A Minimal Supersymmetric SU(5) Missing-Partner Model

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Based on J. Ellis, J. L. Evans, N. Nagata, K. A. Olive, Eur. Phys. J. C **81**, 543 (2021).

Grand Unified Theories (GUTs)

H. Georgi and S.L. Glashow, Phys .Rev. Lett. **32**, 438 (1974).

- Unification of Standard Model gauge groups:

$$\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{SU}(5)$$

Gauge coupling unification

$$g_1(M_{\mathrm{GUT}}) = g_2(M_{\mathrm{GUT}}) = g_3(M_{\mathrm{GUT}})$$

- Unification of quarks and leptons

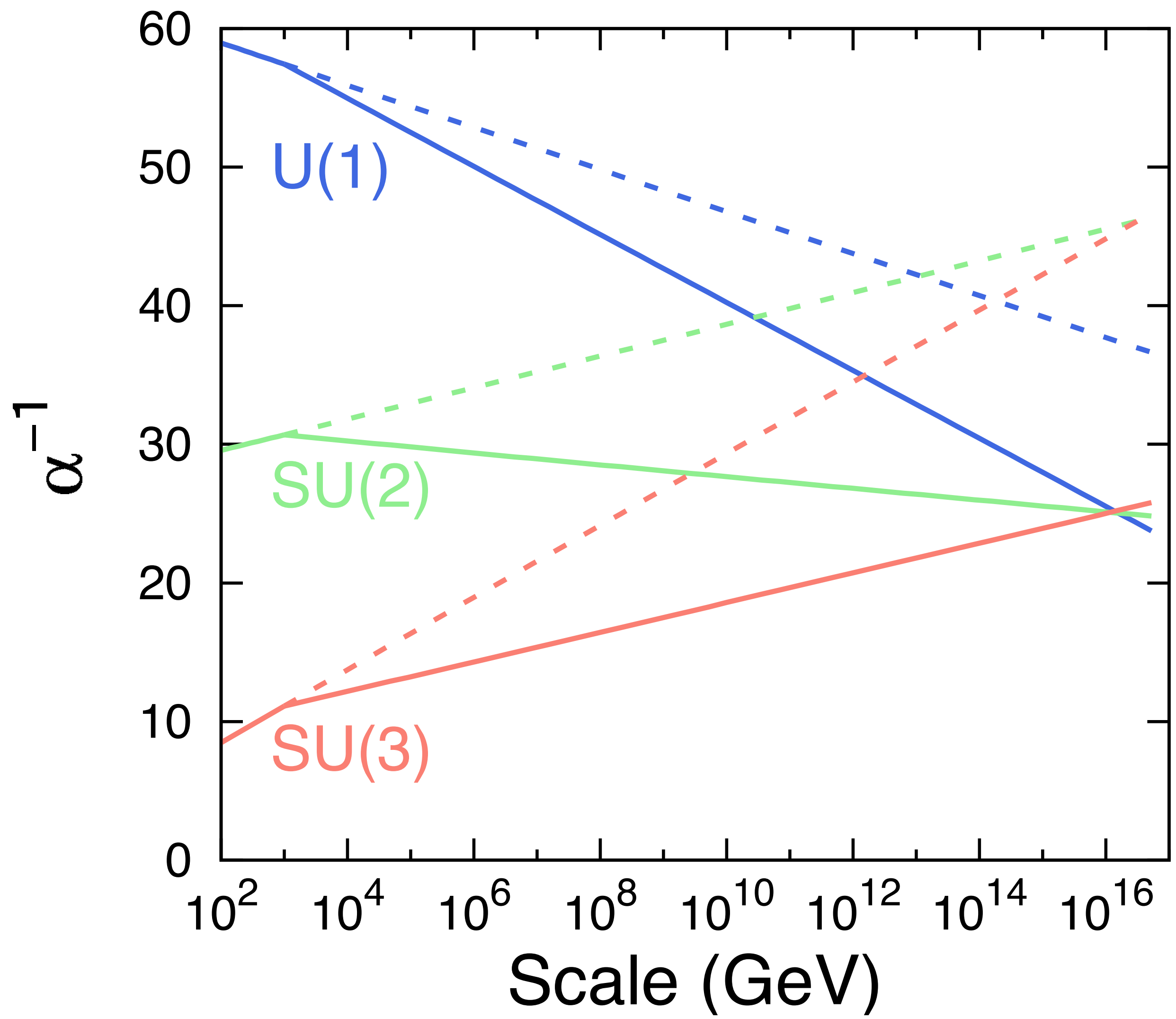
$$\overline{\mathbf{5}} = \begin{pmatrix} \bar{D}_1 \\ \bar{D}_2 \\ \bar{D}_3 \\ E \\ -N \end{pmatrix} \qquad \mathbf{10} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{U}_3 & -\bar{U}_2 & U^1 & D^1 \\ -\bar{U}_3 & 0 & \bar{U}_1 & U^2 & D^2 \\ \bar{U}_2 & -\bar{U}_1 & 0 & U^3 & D^3 \\ -U^1 & -U^2 & -U^3 & 0 & \bar{E} \\ -D^1 & -D^2 & -D^3 & -\bar{E} & 0 \end{pmatrix}$$

SUSY GUTs

S. Dimopoulos and H. Georgi, Nucl. Phys. B**193**, 150 (1981);
N. Sakai, Z. Phys. C**11**, 153 (1981).

Supersymmetry (SUSY) and GUTs go well together.

- Gauge hierarchy problem
- Gauge coupling unification



Solid : SM
Dashed : MSSM

(SUSY scale: 1 TeV)

Doublet triplet splitting

The Higgs fields are accompanied by color-triplet fields:

$$H = \begin{pmatrix} H_C^1 \\ H_C^2 \\ H_C^3 \\ H_u^+ \\ H_u^0 \end{pmatrix}, \quad \overline{H} = \begin{pmatrix} \overline{H}_{C1} \\ \overline{H}_{C2} \\ \overline{H}_{C3} \\ H_d^- \\ -H_d^0 \end{pmatrix},$$

Color-triplet Higgs

MSSM Higgs

- Color triplets need to be heavy: proton decay limits
- MSSM Higgs fields must be light

In the minimal SU(5) GUT, this mass splitting is realized with fine-tuning.

→ Doublet-triplet splitting problem

Missing partner models (MPMs)

In the minimal SU(5), SU(5) is broken by an adjoint Higgs field (**24**).

Instead, we use a **75** representation Σ to break SU(5).

Superpotential

$$W = \lambda_\Theta \bar{H}\Sigma\Theta + \lambda_{\bar{\Theta}} H\Sigma\bar{\Theta} + M_\Theta \Theta\bar{\Theta} + \dots \qquad \Theta, \bar{\Theta} : \mathbf{50}, \overline{\mathbf{50}}$$

$$\mathbf{50} = (\mathbf{1}, \mathbf{1}, -2) \oplus (\mathbf{3}, \mathbf{1}, -1/3) \oplus (\overline{\mathbf{3}}, \mathbf{2}, -7/6) \oplus (\overline{\mathbf{6}}, \mathbf{3}, -1/3) \oplus (\mathbf{6}, \mathbf{1}, 4/3) \oplus (\mathbf{8}, \mathbf{2}, 1/2)$$

$$\overline{H} = \begin{pmatrix} \overline{H}_{C1} \\ \overline{H}_{C2} \\ \overline{H}_{C3} \\ H_d^- \\ -H_d^0 \end{pmatrix}$$

$(\mathbf{3}, \mathbf{1}, -1/3)$ and $\overline{H}_{C1}, \overline{H}_{C2}, \overline{H}_{C3}$: Form a massive vector-like field after Σ gets a VEV.

$H_d^-, -H_d^0$: Remains massless because there is no partner.

➡ Missing partner mechanism

A. Masiero, D. V. Nanopoulos, K. Tamvakis, and T. Yanagida, Phys. Lett. B**115**, 380 (1982);
B. Grinstein, Nucl. Phys. B**206**, 387 (1982).

Difficulty in Minimal SU(5) MPM

There are challenges in the minimal SU(5) MPM:

- Perturbativity above the GUT scale.

  Large representations ($\mathbf{50}, \overline{\mathbf{50}}, \mathbf{75}$) make the gauge coupling blow up just above $M_{\rm GUT}$.

  → Should take M_Θ to be very large to have a cut-off scale much larger than $M_{\rm GUT}$.

- Rapid proton decay

  Lighter color-triplet mass: $$M_{H_C} = \lambda_\Theta \lambda_{\bar{\Theta}} \frac{(2V)^2}{M_\Theta}$$ V: Σ VEV

  Large M_Θ → Small M_{H_C} → Rapid proton decay

Difficulty in Minimal SU(5) MPM

- Challenges perturbative gauge coupling unification

S. Pokorski, K. Rolbiecki, G. G. Ross, K. Sakurai, JHEP **1904**, 161 (2019).

By examining the 2-loop RGEs + 1-loop threshold corrections, the authors concluded that it is not possible to achieve the perturbative gauge coupling unification.

Is the minimal MPM excluded?

Our setup

We study the minimal SU(5) MPM with

- Dimension-5 operator:

$$W_{\rm eff} = \frac{c}{M_P}\mathcal{W}_A^C\mathcal{W}_B^D\Sigma_{CD}^{AB}$$

affects the matching conditions.

- Super-GUT version of Constrained MSSM

Universality condition is imposed at $M_{\rm in} > M_{\rm GUT}$

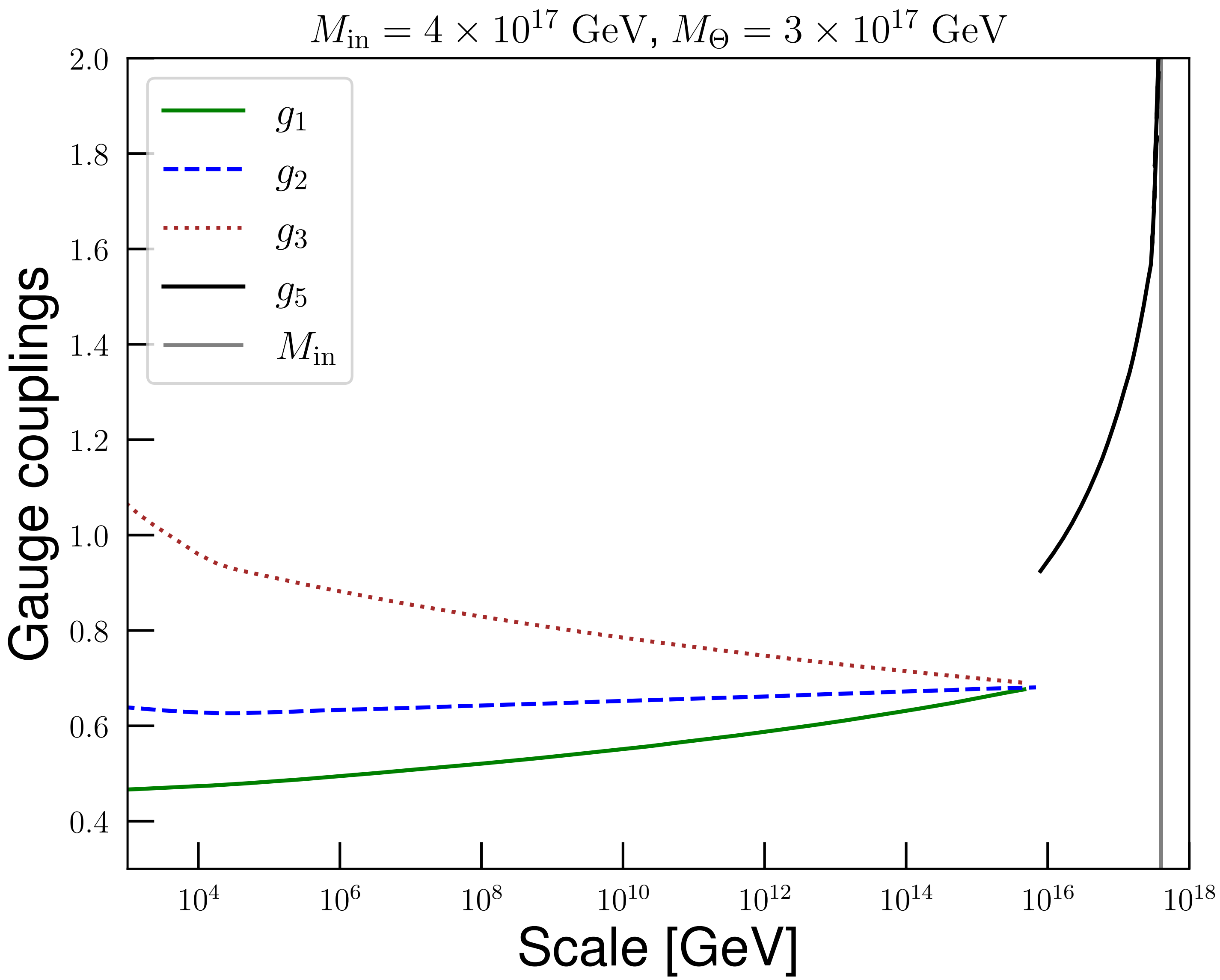
$$m_0,\quad m_{1/2},\quad A_0,\quad B_0,\quad \tan\beta,\quad {\rm sign}(\mu),\quad M_{\rm in}$$

+ MSP parameters

$$M_\Theta,\ \lambda_{\Theta,\bar{\Theta}},\ \lambda'$$

We require perturbativity up to $M_{\rm in}$

Gauge coupling unification



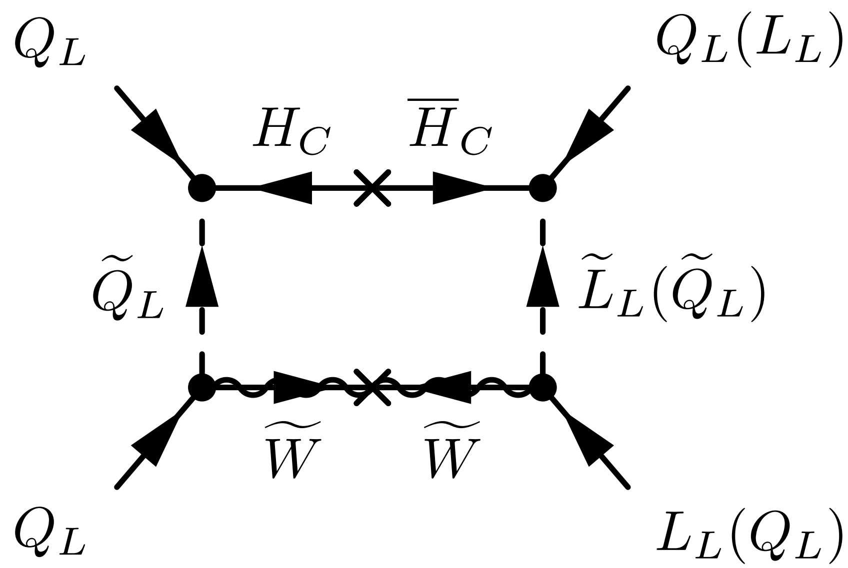
$$M_{\rm in} = 4 \times 10^{17}\ \mathrm{GeV}$$

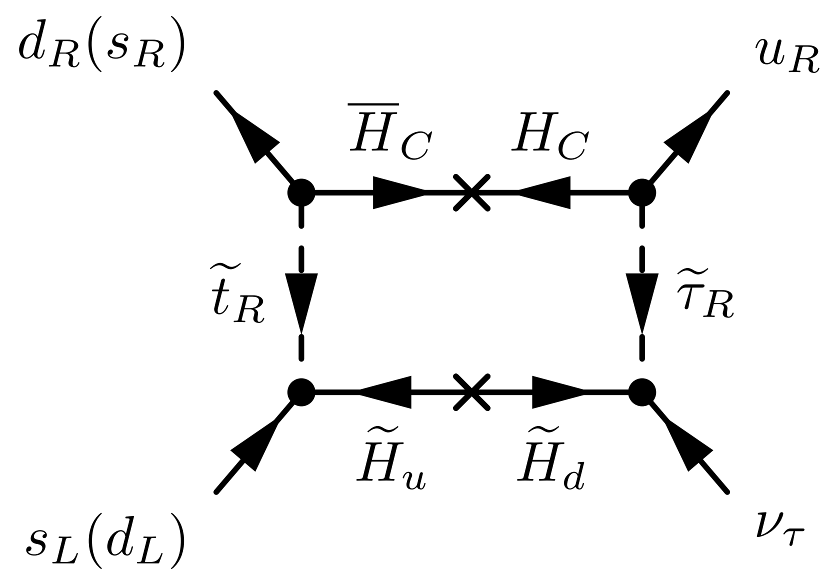
$$M_\Theta = 3 \times 10^{17}\ \mathrm{GeV}$$

- Perturbative gauge coupling unification is achieved.
- Large threshold effect at the GUT scale.
- Perturbativity is (barely) maintained up to the input scale.

Proton decay

Exchange of color-triplet Higgs induces proton decay.





$$\propto \frac{1}{M_{H_C} M_{\rm SUSY}}$$

In our setup, M_{H_C} tends to be suppressed (for large M_Θ):

$$M_{H_C} = \lambda_\Theta \lambda_{\bar{\Theta}} \frac{(2V)^2}{M_\Theta}$$

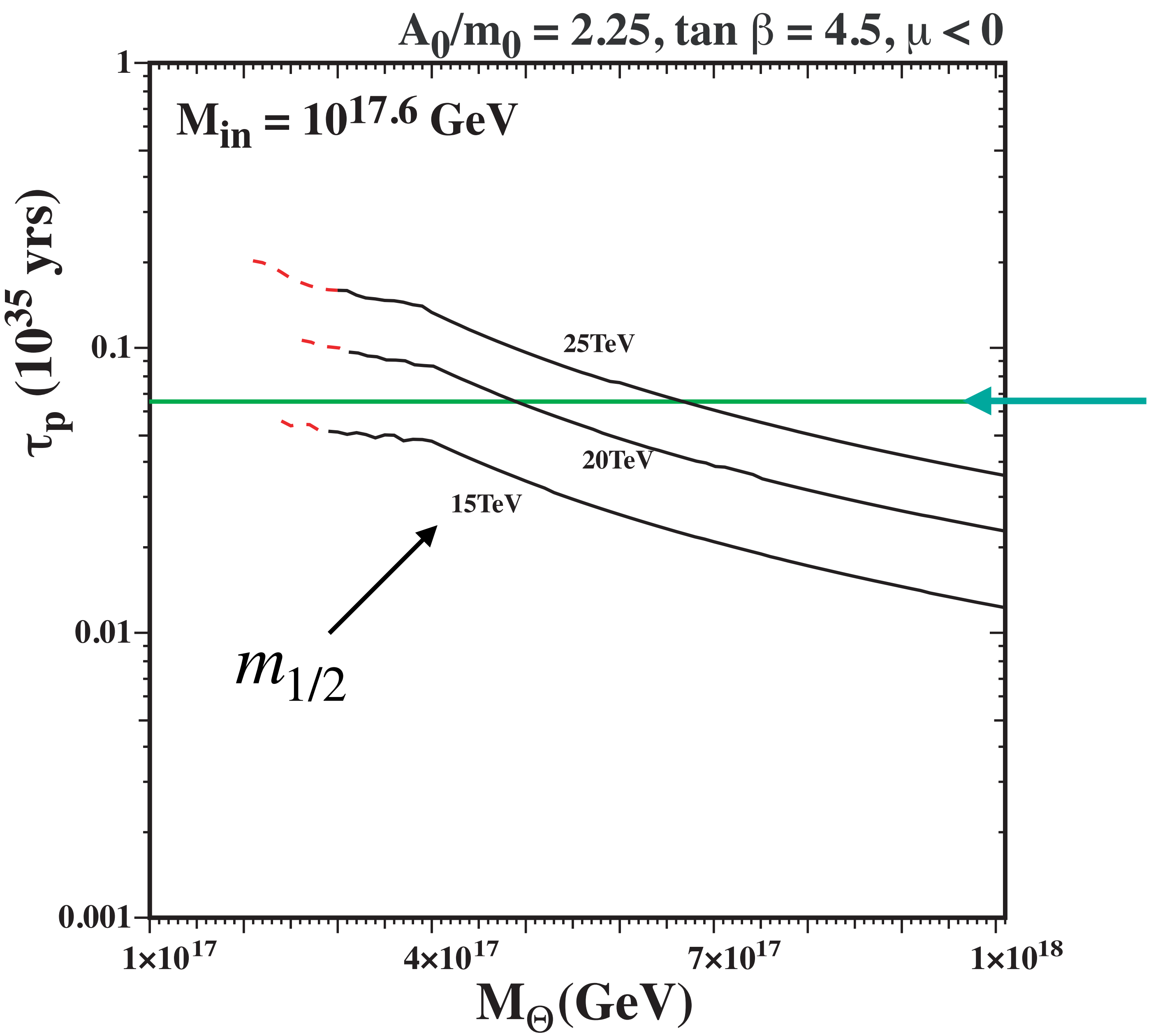
(V: Σ VEV)

This results in rather short proton lifetime.

A high SUSY scale helps.

J. Hisano, D. Kobayashi, T. Kuwahara, N. Nagata, JHEP **1307**, 038 (2013).

Proton lifetime



$$\tau(p \to K^+\bar{\nu}) > 6.6 \times 10^{33} \text{ years}$$

(Super-Kamiokande)

- Limit from the proton-decay bound is relaxed for large SUSY-breaking scale.
- We cannot make the mass of $\mathbf{50}, \overline{\mathbf{50}}$ very large.

J. Ellis, J. L. Evans, N. Nagata, K. A. Olive, Eur. Phys. J. C **81**, 543 (2021).

SUSY scale?

Proton decay limit favors a high SUSY-breaking scale.

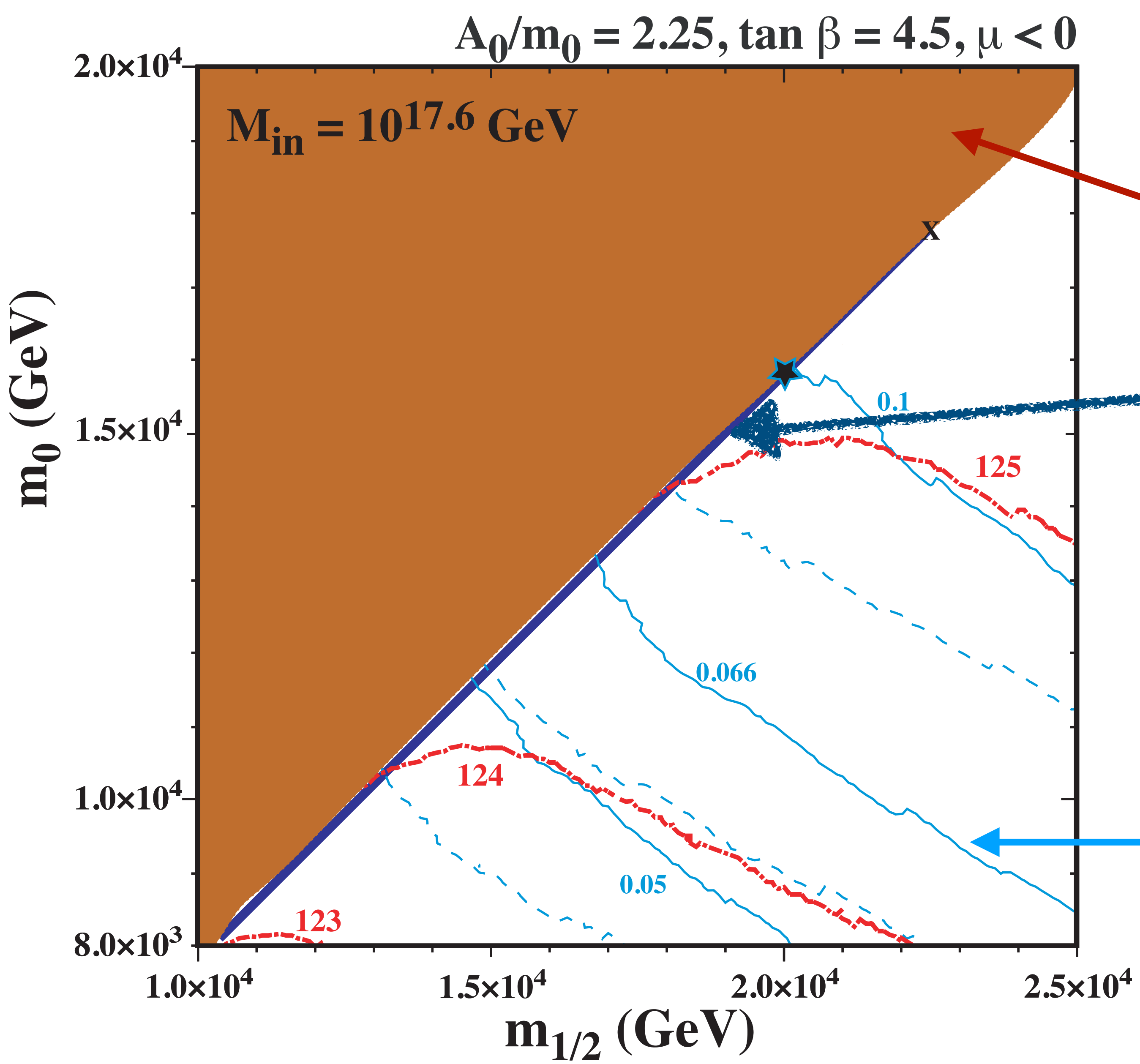
Requirement to explain

- The observed Higgs mass: ~ 125 GeV
- Dark matter relic abundance

See H. Fukuda's talk

makes the parameter space finite.

Result



Stop LSP

$\Omega_{DM} h^2 = 0.12$ (stop coannihilation)

Higgs mass [GeV]

Proton lifetime [10^{35} yrs]

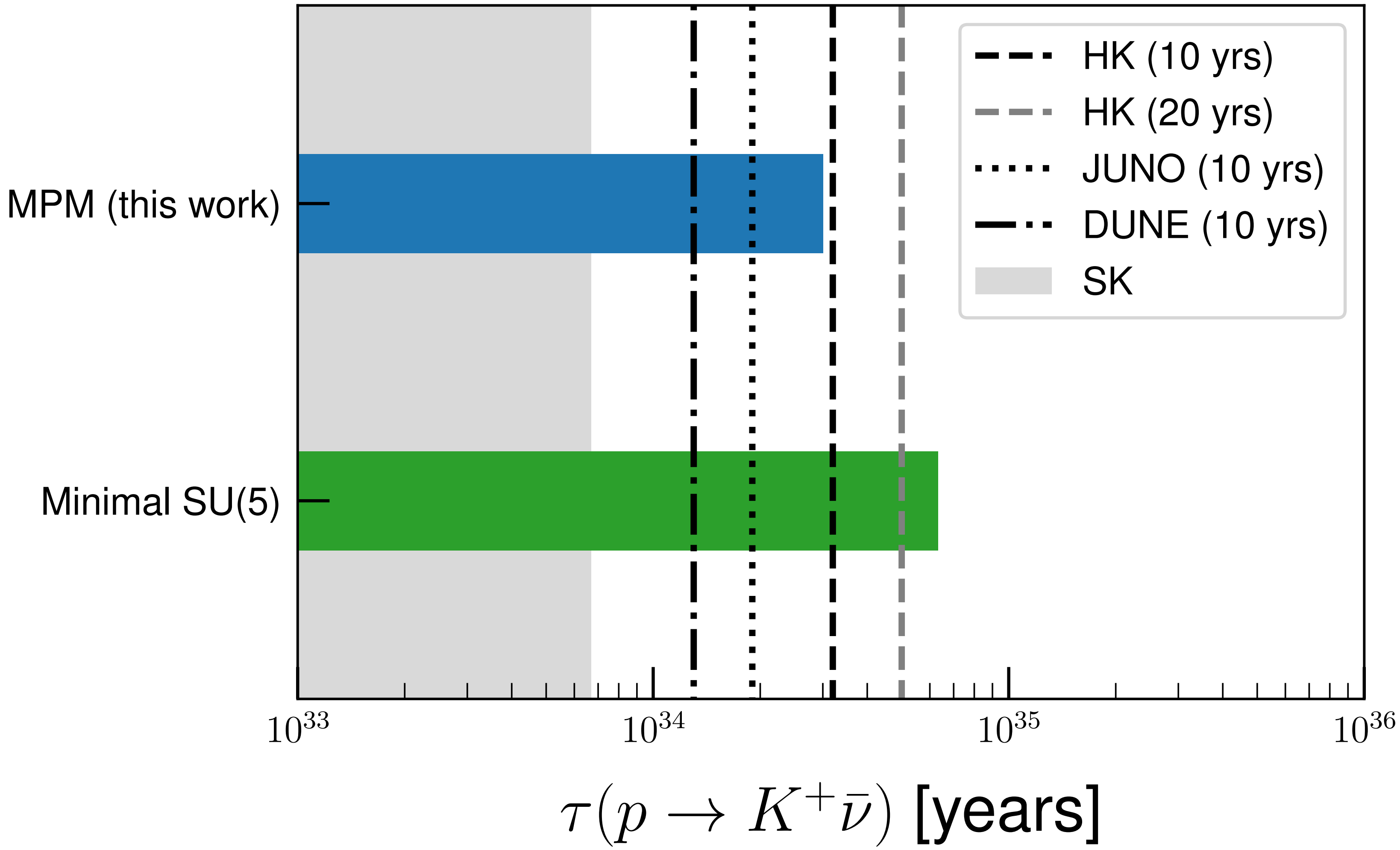
$$\tau(p \to K^+ \bar{\nu}) > 6.6 \times 10^{33} \text{ years}$$

(Super-Kamiokande)

We find a region where dark matter abundance and Higgs mass are consistent with the observed values.

J. Ellis, J. L. Evans, N. Nagata, K. A. Olive, Eur. Phys. J. C **81**, 543 (2021).

Future prospects



For Minimal SU(5), see J. Ellis, J. L. Evans, N. Nagata, K. A. Olive, L. Velasco-Sevilla, Eur. Phys. J. C **80**, 332 (2020).

- Proton lifetimes in MPM are predicted to be shorter than those in Minimal SU(5).
- The prediction can be tested in Hyper-Kamiokande.

J. Ellis, J. L. Evans, N. Nagata, K. A. Olive, Eur. Phys. J. C **81**, 543 (2021).

Summary

- We revisited the minimal Missing Partner Model.
- Thanks to the dim-5 operator, perturbative gauge coupling unification is achieved.
- Dark matter abundance and Higgs mass can be explained while proton decay limit is evaded.
- This model can be tested in Hyper-Kamiokande.

Backup

Superpotential

$$W_5 = \frac{\mu_\Sigma}{2}\,\Sigma^{AB}_{CD}\Sigma^{CD}_{AB} - \frac{1}{3}\lambda'\,\Sigma^{AB}_{CD}\Sigma^{CD}_{EF}\Sigma^{EF}_{AB} + \lambda_\Theta \overline{H}_A \Sigma^{DE}_{BC}\Theta^{ABC}_{DE} + \lambda_{\bar\Theta} H^A \Sigma^{BC}_{DE}\bar\Theta^{DE}_{ABC}$$
$$+ M_\Theta \Theta^{ABC}_{DE}\bar\Theta^{DE}_{ABC} + (h_{\mathbf{10}})_{ij}\,\epsilon_{ABCDE}\Psi^{AB}_i\Psi^{CD}_j H^E + (h_{\bar{\mathbf{5}}})_{ij}\,\Psi^{AB}_i \Phi_{jA}\overline{H}_B\,,$$

VEV of Σ

$$\langle \Sigma^{\alpha\beta}_{\gamma\delta}\rangle_0 = \frac{3}{2}V\left(\delta^\alpha_\gamma\delta^\beta_\delta - \delta^\alpha_\delta\delta^\beta_\gamma\right)\,,$$
$$\langle \Sigma^{ab}_{cd}\rangle_0 = \frac{1}{2}V\left(\delta^a_c\delta^b_d - \delta^a_d\delta^b_c\right)\,,$$
$$\langle \Sigma^{\alpha b}_{\gamma d}\rangle_0 = \langle \Sigma^{b\alpha}_{d\gamma}\rangle_0 = -\langle \Sigma^{b\alpha}_{\gamma d}\rangle_0 = -\langle \Sigma^{\alpha b}_{d\gamma}\rangle_0 = -\frac{1}{2}V\delta^\alpha_\gamma\delta^a_d\,,$$

$$V = \frac{3}{4}\frac{\mu_\Sigma}{\lambda'}$$

$\alpha, \beta, \ldots :\ \mathrm{SU}(2), \quad a, b, \ldots :\ \mathrm{SU}(3)$

Mass spectrum

SU(5) gauge field

$$M_X = \sqrt{24} g_5 V$$

Components in 75 fields

$$M_{\Sigma_{(\mathbf{1},\mathbf{1},0)}} = -\frac{4}{3}\lambda' V \ , \qquad M_{\Sigma_{(\mathbf{3},\mathbf{1},5/3)}} = -\frac{8}{3}\lambda' V \ , \qquad M_{\Sigma_{(\mathbf{3},\mathbf{2},-5/6)}} = 0 \ ,$$
$$M_{\Sigma_{(\mathbf{6},\mathbf{2},5/6)}} = \frac{4}{3}\lambda' V \ , \qquad M_{\Sigma_{(\mathbf{8},\mathbf{1},0)}} = \frac{2}{3}\lambda' V \ , \qquad M_{\Sigma_{(\mathbf{8},\mathbf{3},0)}} = \frac{10}{3}\lambda' V \ .$$

Color-triplet Higgs

$$M_{H_C} = \lambda_\Theta \lambda_{\bar{\Theta}} \frac{(2V)^2}{M_\Theta}$$

Benchmark point parameters

Inputs		
$m_{1/2} = 20$ TeV	$m_0 = 15.9$ TeV	$A_0/m_0 = 2.25$
$\tan\beta = 4.5$	$M_{\rm in} = 10^{17.6}$ GeV	$M_\Theta = 3 \times 10^{17}$ GeV
$\lambda' = 0.005$	$\lambda_{\Theta,\bar{\Theta}} = 1$	$B_0 = A_0 - m_0$
GUT-scale parameters (masses in units of 10^{16} GeV)		
$M_{\rm GUT} = 0.692$	$M_{H_C} = 5.53$	$M_\Sigma = 0.0215$
$M_G = 2.95$	$M_X = 28.6$	$V = 6.46$
$g_5 = 0.907$	$d = 0.24$	
MSSM parameters (masses in units of TeV)		
$m_\chi = 4.2$	$m_{\tilde{t}_1} = 4.2$	$m_{\tilde{g}} = 17.7$
$m_{\chi_2} = 8.5$	$m_{\tilde{H}} = 24.1$	$\mu = -23.5$
$m_{\tilde{l}_L} = 21.5$	$m_{\tilde{l}_R} = 23.2$	$m_{\tilde{\tau}_1} = 20.5$
$m_{\tilde{q}_L} = 26.6$	$m_{\tilde{d}_R} = 24.4$	$m_{\tilde{t}_2} = 18.1$
$A_t = 32.7$	$A_d = 80.9$	$B = -14.6$
$c_K = -0.043$	$c_W = -1.44$	
Observables		
$\Omega_\chi h^2 = 0.125$	$m_h = 125.3$ GeV	$\tau_p = (0.099 \pm 0.026) \times 10^{35}$ yrs

Matching conditions (gauge couplings)

$$\frac{1}{g_1^2(Q)} = \frac{1}{g_5^2(Q)} + \frac{1}{8\pi^2}\left[10\ln\left(\frac{Q}{M_{\Sigma_{(\mathbf{3},\mathbf{1},5/3)}}}\right) + 10\ln\left(\frac{Q}{M_{\Sigma_{(\mathbf{6},\mathbf{2},5/6)}}}\right) + \frac{2}{5}\ln\left(\frac{Q}{M_{H_C}}\right) - 10\ln\left(\frac{Q}{M_X}\right)\right] + \frac{5}{2}\left(\frac{8dV}{M_P}\right) ,$$

$$\frac{1}{g_2^2(Q)} = \frac{1}{g_5^2(Q)} + \frac{1}{8\pi^2}\left[6\ln\left(\frac{Q}{M_{\Sigma_{(\mathbf{6},\mathbf{2},5/6)}}}\right) + 16\ln\left(\frac{Q}{M_{\Sigma_{(\mathbf{8},\mathbf{3},0)}}}\right) - 6\ln\left(\frac{Q}{M_X}\right)\right] - \frac{3}{2}\left(\frac{8dV}{M_P}\right) ,$$

$$\frac{1}{g_3^2(Q)} = \frac{1}{g_5^2(Q)} + \frac{1}{8\pi^2}\left[10\ln\left(\frac{Q}{M_{\Sigma_{(\mathbf{6},\mathbf{2},5/6)}}}\right) + \ln\left(\frac{Q}{M_{\Sigma_{(\mathbf{3},\mathbf{1},5/3)}}}\right) + 3\ln\left(\frac{Q}{M_{\Sigma_{(\mathbf{8},\mathbf{1},0)}}}\right) + 9\ln\left(\frac{Q}{M_{\Sigma_{(\mathbf{8},\mathbf{3},0)}}}\right) + \ln\left(\frac{Q}{M_{H_C}}\right) - 4\ln\left(\frac{Q}{M_X}\right)\right] - \frac{1}{2}\left(\frac{8dV}{M_P}\right) .$$

$$\frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} - \frac{1}{g_1^2(Q)} = -\frac{3}{10\pi^2}\ln\left(\frac{Q}{M_{H_C}}N_{H_C}\right) - \frac{48dV}{M_P} ,$$

$$\frac{5}{g_1^2(Q)} - \frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} = -\frac{3}{2\pi^2}\ln\left(\frac{Q^3}{M_X^2 M_{\Sigma_{(\mathbf{8},\mathbf{1},0)}}}N_X\right) + \frac{144dV}{M_P} ,$$

$$\frac{5}{g_1^2(Q)} + \frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} = -\frac{15}{2\pi^2}\ln\left(N_{g_5}\frac{M^2_{\Sigma_{(\mathbf{8},\mathbf{1},0)}}}{M_X Q}\right) + \frac{6}{g_5^2(Q)} + \frac{72dV}{M_P} ,$$

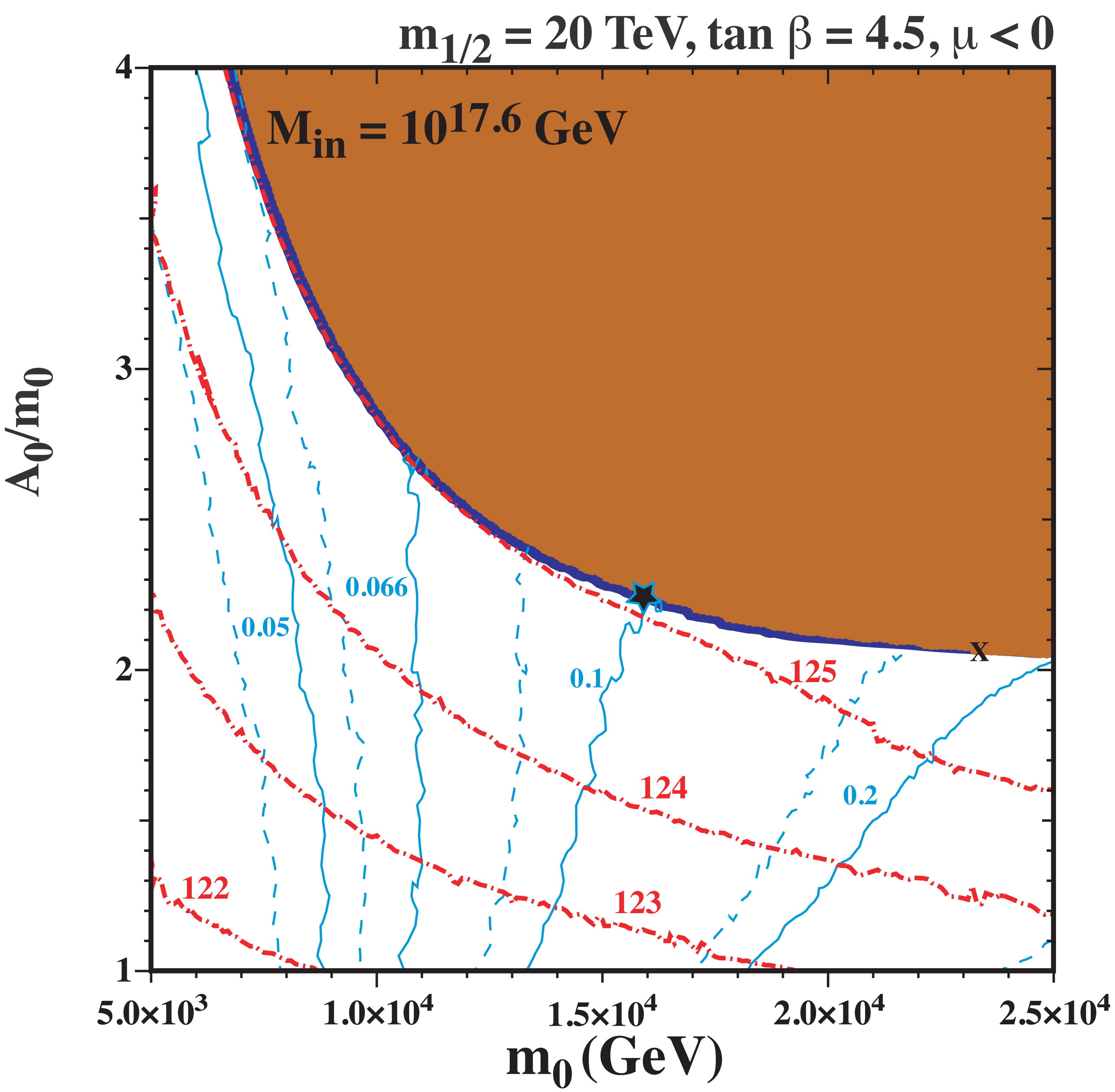
Matching conditions (gaugino masses)

$$M_1 = \frac{g_1^2}{g_5^2} M_5 - \frac{g_1^2}{16\pi^2} \Big[10 M_5 - 10 (A_{\lambda'} - B_\Sigma) - 20 B_\Sigma$$
$$+ \frac{2}{5} \left(B_\Theta - A_\Theta - A_{\bar{\Theta}} + 2 A_\Sigma - 2 B_\Sigma \right) \Big] + \frac{5}{4} \left(\frac{8dV}{M_P} \right) \left(A_{\lambda'} - B_\Sigma \right),$$

$$M_2 = \frac{g_2^2}{g_5^2} M_5 - \frac{g_2^2}{16\pi^2} \left[6 M_5 - 6 \left(A_{\lambda'} - B_\Sigma \right) - 22 B_\Sigma \right] - \frac{3}{4} \left(\frac{8dV}{M_P} \right) \left(A_{\lambda'} - B_\Sigma \right),$$

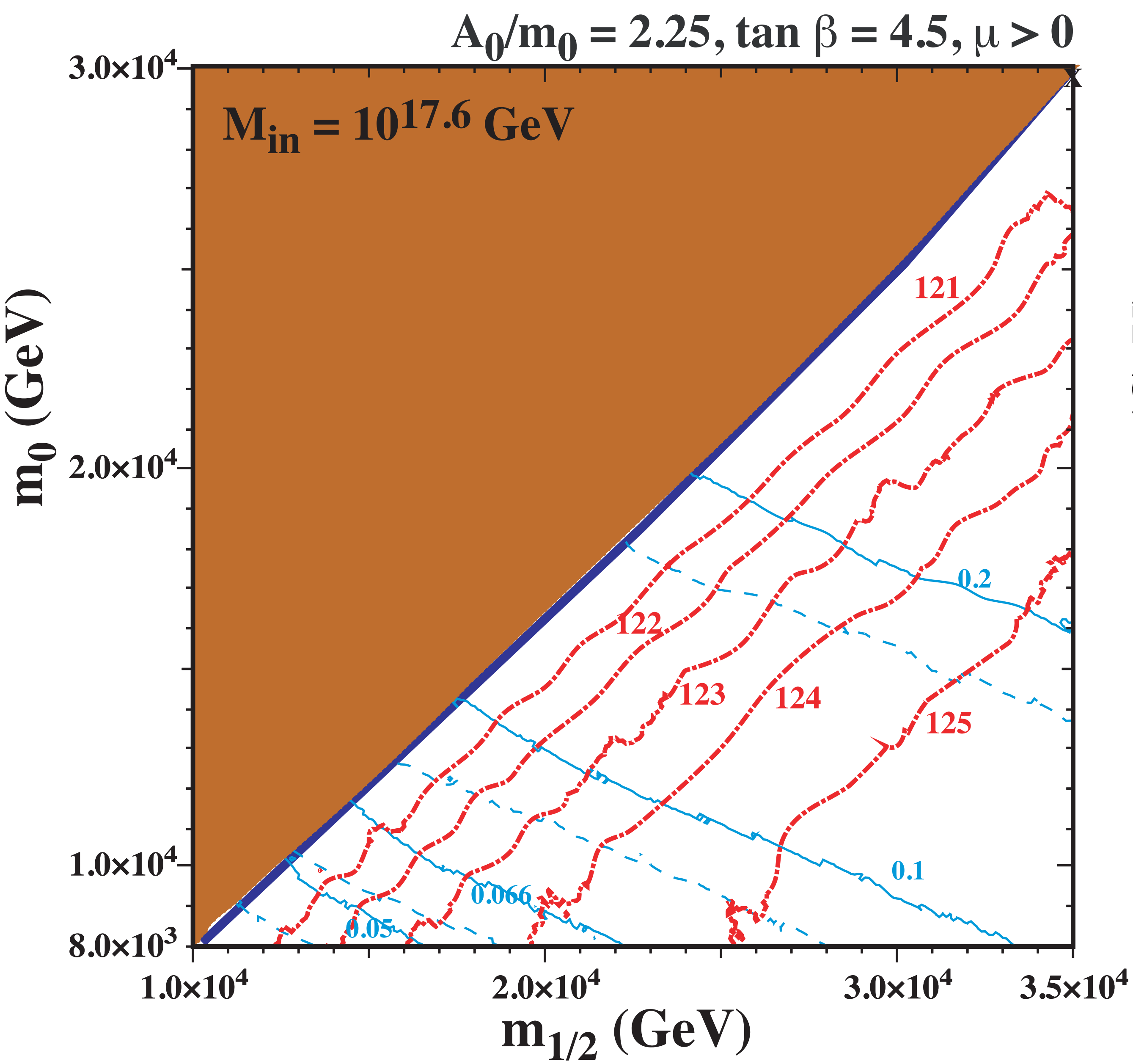
$$M_3 = \frac{g_3^2}{g_5^2} M_5 - \frac{g_3^2}{16\pi^2} \left[4 M_5 - 4 \left(A_{\lambda'} - B_\Sigma \right) - 23 B_\Sigma + B_\Theta - A_\Theta - A_{\bar{\Theta}} + 2 A_\Sigma - 2 B_\Sigma \right]$$
$$- \frac{1}{4} \left(\frac{8dV}{M_P} \right) \left(A_{\lambda'} - B_\Sigma \right) .$$

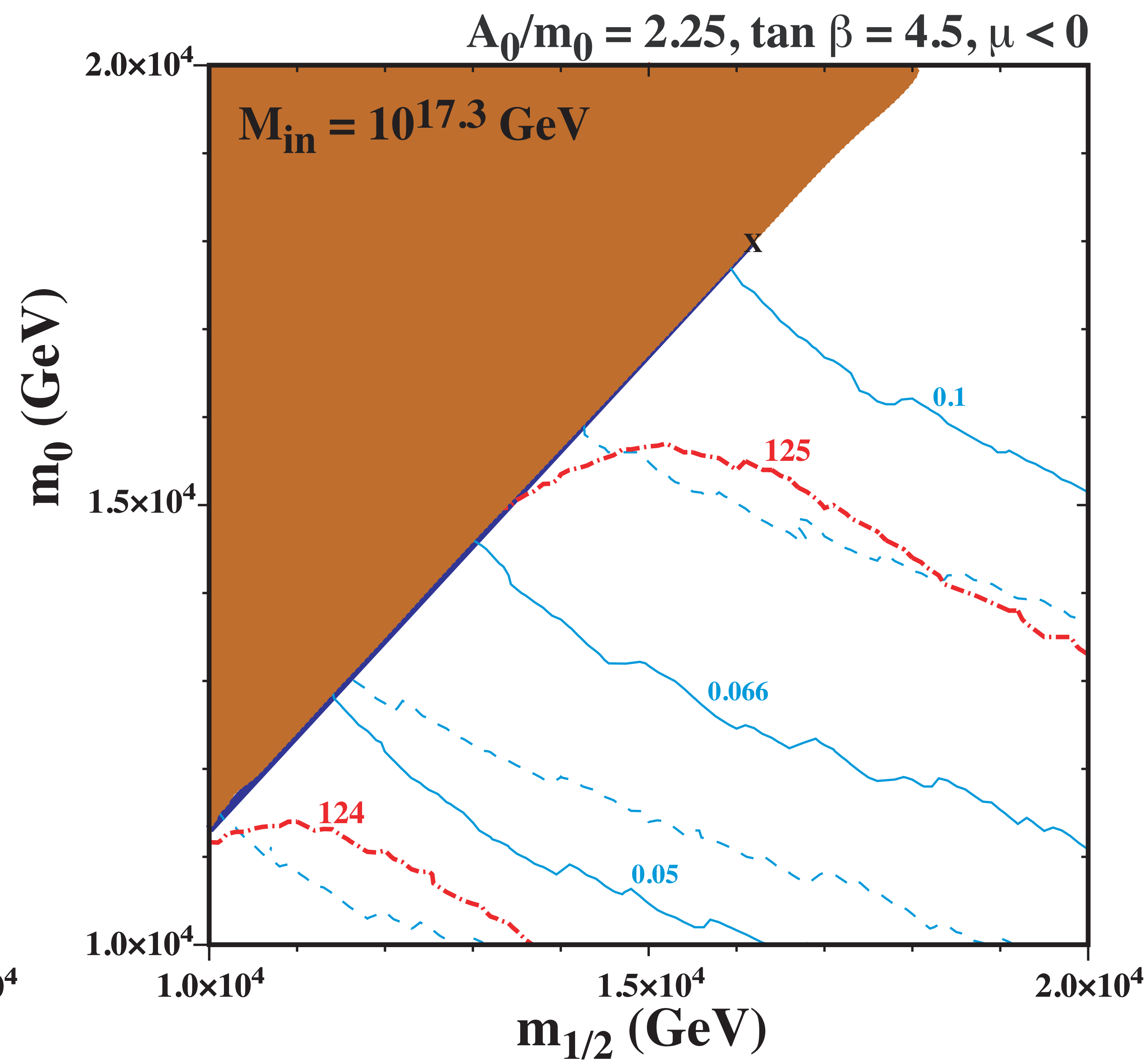
Additional plots



J. Ellis, J. L. Evans, N. Nagata, K. A. Olive, Eur. Phys. J. C **81**, 543 (2021).

Additional plots





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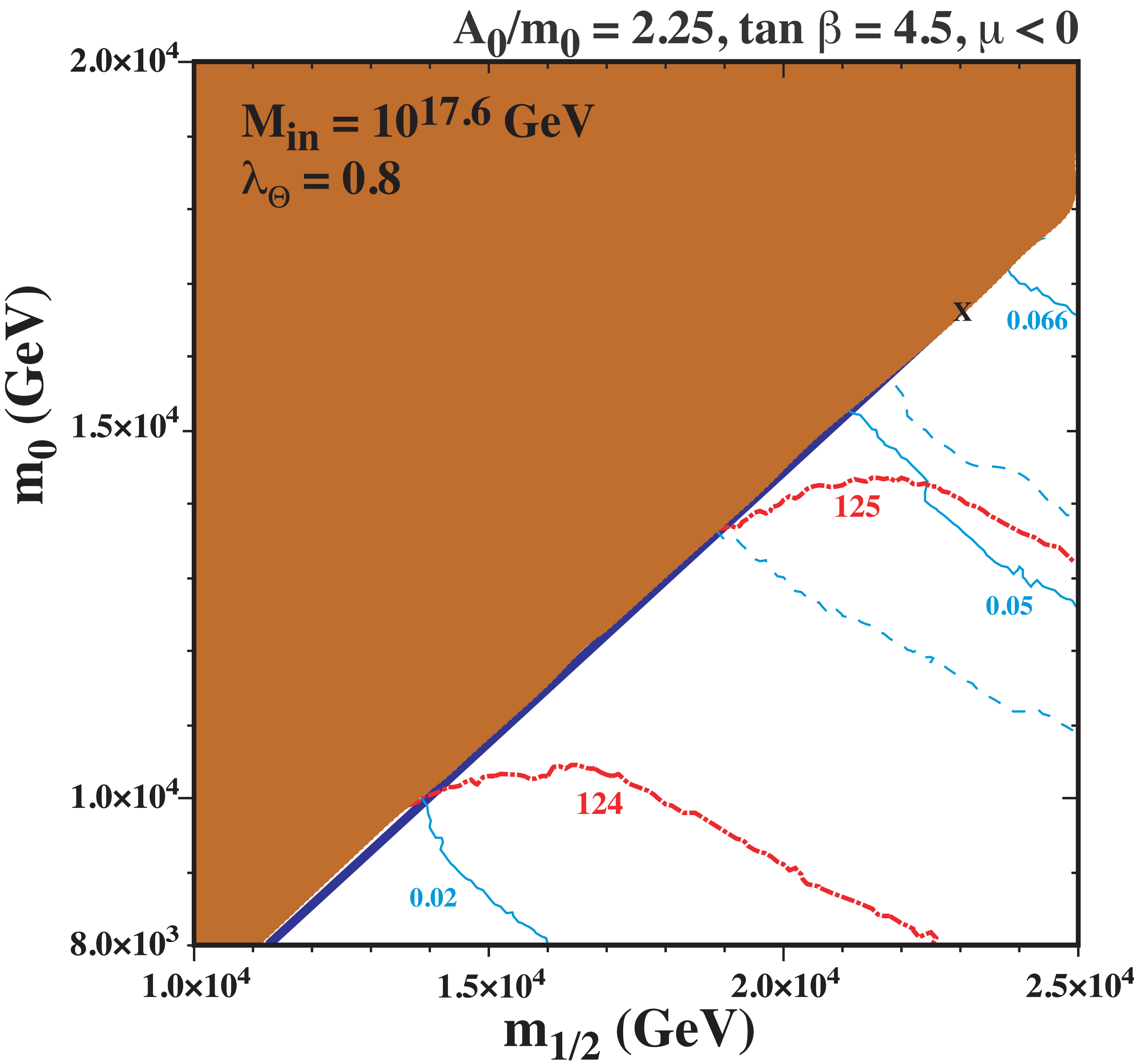
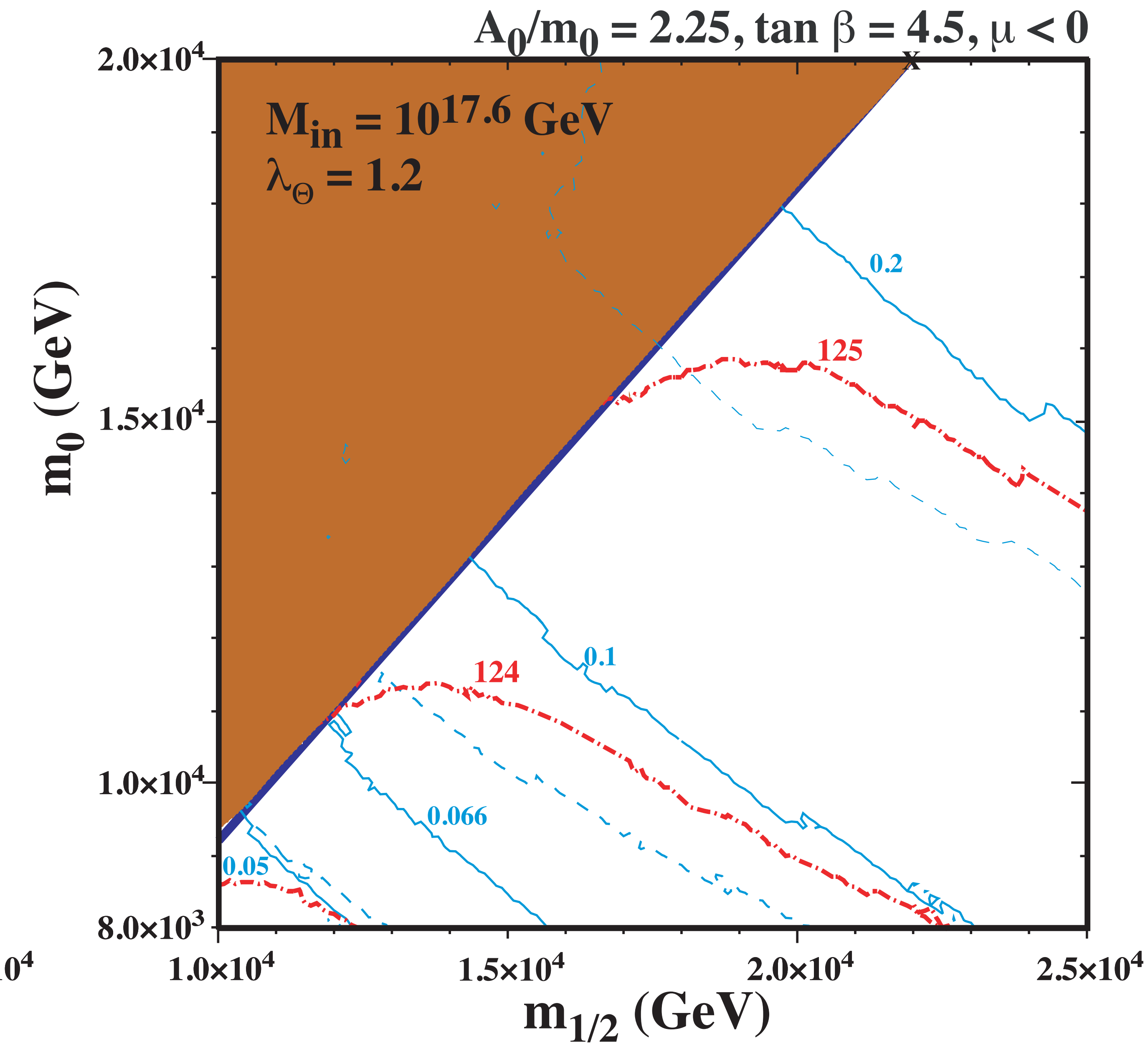
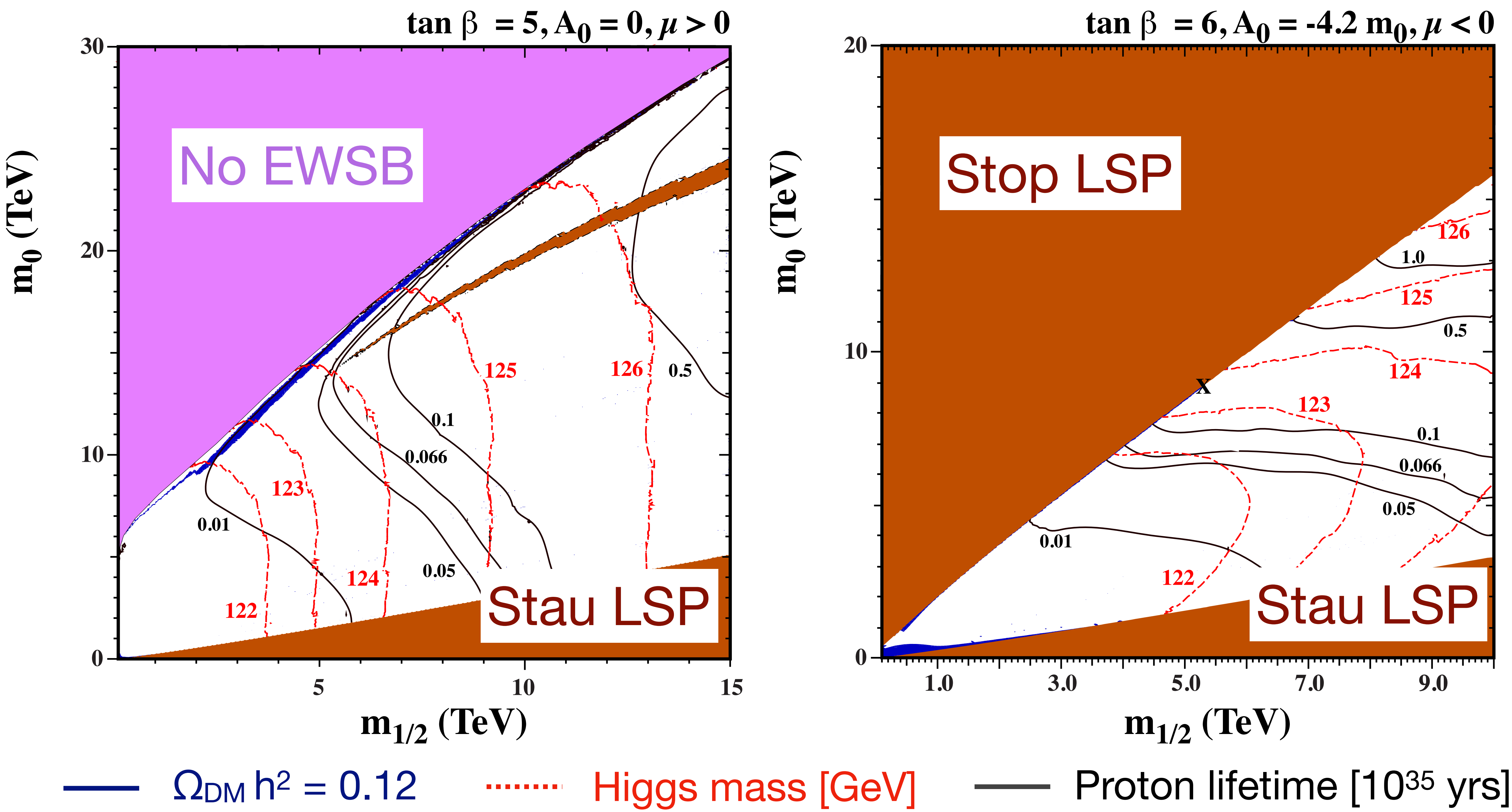




Proton decay in CMSSM



- Proton decay bound can be evaded.
- $p \to K^+ \nu$ decay may be observed in future experiments.

J. Ellis, J. L. Evans, A. Mustafayev, N. Nagata, K. A. Olive, Eur. Phys. J. **C76**, 592 (2016).