A closer look at the extraction of $|V_{ub}|$ from $B \to \pi l \nu$.

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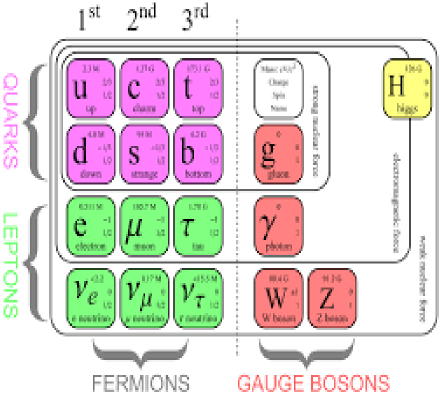
The Standard model

- The theory describing three of the four known fundamental forces.
- Classifies all known elementary particles.



The Standard model

- Charged-current $W^{\pm}$ couplings to quarks in the mass basis -

$$\mathcal{L} = -\frac{g}{\sqrt{2}}\bar{u}_L\gamma^\mu d_L W_\mu^+ + h.c. \to -\frac{g}{\sqrt{2}}\bar{U}_L\gamma^\mu V_{CKM} D_L W_\mu^+ + h.c. \quad (1)$$

- Basis rotation and the fact that one cannot simultaneously diagonalize all of the flavor matrices in the Standard Model → CKM matrix.

- Mixing of flavor through CKM matrix -

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (2)$$

- Parameterized by three mixing angles and the CP-violating phase.

Parameterizations of the CKM matrix

- Precision determinations of CKM elements necessary to probe the quark mixing mechanism of the Standard Model.

- **Wolfenstein parameterization -**

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \tag{3}$$

- To leading order, complex numbers only in the 1-3 and 3-1 mixing elements.

- B hadron decays important probes of CP violation.

- $V_{ub} \rightarrow$ Source of CP violation within the SM.

Measurements of $|V_{ub}|$

- The transition $b \to u l \bar{\nu}$ provides two avenues for determining $|V_{ub}|$ -
  - Inclusive
    (Sum over all possible hadronic states.)
  - Exclusive
    (Decays involving a specific meson in the final state.)
- Experimental and theoretical techniques for these two approaches different and largely independent $\to$ Important cross checks of our understanding.
- Mutual disagreement between exclusive and inclusive measurements.

$$|V_{ub}|^{exc} = (3.70 \pm 0.16) \times 10^{-3}, \quad |V_{ub}|^{inc} = (4.25 \pm 0.12^{+0.15}_{-0.14}) \times 10^{-3}, \quad (4)$$

differ by $\geq 2.2\ \sigma$.

$|V_{ub}|$ from inclusive decays

- The theoretical description of inclusive $\bar{B} \to X_u l \bar{\nu}$ decays based on the Heavy Quark Expansion.
- Total decay rate hard to measure due to the large background from $\bar{B} \to X_c l \bar{\nu}$ transitions $\to$ experimental cuts are necessary.
- In regions of phase space where $\bar{B} \to X_c l \bar{\nu}$ decays are suppressed, can't use HQE $\to$ introduce non-perturbative distribution functions(SF).
- Different approaches to model the shape function $\to$ extracted values of $|V_{ub}|$ model dependent.
- Recent analysis of the inclusive spectra with hadronic-tagging by Belle -

$$|V_{ub}|^{inc} = (4.10 \pm 0.09 \pm 0.22 \pm 0.15) \times 10^{-3} \,. \tag{5}$$

$|V_{ub}|$ from exclusive decays

- Exclusive determinations require knowledge of the form factors.

$$\langle \pi(p_\pi)|V_\mu|B(p_B)\rangle = f_+(q^2)\left[p_B^\mu + p_\pi^\mu - \frac{m_B^2 - m_\pi^2}{q^2}q^\mu\right] + f_0(q^2)\frac{m_B^2 - m_\pi^2}{q^2}q^\mu \tag{6}$$

- $f_+(q^2 = 0) = f_0(q^2 = 0) \rightarrow$ cancel the divergence at $q^2 = 0$.

$$\begin{aligned}\frac{d\Gamma}{dq^2}\left(\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l\right) =& \frac{G_F^2|V_{ub}|^2}{24\pi^3 m_{B^0}^2 q^4}\left(q^2 - m_l^2\right)^2 \left|p_\pi(m_{B^0}, m_{\pi^+}, q^2)\right| \times \\ & \left[\left(1 + \frac{m_l^2}{2q^2}\right) m_{B^0}^2 \left|p_\pi(m_{B^0}, m_{\pi^+}, q^2)\right|^2 \left|f_+\left(q^2\right)\right|^2 \right. \\ & \left. + \frac{3m_l^2}{8q^2}\left(m_{B^0}^2 - m_{\pi^+}^2\right)^2 \left|f_0\left(q^2\right)\right|^2\right.\end{aligned} \tag{7}$$

- Model-independent parametrization based on general properties of analyticity, unitarity, and crossing symmetry.

Form factor parametrization

- The z expansion → mapping the variable q^2 to a new variable z.

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \tag{8}$$

- Choosing $t_0 = (M_B + M_\pi)\,(\sqrt{M_B} - \sqrt{M_\pi})^2$ restricts z to $|z| < 0.28$ $\longrightarrow$ rapid convergence of the expansion.
- BCL parametrization -

$$f_+(z) = \frac{1}{1 - q^2/m_{B^*}^2} \sum_{n=0}^{N_z - 1} b_n^+ \left[z^n - (-1)^{n-N_z} \frac{n}{N_z} z^{N_z}\right], \tag{9}$$

$$f_0(z) = \sum_{n=0}^{N_z - 1} b_n^0 z^n . \tag{10}$$

- BSZ parametrization -

$$f_i(q^2) = \frac{1}{1 - q^2/m_{R,i}^2} \sum_{k=0}^{N} a_k^i \left[z(q^2) - z(0)\right]^k \tag{11}$$

Inputs for extraction of $|V_{ub}|$

- $\bar{B} \to \pi l \bar{\nu}_l \to$ the most promising decay mode for both experiment and theory.

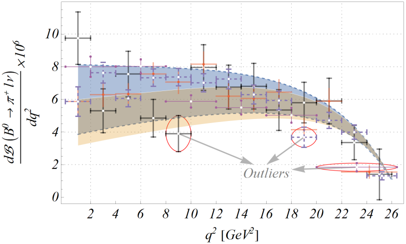
  Four most precise measurements by BABAR and Belle -
- BABAR untagged $B^0 + B^+$ (6 q^2 bins) [arXiv:1005.3288v2] $\to$ BaBar(11)
- BABAR untagged $B^0 + B^+$ (12 q^2 bins) [arXiv:1201.1253] $\to$ BaBar(12)
- Belle untagged B^0 [arXiv:1012.0090] $\to$ Belle(11)
- Belle hadronic tagged B^0 and B^+ [arXiv:1306.2781] $\to$ Belle(13)

- Non-perturbative methods for the calculation of the form factors -
  Lattice QCD (LQCD) $\to$ high momentum transfer q^2 to leptons. (RBC/UKQCD and MILC)
  Light-cone sum rules (LCSR) $\to$ low q^2 region. (arXiv:1811.00983)

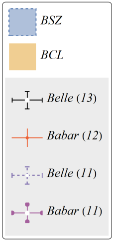
Form-factors extracted only from the LCSR and lattice inputs

BSZ			
χ^2_{min}/DOF	p-value(%)	Parameters	Values
4.48/15	99.6	a_0^+	0.213(22)
		a_1^+	-0.65(14)
		a_2^+	0.263(425)
		a_3^+	0.67(31)
		a_1^0	0.41(17)
		a_2^0	1.46(51)
		a_3^0	1.78(49)
BCL			
χ^2_{min}/DOF	p-value(%)	Parameters	Values
12.88/15	61	b_0^+	0.396(13)
		b_1^+	-0.707(70)
		b_2^+	-0.36(18)
		b_3^+	0.77(32)
		b_0^0	0.521(17)
		b_1^0	-1.756(78)
		b_2^0	1.15(16)

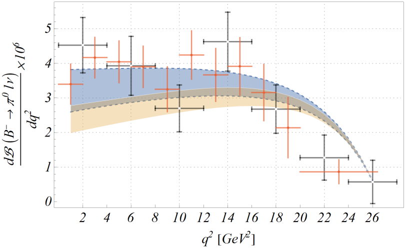
Binned differential branching fraction plots



(a)



(b)



(c)

Different scenarios

- ***Fit 1***: B^0 decays from Belle (2011) and Belle (2013); B^- decays from Belle (2013); the combined modes from BaBar (2011) and BaBar (2012).
  - *Fit 1A*: Experimental data (Fit 1) + synthetic Lattice data points,
  - *Fit 1B*: Experimental data (Fit 1) + synthetic Lattice data points + LCSR.
- ***Fit 2***: B^0 decays from Belle (2011), BaBar (2012), and Belle (2013); B^- decays from BaBar (2012) and Belle (2013).
  - *Fit 2A*: Experimental data (Fit 2) + synthetic Lattice data points,
  - *Fit 2B*: Experimental data (Fit 2) + synthetic Lattice data points + LCSR.
- ***Fit 3***: The combined modes from BaBar (2011) along with the *Fit 2* dataset.
  - *Fit 3A*: Experimental data (Fit 3) + synthetic Lattice data points,
  - *Fit 3B*: Experimental data (Fit 3) + synthetic Lattice data points + LCSR.

Different scenarios

BSZ Parametrization						
Run Name	Full			Dropped Pull > 2		
	χ^2_{min}/DOF	p-value(%)	$V_{ub} \times 10^3$ Frequentist	χ^2_{min}/DOF	p-value(%)	$V_{ub} \times 10^3$ Frequentist
Fit 1A	73.4/56	5.92	3.69(14)	46.6/52	68.68	3.79(15)
Fit 1B	77./65	14.57	3.74(13)	49.3/61	85.77	3.83(14)
Fit 2A	59.5/61	53.17	3.81(14)	46/59	89.26	3.86(15)
Fit 2B	62./70	74.23	3.85(14)	48.3/68	96.63	3.91(14)
Fit 3A	82.2/67	9.98	3.70(14)	53.3/62	77.56	3.76(14)
Fit 3B	85.9/76	20.54	3.75(13)	62./73	81.79	3.84(14)

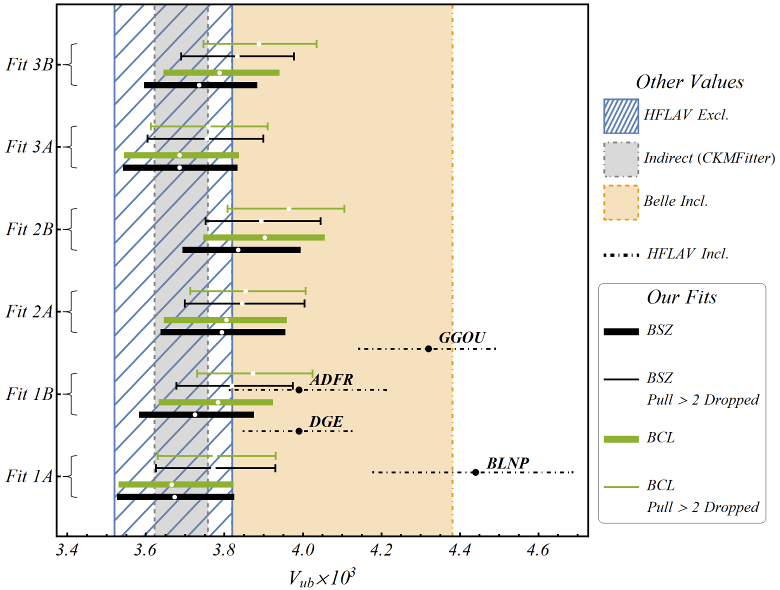
BCL Parametrization						
Run Name	Full			Dropped Pull > 2		
	χ^2_{min}/DOF	p-value(%)	$V_{ub} \times 10^3$ Frequentist	χ^2_{min}/DOF	p-value(%)	$V_{ub} \times 10^3$ Frequentist
Fit 1A	73.5/56	5.84	3.69(14)	46.7/52	68.34	3.79(15)
Fit 1B	92.1/65	1.51	3.79(13)	63.2/61	39.84	3.89(14)
Fit 2A	60.1/61	50.8	3.81(14)	46.5/59	88.19	3.87(15)
Fit 2B	75.9/70	29.42	3.91(14)	58.3/67	76.64	3.96(14)
Fit 3A	82.7/67	9.35	3.70(14)	57.8./63	66.09	3.77(14)
Fit 3B	101.4/76	2.73	3.80(13)	76.3/73	37.27	3.90(14)

Few observations

$$pull_i = \frac{\mathcal{O}_i^{exp} - \mathcal{O}_i^{fit}}{\sigma_i^{exp}} . \quad (12)$$

- BSZ parametrization $\rightarrow$ the quality of fit improves when one includes LCSR. BCL parametrization $\rightarrow$ the fit worsens with the inclusion of LCSR.
- With both Lattice and LCSR data, using the BCL form-factor parametrization results in a slightly larger $|V_{ub}|$ than that obtained from BSZ.
- Extracted $|V_{ub}|$ increases by $\geq 1\%$ with the inclusion of the new LCSR inputs.
- Irrespective of the fit scenario, the extracted $|V_{ub}|$ increases after dropping the data-points with pull > 2.

Comparison of $|V_{ub}|$ results



Comparison of $|V_{ub}|^{exc.}$ obtained in this work

- ***Fit 2B-I***: Input used in *Fit 2B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}(Belle2011)$.
- ***Fit 3B-I***: Input used in *Fit 3B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}(BaBar2011)$.
- ***Fit 3B-II***: Input used in *Fit 3B* without the data on $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}(Belle2011)$ and $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}(BaBar2011)$.

Fit	BSZ			BCL		
Scenario	χ^2/DOF	p-value(%)	$V_{ub} \times 10^3$	χ^2/DOF	p-value(%)	$V_{ub} \times 10^3$
F2B-I	55.4/69	88.14	3.90(14)	68.85/69	48.25	3.96(14)
F3B-I	78.86/75	35.8	3.83(14)	93.6/75	7.19	3.89(14)
F3B-II	72.96/74	51.25	3.88(14)	87.2/74	13.99	3.94(14)

Conclusions

- We have extracted $|V_{ub}|$ analyzing all the available inputs on the exclusive $B \to \pi l \nu$ decays. This includes the data on the partial decay rates, inputs from lattice, and those from LCSR.
- We have identified BaBar(11) data (at least a part of it) as a probable source of bad quality fit. The fit scenarios (Fit 2A and 2B) without that data-set has an appreciable fit-probability.
- We found a very small number of data-points that compromise the fit-quality, and at the same time, influence the extraction of $|V_{ub}|$.
- From the full dataset after dropping $\mathcal{B}(B^0 \to \pi^-)^{[18,20]}$ (Belle(11)) and $\mathcal{B}(B^0 \to \pi^-)^{[20,26.4]}$ (BaBar(11)), the extracted $|V_{ub}|= (3.94(14)) \times 10^{-3}$. $\to$ Consistent with the recent one extracted from inclusive $B \to X_u \ell \nu_\ell$ decay by Belle within 1 σ.

THANK YOU