$\mathcal{N} = 1$ trinification from dimensional reduction of $\mathcal{N} = 1$, 10D E_8 over $SU(3)/U(1) \times U(1) \times Z_3$ and its phenomenological consequences

George Manolakos

The XXVIII International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY 2021)

Phys.Lett.B 813 (2021) 136031 • e-Print: 2009.07059 [hep-ph]

joint work with G. Patellis and G. Zoupanos

Table of contents

Motivation

- *Unified description of Nature*
  - Extra dimensions (HD Gauge sector=4D gauge+Higgs)
  - Supersymmetry (inclusion of fermions)

    → Everything in a HD vector supermultiplet
  - Extra gauge symmetry (GUTs)
  - Less free parameters

- *Upshot: From higher dimensions to a (Supersymmetric) extension of the Standard Model*

Theoretical Part

Coset Space Dimensional Reduction (CSDR)

1. Compactification
   B - compact space
   $\dim B = D - 4 = d$
   D dims $\to$ 4 dims

$$M^D \to M^4 \times B$$
$$x^M \quad x^\mu \quad y^a$$

2. Dimensional Reduction: HD $\mathcal{L}$ independent of the extra coords y^a:
   - "Naive" way: No field dependence on y^a coords (cylinder condition)
   - Elegant way: Allow field dependence on y^a
     $\to$ employ a symmetry of $\mathcal{L}$ to compensate $\to$ Gauge Symmetry

3. CSDR: Special case: $B = S/R$ *Witten (1977); Forgacs, Manton (1980); Chapline, Slansky (1982); Kapetanakis, Zoupanos - Phys.Rept. (1992) Kubyshin, Mourao, Rudolph, Volobujev - Book (1989)*
   – Allow a non-trivial field dependence on y^a
   – But impose the condition that a symmetry transformation by an element of the isometry group S of B (coord transformation) is *counterbalanced* by a gauge transformation of the gauge group G

   $\to$ $\mathcal{L}$ is independent of y^a because it is *gauge invariant* !

Reduction of a D-dim Y-M Lagrangian

Consider a Yang-Mills-Dirac theory in D dims based on group G defined on $M^D \to M^4 \times S/R$, $D = 4 + d$. Due to compactification:

$$g^{MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{ab} \end{pmatrix} \qquad \eta^{\mu\nu} = diag(1, -1, -1, -1)$$

$$d = dimS - dimR \qquad g^{ab} - \text{coset space metric}$$

$$S = \int d^4x d^d y \sqrt{-g}\left[-\frac{1}{4}\mathrm{Tr}(F_{MN}F_{K\Lambda})g^{MK}g^{N\Lambda} + \frac{i}{2}\overline{\psi}\Gamma^M D_M \psi \right]$$

where:

$$D_M = \partial_M - \theta_M - igA_M$$ the covariant derivative of M^D

$$\theta_M = \frac{1}{2}\theta_{MN\Lambda}\Sigma^{N\Lambda}$$ the spin connection of M^D

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig[A_M, A_N]$$ the field strength tensor of A_M

while ψ is in rep F of G and A_M in the adjoint of G.

The Constraints

Demand: *any transformation by an element of* S *acting on* S/R
is compensated by gauge transformations

→ Constraints on the fields of the theory:

$$\delta_A A_\alpha = \xi_A^{\ \beta}\partial_\beta A_\alpha + \partial_\alpha \xi_A^{\ \beta} A_\beta \stackrel{!}{=} \partial_\alpha W_A - [W_A, A_\alpha]$$

$$\delta_A \psi = \xi_A^{\ \alpha}\partial_\alpha \psi - \frac{1}{2} G_{Abc}\Sigma^{bc}\psi \stackrel{!}{=} D(W_A)\psi$$

– ξ_A^α are the Killing vectors of S/R,
– δ_A is an infinitessimal coord transformation
– $D(W_A)$ is a gauge transformation in the appropriate rep

Solution of the constraints:

- Remaining gauge invariance $H = C_G(R_G)$ i.e. $G \supset R_G \times H$
- Surviving fields in 4 dims:
  - Scalars: $S \supset R$ (compare adj$G$ and adj$S$ rep decomps)
  - Fermions: $SO(d) \supset R$ (compare F of G and σ_d rep decomps)
- Obtain the expression of the 4-dim scalar potential
- Note: 4-dim chiral theories only if $D = 4n + 2$ (Weyl+Majorana)

The 4-dim Theory

Integrate out the extra coordinates (+ take into account constraints):

$$S = C \int d^4x \, \mathrm{tr} \left[-\frac{1}{8} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (D_\mu \phi_a)(D^\mu \phi^a) \right]$$

$$+ V(\phi) + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} \bar{\psi} \Gamma^a D_a \psi$$

– $D_\mu = \partial_\mu - igA_\mu$ – C: coset space volume

– $D_a = \partial_a - \theta_a - ig\phi_a$ – $\phi_a \equiv A_a$

$$V(\phi) = -\frac{1}{8} g^{ac} g^{bd} \mathrm{Tr} \left\{ (f_{ab}^{C} \phi_C - ig[\phi_a, \phi_b])(f_{cd}^{D} \phi_D - ig[\phi_c, \phi_d]) \right\}$$

- Last: in case $G \supset S \Rightarrow H$ breaks to $K = C_G(S)$:

$$G \supset S \times K \leftarrow \text{ gauge group after SSB}$$

$$\cup \quad \cap$$

$$G \supset R \times H \leftarrow \text{gauge group in 4 dims}$$

Harnad, Shnider, Tafel (1980)

Reduction of 10-dim, $\mathcal{N}=1$, E_8 over $S/R = SU(3)/U(1)\times U(1)$

Manousselis, Zoupanos (2001-2004)

The non-symmetric coset space $SU(3)/U(1)\times U(1)$:

- 6-dim (nearly-Kähler) compact manifold
- admits torsion and may have different radii
- *Nice feature:* produces *soft supersymmetry breaking terms*

Therefore in this case the setup is:

- 10-dim gauge group $G = E_8$
- Isometry group of coset space $S = SU(3)$
- Homotopy group of coset space $R = U(1)_A \times U(1)_B$
- Using the decomposition:

$$E_8 \supset E_6 \times SU(3) \supset E_6 \times U(1)_A \times U(1)_B$$

→ the 4-dim gauge group is
$H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B$

- Since $S \subset G$, H breaks to

$$K = C_G(S) = E_6 \times [U(1)\times U(1)]_{\text{global}}$$

The 4-dim multiplets and scalar potential

After the dimensional reduction:

- Surviving gauge fields in $\mathcal{N} = 1$ vector supermultiplet
- Surviving matter fields in six $\mathcal{N} = 1$ chiral supermultiplets
  - 3 of them are E_6 singlets
    $$A : 1_{(3,1/2)} \,,\; B : 1_{(-3,1/2)} \,,\; C : 1_{(0,-1)}$$
  - 3 of them are in fundamental of E_6
    $$A^i : 27_{(3,1/2)} \,,\; B^i : 27_{(-3,1/2)} \,,\; C^i : 27_{(0,-1)}$$
- Gaugino mass $M = (1 + 3\tau)\frac{R_1^2+R_2^2+R_3^2}{8\sqrt{R_1^2 R_2^2 R_3^2}}$
- The scalar potential expression admits the following identification

$$V = c(R_i) + V_F + V_D + V_{\text{soft}}$$

- Singlets α and β are chosen to acquire vevs $\rightarrow$
  $E_6 \times U(1)_A \times U(1)_B \rightarrow E_6$
  $\rightarrow U(1)_A \times U(1)_B$ remain as *global symmetries*

Need for another GUT gauge group

- 10-dim $\mathcal{N} = 1$ E_8 gauge theory $\rightarrow$ 4-dim (broken) $\mathcal{N} = 1$ E_6 GUT with $U(1)_A \times U(1)_B$ global symmetry
- *Remarkable virtue*: SUSY already broken (unlike the CY case) + explanation of the origin of the SSB sector
- BUT E_6 is not a nice gauge group for a GUT to accommodate Standard Model or its extensions
- We need to break it down to a more appropriate one
- Employ the *Wilson Flux Breaking mechanism*

Wilson Flux Breaking Mechanism

Hosotani (1983); Witten (1985); Zoupanos (1988); Kozimirov, Kuzmin, Tkachev (1989); Kapetanakis, Zoupanos (1989)

- So far CSDR occurs over the *simply connected* manifold $B_0 = SU(3)/U(1) \times U(1)$
- On simply connected manifolds: $F = 0 \Rightarrow A = 0$
- Instead of B_0 employ $B = B_0/F^{S/R}$, $F^{S/R}$ a freely acting discrete symmetry $\longrightarrow$ *multiply-connected*
- A flat potential $F = 0$ can have non-trivial physical effects (Bohm-Aharonov)
- For every $g \in F^{S/R}$ an element $U_g \in H$ is corresponded - a *Wilson line*
- The "multiplication" of two loops is a homomorphism of $F^{S/R}$ in H with image $T^H = \{U_g\}$
- H breaks to the subgroup $C_H(T^H)$ *Green, Schwarz, Witten (1987)*
- The surviving matter fields are the ones invariant under $F^{S/R} \oplus T^H$

Irges, Zoupanos (2011)

- In our case $F^{S/R} = \mathbb{Z}_3$ and $B = SU(3)/U(1) \times U(1) \times \mathbb{Z}_3$
- The gauge group $H = E_6$ breaks to $SU(3)_c \times SU(3)_L \times SU(3)_R$

$$E_6 \supset SU(3)_c \times SU(3)_L \times SU(3)_R \qquad 27 = (1,3,\bar{3}) \oplus (3,\bar{3},1) \oplus (\bar{3},1,3)$$

Kephart, Vaughn (1981)

Surviving matter content of the projected theory:

- $\alpha_3 \equiv \Psi_1 \equiv q^c \sim (\bar{3},1,3)_{(3,\frac{1}{2})}, \quad \beta_2 \equiv \Psi_2 \equiv Q \sim (3,\bar{3},1)_{(-3,\frac{1}{2})},$
  $\gamma_1 \equiv \Psi_3 \equiv L \sim (1,3,\bar{3})_{(0,-1)}, \quad \gamma \equiv \theta \sim (1,1,1)_{(0,-1)}$

Non-trivial monopole charges in $R \to$ three generations:
$\Psi_1^{(l)},\ \Psi_2^{(l)},\ \Psi_3^{(l)},\ \gamma^{(l)}$

Dolan (2003)

$$q^c = \begin{pmatrix} d_R^{c1} & u_R^{c1} & D_R^{c1} \\ d_R^{c2} & u_R^{c2} & D_R^{c2} \\ d_R^{c3} & u_R^{c3} & D_R^{c3} \end{pmatrix}, Q = \begin{pmatrix} -d_L^1 & -d_L^2 & -d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}, L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & S \end{pmatrix}$$

The components of the scalar potential are (one generation)

$$\frac{2}{g^2}V_D = \frac{1}{2}D^A D^A + \frac{1}{2}D_1 D_1 + D\frac{1}{2}D_2 D_2\,, \quad D^A = \frac{1}{\sqrt{3}}\langle\Psi_i|G^A|\Psi_i\rangle$$

$$D_1 = 3\sqrt{\frac{10}{3}}(\mathrm{Tr}(q^{c\dagger}q^c) - \mathrm{Tr}(Q^\dagger Q))$$

$$D_2 = \sqrt{\frac{10}{3}}(\mathrm{Tr}(q^{c\dagger}q^c) + \mathrm{Tr}(Q^\dagger Q) - 2\mathrm{Tr}(L^\dagger L) - 2|\theta|^2)$$

$$\frac{2}{g^2}V_F = 360\mathrm{tr}(\hat{q}^{c^\dagger}\hat{q}^c + \hat{Q}^\dagger\hat{Q} + \hat{L}^\dagger\hat{L})$$

$$\begin{aligned}\frac{2}{g^2}V_{soft} &= \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2}\right)\langle\Psi_1|\Psi_1\rangle + \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2}\right)\langle\Psi_2|\Psi_2\rangle \\ &+ \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2}\right)(\langle\Psi_3|\Psi_3\rangle + |\theta|^2) \\ &+ 80\sqrt{2}\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_1R_2}\right)(d_{abc}\Psi_1^a\Psi_2^b\Psi_3^c + h.c) \\ &= m_1^2\langle\Psi_1|\Psi_1\rangle + m_2^2\langle\Psi_2|\Psi_2\rangle + m_3^2\Big(\langle\Psi_3|\Psi_3\rangle + |\theta|^2\Big) + (\alpha_{abc}\Psi_1^a\Psi_2^b\Psi_3^c\end{aligned}$$

Note: SUSY is broken by D, F besides SSB terms

Further Gauge Breaking of $SU(3)^3$

Babu, He, Pakvasa (1986); Ma, Mondragon, Zoupanos (2004); Leontaris, Rizos (2006); Sayre, Wiesenfeldt, Willenbrock (2006)

- Two generations of L acquire vevs that break the GUT:

$$\langle L_s^{(3)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix}, \quad \langle L_s^{(2)} \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix}$$

- Each one alone not enough to produce the (MS)SM gauge group:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)$$
$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times SU(2)'_R \times U(1)'$$

- Their *combination* gives the desired breaking:

$$SU(3)_c \times SU(3)_L \times SU(3)_R \to SU(3)_c \times SU(2)_L \times U(1)_Y$$

- The potential after GUT breaking allows the following configuration for θ's vevs:

$$\to \quad \langle \theta^{(3)} \rangle \sim \mathcal{O}(TeV)\ , \quad \langle \theta^{(1,2)} \rangle \sim \mathcal{O}(M_{GUT})$$

- Electroweak breaking then proceeds by: $\langle L_s^{(3)} \rangle = \mathrm{diag}(v_d, v_u, 0)$

Theoretical Recap

- Begin with 10-dim, $\mathcal{N} = 1$ E_8 YM-D theory
- Compactification of the extra dimensions
- Dimensional reduction over $SU(3)/U(1) \times U(1) \times \mathbb{Z}_3$
- Wilson Flux Breaking mechanism leads to an $SU(3)^3$ GUT with specific particle spectrum (three generations)
- GUT symmetry breaking leads to a (broken) supersymmetric extension of the SM
- The scalar potential includes F-terms, D-terms and SSB terms

Phenomenological Part

Choice of Radii → Split-like scenario

Manolakos, Patellis, Zoupanos (2020)

- Small and equal R_i → high scale SUSY breaking
- Small $R_i \sim \frac{1}{M_{GUT}}$ with R_3 slightly different in a specific configuration

  → Split-like SUSY scenario *Arkani-Hamed, Dimopoulos (2004); Giudice, Romanino (2004); Arkani-Hamed, Dimopoulos, Giudice, Romanino (2004)*

$$\rightarrow \quad m_3^2 \sim -\mathcal{O}(TeV^2), \quad m_{1,2}^2 \sim -\mathcal{O}(M_{GUT}^2), \quad a_{abc} \gtrsim M_{GUT}$$

- supermassive squarks
- TeV-scale sleptons
- TeV-scale soft Higgs squared masses

Reminder: in this scenario $M_C = M_{GUT}$

Lepton Yukawas and μ terms

- The two global $U(1)$s forbid Yukawa terms for leptons
  $\rightarrow$ introduce higher-dimensional operators: $L\bar{e}H_d\left(\frac{\overline{K}}{M}\right)^3$
- μ terms for each generation of Higgs doublets are absent
  $\rightarrow$ solution through higher-dimensional operators: $H_u^{(i)}H_d^{(i)}\overline{\theta}^{(i)}\frac{\overline{K}}{M}$

– $\overline{K}$ is the vev of the conjugate scalar component of either $S^{(i)}$, $\nu_R^{(i)}$ or $\theta^{(i)}$, or any combination of them

The configuration:

$$\langle\theta^{(3)}\rangle \sim \mathcal{O}(TeV)\ , \quad \langle\theta^{(1,2)}\rangle \sim \mathcal{O}(M_{GUT})$$

leads to a 2HDM below GUT scale

Approximate Scale of Parameters

Parameter	Scale
soft trilinear couplings	$\mathcal{O}(GUT)$
squark masses	$\mathcal{O}(GUT)$
slepton masses	$\mathcal{O}(TeV)$
soft Higgs masses	$\mathcal{O}(TeV)$
$\mu^{(3)}$	$\mathcal{O}(TeV)$
$\mu^{(1,2)}$	$\mathcal{O}(GUT)$
unified gaugino mass M_U	$\mathcal{O}(TeV)$

Gauge Unification

Since many SUSY parameters are comparable to M_{GUT}, we consider them to decouple at an intermediate scale M_{int}

The 1-loop gauge β-functions are

$$2\pi\beta_i = b_i\alpha_i^2$$

Scale	b_1	b_2	b_3
M_{EW}-M_{TeV}	$\frac{21}{5}$	-3	-7
M_{TeV}-M_{int}	$\frac{11}{2}$	$-\frac{1}{2}$	-5
M_{int}-M_{GUT}	$\frac{39}{5}$	3	-3

– b_i depends on particle content

- $\alpha_{1,2}$ are used as input to determine M_{GUT} - α_3 to confirm
- 0.3% uncertainty at the unification boundary

→ α_3 is predicted within 2σ of the experimental value

$$a_s(M_Z) = 0.1218$$

$$a_s^{EXP}(M_Z) = 0.1187 \pm 0.0016$$

Scale	GeV
M_{GUT}	$\sim 1.7 \times 10^{15}$
M_{int}	$\sim 9 \times 10^{13}$
M_{TeV}	~ 1500

✓No proton decay problem: $U(1)_A = -\frac{1}{9}B$

Higgs Potential - All contributions

The analysis is restricted to the third generation

$$V_{Higgs} = \left(3|\mu^{(3)}|^2 + m_3^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) + \left(3|\mu^{(3)}|^2 + m_3^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right)$$
$$+ b^{(3)}\left[(H_u^+ H_D^- - H_u^0 H_D^0) + c.c.\right]$$
$$+ \frac{10}{3} g^2 \Big[|H_d^0|^4 + |H_d^-|^4 + |H_u^0|^4 + |H_u^+|^4 +$$
$$2|H_d^0|^2|H_d^-|^2 + 2|H_d^-|^2|H_u^0|^2 + 2|H_d^0|^2|H_u^+|^2 + 2|H_u^0|^2|H_u^+|^2 \Big]$$
$$+ \frac{20}{3} g^2 \left[|H_d^0|^2|H_u^0|^2 + |H_d^-|^2|H_u^+|^2\right] - 20g^2\left[\overline{H_d^0} H_d^- \overline{H_u^0} H_u^+ + c.c.\right]$$

$\rightarrow$ Comparison with standard 2 Higgs doublet potential gives:

Gunion, Haber (1986); Quiros (1997);
Branco, Ferreira, Lavoura, Rebelo, Sher, Silva (2012)

- $\lambda_1 = \lambda_2 = \lambda_3 = \frac{20}{3} g^2$
- $\lambda_4 = 20 g^2$
- $\lambda_5 = \lambda_6 = \lambda_7 = 0$

– $\lambda_{5,6,7} = 0$ as expected in a SUSY theory
– These relations are boundary conditions at M_{GUT}

Boundary Conditions and Uncertainties

At the unification scale we have the following boundary conditions and their respective uncertainties due to threshold corrections (such uncertainties also appear at the TeV boundary):

Kubo, Mondragon, Olechowski, Zoupanos (1996)

GUT BC	GUT Unc.	TeV Unc.
$g_3 = g$	0.3%	
$Y_{t,b} = g$	6%	2%
$\lambda_{1,2} = \frac{20}{3} g^2$	8%	8%
$\lambda_3 = \frac{20}{3} g^2$	7%	5%
$\lambda_4 = 20 g^2$	7%	5%

The τ lepton Yukawa emerges from a higher-dimensional operator and has significantly wider freedom. The standard τ lepton mass is used as input.

1-loop Results

1-loop β-functions used throughout the analysis that change between the three landmark scales M_{GUT}, M_{int} and M_{TeV}.

$\rightarrow$ $m_b(M_Z)$ and $\hat{m}_t$ are predicted within 2σ of the experimental values

- $m_b(M_Z) = 3.00\ GeV$ $\quad$ $m_b^{EXP}(M_Z) = 2.83 \pm 0.10\ GeV$
- $\hat{m}_t = 171.6\ GeV$ $\quad$ $\hat{m}_t^{EXP} = 172.4 \pm 0.7\ GeV$

$\rightarrow$ m_h is predicted within 1σ of the experimental value

- $m_h = 125.18\ GeV$ $\quad$ $m_h^{EXP} = 125.10 \pm 0.14\ GeV$

– Large $\tan\beta \sim 48$
– $M_A \sim 2000 - 3000\ GeV$ ✓
– LSP $\sim 1500\ GeV$

Work in preparation/planned

- 2-loop analysis
- Full (light) SUSY spectrum
- Application of B-physics constraints

o Calculation of CDM relic density

o Investigation of discovery potential at existing and future colliders

– Examination of higher-dimensional potential $\rightarrow$ test agreement with observed value of cosmological constant

Thank you for your attention!