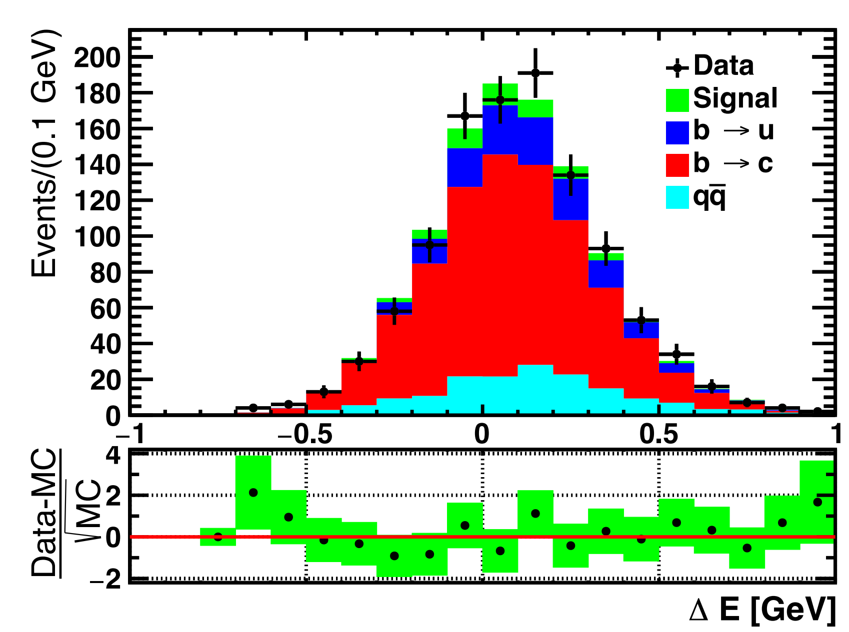
(c) $M_{\rm bc}(\eta'\to\pi^+\pi^-\eta(\gamma\gamma))$



(d) $\Delta E(\eta\to\gamma\gamma)$



(e) $\Delta E(\eta\to\pi^+\pi^-\pi^0)$



(f) $\Delta E(\eta'\to\pi^+\pi^-\eta(\gamma\gamma))$

$$\mathcal{B}(B^+\to\eta\ell^+\nu) = (2.83\pm0.55\pm0.34)\times10^{-5}$$

$$\mathcal{B}(B^+\to\eta'\ell^+\nu) = (2.79\pm1.29\pm0.30)\times10^{-5}$$

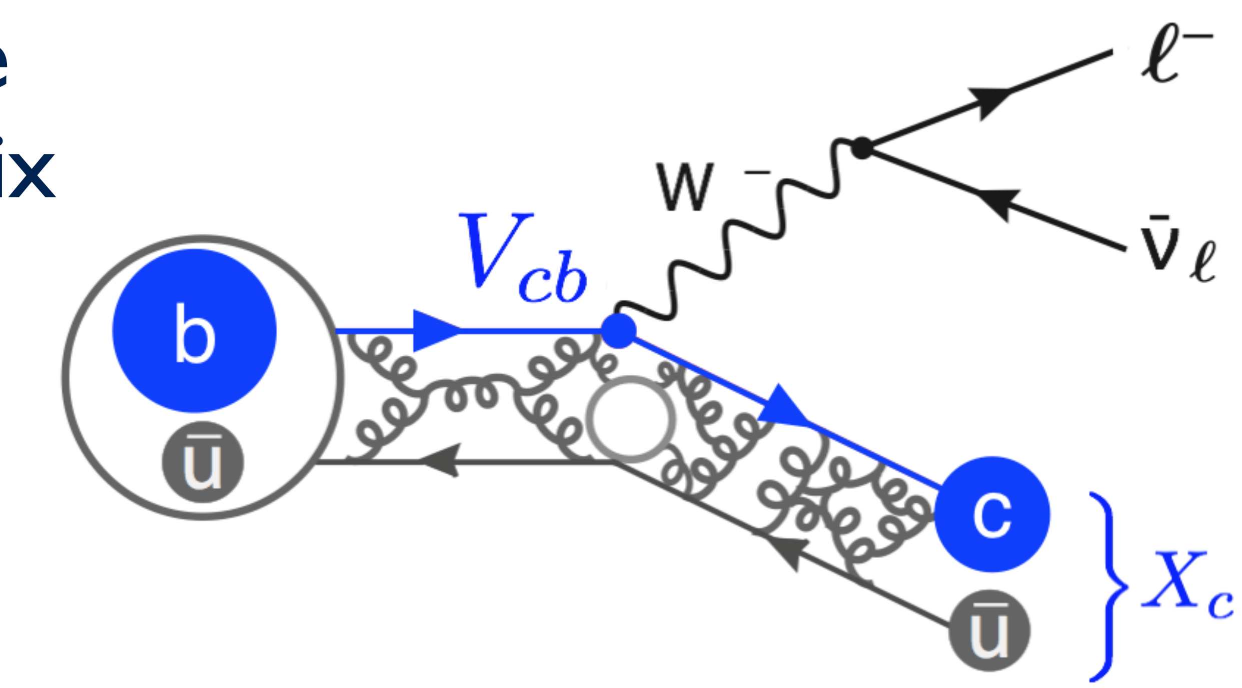
still dominated by stat. error

preliminary

q^2 moments in $B \to X_c \ell^+ \nu_\ell$

Motivations

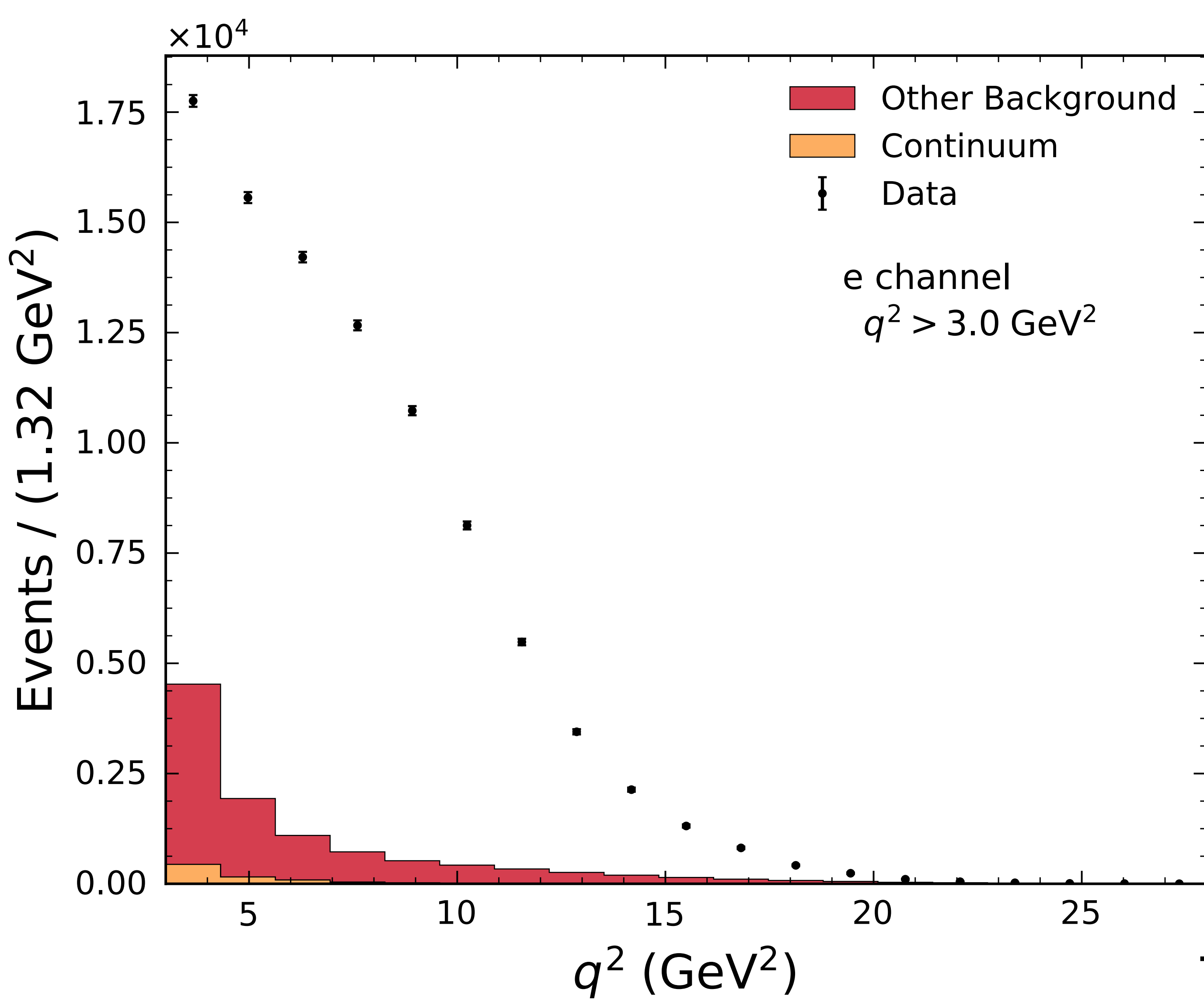
- $\exists$ tension between exclusive & inclusive measurements of $|V_{cb}|$
- inclusive approach for $|V_{cb}|$
  - free from uncertainties of form factor shape and normalization
  - exploits HQE — total decay rate and spectral moments can be expanded into a manageable number of non-perturbative matrix elements
- measure q^2 moments
  - a novel approach by Fael, Manel, Keri Vos [JHEP 02, 177 (2019)]
  - use "reparametrization invariance"
  - data-driven method for $|V_{cb}|$ up to $1/m_b^4$



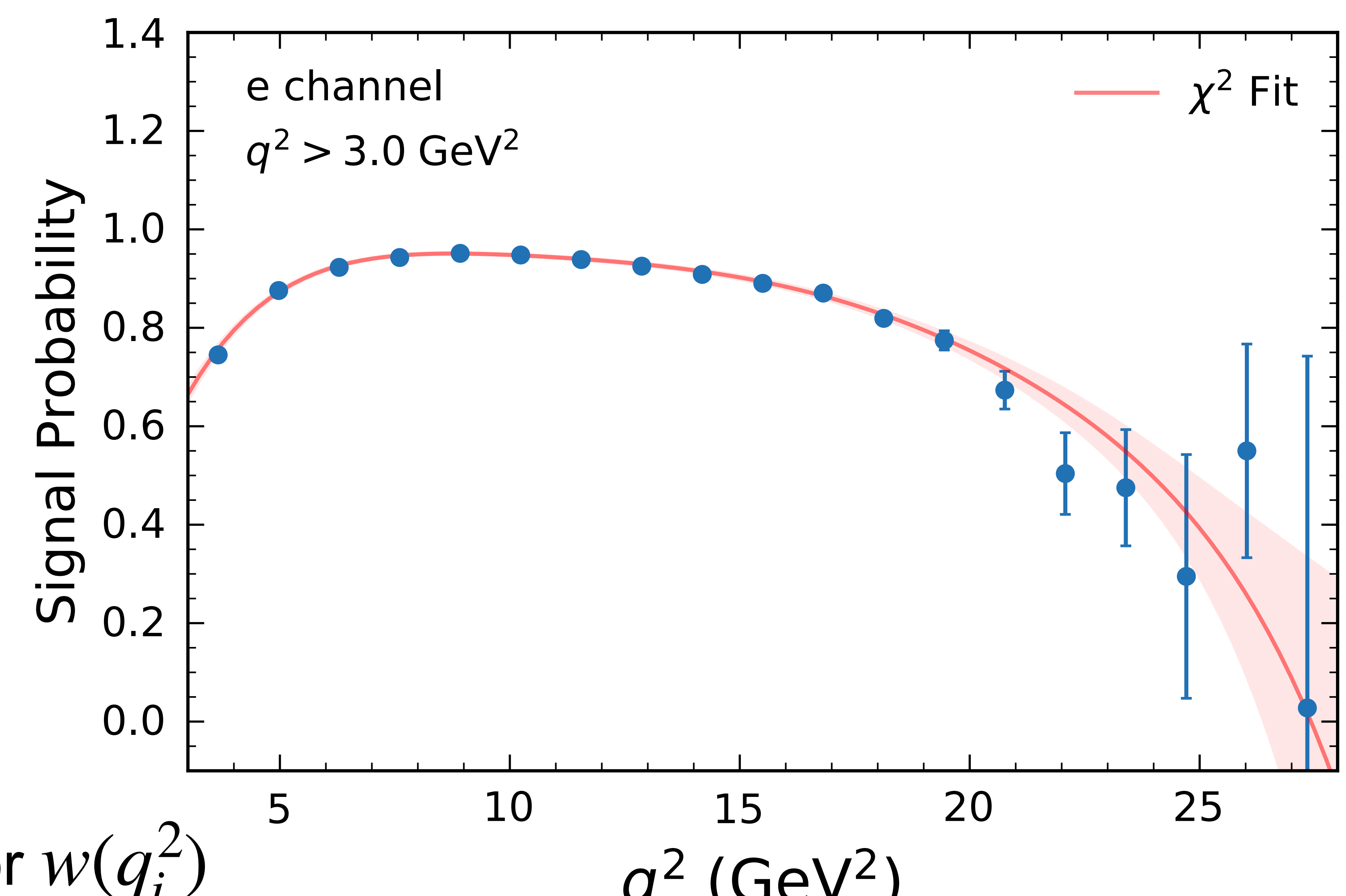
q^2 moments in $B \to X_c \ell^+ \nu_\ell$

- Analysis features
  - use full reconstruction tagging (of the accompanying B) — B_{tag}
  - Remainder (after B_{tag}) of the signal event (ℓ^+, X_c and a missing ν) — measure M_X, q^2
- The moments — how to

$$\langle q^{2m} \rangle = \frac{C_{\text{cal}} \cdot C_{\text{acc}}}{\sum_i^{\text{events}} w(q_i^2)} \times \sum_i^{\text{events}} w(q_i^2) \cdot q_{\text{cal}\, i}^{2m}$$



for $w(q_i^2)$



preliminary

q^2 moments in $B \to X_c \ell^+ \nu_\ell$

- Analysis features
  - use full reconstruction tagging (of the accompanying B) — B_{tag}
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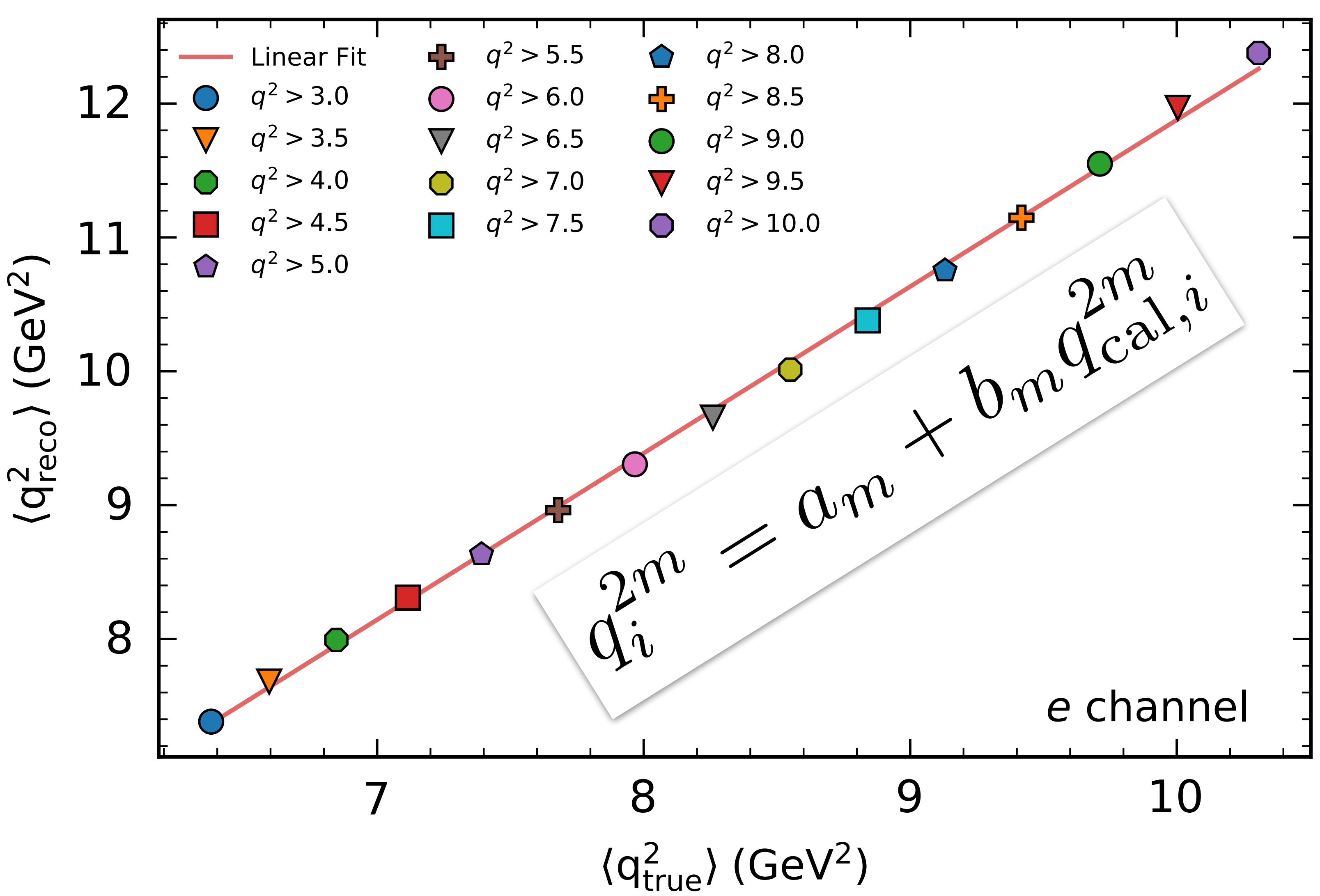
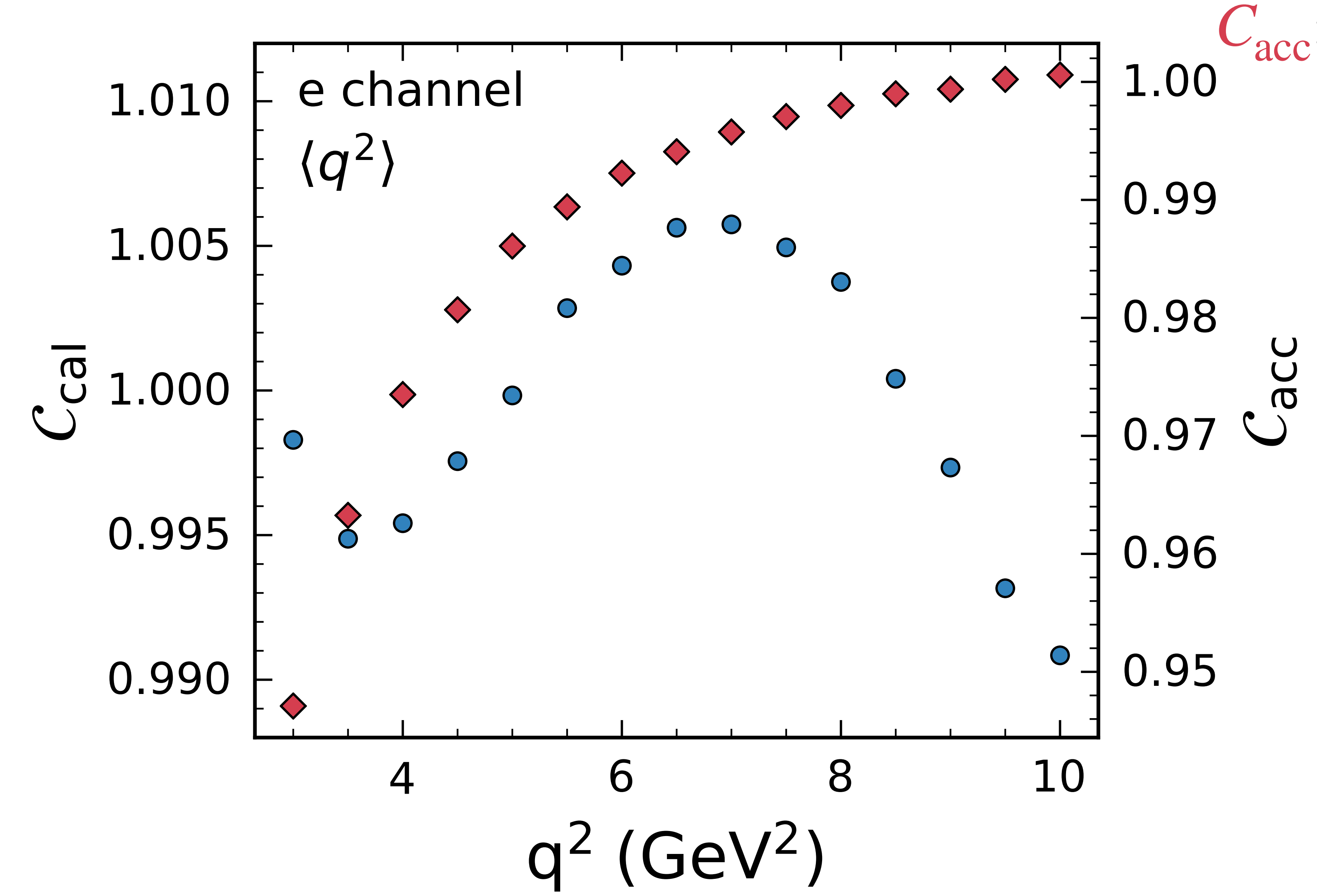
$$\langle q^{2m} \rangle = \frac{C_{\rm cal} \cdot C_{\rm acc}}{\sum_i^{\rm events} w(q_i^2)} \times \sum_i^{\rm events} w(q_i^2) \cdot q_{{\rm cal}\, i}^{2m}$$

$C_{\rm cal}$: residual bias correction

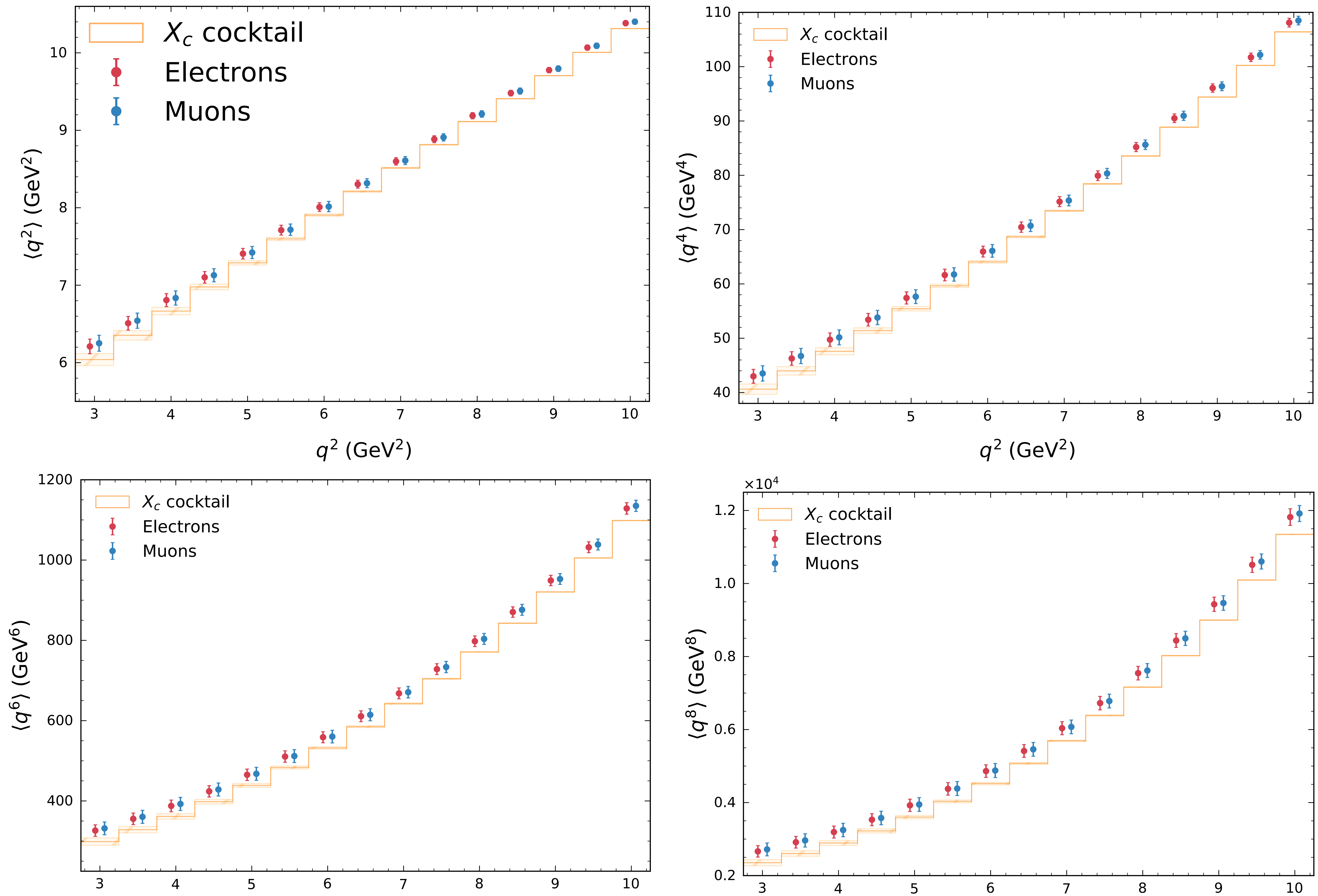
$C_{\rm acc}$: acceptance correction



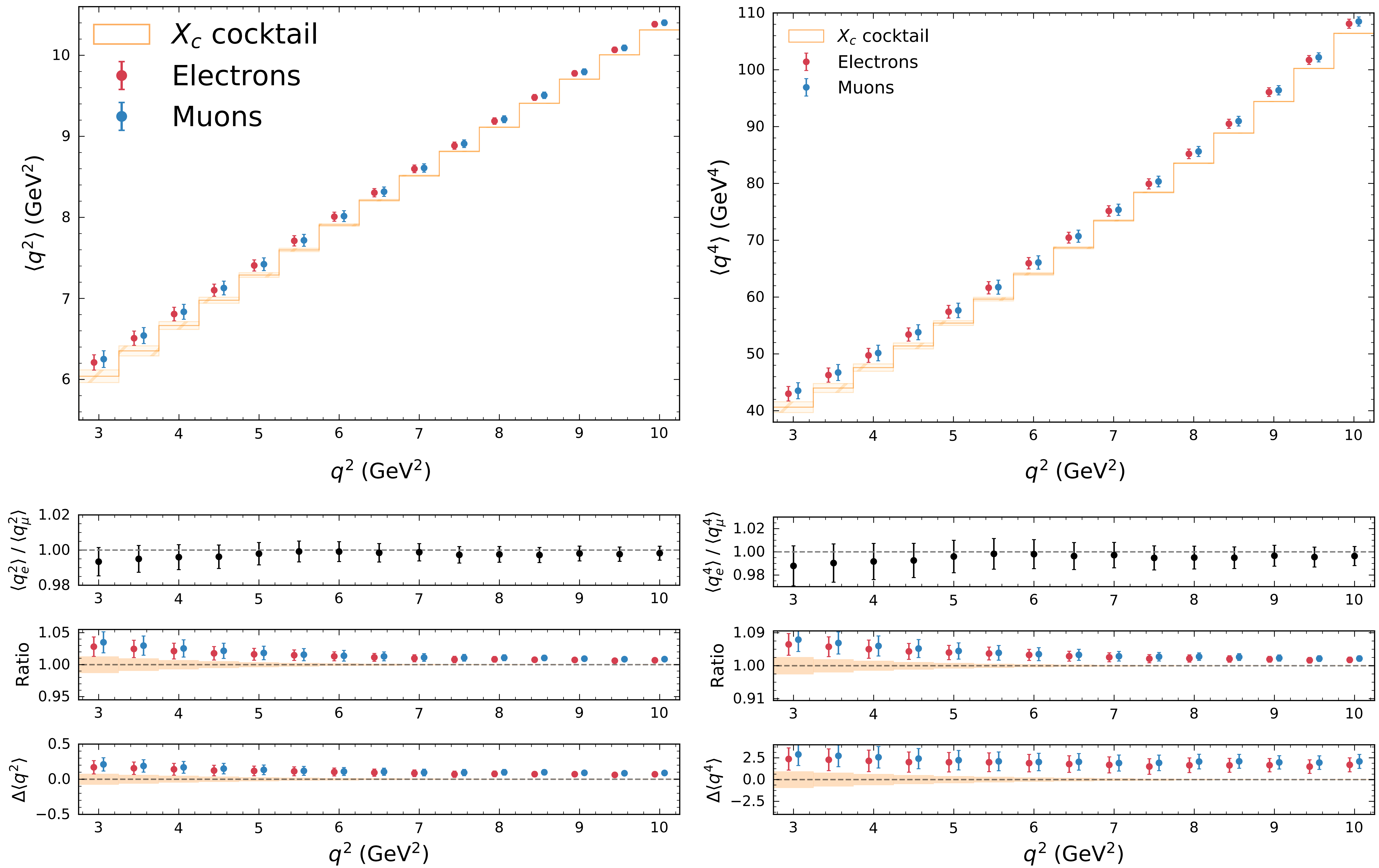


preliminary

q^2 moments in $B \to X_c \ell^+ \nu_\ell$



q^2 moments in $B \to X_c \ell^+ \nu_\ell$



Inclusive $B \to X_u \ell^+ \nu_\ell$ and $|V_{ub}|$

Features of the analysis

- extend the analysis region as much as possible into the $B \to X_c \ell^+ \nu_\ell$ dominated region of phase space
- use "full reconstruction" (of the accompanying B) or "B-tagging" method
- machine learning to (i) reconstruct $B_{\rm tag}$ (NN), and (ii) to separate $B \to X_u \ell^+ \nu_\ell$ from the dominant $B \to X_c \ell^+ \nu_\ell$ background (BDT)
- we focus on integrated (over phase space) measurement of $\Delta\mathscr{B}$

main improvement

- more efficient B tagging using NN ➔ effective increase of statistics ($\times\, 1.8$)
- improved modeling of $B \to X_c \ell^+ \nu_\ell$ background (e.g. for "gap modes") as well as signal

Simulation of signal decays

The DFN model (De Fazio & Neubert, JHEP 06, 017 (1999))

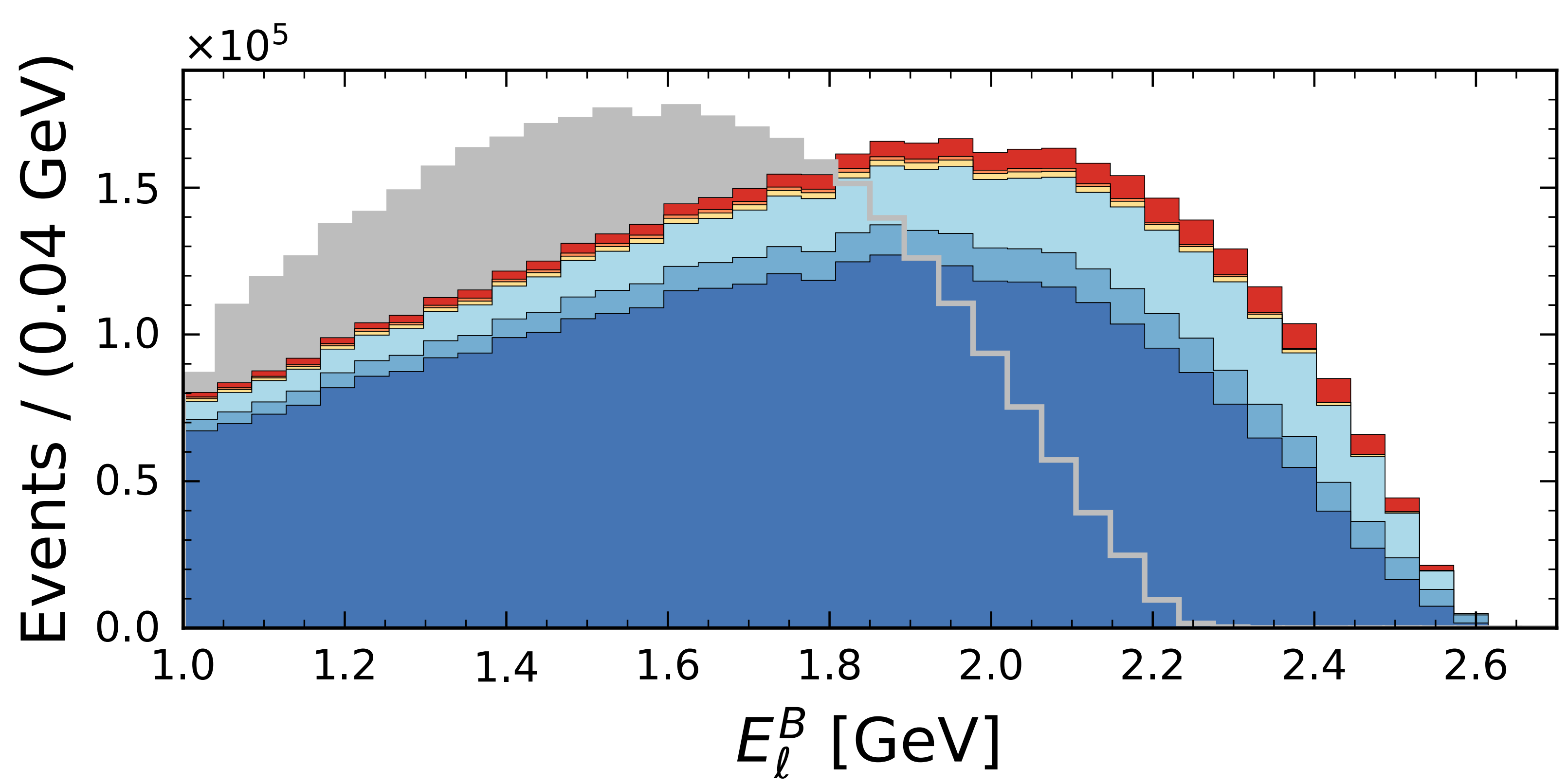
- triple differential rate on (q^2, E_ℓ^B, M_X^2),
- convolved with a non-perturbative shape function using ad-hoc model
- free parameters of the model
  - m_b^{KN} in Kagan-Neubert scheme[52], and another parameter a_{KN}
  - determined from a fit to $B \to X_c \ell^+ \nu_\ell$ and $B \to X_s \gamma$ decays[53]

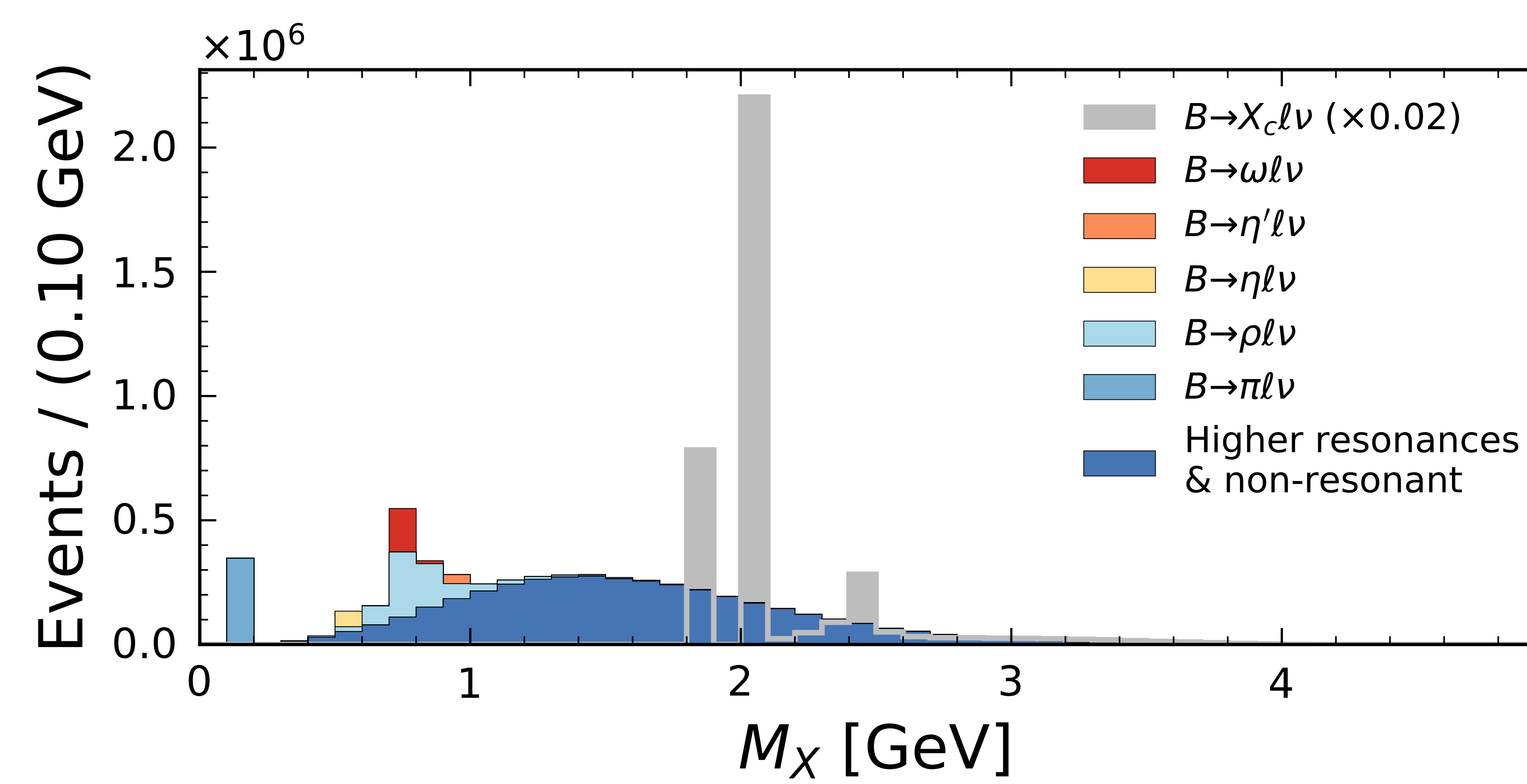
[52] A. L. Kagan and M. Neubert, Eur. Phys. J. C **7**, 5 (1999), arXiv:hep-ph/9805303.
[53] O. Buchmuller and H. Flacher, Phys. Rev. D **73**, 073008 (2006), arXiv:hep-ph/0507253.
[54] T. Sjöstrand, Comput. Phys. Commun. **82**, 74 (1994).
[55] C. Ramirez, J. F. Donoghue, and G. Burdman, Phys. Rev. **D 41**, 1496 (1990).
[56] M. Prim *et al.* (Belle Collaboration), Phys. Rev. D **101**, 032007 (2020), arXiv:1911.03186 [hep-ex].
[58] B. O. Lange, M. Neubert, and G. Paz, Phys. Rev. D **72**, 073006 (2005), arXiv:hep-ph/0504071.

the "hybrid" approach

- to combine the exclusive and inclusive $B \to X_u \ell^+ \nu_\ell$
- original suggestion[55]; implementation[56]; check w/ BLNP[58] for model dependence

$$\Delta\mathcal{B}_{ijk}^{\mathrm{incl}} = \Delta\mathcal{B}_{ijk}^{\mathrm{excl}} + w_{ijk} \times \Delta\mathcal{B}_{ijk}^{\mathrm{incl}} ,$$



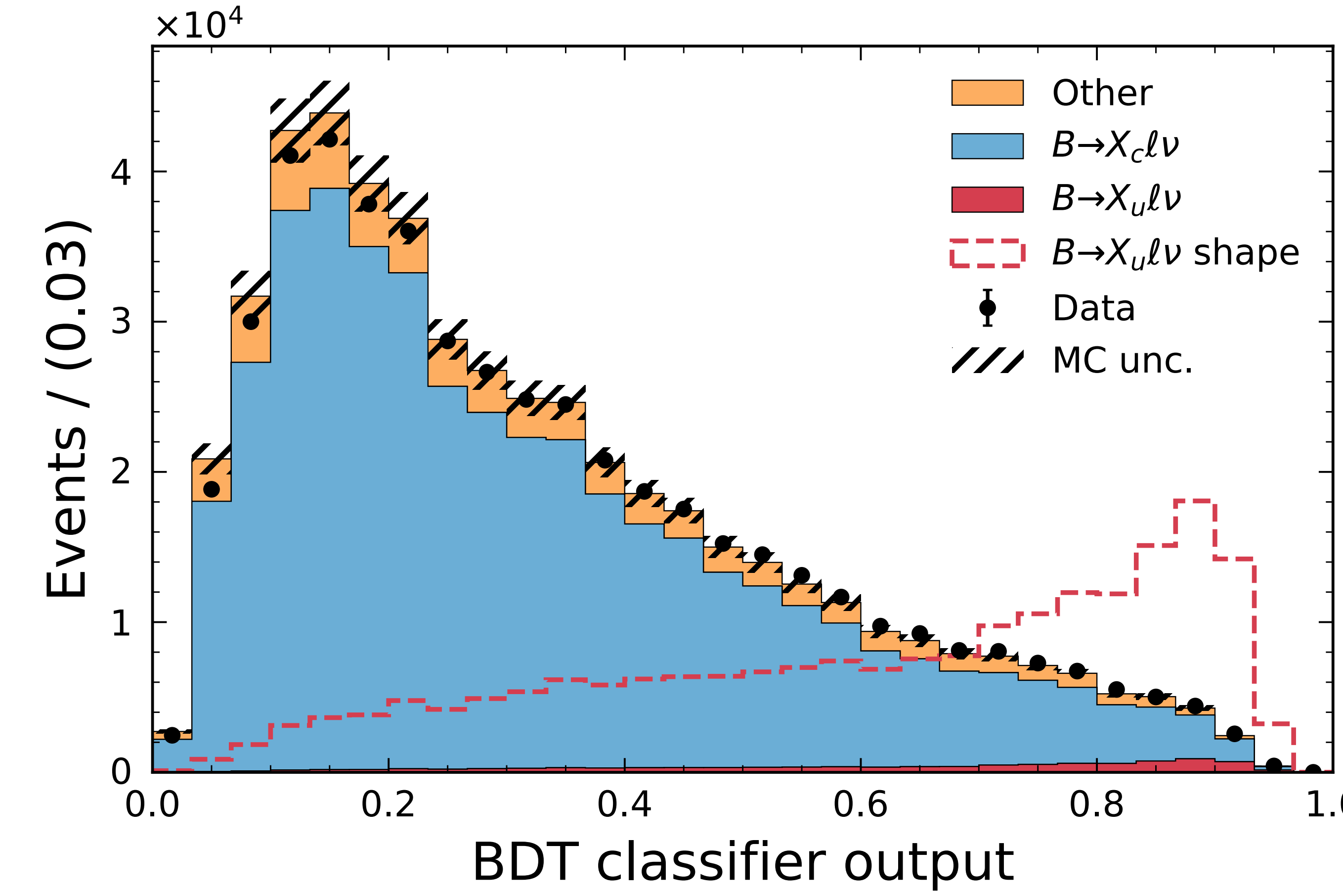


generator-level distributions of $B \to X_u \ell^+ \nu_\ell$ (scaled, x50) and $B \to X_c \ell^+ \nu_\ell$ after B-tag selection

Background suppression using BDT

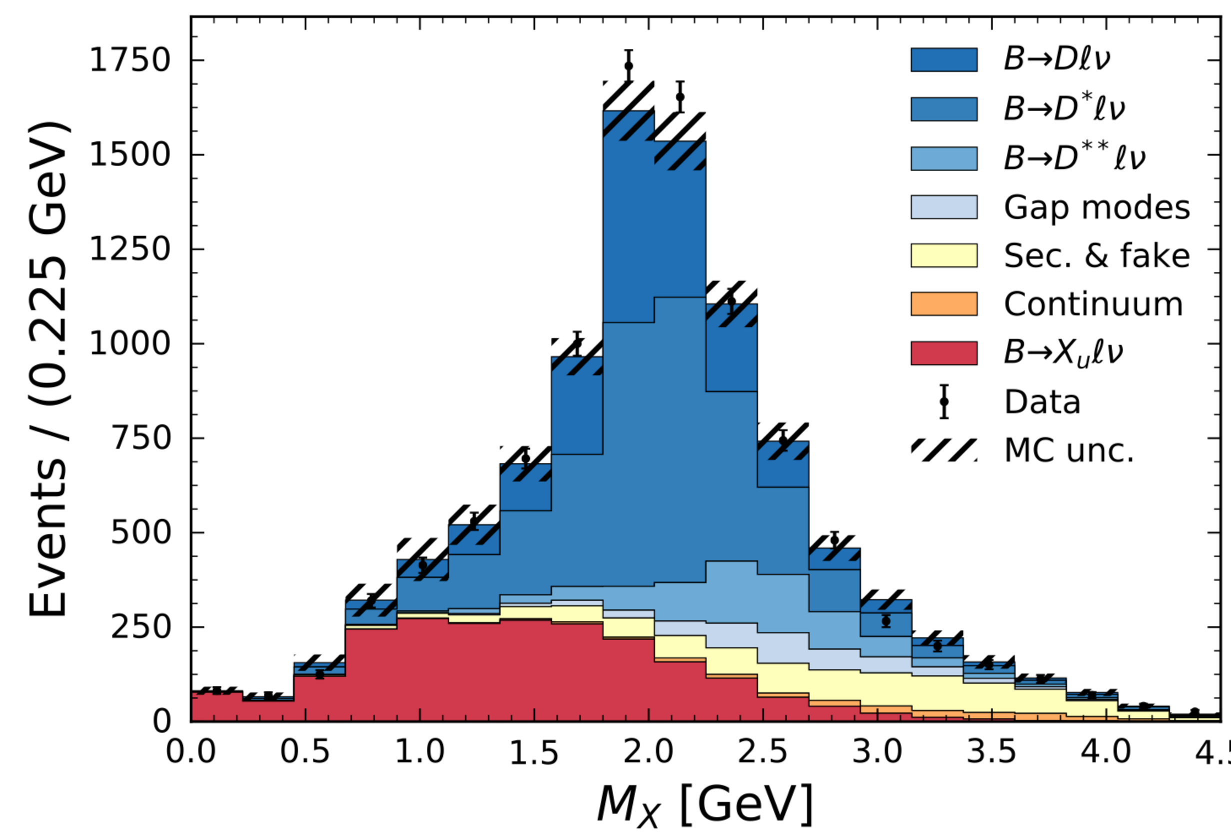
- BDT training ($B \to X_u \ell^+ \nu_\ell$ against $B \to X_c \ell^+ \nu_\ell$) using
  - M^2_{miss} (larger multiplicity of X_c → more broadening)
  - D^* veto — use slow-π kinematics
  - # of kaons: $K^\pm$, K^0_S
  - B_{sig} vertex fit
  - sum(charges)

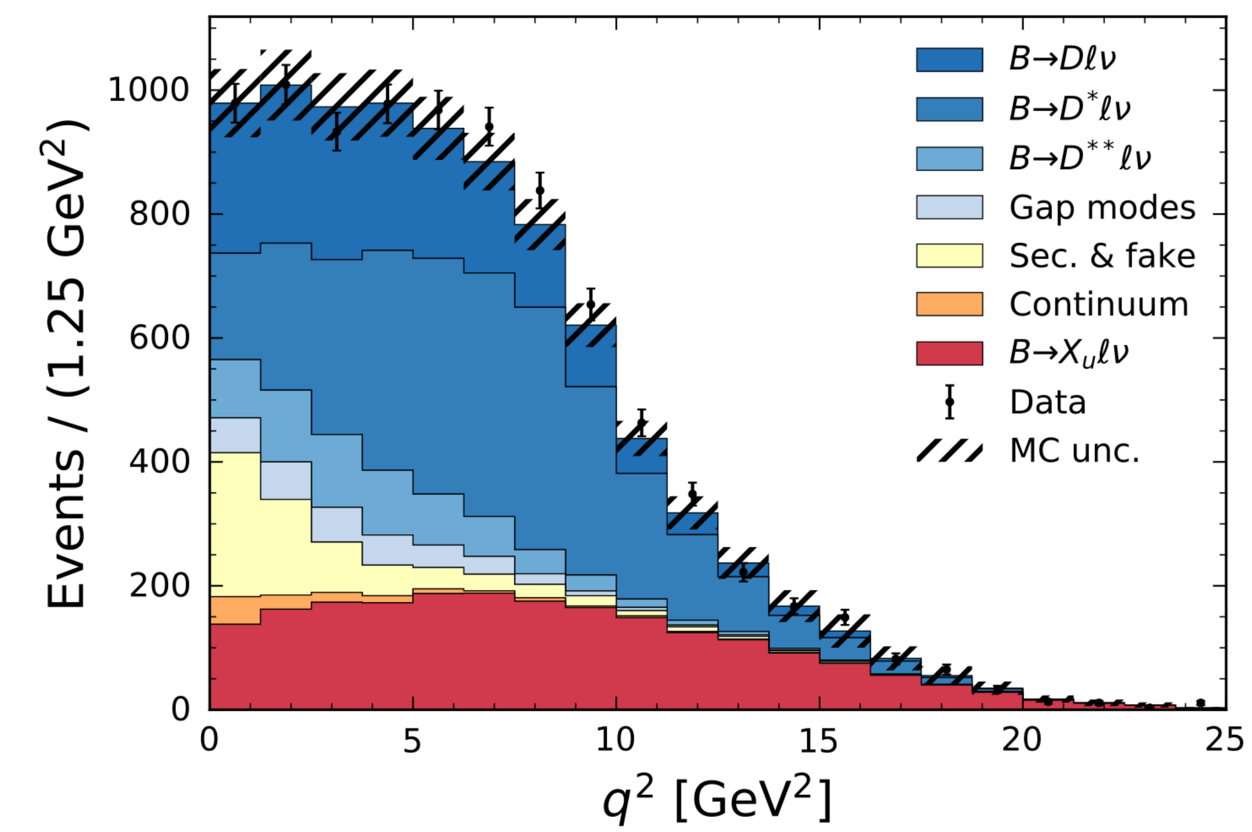
Selection	$B \to X_u \ell^+ \nu_\ell$	$B \to X_c \ell^+ \nu_\ell$	Data
$M_{\text{bc}} > 5.27$ GeV	84.8%	83.8%	80.2%
$\mathcal{O}_{\text{BDT}} > 0.85$	18.5%	1.3%	1.6%
$\mathcal{O}_{\text{BDT}} > 0.83$	21.9%	1.7%	2.1%
$\mathcal{O}_{\text{BDT}} > 0.87$	14.5%	0.9%	1.1%



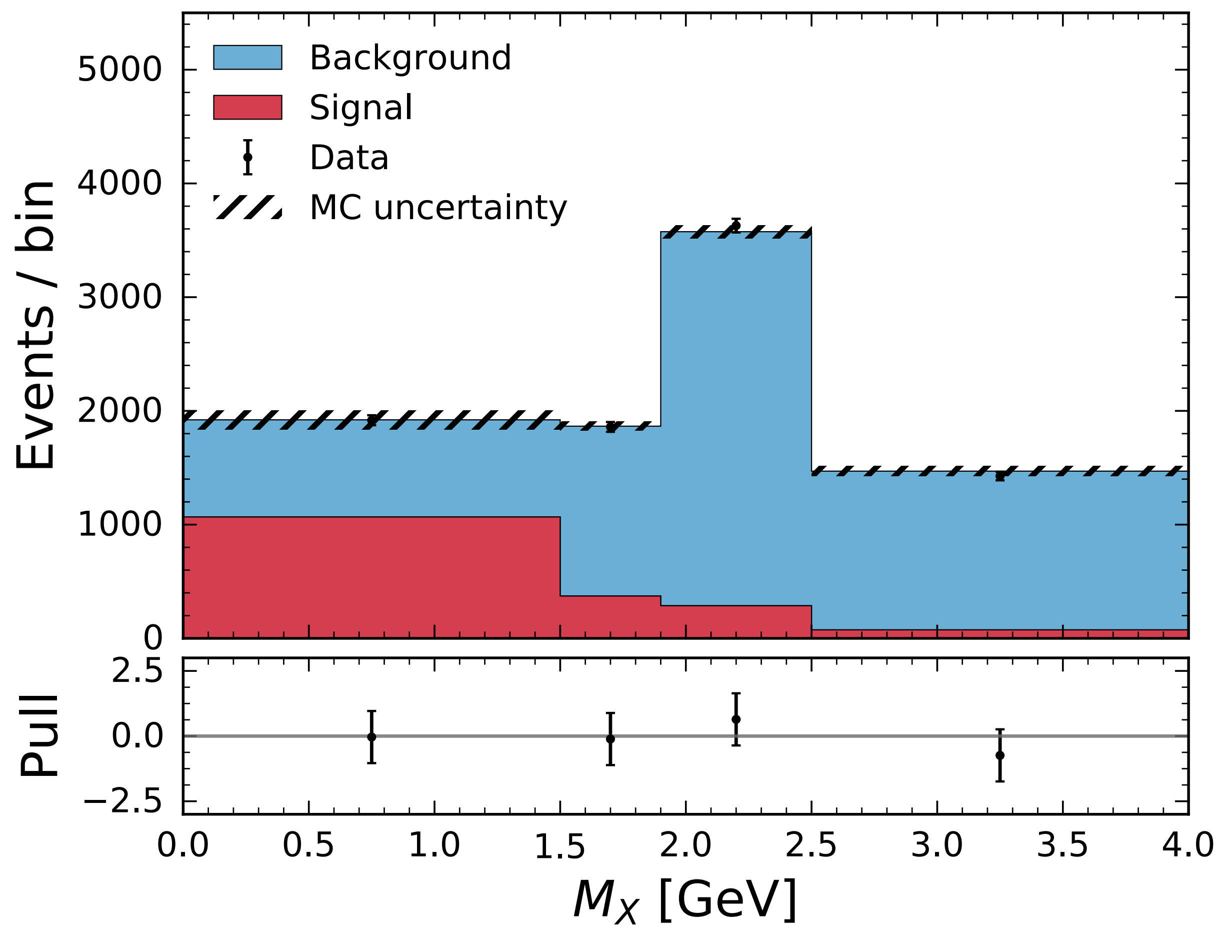
- maximize the expected significance of the most inclusive partial BF, with systematic error

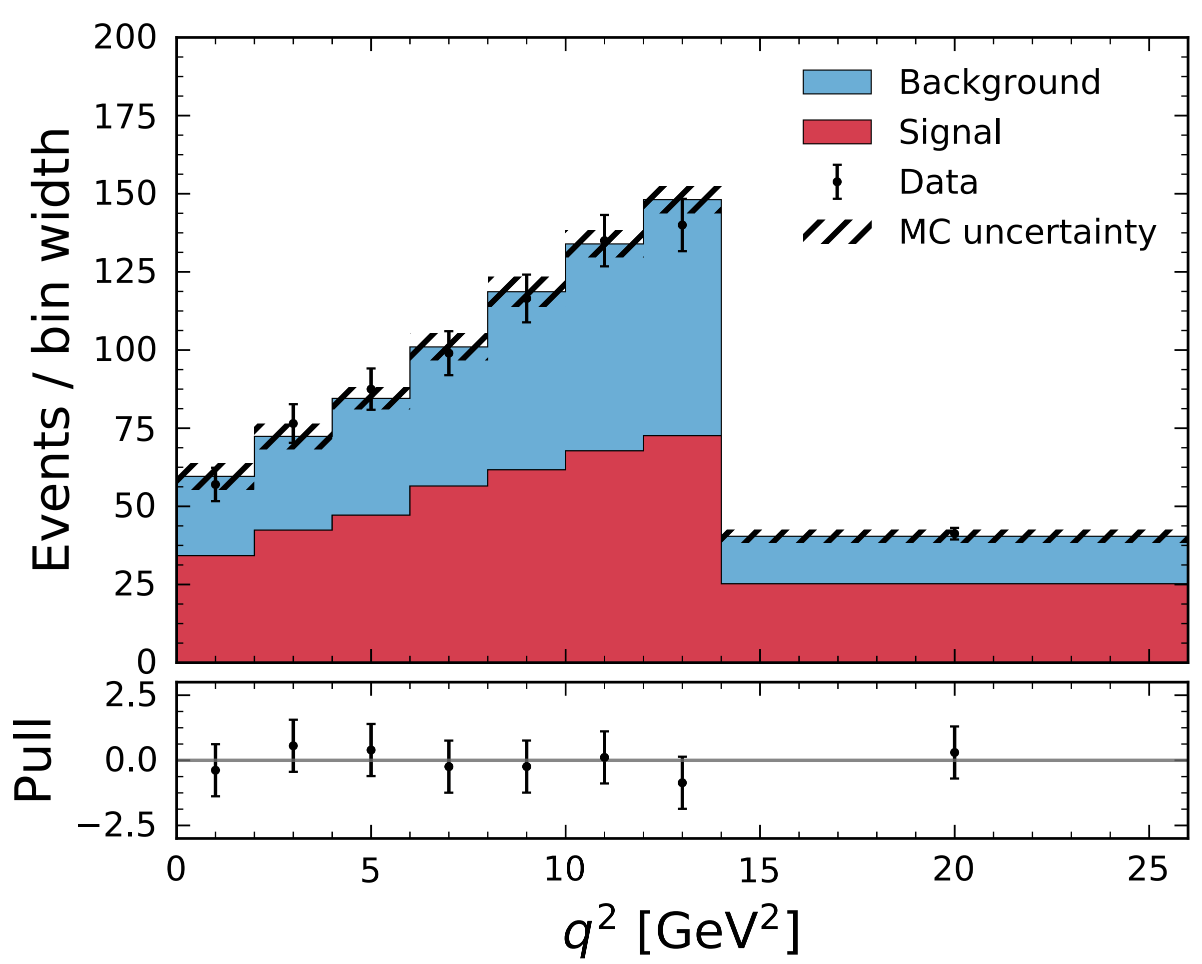
- Signal & bkgd. distribution after BDT
  - $B \to X_u \ell^+ \nu_\ell$ is scaled to world average BF (before fitting)
  - data & MC, in good agreement
  - $M_X < 1.7$ GeV, to reduce the dependence on the $B \to X_c \ell^+ \nu_\ell$ modeling





Partial branching fraction





- 2D fit on (M_X, q^2) with $E_\ell^B > 1.0$ GeV
  - 85% of $B \to X_u \ell^+ \nu_\ell$ phase space,
  - for the most precise determination
  - [Left] post-fit distributions from the fit, shown for the signal-enriched region $M_X < 1.5$ GeV *(for the q^2 plot)*

$$\Delta\mathcal{B}(B \to X_u \ell^+ \nu_\ell) = (1.59 \pm 0.07 \pm 0.17) \times 10^{-3}$$

  - in good agreement with existing BaBar results

Phys. Rev. D **86**, 032004 (2012)

- comparison with Belle's previous inclusive $B \to X_u \ell^+ \nu_\ell$ (PRL, 2010)
  - consistent within 1.7σ (exact stat. correlation difficult to assess)
  - main difference in modeling of signal and bkgd.
  - with improved understanding, along with more precise measurements of BFs and form factors
  - this (2021) work supersedes (2010)

Determination of $|V_{ub}|$

- by the relation

$$|V_{ub}| = \sqrt{\frac{\Delta\mathcal{B}(B \to X_u \ell^+ \nu_\ell)}{\tau_B \cdot \Delta\Gamma(B \to X_u \ell^+ \nu_\ell)}}$$

  - $\Delta\Gamma$ is the theory calculation of width, not including the CKM factor

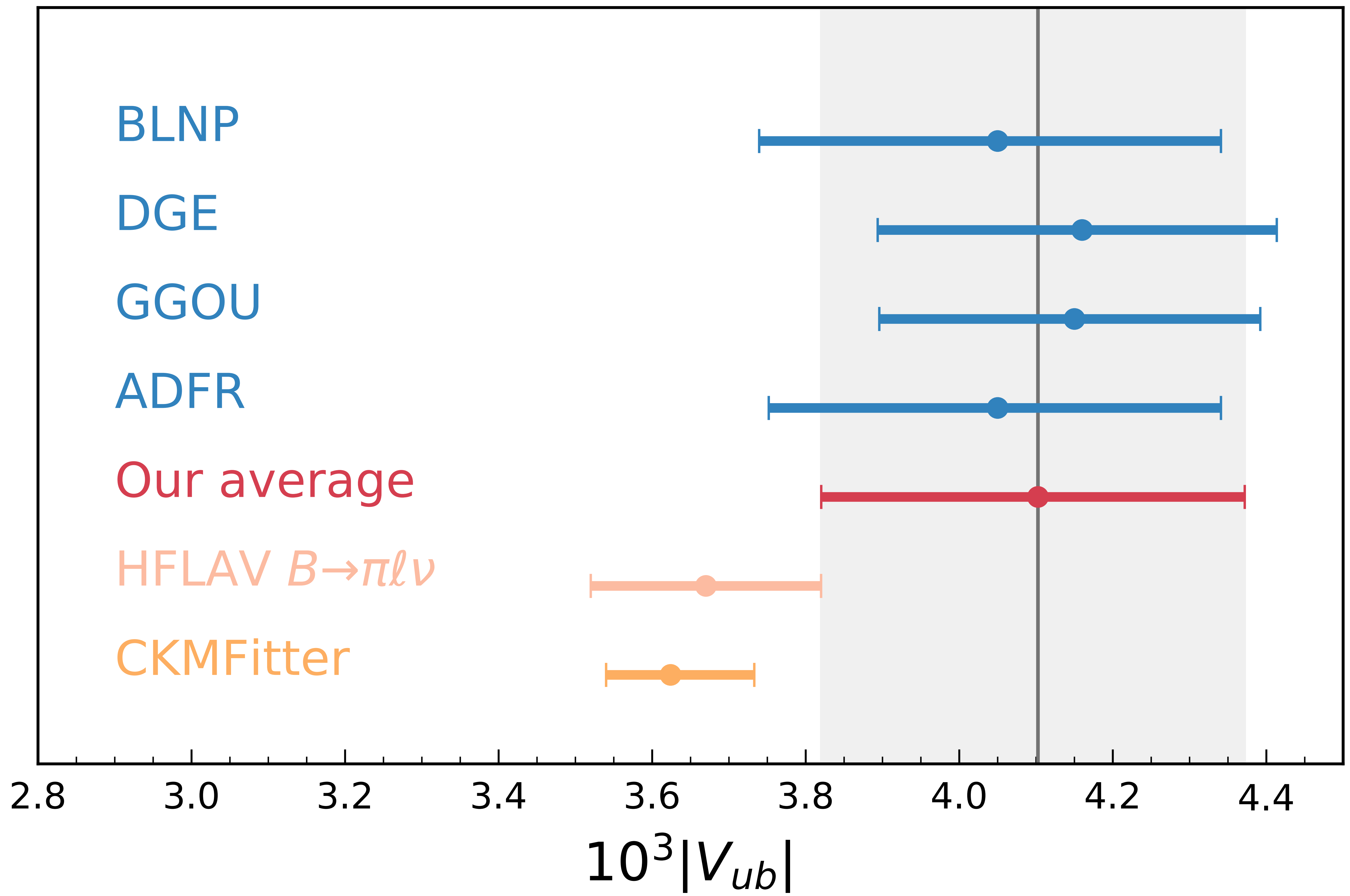
- use Four theory predictions
  - BLNP[17]
  - DGE[19,20]
  - GGOU[18]
  - ADFR[21,22]

[17] B. O. Lange, M. Neubert, and G. Paz, Phys. Rev. D **72**, 073006 (2005), arXiv:hep-ph/0504071.
[18] P. Gambino, P. Giordano, G. Ossola, and N. Uraltsev, JHEP **10**, 058 (2007), arXiv:0707.2493 [hep-ph].
[19] J. R. Andersen and E. Gardi, JHEP **01**, 097 (2006), arXiv:hep-ph/0509360.
[20] E. Gardi, Frascati Phys. Ser. **47**, 381 (2008), arXiv:0806.4524 [hep-ph].
[21] U. Aglietti, F. Di Lodovico, G. Ferrera, and G. Ricciardi, Eur. Phys. J. C **59**, 831 (2009), arXiv:0711.0860 [hep-ph].
[22] U. Aglietti, G. Ferrera, and G. Ricciardi, Nucl. Phys. B **768**, 85 (2007), arXiv:hep-ph/0608047.

$\Delta\Gamma(B \to X_u \ell^+ \nu_\ell)$ from various theory calculations (ps^{-1})

Phase-space region	BLNP [17]	DGE [19, 20]	GGOU [18]	ADFR [21, 22]
$M_X < 1.7\,\mathrm{GeV}$	$45.2^{+5.4}_{-4.6}$	$42.3^{+5.8}_{-3.8}$	$43.7^{+3.9}_{-3.2}$	$52.3^{+5.4}_{-4.7}$
$M_X < 1.7\,\mathrm{GeV},\ q^2 > 8\,\mathrm{GeV}^2$	$23.4^{+3.4}_{-2.6}$	$24.3^{+2.6}_{-1.9}$	$23.3^{+3.2}_{-2.4}$	$31.1^{+3.0}_{-2.6}$
$E_\ell^B > 1\,\mathrm{GeV}$	$61.5^{+6.4}_{-5.1}$	$58.2^{+3.6}_{-3.0}$	$58.5^{+2.7}_{-2.3}$	$61.5^{+5.8}_{-5.1}$

Determination of $|V_{ub}|$



- $|V_{ub}|$ from inclusive $B \to X_u \ell^+ \nu_\ell$
- compatible with exclusive determination by 1.3σ
- compatibility with CKM unitarity[73] : 1.6σ
- differential shapes and other properties — left for future work

[73] J. Charles *et al.* (CKMfitter Group), Eur. Phys. J. C **41**, 1 (2005), arXiv:hep-ph/0406184.

$$|V_{ub}| = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3}$$

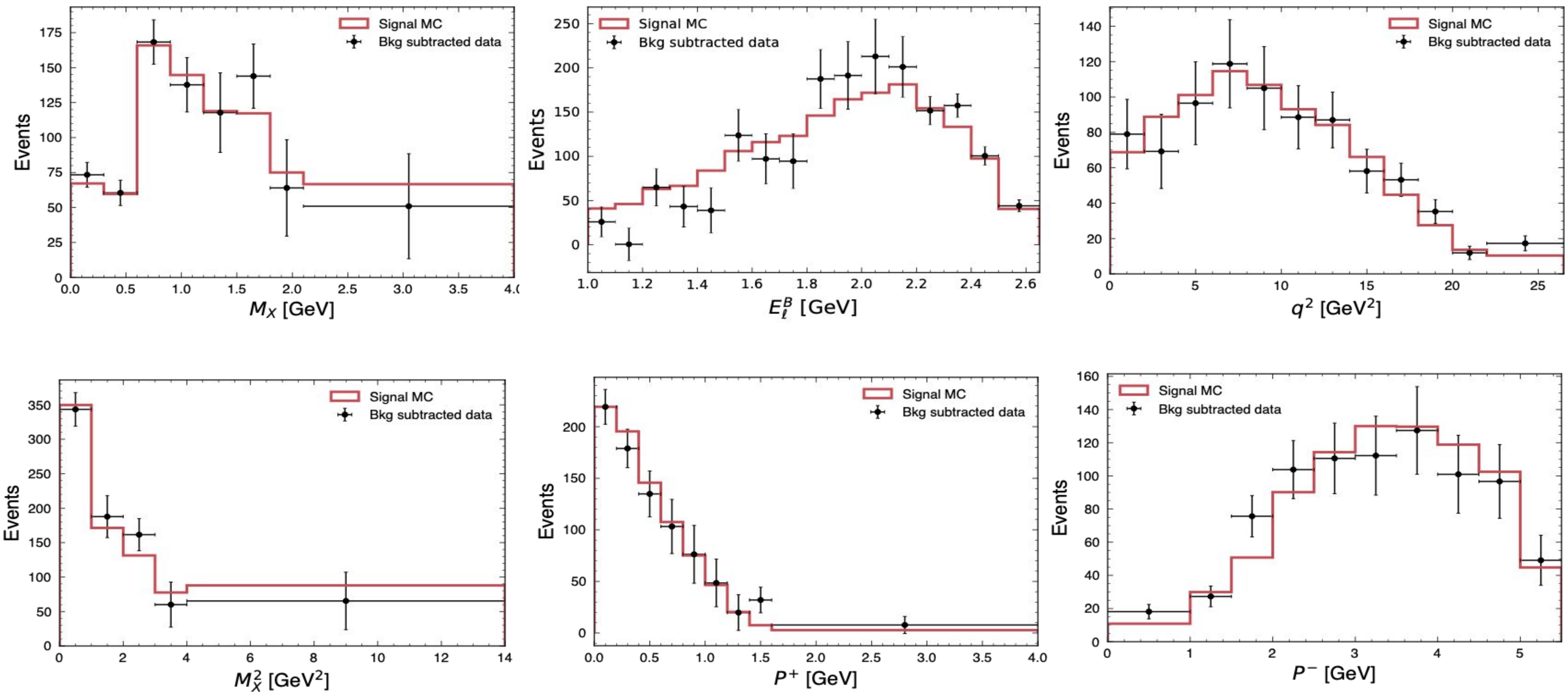
to quote a single value, we take a simple arithmetic avg. of the most inclusive results (2D fits)

Differential $\Delta\mathscr{B}(B \to X_u \ell^+ \nu)$

- Measure kinematic variables in the partial phase space with $E_\ell^B > 1$ GeV
  - $q^2,\ E_\ell^B,\ M_X,\ M_X^2,\ P_+,\ P_- \quad (P_\pm = E_X \mp |p_X|)$
- Basic selection & reconstruction following integrated partial BF analysis (PRD 104, 012008 (2021))
- additionally, require $|E_{\text{miss}} - p_{\text{miss}}| < 0.1$ GeV, $M_X < 2.4$ GeV to improve resolution and background shape uncertainty
- Background subtraction via M_X fit

Key distributions for Differential $\Delta\mathcal{B}$

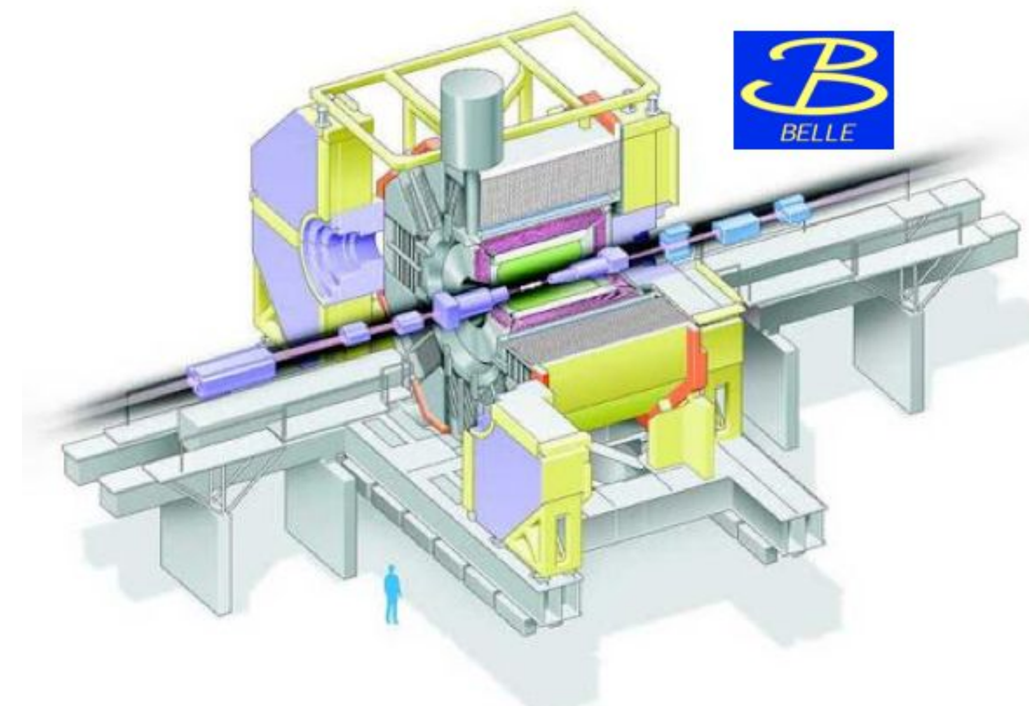
Preliminary



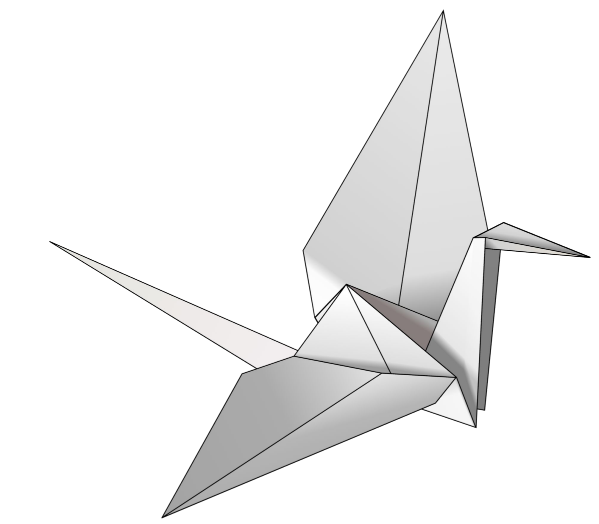
arXiv:2107.13855
submitted to PRL

Unfolding for Differential $\Delta\mathscr{B}$

X : true distribution

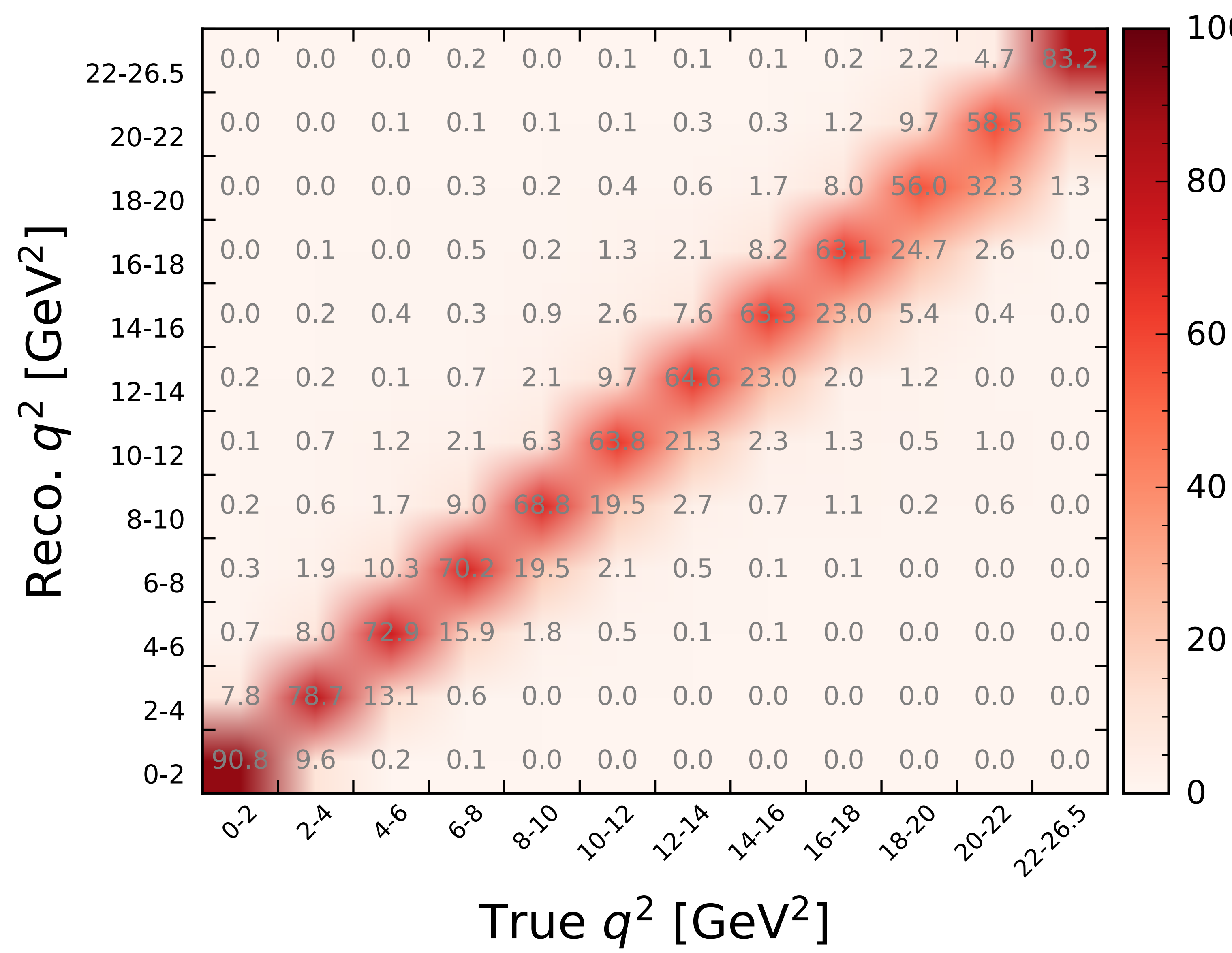


M : detector response



Y : measured distribution

- The migration matrix M as a representation of detector response (Fig. for q^2)
- $M(i,j)$ = prob (%) to observe an event in bin i if it had a true value in bin j



Reco. \ True	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18	18-20	20-22	22-26.5
22-26.5	0.0	0.0	0.0	0.2	0.0	0.1	0.1	0.1	0.2	2.2	4.7	83.2
20-22	0.0	0.0	0.1	0.1	0.1	0.1	0.3	0.3	1.2	9.7	58.5	15.5
18-20	0.0	0.0	0.0	0.3	0.2	0.4	0.6	1.7	8.0	56.0	32.3	1.3
16-18	0.0	0.1	0.0	0.5	0.2	1.3	2.1	8.2	63.1	24.7	2.6	0.0
14-16	0.0	0.2	0.4	0.3	0.9	2.6	7.6	63.3	23.0	5.4	0.4	0.0
12-14	0.2	0.2	0.1	0.7	2.1	9.7	64.6	23.0	2.0	1.2	0.0	0.0
10-12	0.1	0.7	1.2	2.1	6.3	63.8	21.3	2.3	1.3	0.5	1.0	0.0
8-10	0.2	0.6	1.7	9.0	63.8	19.5	2.7	0.7	1.1	0.2	0.6	0.0
6-8	0.3	1.9	10.3	70.2	19.5	2.1	0.5	0.1	0.1	0.0	0.0	0.0
4-6	0.7	8.0	72.9	15.9	1.8	0.5	0.1	0.1	0.0	0.0	0.0	0.0
2-4	7.8	78.7	13.1	0.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0-2	90.8	9.6	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

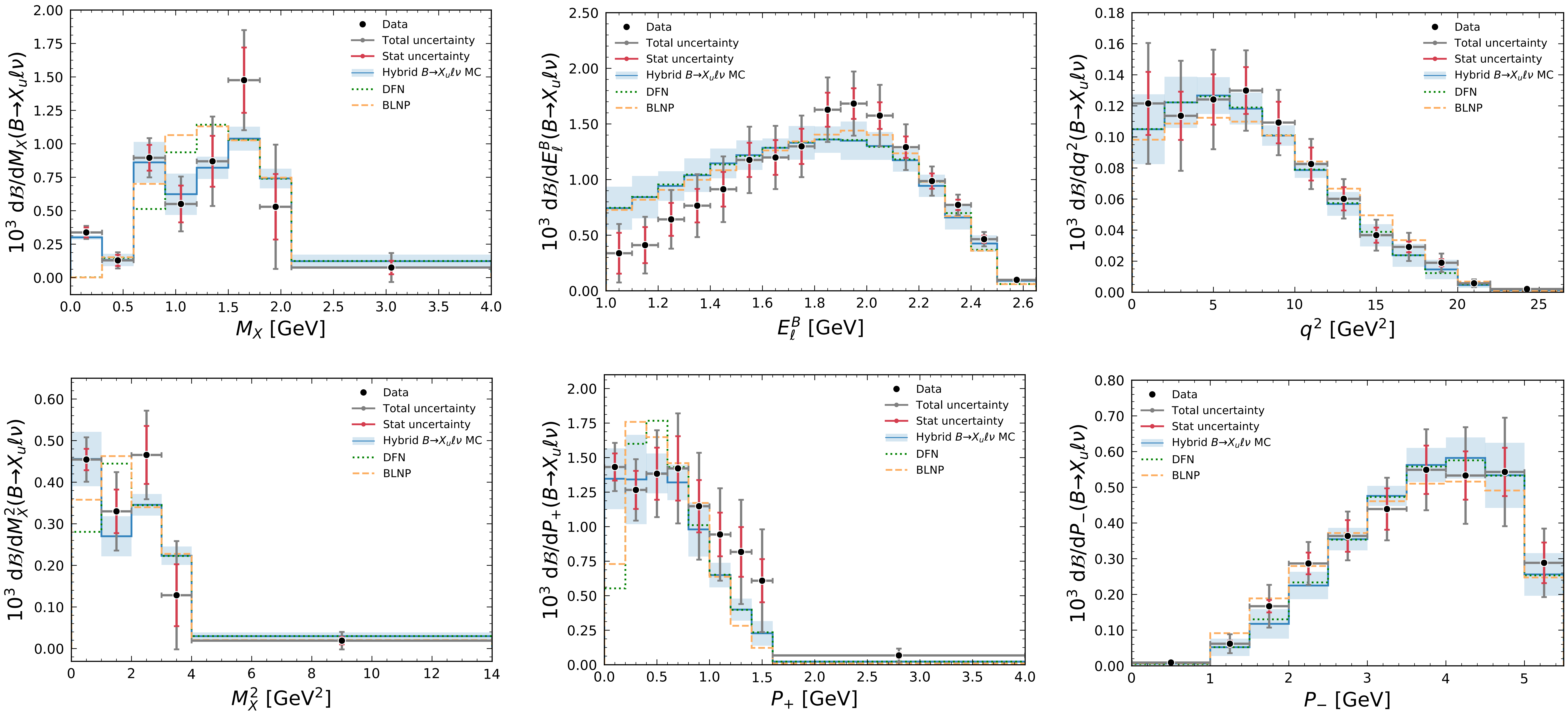
$Y = MX$ or $X = M^{-1}Y$

- We use Singular-Value-Decomposition (SVD) method for unfolding
[NIMA 372, 469 (1996)]

arXiv:2107.13855
submitted to PRL

Differential $\Delta\mathscr{B}(B \to X_u \ell^+ \nu)$ — Results

- Measure differential $\Delta\mathscr{B}(B \to X_u \ell^+ \nu)$ with $E_\ell^B > 1$ GeV (Figs. below)
- All MC shapes are normalized to $\Delta\mathscr{B} = 1.59 \times 10^{-3}$
- Useful input for future model-independent determination of $|V_{ub}|$



Study of $\overline{B}^0 \to D^+h^-$ $(h = K, \pi)$

Test of factorization

- for a better understanding of QCD effects :

  $R^D = \mathcal{B}(B \to DK)/\mathcal{B}(B \to D\pi) \simeq \tan^2\theta_C(f_K/f_\pi)^2$
  can provide crucial info.

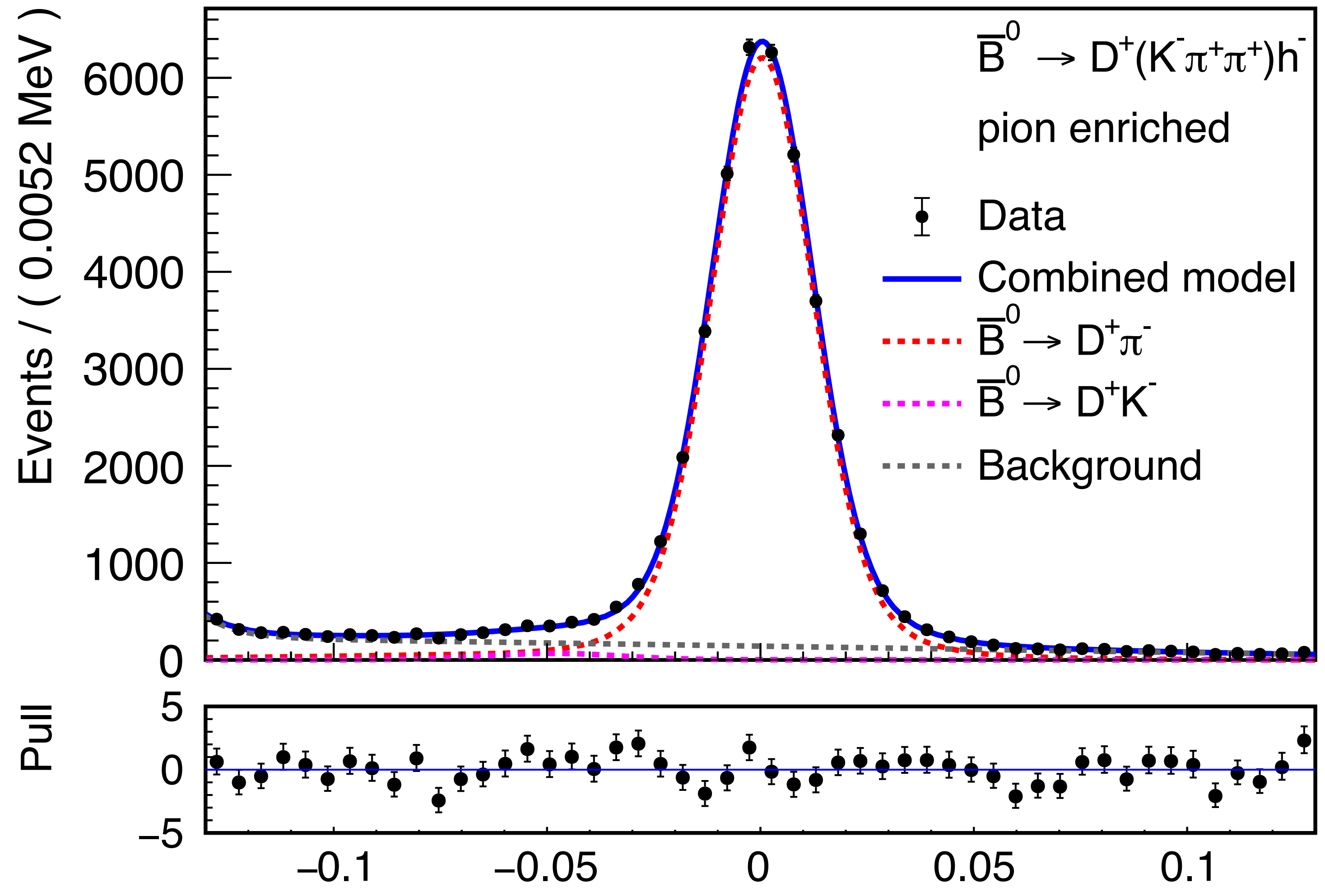
- Comparing $\overline{B} \to D^{(*)}\pi$ with $\overline{B} \to D^{(*)}\ell^-\overline{\nu}$ can be another good test of factorization (hence "*related*")

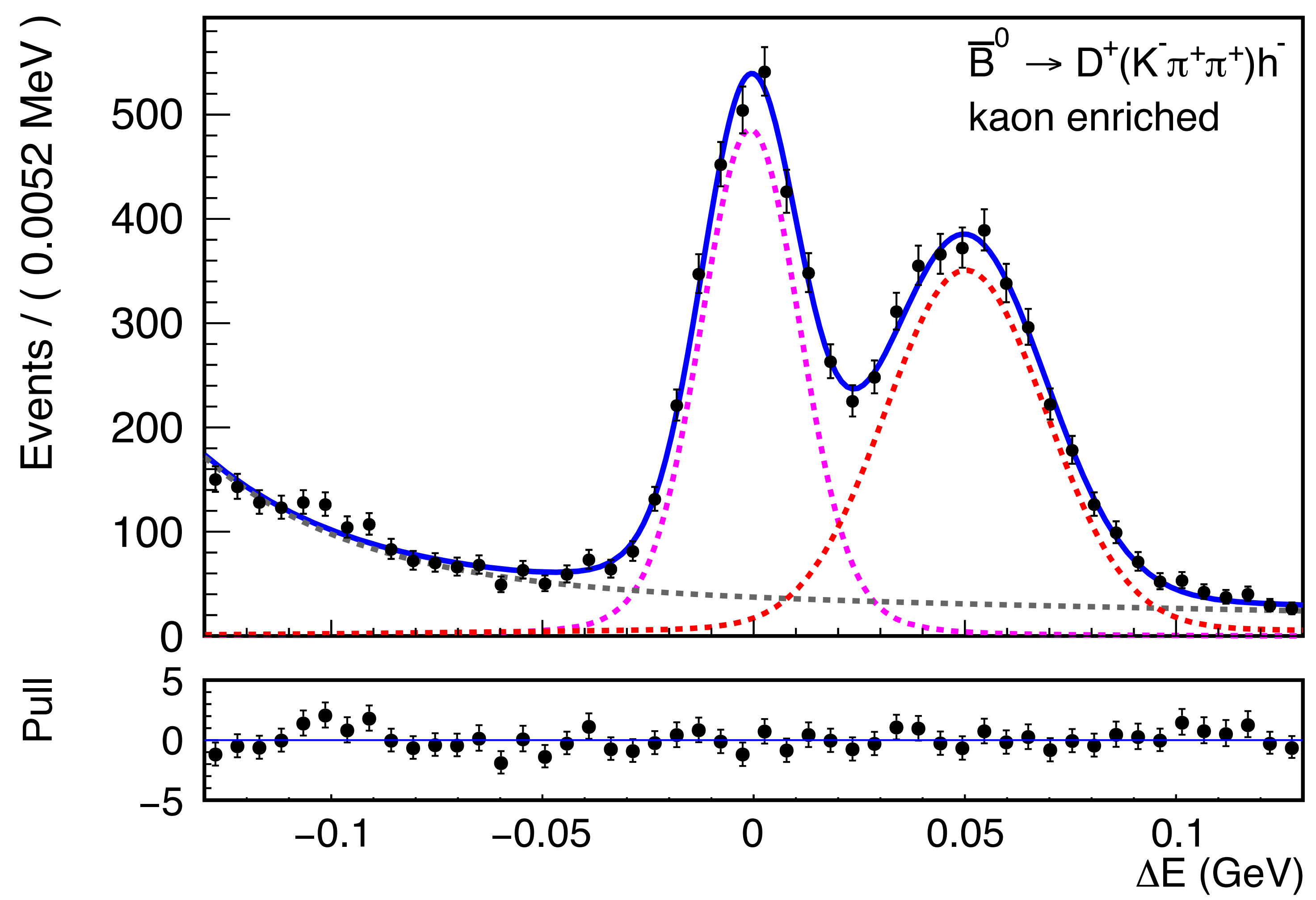
Results

$$R^D = 0.082 \pm 0.002 \pm 0.002$$

$$\mathcal{B}[\overline{B}^0 \to D^+(\to K^-\pi^+\pi^+)\pi^-] = (2.50 \pm 0.01 \pm 0.10 \pm 0.04) \times 10^{-3}$$

$$\mathcal{B}[\overline{B}^0 \to D^+(\to K^-\pi^+\pi^+)K^-] = (2.05 \pm 0.05 \pm 0.08 \pm 0.04) \times 10^{-4}$$





Epilogue

- Belle has made immense contributions to flavor physics and CP-violation, in particular, in the CKM unitarity triangle.
- Semileptonic B-meson decays provide great probe for CKM elements, $|V_{ub}|, |V_{cb}|$
- The exploration of semileptonic B decays continues at Belle, and will do so until the torch is passed on to Belle II.

"*There must be something in the flavors. We just don't know where we can find it and what its scale is.*"*

"We shall not cease from exploration"†

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* In a private conversation with Tao Han

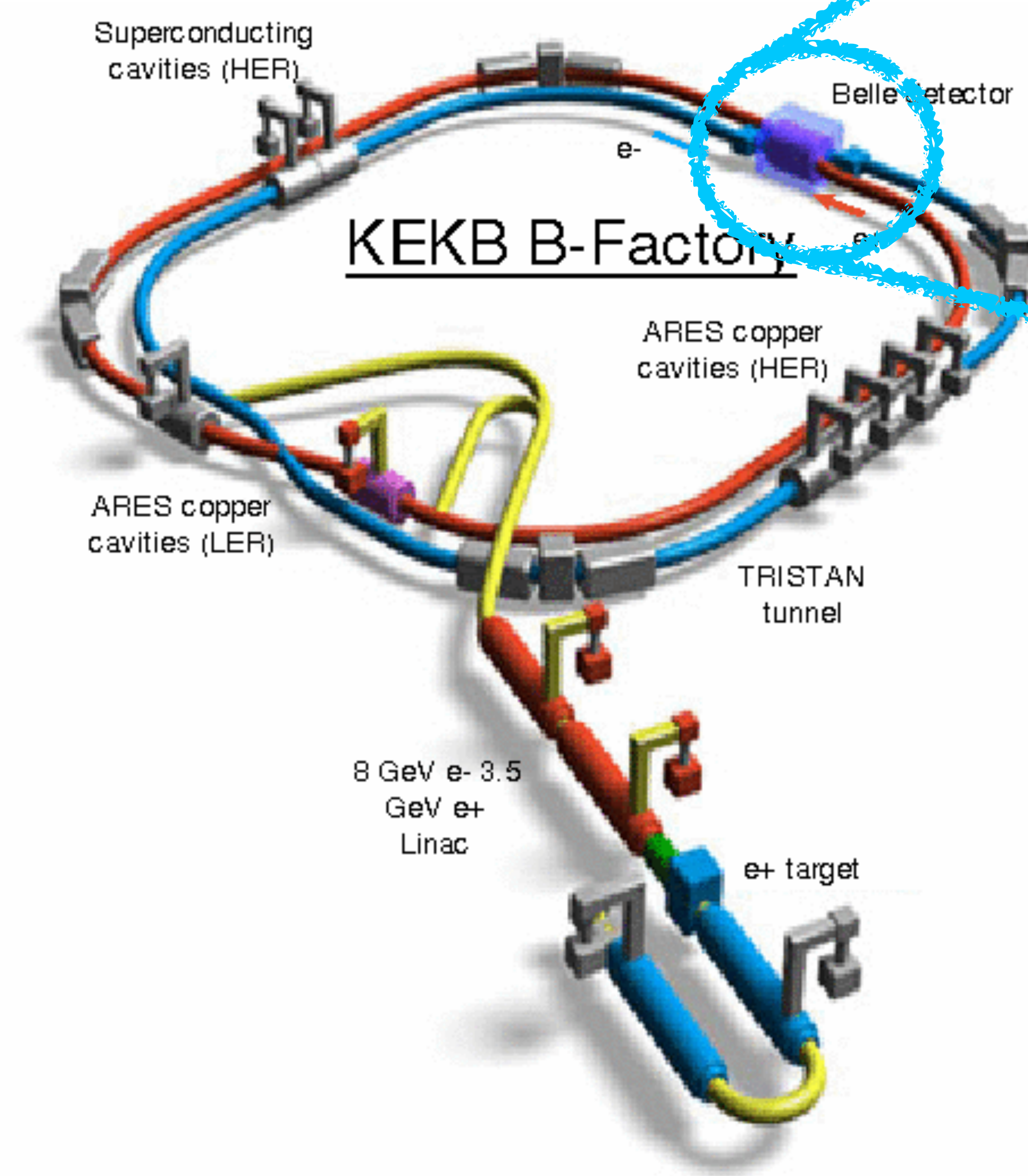
† T. S. Eliot

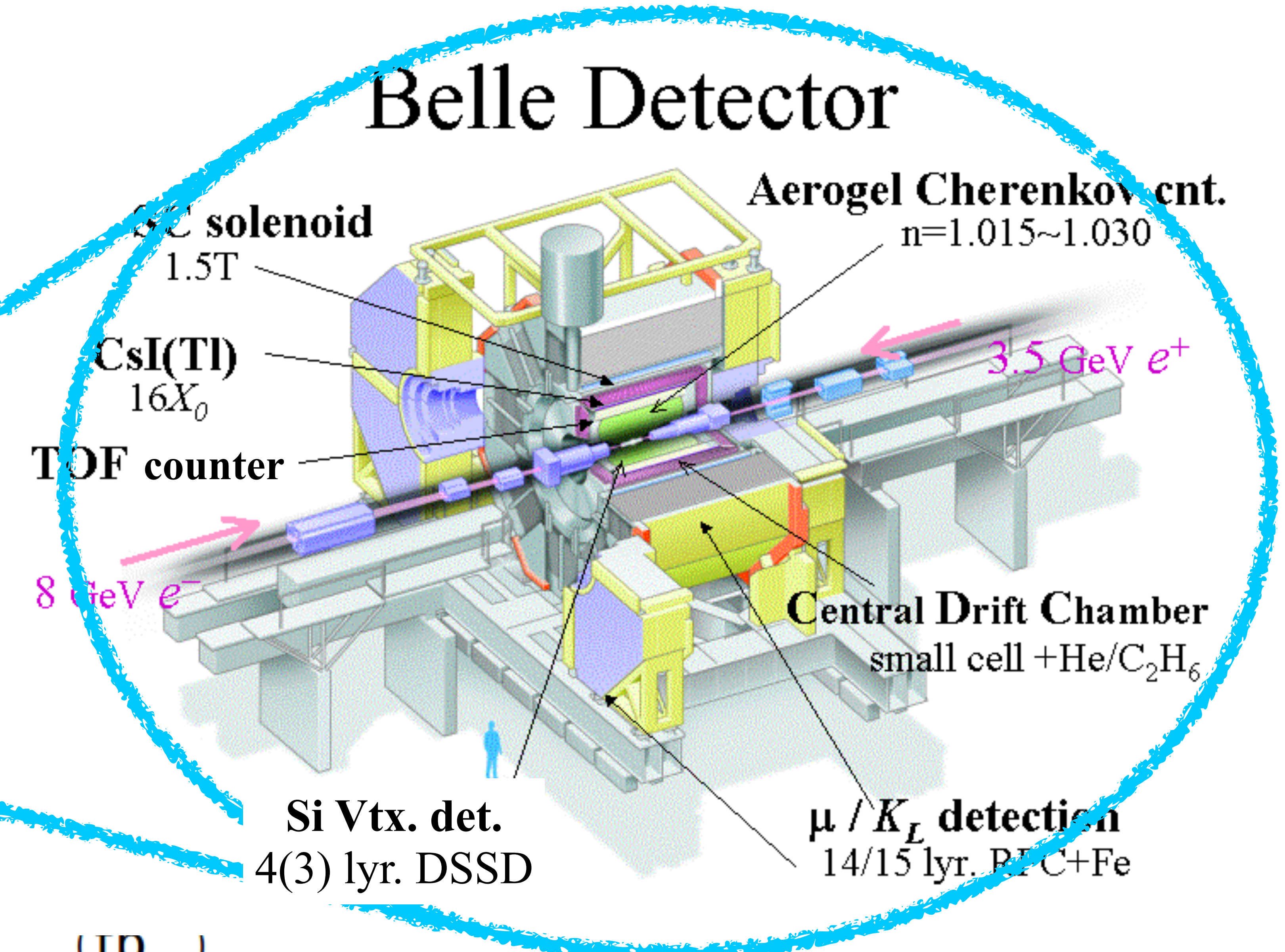
Thank you!

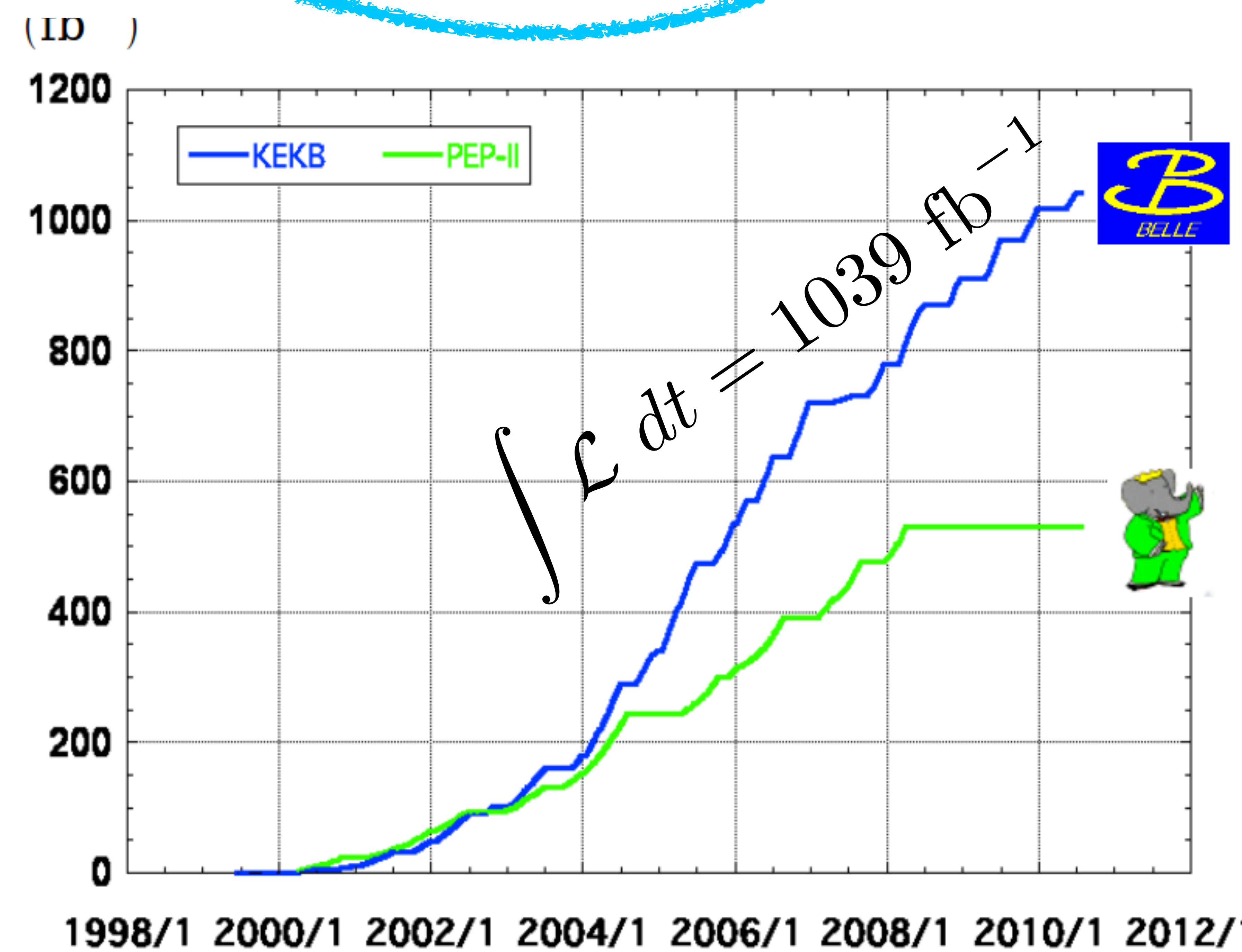
Back-up materials

22 countries
100 institutions
~450 members

$$\mathcal{L}_{\text{peak}} = 21.1\ \text{nb}^{-1}\text{s}^{-1}$$







> 1 ab^{-1}
On resonance:
$\Upsilon(5S)$: 121 fb^{-1}
$\Upsilon(4S)$: 711 fb^{-1}
$\Upsilon(3S)$: 3 fb^{-1}
$\Upsilon(2S)$: 25 fb^{-1}
$\Upsilon(1S)$: 6 fb^{-1}
Off reson./scan:
~ 100 fb^{-1}

~ 550 fb^{-1}
On resonance:
$\Upsilon(4S)$: 433 fb^{-1}
$\Upsilon(3S)$: 30 fb^{-1}
$\Upsilon(2S)$: 14 fb^{-1}
Off resonance:
~ 54 fb^{-1}

$$e^- \xrightarrow{\textbf{8 GeV}} (\star) \xleftarrow{\textbf{3.5 GeV}} e^+$$

$B^+ \to \pi^+\pi^-\ell^+\nu_\ell$

3 fit results and branching fractions

Bin	$M_{\pi\pi}$ [GeV]	Signal
1	$M_{\pi\pi} < 0.46$	$7.1^{+4.1}_{-3.2}$
2	$0.46 \leq M_{\pi\pi} < 0.54$	$10.0^{+4.4}_{-3.5}$
3	$0.54 \leq M_{\pi\pi} < 0.62$	$10.6^{+4.3}_{-3.5}$
4	$0.62 \leq M_{\pi\pi} < 0.70$	$23.3^{+6.2}_{-5.4}$
5	$0.70 \leq M_{\pi\pi} < 0.78$	$90.3^{+10.7}_{-10.0}$
6	$0.78 \leq M_{\pi\pi} < 0.86$	$50.5^{+8.1}_{-7.4}$
7	$0.86 \leq M_{\pi\pi} < 0.94$	$29.6^{+6.4}_{-5.7}$
8	$0.94 \leq M_{\pi\pi} < 1.02$	$10.2^{+4.2}_{-3.4}$
9	$1.02 \leq M_{\pi\pi} < 1.10$	$8.9^{+3.7}_{-3.0}$
10	$1.10 \leq M_{\pi\pi} < 1.18$	$5.7^{+3.1}_{-2.4}$
11	$1.18 \leq M_{\pi\pi} < 1.26$	$15.7^{+5.0}_{-4.2}$
12	$1.26 \leq M_{\pi\pi} < 1.34$	$11.8^{+4.2}_{-3.4}$

$$\mathcal{B}(B^+ \to \pi^+\pi^-\ell^+\nu_\ell)[\text{1D}(M_{\pi\pi})] = (22.3^{+2.0}_{-1.8}(\text{stat}) \pm 3.9(\text{syst})) \times 10^{-5}$$

Bin	q^2 [GeV^2]	Signal
1	$q^2 < 1$	$16.5^{+6.8}_{-6.1}$
2	$1 \leq q^2 < 2$	$11.4^{+6.0}_{-5.1}$
3	$2 \leq q^2 < 3$	$13.0^{+5.6}_{-4.7}$
4	$3 \leq q^2 < 4$	$16.0^{+6.6}_{-5.8}$
5	$4 \leq q^2 < 5$	$24.3^{+7.1}_{-6.3}$
6	$5 \leq q^2 < 6$	$12.2^{+5.7}_{-4.9}$
7	$6 \leq q^2 < 7$	$10.8^{+5.1}_{-4.1}$
8	$7 \leq q^2 < 8$	$21.4^{+6.5}_{-5.7}$
9	$8 \leq q^2 < 9$	$9.6^{+4.7}_{-3.9}$
10	$9 \leq q^2 < 10$	$30.8^{+6.7}_{-6.0}$
11	$10 \leq q^2 < 11$	$11.6^{+5.0}_{-4.1}$
12	$11 \leq q^2 < 12$	$16.3^{+4.9}_{-4.1}$
13	$12 \leq q^2 < 13$	$19.4^{+5.3}_{-4.5}$
14	$13 \leq q^2 < 14$	$15.4^{+4.7}_{-4.0}$
15	$14 \leq q^2 < 15$	$15.1^{+4.9}_{-4.1}$
16	$15 \leq q^2 < 16$	$10.8^{+4.0}_{-3.2}$
17	$16 \leq q^2 < 17$	$12.3^{+4.4}_{-3.7}$
18	$q^2 \geq 18$	$32.3^{+6.8}_{-6.1}$

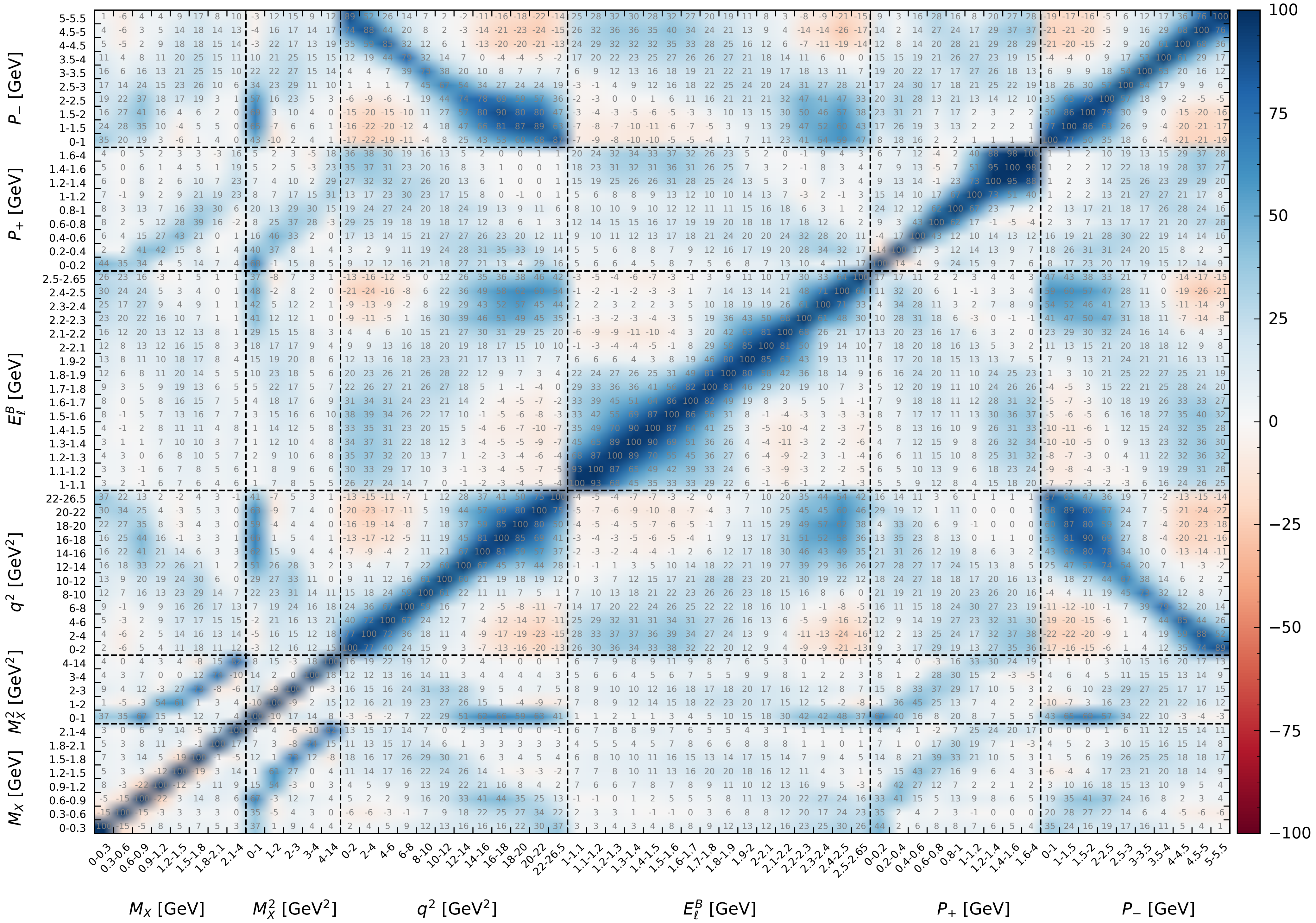
Bin	$M_{\pi\pi}$ [GeV]	q^2 [GeV^2]	Signal
1	$M_{\pi\pi} \leq 0.6$	$q^2 \leq 8$	$9.8^{+4.6}_{-3.7}$
2	$M_{\pi\pi} \leq 0.6$	$8 < q^2$	$15.8^{+5.5}_{-4.6}$
3	$0.6 < M_{\pi\pi} \leq 0.9$	$q^2 \leq 4$	$29.5^{+6.4}_{-5.7}$
4	$0.6 < M_{\pi\pi} \leq 0.9$	$4 < q^2 \leq 8$	$34.8^{+7.0}_{-6.2}$
5	$0.6 < M_{\pi\pi} \leq 0.9$	$8 < q^2$	$116.2^{+12.2}_{-11.5}$
6	$0.9 < M_{\pi\pi} \leq 1.2$	$q^2 \leq 4$	$8.0^{+3.7}_{-2.9}$
7	$0.9 < M_{\pi\pi} \leq 1.2$	$4 < q^2 \leq 8$	$9.2^{+4.0}_{-3.2}$
8	$0.9 < M_{\pi\pi} \leq 1.2$	$8 < q^2$	$27.6^{+6.4}_{-5.6}$
9	$1.2 < M_{\pi\pi} \leq 1.5$	$q^2 \leq 4$	$11.3^{+4.3}_{-3.5}$
10	$1.2 < M_{\pi\pi} \leq 1.5$	$4 < q^2 \leq 8$	$9.7^{+4.3}_{-3.5}$
11	$1.2 < M_{\pi\pi} \leq 1.5$	$8 < q^2$	$13.2^{+4.4}_{-3.7}$

$$\mathcal{B}(B^+ \to \pi^+\pi^-\ell^+\nu_\ell)[\text{2D}] = (23.0^{+1.9}_{-1.7}(\text{stat}) \pm 3.0(\text{syst})) \times 10^{-5}$$

$$\boxed{\mathcal{B}(B^+ \to \pi^+\pi^-\ell^+\nu_\ell)[\text{1D}(q^2)] = (22.7^{+1.9}_{-1.6}(\text{stat}) \pm 3.4(\text{syst})) \times 10^{-5}}$$

Differential $\Delta\mathcal{B}(B \to X_u \ell^+ \nu)$ — extra

Correlations among measured values



Compatibility check