4D effective action from non-Abelian DBI action with magnetic flux background

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Based on the collaboration w/

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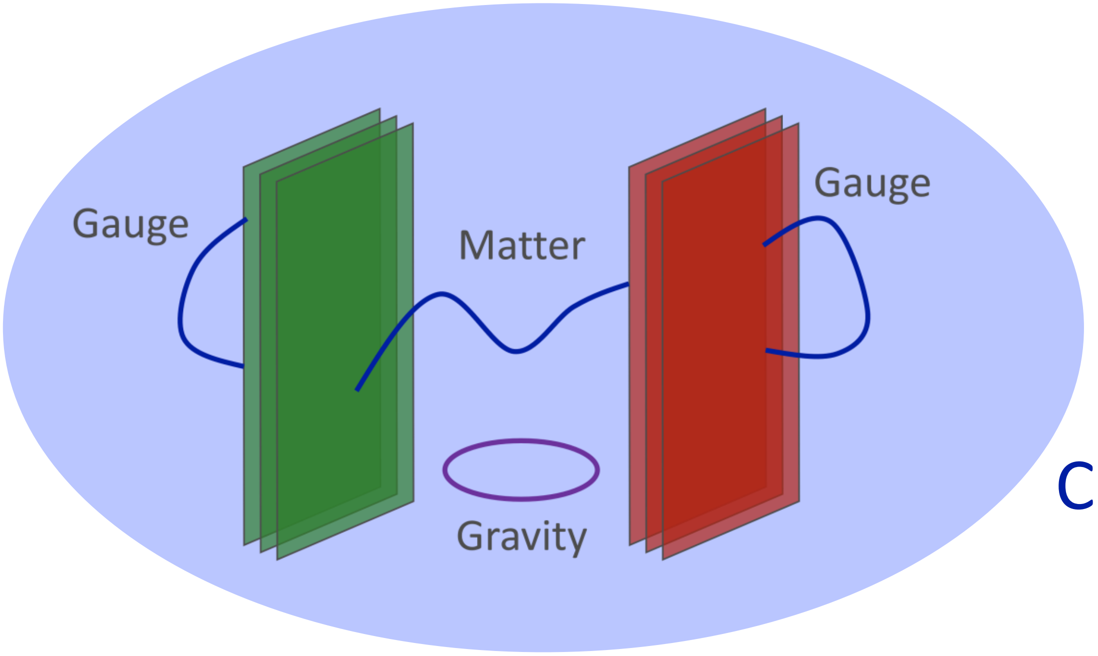
String theory as ultimate theory

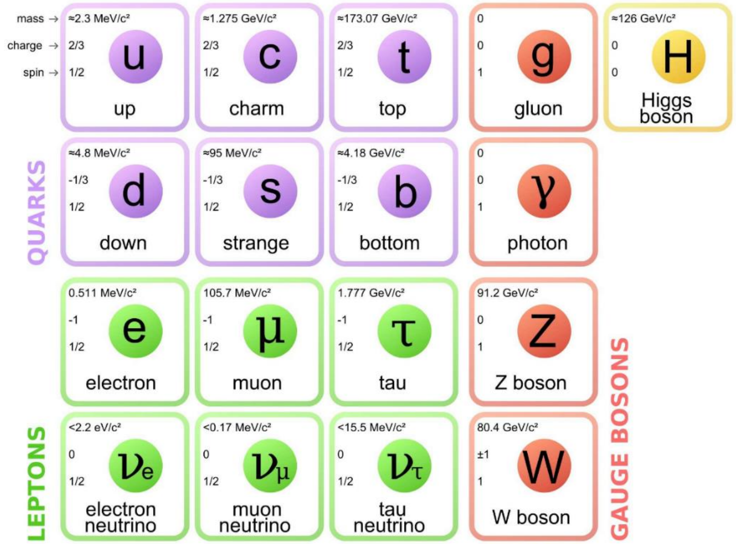
- String theory is an attractive candidate of the ultimate theory
- String theory → Standard Model w/

  $SU(3)_C \times SU(2)_L \times U(1)_Y$, chiral 3-generations and flavor structure

  + dark sectors



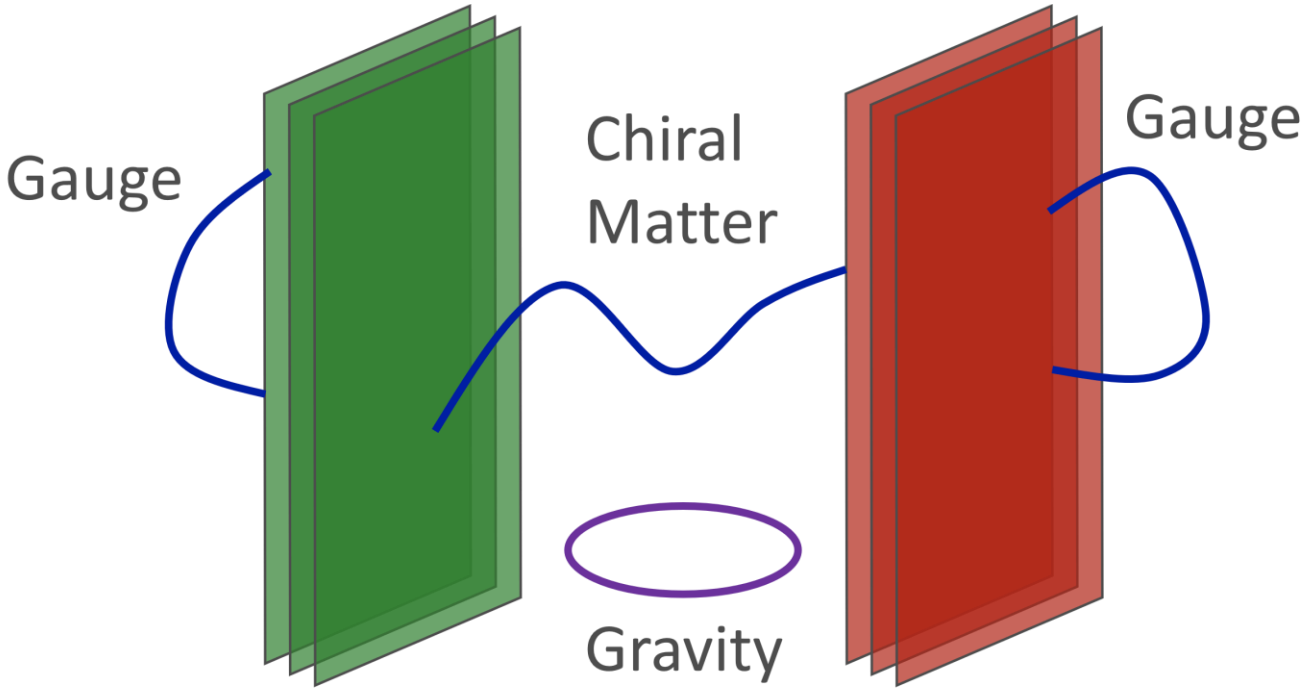
Compactification



Magnetized D-branes

- Background magnetic flux
  - realizing chiral fermion via index theorem
  - Zero-mode degeneracy → generation number
- Leading terms of the QFT on the magnetized D-branes

→ The Super Yang-Mills theory on the magnetized extra dimensions

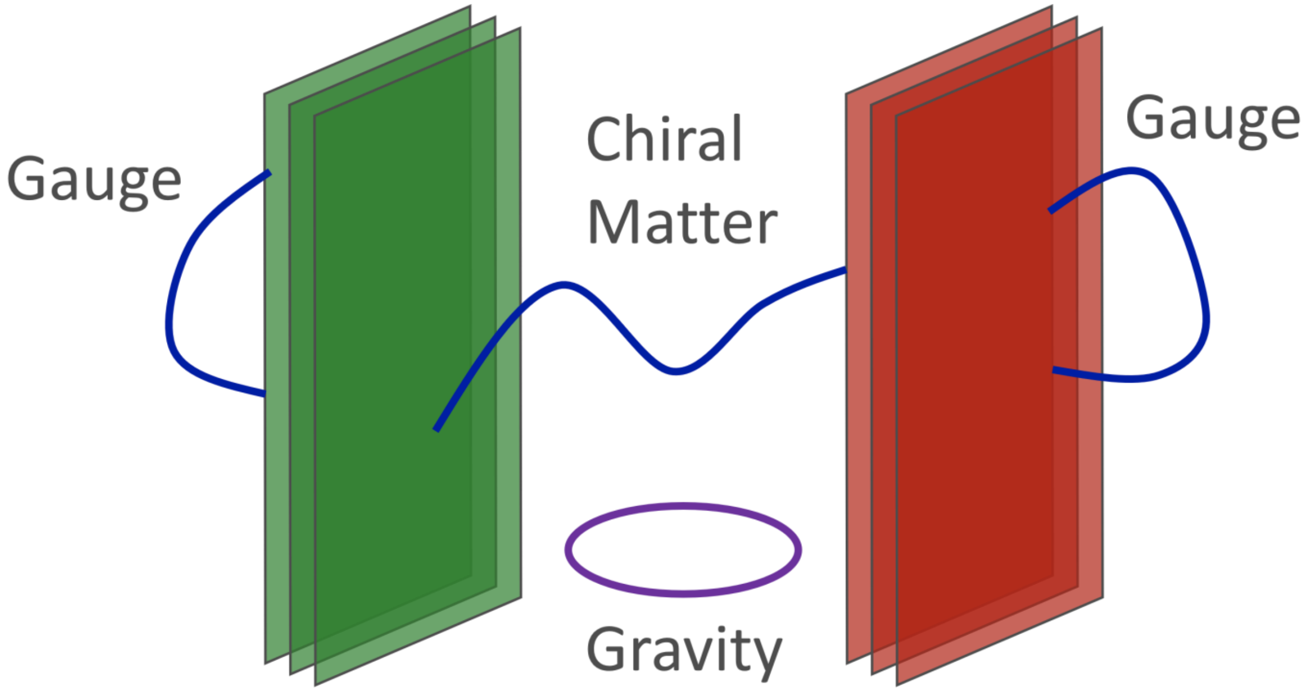


This talk

What is the stringy corrections?

→ We derive 4D $\mathcal{N} = 1$ SUSY EFT from non-Abelian Dirac-Born-Infeld action

- We consistently treat not only the gauge couplings but also **the matter Kähler metrices** in $\mathcal{O}(F^4)$
- Check the F-term potential comparing the SUGRA formula



Effective action for D-brane

- Dirac-Born-Infeld (DBI) action is an effective action of Dp-brane:

$$S_{\mathrm{DBI}} = -T_p \int_{\mathcal{W}} d^{p+1}\xi e^{-\varphi} \sqrt{-\det_{p+1}(g + 2\pi\alpha' F)}$$

- DBI action in the non-abelian gauge group is non-trivial.
  → Non-abelian DBI action

[Tseytlin '97,...]

$$S_{\mathrm{NDBI}} = -T_p \int d^{p+1}\xi \, e^{-\varphi} \, \boxed{\mathrm{str}} \sqrt{-\det_{p+1}(g + 2\pi\alpha' F)}$$

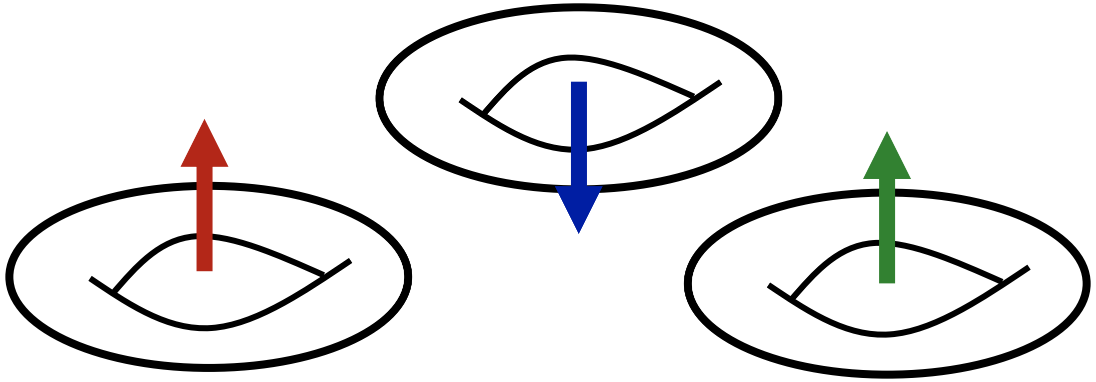
- Symmetrized trace prescription

$$\mathrm{str}(M_1 \cdots M_n) = \frac{1}{n!} \mathrm{tr}(M_1 \cdots M_n + \mathrm{permutations})$$

NDBI action on torus

- We will consider the space-filling D9-brane $\times 3$ with magnetized flux
- For the simplicity, the extra six-dimensional space is factorized torus
- The background fluxes are Abelian

$$\int_{\mathbb{T}^2} \hat{F}_i = 2\pi M^{(i)} = 2\pi \begin{pmatrix} M_a^{(i)} & & \\ & M_b^{(i)} & \\ & & M_c^{(i)} \end{pmatrix}$$



gauge symmetry

$$U(3) \to U(1)_a \times U(1)_b \times U(1)_c$$

- SUSY condition

$$\sum_{i=1}^{3} \frac{M_\alpha^{(i)}}{\mathcal{A}^{(i)}} = 0, \quad \alpha = a, b, c$$

Wave function of extra dimensions

[Cremades-Ibanez-Marchesano '04]

- Dirac equations: $$(\partial_x + i\partial_y - iq\pi M y)\psi_+ = 0$$ $$(\partial_x - i\partial_y - iq\pi M y)\psi_- = 0$$

normalizable
or unnormalizable

- Solution of Dirac eq. w/ background flux gives the profile of the wavefunction

$$\Theta^{j,M}(z) = \left(\frac{2\tau_I |M|}{\mathcal{A}^2}\right)^{1/4} \exp\left[i\pi M \frac{\mathrm{Im}\, z}{\tau_I} z\right] \vartheta \begin{bmatrix} \frac{j}{M} \\ 0 \end{bmatrix} (Mz, M\tau)$$

Normalization of wavefunction

Gaussian factor

Periodicity

$$\int_{\mathbb{T}^2} d^2y \sqrt{g_{\mathbb{T}^2}}\, \overline{\Theta^{i,M}(z)}\, \Theta^{j,M}(z) = \delta_{i,j}$$

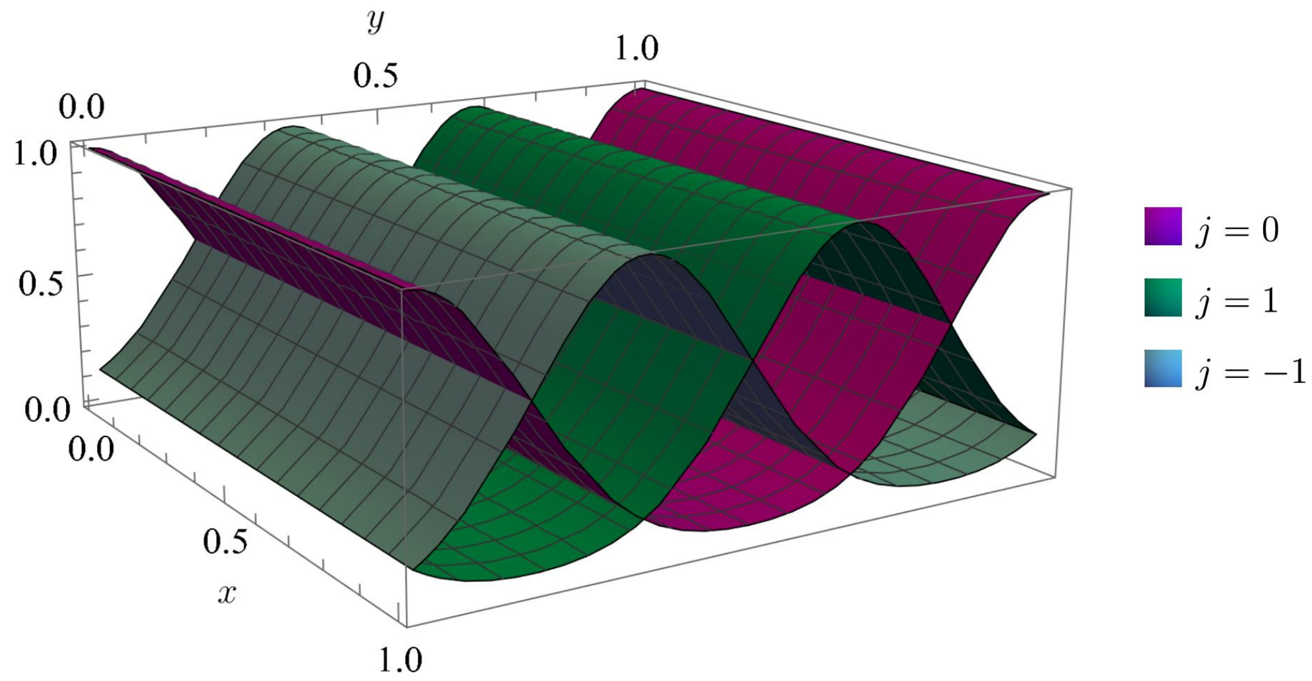
$$j = 0, 1, \ldots, |M| - 1$$

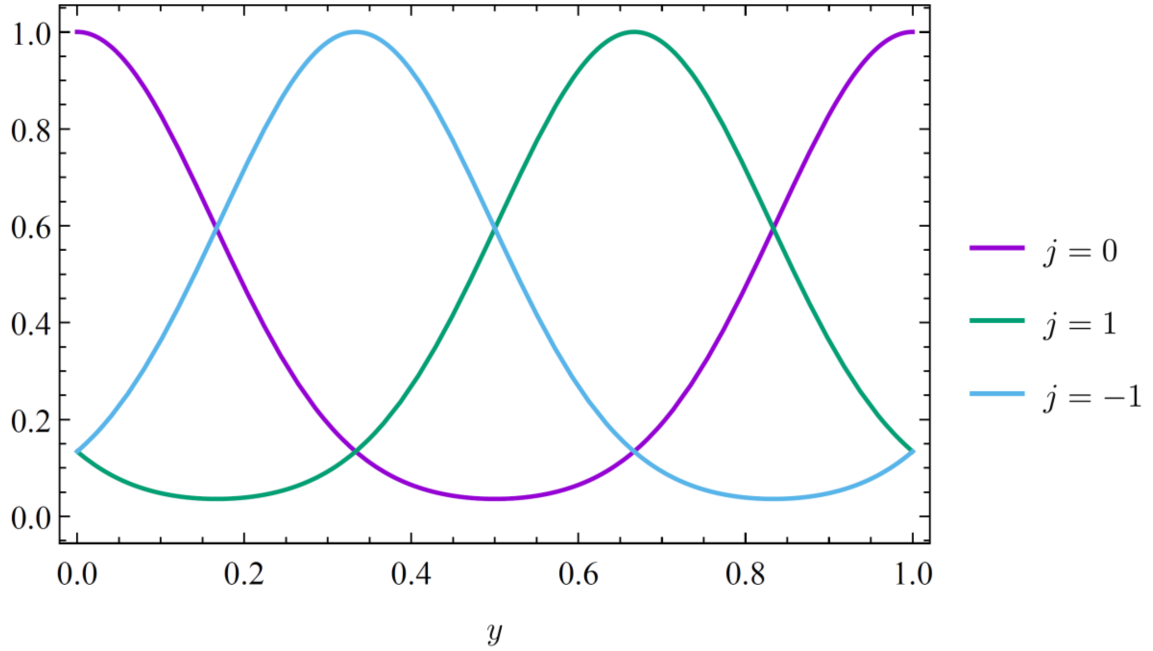
$$\Theta(z) \sim \frac{1}{\sqrt{\mathcal{A}}}$$

Wave function of extra dimensions

- Profiles of $\left|\Theta^{j,M}(z)\right|^2$

$$M = 3, \quad \tau = i$$





- Overlap integral of wavefunctions gives the Yukawa coupling

Strategy

- Expand the NDBI action $\mathcal{O}(F^4)$ and use the background field method

- 4D fields are parametrized by

$$a_\mu = \begin{pmatrix} a_\mu^a(x) & & \\ & a_\mu^b(x) & \\ & & a_\mu^c(x) \end{pmatrix} \quad a_{z_i} = \begin{pmatrix} & A_i(x)\underline{\phi_i^{ab}(y)}\delta_{i1} & \\ & & B_i(x)\underline{\phi_i^{bc}(y)}\delta_{i2} \\ C_i(x)\underline{\phi_i^{ca}(y)}\delta_{i3} & & \end{pmatrix}$$

- The flux is chosen so that these bi-fundamental scalar arise
- Closed string moduli are defined by

[Cremades-Ibanez-Marchesano ’04]

$$s = e^{-\varphi}\,\mathrm{Vol}(\mathbb{T}^6), \quad t_i = e^{-\varphi}\,\mathrm{Vol}(\mathbb{T}_i^2), \quad U_i = i\overline{\tau^{(i)}}$$

$$K^{(0)} = -\log(S+\overline{S}) - \sum_{i=1}^{3}\log(T_i+\overline{T_i}) - \sum_{i=1}^{3}\log(U_i+\overline{U_i})$$

Strategy

- Expansion of the NDBI action

$$ds_{10}^2 = e^{2\Phi} ds_4^2 + \ell_s^2 \sum_{i=1}^{3} e^{2\sigma_i} |dx_i + \tau^{(i)} dy_i|^2, \quad e^{2\Phi} = \frac{e^{\varphi}}{\prod_{r=1}^{3} \mathrm{Vol}(\mathbb{T}_r^2)}$$

$$S_{\mathrm{NDBI}} \approx -T_9 \ell_s^6 \int d^{10}X \sqrt{-\det g_4} e^{4\Phi} \left(\prod_{r=1}^{3} \int_{\mathbb{T}_r^2} d^2 y_r \sqrt{g_r} \right) e^{-\varphi} \frac{(2\pi\alpha')^2}{4} \mathrm{tr} \Big[F_{MN} F_{MN} \\ - \frac{(2\pi\alpha')^2}{3} \left(F_{KL} F_{LM} F_{NK} F_{MN} + \frac{1}{2} F_{KL} F_{LM} F_{MN} F_{NK} - \frac{1}{4} F_{KL} F_{KL} F_{MN} F_{MN} - \frac{1}{8} F_{KL} F_{MN} F_{KL} F_{MN} \right) + \mathcal{O}(F^6) \Big]$$

- Background field method

$$A_M = \hat{A}_M \delta_{Mm} + a_M, \quad F_{MN} = \hat{F}_{MN} + f_{MN}$$

- Field strength of the fluctuations

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu, \quad f_{\mu i} = \partial_\mu a_{z_i} + i[a_\mu, a_{z_i}],$$
$$f_{ij} = i[a_i, a_j], \quad f_{i\bar{j}} = i[a_i, a_{\bar{j}}]$$

Gauge coupling

- Gauge couplings ~ volume of the extra dimension including fluxes

$$2\pi S_{\mathrm{NDBI}} \sim -\frac{1}{4}\int_{M_4} d^4x\,\mathrm{tr}\, e^{-\varphi}\left(\int_{\mathbb{T}^6} d^6y\sqrt{g_6}\left[1+\hat{F}_{i\bar{j}}\hat{F}^{\bar{j}i}\right]\right) f_{\mu\nu}f^{\mu\nu}$$

Gauge coupling

- Gauge coupling for $U(1)_a$

[Lust-Mayr-Richter Stieberger '04, Blumenhagen-Kors-Lust-Stieberger '07]

$$\frac{1}{g_a^2} = s - t_1 M_a^{(2)} M_a^{(3)} - t_2 M_a^{(1)} M_a^{(3)} - t_3 M_a^{(1)} M_a^{(2)}$$

10D dilaton × total Volume

Flux^2 × Volume

Matter Kähler metric

- Read the kinetic term of bi-fundamental scalar

$$\int d^6y\sqrt{g_6}e^{2\Phi-\varphi}\,\mathrm{tr}\left(f_{\mu i}f^{\mu i}+\mathsf{c}\hat{F}_{j\bar{k}}\bar{F}^{\bar{k}j}f_{\mu i}f^{\mu i}\right)\sim\frac{2u_i}{t_i\,\mathrm{Vol}(\mathbb{T}^6)}(1+\mathsf{c}m^2)|\partial_\mu A_i|^2$$

- In addition, let us rescale : $A_i\mapsto\alpha_{ab}^{(i)}\,A_i$

[Abe-Kobayashi-Ohki-Sumita '12]

$$\alpha_{ab}^{(i)}=\frac{1}{\sqrt{2^2u_i}}\frac{\sqrt{\mathrm{Vol}(\mathbb{T}^6)}}{(2^3u_1u_2u_3)^{1/4}}\left(\frac{I_{ab}^{(i)}}{\prod_{r\neq i}|I_{ab}^{(r)}|}\right)^{1/4}$$

- Matter Kähler

$$\mathcal{Z}_{ab}^{i}=Z_{ab}^{i}\times\left[1-\frac{t_i}{6s}\left(2M_a^{(j)}M_a^{(k)}+2M_b^{(j)}M_b^{(k)}+M_a^{(j)}M_b^{(k)}+M_b^{(j)}M_a^{(k)}\right)\right]$$

$$Z_{ab}^{i}=\frac{1}{2t_i}\left(\prod_{k=1}^{3}\frac{1}{2u_k}\right)\sqrt{\frac{|I_{ab}^{(i)}|}{\prod_{j\neq i}|I_{ab}^{(j)}|}}$$

Gauge couplings and matter Kähler metrices

- Results

$$\frac{1}{g_a^2} = s - t_1 M_a^{(2)} M_a^{(3)} - t_2 M_a^{(1)} M_a^{(3)} - t_3 M_a^{(1)} M_a^{(2)}$$

$$\mathcal{Z}_{ab}^i = Z_{ab}^i \times \left[1 - \frac{t_i}{6s}\left(2M_a^{(j)} M_a^{(k)} + 2M_b^{(j)} M_b^{(k)} + M_a^{(j)} M_b^{(k)} + M_b^{(j)} M_a^{(k)}\right)\right]$$

- Conditions for a good description of an EFT

  → Existence of the upper bound on the flux

  ~ If the flux is too large, the current expansion is not valid

$$s > t_i |M^{(j)} M^{(k)}|$$

F-term scalar potential

- Is the matter Kähler metric is consistent with SUGRA?

  → We will check this via F-term scalar potential derived from NDBI

Flux contributions in the F-term scalar potential

$$V_F \ni 2\frac{e^{3\varphi}}{(\mathrm{Vol}(\mathbb{T}^6))^2}\frac{1}{g_{1\bar{1}}g_{2\bar{2}}}\left[1+\frac{1}{6}\left(2\frac{M_a^{(1)}}{\mathcal{A}^{(1)}}\frac{M_a^{(2)}}{\mathcal{A}^{(2)}}+2\frac{M_a^{(1)}}{\mathcal{A}^{(1)}}\frac{M_c^{(2)}}{\mathcal{A}^{(2)}}+\frac{M_a^{(1)}}{\mathcal{A}^{(1)}}\frac{M^{(2)}}{\mathcal{A}^{(2)}}+\frac{M_c^{(1)}}{\mathcal{A}^{(1)}}\frac{M_a^{(2)}}{\mathcal{A}^{(2)}}\right)\right]$$

$$\times\left(\alpha_{ab}^{(1)}\right)^2\times\left(\alpha_{bc}^{(2)}\right)^2\times A_1^{\mathbb{A}}B_2^{\mathbb{B}}\overline{A_1^{\mathbb{A}'}B_2^{\mathbb{B}'}}\times\left(\int d^6\sqrt{g_6}\,\phi_1^{\mathbb{A},ab}\phi_2^{\mathbb{B},bc}\overline{\phi_1^{\mathbb{A}',ab}\phi_2^{\mathbb{B}',bc}}\right)$$

$$=A_1^{\mathbb{A}}B_2^{\mathbb{B}}\overline{A_1^{\mathbb{A}'}B_2^{\mathbb{B}'}}\times\boxed{\frac{2Z_{ca}^3}{\mathcal{Z}_{ca}^3}}\times\frac{e^{3\varphi}}{(\mathrm{Vol}(\mathbb{T}^6))^2}\frac{1}{g_{1\bar{1}}g_{2\bar{2}}}\left(\alpha_{ab}^{(1)}\right)^2\left(\alpha_{bc}^{(2)}\right)^2\left(\int d^6y\sqrt{g_6}\phi_1^{\mathbb{A},ab}\phi_2^{\mathbb{B},bc}\overline{\phi_1^{\mathbb{A}',ab}\phi_2^{\mathbb{B}',bc}}\right)$$

The flux dependence in the F-term potential
→ controlled **by the matter Kähler metric**

F-term scalar potential

- Let us introduce the holomorphic Yukawa coupling by

[Cremades-Ibanez-Marchesano '04, Abe-Kobayashi-Ohki-Sumita '12]

$$W_{\mathbb{ABC}} = \sqrt{2} e^{-K^{(0)}/2} e^{3\Phi-\varphi} \frac{\alpha_{ab}^{(1)} \alpha_{bc}^{(2)} \alpha_{ca}^{(3)}}{\sqrt{g_{1\bar{1}} g_{2\bar{2}} g_{3\bar{3}}}} \int_{\mathbb{T}^6} d^6 y \sqrt{g_6} \phi_1^{\mathbb{A},ab} \phi_2^{\mathbb{B},bc} \phi_3^{\mathbb{C},ca}$$

this can be rewritten as the product of Jacobi theta function

- Using this coupling, the F-term potential becomes

$$V_F \ni \frac{e^{K_{(0)}}}{\mathcal{Z}_{ca}^3} \times A_1^{\mathbb{A}} B_2^{\mathbb{B}} \overline{A_1^{\mathbb{A}'} B_2^{\mathbb{B}'}} \times \sum_{\mathbb{C}} W_{\mathbb{ABC}} \overline{W_{\mathbb{A}'\mathbb{B}'\mathbb{C}}}$$

- This is consistent with the SUGRA w/ following superpotetial

$$W = \sum_{\mathbb{A},\mathbb{B},\mathbb{C}} W_{\mathbb{ABC}} A_1^{\mathbb{A}} B_2^{\mathbb{B}} C_3^{\mathbb{C}}$$

Yukawa coupling

- Quartic couplings in the bosonic sector + holomorphy
  → Yukawa couplings can be read

- Canonical normalization gives the physical Yukawa couplings

$$A_i \mapsto A_i/\sqrt{\mathcal{Z}^i_{ab}}$$

$$\hat{Y}_{\mathbb{ABC}} = \frac{W_{\mathbb{ABC}}}{\sqrt{\mathcal{Z}^1_{ab}\mathcal{Z}^2_{bc}\mathcal{Z}^3_{ca}}}$$

- Flux dependence on the Kähler metric may give the difference of the quark/lepton Yukawa couplings

Summary

- We use the non-abelian DBI action as the effective action of the D-brane and derive the higher-order flux contributions to the EFT

- The gauge coupling constants, the matter Kähler metrices, and the F-term scalar potential are directly read from the expansion of NDBI action

- Comparing with SUGRA formulae and it is confirmed that NDBI results are consistent

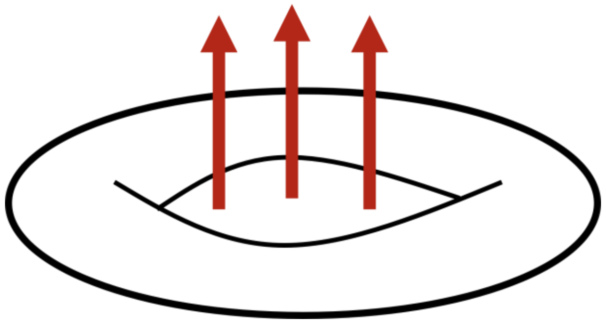
→ horomorphy is important in the F-term and flux corrections are controlled by matter Kähler metrices

Back up

Magnetized torus

[Cremades-Ibanez-Marchesano '04,...]

- $\mathcal{N}=1$ SYM on the magnetized torus
- Background flux is quantized



$$\int_{\mathbb{T}^2} \hat{F}_2 = 2\pi M$$

- Dimensional reduction: $$\lambda(x^\mu, y^m) = \sum_n \chi_n(x^\mu) \otimes \psi_n(y_m), \quad \psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$
- Dirac equation

$$(\partial_x + \underline{i\partial_y - iq\pi M y})\psi_+ = 0$$

$$(\partial_x \underline{- i\partial_y - iq\pi M y})\psi_- = 0$$

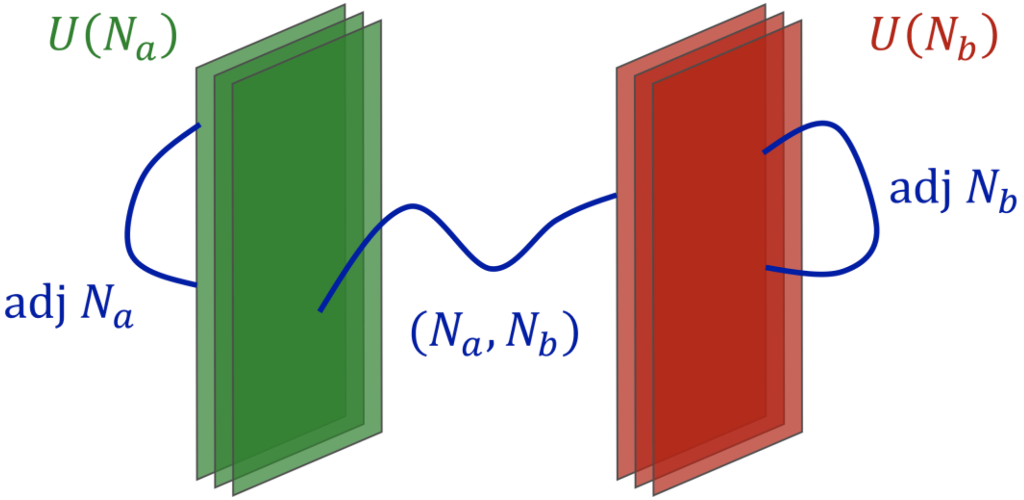
We can get the chiral fermion
as the normalizable solution of Dirac eq.
due to the background flux

Magnetized Extra Dimensions

- Background flux breaks the gauge group

$$U(N = N_a + N_b) \to U(N_a) \times U(N_b)$$

$$\bar{F}_{45} = 2\pi \begin{pmatrix} M_a \mathbf{1}_{N_a} & \\ & M_b \mathbf{1}_{N_b} \end{pmatrix}$$



- Representation of the field contents

$$\lambda = \begin{pmatrix} \lambda^{aa} & \lambda^{ab} \\ \lambda^{ba} & \lambda^{bb} \end{pmatrix}, \quad A_M = \begin{pmatrix} A_M^{aa} & A_M^{ab} \\ A_M^{ba} & A_M^{bb} \end{pmatrix}$$

  - (a,a) and (b,b) are adj. fields of unbroken gauge

$$\text{adj}\, N_a, \quad \text{adj}\, N_b$$

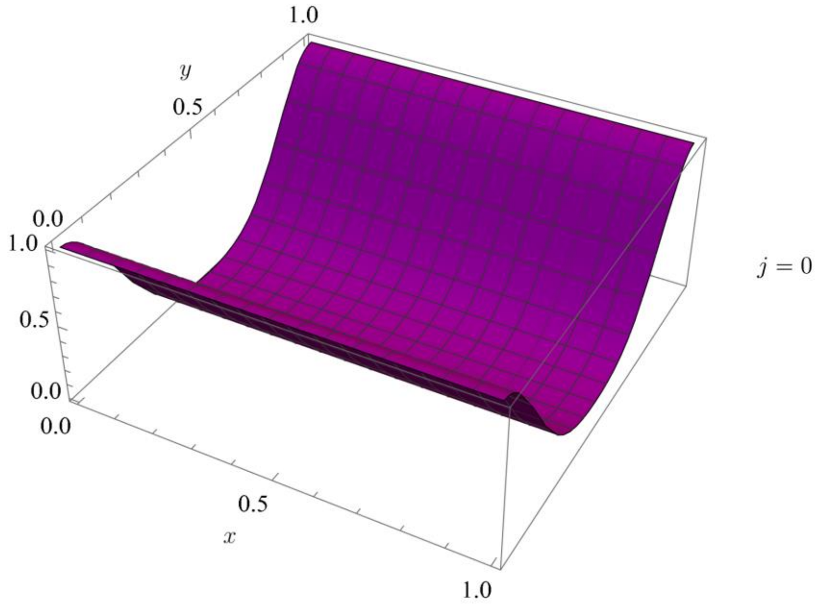
  - (a,b) and (b,a) are bi-fundamental matters

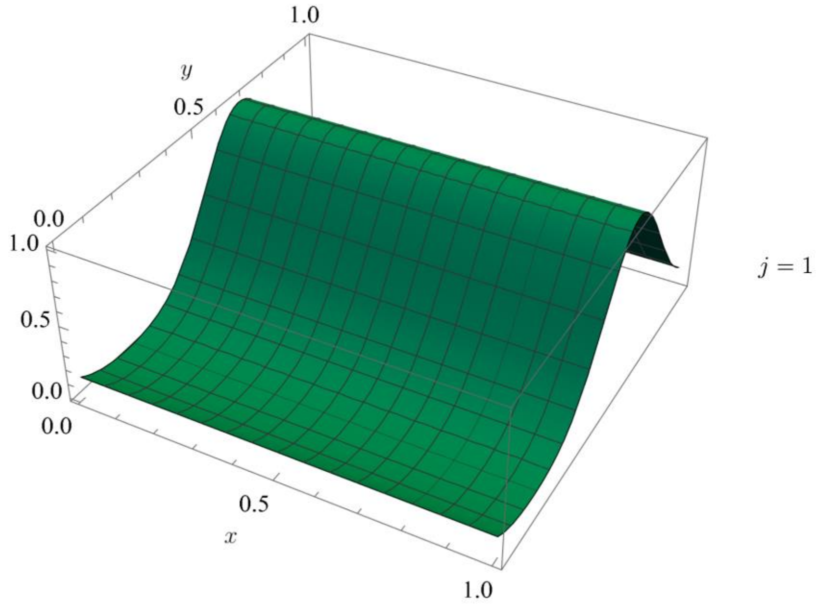
$$(N_a, \bar{N}_b), \quad (\bar{N}_a, N_b)$$

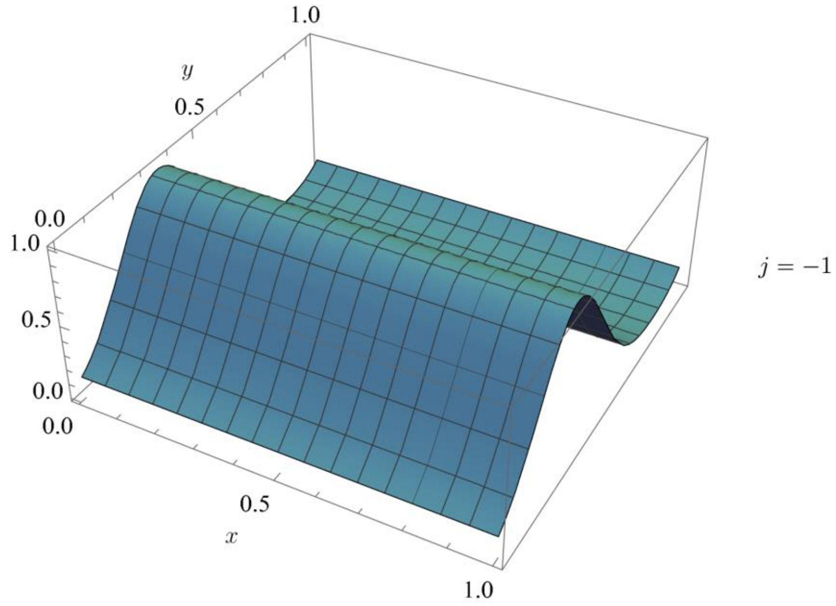
Wave function of extra dimensions

- $\left|\Theta^{j,M}(z)\right|^2$

$$M = 3, \quad \tau = i$$







Closed string zero modes

- Dimensional reduction

$$S_{\text{gravity}} = \frac{1}{(2\pi)^7\alpha'^4}\int d^{10}\sqrt{-g_{10}}e^{-2\varphi}\left[\mathcal{R}_{(10)} + 4(\partial_M\varphi)^2\right]$$
$$= M_P^2\int d^4x\sqrt{-g_4}\left[\frac{1}{2}\mathcal{R}_{(4)} - \frac{(\partial s)^2}{4s^2} - \sum_k\left(\frac{(\partial t_k)^2}{4t_k^2} + \frac{|\partial U_k|^2}{(U_k+\overline{U_k})^2}\right)\right]$$

- Closed string moduli

$$s = e^{-\varphi}\,\mathrm{Vol}(\mathbb{T}^6), \quad t_k = e^{-\varphi}\mathcal{A}^{(k)}, \quad U_k = i\overline{\tau^{(k)}} = u_k + i\,\mathrm{Im}(U_k)$$

$$\mathrm{Vol}(\mathbb{T}^6) = \prod_{i=1}^{3}\mathcal{A}^{(i)}$$

$\tau^{(k)}$: complex structure of k-the torus

Wavefunctions of NDBI expansion

- The wavefunctions of the bi-fundamental scalars

$$\phi_1^{\mathbb{A},ab} = \phi_1^{\mathbb{A},I_{ab}} = \Theta^{A^{(1)},I_{ab}^{(1)}}(z_1) \otimes \overline{\Theta^{A^{(2)},|I_{ab}^{(2)}|}(z_2)} \otimes \overline{\Theta^{A^{(3)},|I_{ab}^{(3)}|}(z_3)}$$

$$\phi_2^{\mathbb{B},bc} = \phi_2^{\mathbb{B},I_{bc}} = \overline{\Theta^{B^{(1)},|I_{bc}^{(1)}|}(z_1)} \otimes \Theta^{B^{(2)},I_{bc}^{(2)}}(z_2) \otimes \overline{\Theta^{B^{(3)},|I_{bc}^{(3)}|}(z_3)}$$

$$\phi_3^{\mathbb{C},ca} = \phi_3^{\mathbb{C},I_{ca}} = \overline{\Theta^{C^{(1)},|I_{ca}^{(1)}|}(z_1)} \otimes \overline{\Theta^{C^{(2)},|I_{ca}^{(2)}|}(z_2)} \otimes \Theta^{C^{(3)},I_{ca}^{(3)}}(z_3)$$

- Choice of the intersection number

$$\begin{aligned} I_{ab}^{(1)} &> 0, \quad I_{ab}^{(2),(3)} < 0 \\ I_{bc}^{(2)} &> 0, \quad I_{bc}^{(1),(3)} < 0 \\ I_{ca}^{(3)} &> 0, \quad I_{ca}^{(1),(2)} < 0 \end{aligned}$$

$$I_{\alpha\beta}^{(i)} = M_\alpha^{(i)} - M_\beta^{(i)},$$
$$(\alpha, \beta = 1, 2, 3, \ i = 1, 2, 3)$$

Holomorphic Yukawa coupling = Jacobi theta function

$$W_{\mathbb{ABC}} = \sqrt{2} e^{-K_0/2} \alpha_{ab}^{(1)} \alpha_{bc}^{(2)} \alpha_{ca}^{(3)} \frac{e^{3\Phi - \varphi}}{\sqrt{g_{1\bar{1}} g_{2\bar{2}} g_{3\bar{3}}}} \int_{\mathbb{T}^6} d^6 y \sqrt{g_6} \phi_1^{\mathbb{A},ab} \phi_2^{\mathbb{B},bc} \phi^{\mathbb{C},ca}$$

$$= 2 \prod_{r=1}^{3} W_{A^{(r)} B^{(r)} C^{(4)}}$$

$$W_{A^{(1)} B^{(1)} C^{(1)}} = \overline{\vartheta \left[\frac{B^{(1)} |I_{ca}^{(1)}| - C^{(1)} |I_{bc}^{(1)}| + m^{(1)} I_{bc}^{(1)} I_{ca}^{(1)}}{|I_{ab}^{(1)} I_{bc}^{(1)} I_{ca}^{(1)}|} \right] (0, i\overline{U_1} |I_{ab}^{(1)} I_{bc}^{(1)} I_{ca}^{(1)}|)}$$

$$W_{A^{(2)} B^{(2)} C^{(2)}} = \overline{\vartheta \left[\frac{C^{(2)} |I_{ab}^{(2)}| - A^{(2)} |I_{ca}^{(2)}| + m^{(2)} I_{ab}^{(2)} I_{ca}^{(2)}}{|I_{ab}^{(2)} I_{bc}^{(2)} I_{ca}^{(2)}|} \right] (0, i\overline{U_2} |I_{ab}^{(2)} I_{bc}^{(2)} I_{ca}^{(2)}|)}$$

$$W_{A^{(3)} B^{(3)} C^{(3)}} = \overline{\vartheta \left[\frac{A^{(3)} |I_{bc}^{(3)}| - B^{(3)} |I_{ab}^{(3)}| + m^{(3)} I_{ab}^{(3)} I_{bc}^{(3)}}{|I_{ab}^{(3)} I_{bc}^{(3)} I_{ca}^{(3)}|} \right] (0, i\overline{U_3} |I_{ab}^{(3)} I_{bc}^{(3)} I_{ca}^{(3)}|)}$$

$$A^{(1)} = B^{(1)} + C^{(1)} + m^{(1)} |I_{bc}^{(1)}|, \ m^{(1)} = 0, 1 \ldots, I_{ab}^{(1)} - 1$$
$$B^{(2)} = A^{(2)} + C^{(2)} + m^{(2)} |I_{ca}^{(2)}|, \ m^{(2)} = 0, 1 \ldots, I_{ca}^{(1)} - 1$$
$$C^{(3)} = A^{(3)} + B^{(3)} + m^{(3)} |I_{ab}^{(3)}|, \ m^{(3)} = 0, 1 \ldots, I_{ca}^{(3)} - 1$$