The Landscape and the Scale of SUSY Breaking

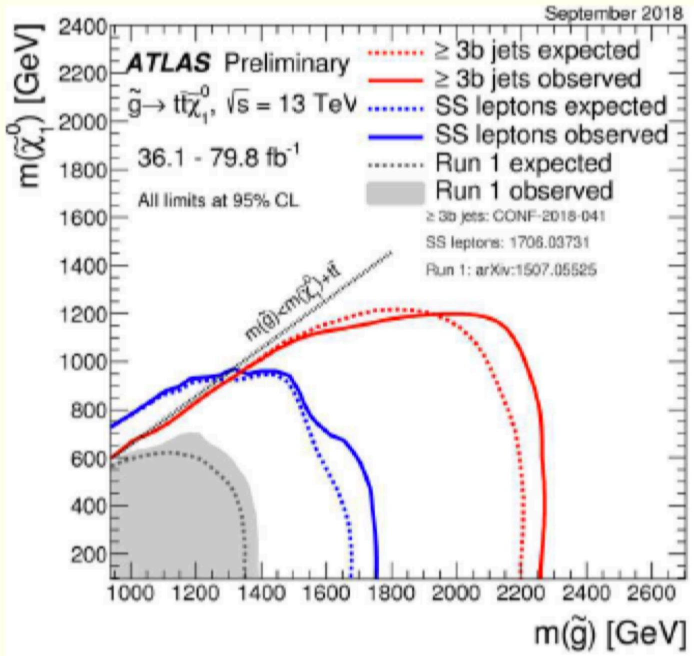
Kuver Sinha
University of Oklahoma

Based on work with
Michele Cicoli, Anshuman Maharana, Howie Baer, Gordy Kane, …

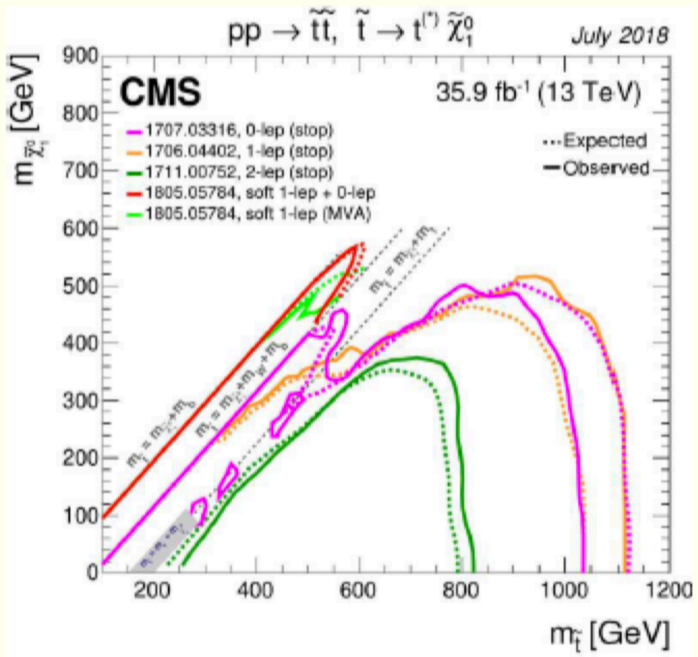
Where we are

No superpartners at LHC

ATLAS



CMS



$m_{\tilde{g}} > 2.25$ TeV

$m_{\tilde{t}_1} > 1.1$ TeV

Summary

Statistical arguments + weak-scale anthropic requirements may explain lack of super partners + give predictions for where they are

Predicated upon the existence of the string landscape

A Prediction from 2012

Mike Douglas

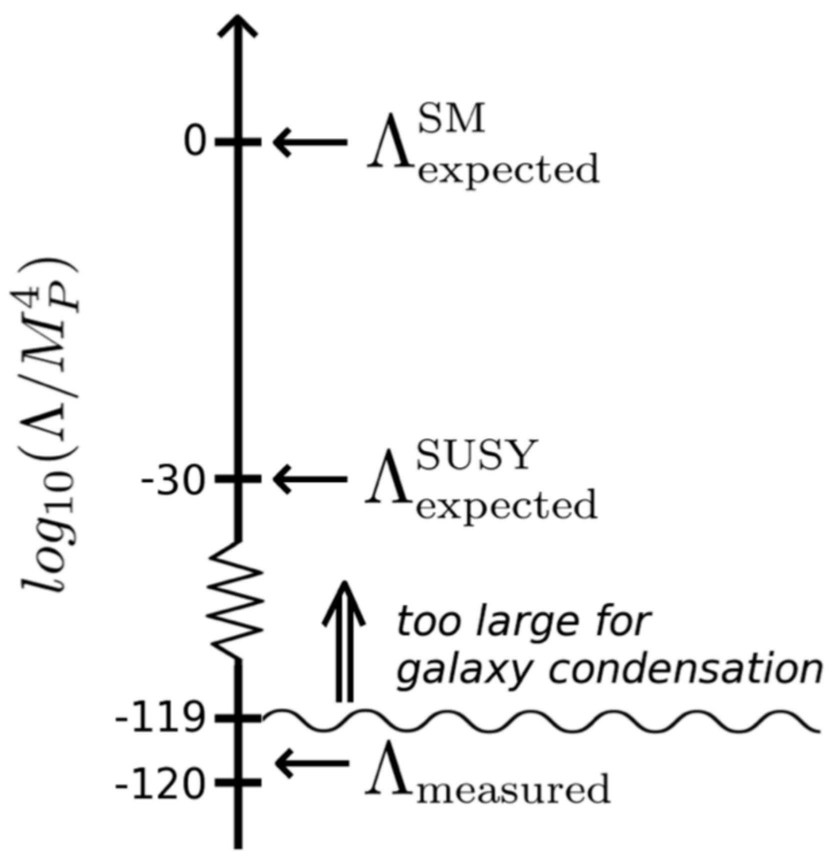
still try to see as far as he or she can, I am going to go out on a limb and argue that

> String/M theory will predict that our universe has supersymmetry, broken at the $30 - 100\,\mathrm{TeV}$ scale. If at the lower values, we may see gluinos at LHC, while if at the higher values, it will be very hard to see any evidence for supersymmetry.

- String theory and low-energy supersymmetry (arXiv:1204.6626)

The arguments relied on statistics of flux vacua and the string landscape

A Prediction from 1988



Steven Weinberg

the great predictions of the latter 20th century [4]: namely that given a multiverse which includes a vast assortment of pocket-universes with varying cosmological constants, then it may not be surprising to find $\Lambda \sim 10^{-120} m_P^4$ since if it was much bigger, then galaxy condensation would not occur and we would not even be here to discuss the issue. The situation is portrayed in Fig. 1 which depicts the fact that the cosmological constant ought to be at its most natural value *subject to the constraint that we can exist to observe it.* Such anthropic reasoning relies on the existence of a vast landscape of possibilities that is provided for by the discretuum of flux vacua from string theory [1–3, 5].

Landscape Statistics

string theory yields a landscape of 4D vacua

(i) are they actual solutions?
(ii) how are they connected?
(iii) is there a selection principle?

2 approaches:

(i) focus on a specific vacuum (pros: explicit construction; cons: lamppost)

(ii) study statistics (pros: find generic features; cons: are results trustworthy?)

Focus on type IIB flux landscape

(i) most well-understood compactifications with moduli stabilization and SUSY breaking

(ii) Standard Model-like constructions with D-branes

(iii) huge number of vacua

Type IIB Vacuum Statistics

- String/M theory has many consistent vacuum states which at least roughly match the Standard Model, and might be candidates to describe our world.

- The number of vacua is so large that the problems of reproducing the Standard Model in detail, and the classic problems of "beyond the Standard Model physics" such as the hierarchy problem and cosmological constant problem, might admit statistical solutions. The basic example is that in an ensemble of N vacua which differ only in a parameter Λ (say the c.c. as in [6]), and in which Λ is uniformly distributed, it is likely that a quantity which appears fine tuned by an amount $\epsilon > 1/N$ (for the c.c., 10^{-120} in a generic nonsupersymmetric theory) will be realized by at least one vacuum, just on statistical grounds.

- No single vacuum is favored by the theory. Although selection principles might be found, they will not determine a unique vacuum *a priori*, but rather cut down the possibilities in a way which is useful only when combined with other information.

Douglas, 2004: Statistical analysis of the SUSY breaking scale, hep-th/0405279

Framework

background fluxes

$$W_{\text{tree}} = \int_X G_3 \wedge \Omega(U)$$

cplx. str. moduli

$$K_{\text{tree}} = -2\ln\mathcal{V} - \ln\left(S+\bar{S}\right) - \ln\left(-\mathrm{i}\int_X \Omega(U)\wedge\bar{\Omega}(\bar{U})\right)$$

axio-dilaton

where $\mathcal{V}$ is the dimensionless volume of the internal manifold expressed in units of the string length $\ell_s = 2\pi\sqrt{\alpha'} = M_s^{-1}$. The Calabi-Yau volume $\mathcal{V}$ is also a function of the real parts of the Kähler moduli $T_i = \tau_i + \mathrm{i}\theta_i$ (with $i = 1, ..., h^{1,1}(X)$) where the τ_i's control the size of internal divisors while the θ_i's are the axions obtained from the dimensional reduction of the RR 4-form C_4 over the same 4-cycles. For the simplest cases with just a single Kähler modulus, $\mathcal{V} = \tau^{3/2}$.

Framework

Scalar potential:

$$V_F = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \overline{W} - 3|W|^2 \right) = K_{i\bar{j}} F^i \overline{F}^{\bar{j}} - 3m_{3/2}^2$$

$$F^i = e^{K/2} K^{i\bar{j}} D_{\bar{j}} \overline{W} \qquad \text{and} \qquad m_{3/2} = e^{K/2} |W| \, .$$

$$V_{\text{tree}} = |F^S|^2 + |F^U|^2 + |F^T|^2 - 3m_{3/2}^2$$

Neglected by Denef-Douglas since T-moduli are not fixed by fluxes at tree level.

Distribution of SUSY-breaking Scale

Distribution of SUSY-breaking vacua

$$dN(F, \hat{\Lambda}) = \prod d^2F^S\, d^2F^U\, d\hat{\Lambda}\, \rho(F, \hat{\Lambda})$$

analysis for the S and U-moduli. The Kähler moduli have been instead neglected since these moduli are not stabilised by fluxes at tree-level, and so the dynamics that fixes them beyond the tree-level approximation has been assumed to give rise just to small corrections to the leading order picture.

Number of flux vacua at Lambda = 0

$$dN_{\Lambda=0}(F) = \prod d^2F^S\, d^2F^U\, d\hat{\Lambda}\, \rho(F, \hat{\Lambda})\, \delta\left(|F^S|^2 + |F^U|^2 - \hat{\Lambda}\right)$$

where $\hat{\Lambda}$ is the depth of the supersymmetric AdS vacuum, $\hat{\Lambda} = 3m_{3/2}^2$

SUSY statistics neglecting Kahler moduli

Assumption: SUSY breaking decoupled from cc=0

$$dN_{\Lambda=0}(F) = d^2F\,\rho(F)$$

$$|F|^2 = 3m_{3/2}^2 \quad \Rightarrow \quad d^2F \simeq |F|\,d|F| \simeq m_{3/2}\,dm_{3/2}$$

$$dN_{\Lambda=0}(m_{3/2}) \simeq \rho(m_{3/2})\,m_{3/2}\,dm_{3/2}$$

Assumption: F-terms are uniformly distributed

$$\rho(m_{3/2}) \sim m_{3/2}^{\beta} \qquad \text{with} \qquad \beta \geq 0$$

Zero is the uniform case

$$\Rightarrow \quad dN_{\Lambda=0}(m_{3/2}) \simeq m_{3/2}^{\beta+1}\,dm_{3/2} \qquad \text{with} \qquad \beta \geq 0$$

SUSY statistics neglecting Kahler moduli

Previous results:

(i) uniform distribution of SUSY breaking scales from uniform distribution of F-terms (Douglas; Denef Douglas)

(ii) logarithmic distribution of SUSY breaking scales from dynamical SUSY breaking (Dine Gorbatov Thomas)

But

Kahler moduli stabilization was ignored

What happens to statistics when moduli stabilization is incorporated?

Importance of Kahler moduli

Kahler moduli do not appear in the tree level super potential

(holomprphy and shift symmetry)

$$F^T = e^{K/2}\,\overline{W}\,K^{T\bar{T}}K_{\bar{T}} \quad \Rightarrow \quad V_{\rm tree} = |F^S|^2 + |F^U|^2 + m_{3/2}^2\left(\underline{K_{\bar{T}}K^{\bar{T}T}K_T} - 3\right)$$

=0

No-scale cancellation
(10 D scale invariance, SUSY, shift symmetry)

Consequences

(1) Runaway when either $D_S W \neq 0 \text{ or } D_U W \neq 0$

$$V_{\rm tree} = |F^S|^2 + |F^U|^2 = \frac{e^{K_{\rm cs}}}{\mathcal{V}^2\left(S+\bar{S}\right)}\left[|D_S W|^2 + |D_U W|^2\right]$$

$\Rightarrow$ Stability requires $F^S = F^U = 0$

or at least $F^S \sim F^U \ll F^T$

Tree-level instability: Very hard to cure by counter-balancing with corrections. Have to incorporate quantum corrections to fix T and obtain gravitino mass.

(2) The gravitino mass is set by the F-terms of the Kahler moduli (due to the no-scale relation). To obtain its distribution, one has to study corrections to the tree-level action

In fact, we'll see that a large number of vacua (all LVS examples) in fact do not have uniform graviton mass distributions and thus beta is not zero for them

Kahler moduli stabilization

Two main stabilization schemes

KKLT

$$W = W_0 + A\, e^{-aT} \quad \Rightarrow \quad V_{KKLT} = \frac{2e^{-2a\tau} a^2 A^2}{3s\mathcal{V}^{2/3}}\left(1+\frac{3}{a\tau}\right) - \frac{2e^{-a\tau} a A W_0}{s\mathcal{V}^{4/3}}$$

Minimization

leading order

correction, small when EFT is trusted: tau >>1

$$e^{a\langle\tau\rangle} = \frac{2Aa\langle\tau\rangle}{3W_0}\left(1+\frac{3}{2a\langle\tau\rangle}\right) \simeq \frac{2Aa\langle\tau\rangle}{3W_0} \quad \Rightarrow \quad \langle\tau\rangle \simeq \frac{1}{a}\,|\ln W_0|$$

(1) $\langle\tau\rangle$ >>1 requires exponentially small W0

(2) SUSY AdS minimum with $F^T = 0$

Add anti-D3 brane: nilpotent super field in 4D EFT

KKLT

New vacuum

$$e^{a\langle\tau\rangle} = \frac{2Aa\langle\tau\rangle}{3W_0}\left(1+\frac{5}{2a\langle\tau\rangle}\right) \Rightarrow \langle\tau\rangle \simeq \frac{1}{a}|\ln W_0|$$

Assumption that SUSY breaking and cc distributions are decoupled is OK

Gravitino mass

$$m_{3/2} \simeq \sqrt{\frac{g_s}{8\pi}\frac{|W_0|}{\langle\mathcal{V}\rangle}} \simeq \frac{\pi\, g_s^{1/2}}{\mathfrak{n}^{3/2}}\frac{|W_0|}{|\ln W_0|^{3/2}}$$

Controlled by W0

$$a = 2\pi/\mathfrak{n}$$

Distribution

$$dm_{3/2} \simeq m_{3/2}\left(\frac{d|W_0|}{|W_0|} + \frac{1}{2}\frac{dg_s}{g_s} - \frac{3}{2}\frac{d\mathfrak{n}}{\mathfrak{n}}\right)$$

$$dN \simeq dg_s$$

Assume uniform distribution of string coupling

$$dN \simeq |W_0|d|W_0|.$$

Assume uniform distribution of W0 as a cplx. variable

$$dN \simeq -\mathfrak{n}^{-r}\, d\mathfrak{n}.$$

Assume phenomenological distribution of n

KKLT: Statistical Distribution

$$dm_{3/2} \simeq m_{3/2}\left(\frac{1}{|W_0|^2} + \frac{1}{2g_s} + \frac{3}{2}\mathfrak{n}^{r-1}\right)dN$$

$$\simeq \frac{M_p^2}{m_{3/2}}\left[\frac{g_s}{\mathfrak{n}^3|\ln W_0|^3} + \frac{\epsilon^2}{2}\left(\frac{1}{g_s} + 3\mathfrak{n}^{r-1}\right)\right]dN$$

$\epsilon \equiv m_{3/2}/M_p$

$\epsilon \ll 1$ To trust EFT

$$\frac{dN}{dm_{3/2}} \simeq \left(\frac{\mathfrak{n}^3|\ln W_0|^3}{g_s}\right)\frac{m_{3/2}}{M_p^2} \simeq \frac{m_{3/2}}{M_p^2}$$

$$N_{KKLT}(m_{3/2}) \sim \left(\frac{m_{3/2}}{M_p}\right)^2$$

$$\boxed{\rho_{KKLT}(m_{3/2}) \sim \frac{1}{M_p^2}\left(\frac{\mathfrak{n}^3|\ln W_0|^3}{g_s}\right) \sim \text{const.}}$$

$\beta = 0$

Large Volume Scenarios

$$\mathcal{V} = \tau_b^{3/2} - \tau_s^{3/2}$$

} Volume of the Calabi-Yau. $\tau_b \gg \tau_s \gg 1$

τ_b is a 'big' divisor controlling the overall volume while τ_s is a 'small' divisor supporting non-perturbative effects

$$K = -2\ln\left(\mathcal{V} + \frac{\xi}{2}\left(\frac{S+\bar{S}}{2}\right)^{3/2}\right)$$

$$W = W_0 + A_s\, e^{-a_s T_s},$$

} Kahler potential and super potential

$$V_{LVS} = \frac{4}{3}\frac{a_s^2 A_s^2 \sqrt{\tau_s} e^{-2a_s\tau_s}}{s\mathcal{V}} - \frac{2a_s A_s |W_0| \tau_s e^{-a_s\tau_s}}{s\mathcal{V}^2} + \frac{3\sqrt{s}\xi|W_0|^2}{8\mathcal{V}^3}$$

} Scalar potential

Minimize

$$\langle\mathcal{V}\rangle \simeq \frac{3\sqrt{\langle\tau_s\rangle}\,|W_0|}{4a_s A_s} e^{a_s\langle\tau_s\rangle} \qquad \langle\tau_s\rangle \simeq \frac{1}{g_s}\left(\frac{\xi}{2}\right)^{2/3}$$

} SUSY breaking AdS vacuum. Uplifting by T-branes, anti-branes, etc. does not change these values at leading order

Large Volume Scenarios

1. In LVS models, it is the smallness of g_s that guarantees that the effective field theory is under control. In fact, if the string coupling is such that perturbation theory does not break down, i.e. $g_s \lesssim 0.1$, stringy corrections to the 4D action can be safely ignored since both τ_b and τ_s are much larger than the string scale. Hence these models can exist for natural values of the flux-generated superpotential W_0 with $W_0 \sim \mathcal{O}(1-10)$.
2. The LVS vacuum is AdS with $V_{\rm LVS} \sim -m_{3/2}^3$ and non-supersymmetric with the largest F-term given by $F^{T_b} \sim \tau_b m_{3/2}$. Hence the Goldstino is the fermionic partner of T_b in the corresponding $N=1$ chiral superfield. This is eaten up by the gravitino which acquires a non-zero mass.

Since uplifting doesn't change the stabilized values by much, the gravitino mass is given by

$$m_{3/2} \simeq \sqrt{\frac{g_s}{8\pi}} \frac{|W_0|}{\langle \mathcal{V} \rangle} \simeq c_1 \frac{g_s}{\mathfrak{n}} e^{-\frac{c_2}{g_s \mathfrak{n}}}$$

Controlled by exp (-1/gs)

LVS: Statistical Distribution

$$dm_{3/2} = \frac{\partial m_{3/2}}{\partial g_s} dg_s + \frac{\partial m_{3/2}}{\partial \mathfrak{n}} d\mathfrak{n} \simeq c_2 \frac{m_{3/2}}{(g_s \mathfrak{n})^2} (\mathfrak{n}\, dg_s + g_s\, d\mathfrak{n})$$

$$\simeq m_{3/2} \left[\ln \left(\frac{M_p}{m_{3/2}} \right) \right]^2 (\mathfrak{n}\, dg_s + g_s\, d\mathfrak{n}) \,, \quad \Big\} \quad \begin{array}{l} dg_s \simeq dN \\ dN \simeq -\mathfrak{n}^{-r}\, d\mathfrak{n} \end{array}$$

$$\Rightarrow \quad dm_{3/2} \simeq \mathfrak{n}\, m_{3/2} \left[\ln \left(\frac{M_p}{m_{3/2}} \right) \right]^2 \left[1 - \frac{c_2\, \mathfrak{n}^{r-2}}{\ln \left(\frac{M_p}{m_{3/2}} \right)} \right] dN$$

$$\frac{dN}{dm_{3/2}} \simeq \frac{1}{\mathfrak{n}\, m_{3/2}} \left[\ln \left(\frac{M_p}{m_{3/2}} \right) \right]^{-2} \quad \Rightarrow \quad N_{LVS}(m_{3/2}) \sim \ln \left(\frac{m_{3/2}}{M_p} \right)$$

$$\boxed{\rho_{LVS}(m_{3/2}) \sim \frac{1}{\mathfrak{n}\, m_{3/2}^2} \left[\ln \left(\frac{M_p}{m_{3/2}} \right) \right]^{-2}}$$

Summary

$$dN_{\Lambda=0}(m_{3/2}) \simeq \rho(m_{3/2})\, m_{3/2} \cdot f_{EWFT} \quad dm_{3/2}$$

$$\rho(m_{3/2}) \sim m_{3/2}^{\beta} \quad \text{with} \quad \beta \begin{cases} = 0 \text{ (KKLT)} \\ = -2 \text{ (LVS)} \end{cases}$$

So which is it?

Relative preponderance of KKLT and LVS vacua

KKLT: Perturbatively Flat Vacua

$$W_0 = \sqrt{\tfrac{2}{\pi}}\,(F - \tau H)^T \cdot \Sigma \cdot \Pi$$

flux flux

$$\Sigma = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \qquad \Pi = \begin{pmatrix} \int_{B^a} \Omega \\ \int_{A_a} \Omega \end{pmatrix} = \begin{pmatrix} \mathcal{F}_a \\ U^a \end{pmatrix}$$

$$\mathcal{F}(U) = \mathcal{F}_{\rm pert}(U) + \mathcal{F}_{\rm inst}(U)$$

$$\mathcal{F}_{\rm pert}(U) = -\frac{1}{3!}\mathcal{K}_{abc}U^aU^bU^c + \frac{1}{2}\mathbf{a}_{ab}U^aU^b + b_aU^a + \xi$$

$$\mathcal{F}_{\rm inst}(U) = \frac{1}{(2\pi i)^3}\sum_{\vec{q}} A_{\vec{q}}\, e^{2\pi i \vec{q}\cdot\vec{U}}$$

Cornell group:
Demirtas et. al.

$$|W_0| \sim \lambda_1\, e^{-2\pi\lambda_2/g_s}$$

⇨

Logarithmic distribution even in KKLT!

But what about statistics?

$$|W_0| \sim \lambda_1 \, e^{-2\pi\lambda_2/g_s}$$

Broeckel, Cicoli, Maharana, Singh, KS

Statistics seems to be limited. Vacua of this kind seem to be a set of measure zero in the full ensemble of KKLT vacua.

(i) the vacua occur in a sub-manifold in cplx. str. moduli space: $\vec{U} = \tau \vec{p}\,,$

where the vector $\vec{p}$ is real and has all positive entries.

subspace of the moduli space which is isomorphic to $\mathcal{M}_\tau \times (\mathbb{R}^+)^{h^{1,2}_-}$

(ii) predict a light cplx. str. modulus with mass ~ $|W_0|^k$ (for some positive k)

If these vacua are dominant, the determinant of the boson mass matrix for cplx. str. moduli should vanish as W0 —> 0. But the Denef-Douglas vacua don't have this property.

Summary

$$dN_{\Lambda=0}(m_{3/2}) \simeq \rho(m_{3/2})\, m_{3/2} \cdot f_{EWFT} \quad dm_{3/2}$$

$$\rho(m_{3/2}) \sim m_{3/2}^{\beta} \qquad \text{with} \qquad \beta \begin{cases} = 0 \text{ (KKLT)} \\ = -2 \text{ (LVS)} \end{cases}$$

The important point is that the landscape has a statistical pull of the SUSY breaking scale to high values. How strong a pull depends on the relative preponderance of KKLT and LVS vacua.

Whither Low-scale SUSY, then?

$$dN_{\Lambda=0}(m_{3/2}) \simeq \rho(m_{3/2})\, m_{3/2} \cdot f_{EWFT}\ \ dm_{3/2}$$

$$\rho(m_{3/2}) \sim m_{3/2}^{\beta} \quad \text{with} \quad \beta \begin{cases} = 0 \text{ (KKLT)} \\ = -2 \text{ (LVS)} \end{cases}$$

$$f_{EWFT} = ?$$

Baer, Sinha, et. al.
Baer's talks at SUSY

Electroweak Fine-tuning

$$f_{EWFT} = ?$$

An effective field theory (or specific coupling or observable) T_1 is more natural in string theory than T_2 if the number of *phenomenologically acceptable* vacua leading to T_1 is larger than the number leading to T_2.

Douglas, 2012

anthropics lies here

Question: can we quantify these ideas into what the LHC is seeing?

Fine-tuning Penalty

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

$$f_{EWFT} \sim m^2_{weak}/m^2_{soft}$$

Even in the event of appropriate EWSB, the factor $f_{EWFT} \sim m^2_{weak}/m^2_{soft}$ penalizes but does not forbid vacua with a weak scale far larger than its measured value. In contrast, Agrawal *et al.* [7] have shown that a weak scale larger than ~ 3 times its measured value would lead to much weaker weak interactions and a disruption in nuclear synthesis reactions, and likely an unlivable universe as we know it. In addition, Susskind posits that an increased weak scale would lead to larger SM particle masses and consequent disruptions in both atomic and nuclear physics. From these calculations, it seems reasonable to *veto* SM-like vacua which lead to a weak scale more than (conservatively) four times its measured value.

Agrawal, Barr, Donoghue, Seckel (1998)

Donoghue Penalty

Minimization of MSSM Higgs potential

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2.$$

Radiative corrections from (s)particles with Yukawa/gauge coupling to Higgs

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8 g_Z^2 (\frac{1}{4} - \frac{2}{3} x_W) \Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right]$$

$$\Delta_{\rm EW} \equiv max_i \left(C_i \right) / (M_Z^2/2)$$

where $C_{H_u} = |-m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)|/$, $C_{H_d} = |m_{H_d}^2/(\tan^2\beta - 1)|/$ and $C_\mu = |-\mu^2|$, along with analogous definitions for $C_{\Sigma_u^u(k)}$ and $C_{\Sigma_d^d(k)}$. Low $\Delta_{\rm EW}$ means less fine-tuning.

Donoghue Penalty

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

$$case\ \mathbf{A}: \quad f_{EWFT} \rightarrow \Theta(30 - \Delta_{\rm EW})$$

Σ_u^u generate large values of the weak scale $m_{weak} \gg 100$ GeV. The value of $\Delta_{\rm EW} < 30$ then corresponds to calculated anthropic requirements from Agrawal *et al.* that the weak scale not deviate by more than a factor of several from its measured value [33]. In this case, $\Delta_{\rm EW} = 30$ corresponds to a Z mass nearly four times its measured value.

$$case\ \mathbf{B}: \quad f_{EWFT} \rightarrow \Delta_{\rm EW}^{-1}$$

$$\Delta_{\rm EW}^{-1} \sim (m_Z^2/2)/max\ [|m^2_{H_u}(weak)|\ or\ \mu^2\ \ or\ \ |\Sigma_u^u(i)|]$$

Instead of placing a generic m^2_{soft} in the denominator of Eq. 11, we place the maximal weak scale contribution to the magnitude of the weak scale. Rather than placing a sharp cutoff on

Conclusions

Landscape statistics predicted a long time ago that the SUSY breaking scale was very likely high.

Does a landscape of dS vacua exist? If it does, SUSY is probably broken at high scale

If you’re willing to temper that draw with weak-scale anthropics, you do have predictions for the collider program.

Backup Slides

Distribution of Condensing Group

As we discuss in App. A, the distribution of the string coupling can be considered as approximately uniform[5], implying $dg_s \simeq dN$. On the other hand, the distribution of the rank of the condensing gauge group in the string landscape is still poorly understood.[6] Ref. [67] estimated the largest value of $\mathfrak{n}$ as a function of the total number of Kähler moduli, counted by the topological number $h^{1,1}$, but did not study how the number of vacua varies in terms of $\mathfrak{n}$. Moreover the F-theory analysis of [67] is based on the assumption that the formation of gaugino condensation in the low-energy 4D theory is not prevented by the appearance of unwanted matter fields.

In fact, as shown in [68, 69], F-theory sets severe constraints on the form of 'non-Higgsable' gauge groups which guarantee that the low-energy theory features a pure super Yang-Mills theory undergoing gaugino condensation. Even if simple gauge groups like $SU(2)$ or $SU(3)$ are allowed, they do not survive in the weak coupling type IIB limit since they arise only from non-trivial (p, q) 7-branes that do not admit a perturbative description in terms of D7-branes. The only type IIB case allowed for pure super Yang-Mills is $SO(8)$ which corresponds to $\mathfrak{n} = 6$. This fits with the fact that all explicit type IIB Calabi-Yau orientifold models which have been constructed so far, feature exactly an $SO(8)$ condensing gauge group [41, 70–73].

Distribution of Condensing Group

A non-perturbative superpotential can however arise also in a hidden gauge group with matter fields, even if there are constraints on the numbers of flavours and colours [74]. Chiral matter can always be avoided by turning off all gauge fluxes on D7-branes but vector-like states are ubiquitous features of type IIB models obtained as the $g_s \to 0$ limit of F-theory constructions. Given that the interplay between vector-like states and the generation of a non-perturbative superpotential has not been studied in the literature so far, it is not clear yet if $\mathfrak{n}$ can only take two values, i.e. $\mathfrak{n} = 1$ for ED3s and $\mathfrak{n} = 6$ for a pure $SO(8)$ theory, or an actual $\mathfrak{n}$-distribution is indeed present in the string landscape. Even if we do not have a definite answer to this question at the moment, we can however argue that, if an actual $\mathfrak{n}$-distribution exists, the number of states N is expected to decrease when $\mathfrak{n}$ increases since D7-tadpole cancellation is easier to satisfy for smaller values of $\mathfrak{n}$. We shall therefore take a phenomenological approach and assume $dN \sim -\mathfrak{n}^{-r}\, d\mathfrak{n}$ with $r > 0$.

KKLT

New vacuum

$$e^{a\langle\tau\rangle} = \frac{2Aa\langle\tau\rangle}{3W_0}\left(1 + \frac{5}{2a\langle\tau\rangle}\right) \Rightarrow \langle\tau\rangle \simeq \frac{1}{a}\,|\ln W_0|$$

Assumption that SUSY breaking and cc distributions are decoupled is OK

Gravitino mass

$$m_{3/2} \simeq \sqrt{\frac{g_s}{8\pi}}\frac{|W_0|}{\langle\mathcal{V}\rangle} \simeq \frac{\pi\, g_s^{1/2}}{\mathfrak{n}^{3/2}}\frac{|W_0|}{|\ln W_0|^{3/2}}$$

Controlled by exponentially small W0

Ensembles of Flux Vacua

d=4, N=1 effective supergravity, Type IIB

$$V = e^{K/m_P^2}\left(g^{i\bar{j}} D_i W D_{\bar{j}} W^* - \frac{3}{m_P^2}|W|^2\right) + \frac{1}{2}\sum_\alpha D_\alpha^2$$

$$= e^{K/m_P^2}\left(\sum_i |F_i|^2 - 3\frac{|W|^2}{m_P^2}\right) + \frac{1}{2}\sum_\alpha D_\alpha^2$$

$$\Lambda_{cc} = m_{hidden}^4 - 3e^{K/m_P^2}|W|^2/m_P^2$$

$$\partial V/\partial z^i = \partial V/\partial \bar{z}^{\bar{i}} = 0.$$

Douglas and Denef, 2004: Distributions of Flux Vacua hep-th/0404116

Ensembles of Flux Vacua

$$dN_{F,metastable}(z) = \sum_i \delta_z(V'(z))\theta(V''(z))$$

where $\theta(V'')$ is 1 when the $2n \times 2n$ real matrix of squared bosonic masses $M = V''$ is positive definite. The derivatives of V appearing here are

$$\begin{aligned}
\partial_a V &= e^{\mathcal{K}}(D_a D_b W \bar{D}^b \bar{W} - 2 D_a W \bar{W}) \\
D_a \partial_b V &= e^{\mathcal{K}}(D_a D_b D_c W \bar{D}^c \bar{W} - D_a D_b W \bar{W}) \\
\bar{D}_{\bar{a}} \partial_b V &= e^{\mathcal{K}}(R^d{}_{c\bar{a}b} D_d W \bar{D}^c \bar{W} + g_{b\bar{a}} D_c W \bar{D}^c \bar{W} - D_b W D_{\bar{a}} \bar{W} \\
&\quad -2 g_{b\bar{a}} W \bar{W} + D_b D_c W \bar{D}_{\bar{a}} \bar{D}^c \bar{W}),
\end{aligned}$$

where R is the curvature of the cotangent bundle, i.e. $R^d{}_{ca\bar{b}}\, X_d \equiv [\nabla_a, \bar{\nabla}_{\bar{b}}] X_c = \bar{\partial}_{\bar{b}}(g^{\bar{e}d}\partial_a g_{c\bar{e}})\, X_d$.

$$\boxed{W(z),\ \mathcal{K}(z,\bar{z})}$$

Douglas and Denef, 2004: Distributions of Flux Vacua hep-th/0404116

Ensembles of Flux Vacua

$$W = \int_X G_4 \wedge \Omega = N^\alpha \Pi_\alpha$$

X: elliptically fibered CY 4-fold. F-theory compactification

$\Pi_\alpha = \int \Sigma_\alpha \wedge \Omega$ are the periods of some basis $\{\Sigma_\alpha\}$ of $H^4(X, \mathbb{Z})$

Weil Petersson metric on complex structure moduli space

$$\mathcal{K} = -\ln \hat{\Pi}_\alpha (\eta^{-1})^{\alpha\beta} \hat{\bar{\Pi}}_\beta$$

Can replace sum over flux quanta by integral

Useful to change variables using the fact that Ω and its derivatives supply a Hodge decomposition basis of $H^4(X, \mathbb{Z})$

Douglas and Denef, 2004: Distributions of Flux Vacua hep-th/0404116

Ensembles of Flux Vacua

Finally left with density per unit volume in moduli space

$$\int_R d^{2n} z \rho(z)$$

A simplified picture of the results is that one can define an "average density of vacua" in the moduli space, which can be integrated over a region of interest and then multiplied by a "total number of allowed choices of flux," to estimate the total number of vacua which stabilize moduli in that region. This estimate becomes exact in the limit of large flux,

Computed explicitly for simple compactifications: T6, conifold, mirror quintic. Based on results, general arguments for landscape were advanced

Douglas and Denef, 2004: Distributions of Flux Vacua [hep-th/0404116]

SUSY Breaking Vacua

$$dN_{vac}[F_i, D_\alpha, \hat{\Lambda}] = \prod dF \; dD \; d\hat{\Lambda}\rho(F_i, D_\alpha, \hat{\Lambda})$$

$$dN_{\Lambda=0}[F_i, D_\alpha] = \prod d^2F_i \; dD_\alpha \; d\hat{\Lambda}\rho(F_i, D_\alpha, \hat{\Lambda}) \; \delta(\sum_i |F_i|^2 + \sum_\alpha D_\alpha^2 - 3e^K|W|^2)$$

$$\Lambda = \sum_i |F_i|^2 + \sum_\alpha D_\alpha^2 - 3e^K|W|^2$$

distribution for SUSY breaking vacua decouples from the cosmological constant problem

$$\underline{dN_{\Lambda=0}[F_i, D_\alpha]} = \prod d^2F_i \; dD_\alpha \; \rho(F_i, D_\alpha)$$

CC Distribution

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

$$m^2_{hidden} = \sum_i |F_i|^2 + \frac{1}{2}\sum_\alpha D^2_\alpha$$

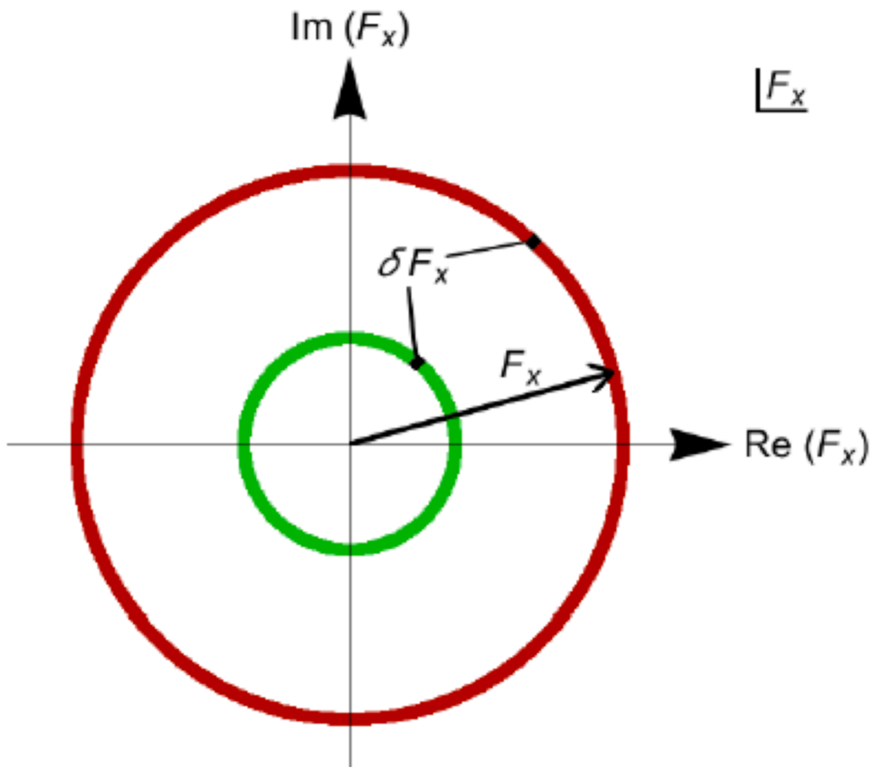
$$m_{weak} \simeq m_{W,Z,h} \simeq 100 \text{ GeV}$$

f_{cc} decouples

Regarding the role of the cosmological constant in determining the SUSY breaking scale, a key observation of Denef and Douglas [10, 11] and Susskind [9] was that W at the minima is distributed uniformly as a complex variable, and the distribution of $e^{K/m_P^2}|W|^2/m_P^2$ is not correlated with the distributions of F_i and D_α. Setting the cosmological constant to nearly zero, then, has no effect on the distribution of supersymmetry breaking scales. Physically, this can be understood by the fact that the superpotential receives contributions from many sectors of the theory, supersymmetric as well as non-supersymmetric. The cosmological fine-tuning penalty is $f_{cc} \sim \Lambda/m^4$ where the above discussion leads to $m^4 \sim m^4_{string}$ rather than $m^4 \sim m^4_{hidden}$, rendering this term inconsequential for determining the number of vacua with a given SUSY breaking scale.

Distribution of Vacua

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \; f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$



$$dN_{\Lambda=0}[F_i, D_\alpha] = \prod d^2F_i \; dD_\alpha \; \rho(F_i, D_\alpha)$$

$$f_{SUSY}(m^2_{hidden}) \sim (m^2_{hidden})^{2n_F + n_D - 1}$$

In words, the total supersymmetry breaking scale is the distance from the origin in the space of supersymmetry breaking parameters, and in a high dimensional space most of the volume is near the boundary. Essentially the same observation

- more complicated regimes or multi-modulus compactifications?

Caveats

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

$$f_{SUSY}(m^2_{hidden}) \sim (m^2_{hidden})^{2n_F + n_D - 1}$$

- enhancement of number of vacua near conifold points: should skew vacua, but still vast majority of vacua do not produce exponentially small scales

- multiple F and D terms will skew distribution to high scales

- partially supersymmetric sectors may skew the distribution to low scales

Douglas, hep-th/0405279

Summary

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

- $f_{cc} \sim \Lambda/m^4$ where DD maintain $m \sim m_{string}$ and not m_{hidden}
- $f_{SUSY}(m^2_{hidden}) \sim (m^2_{hidden})^{2n_F+n_D-1}$ for uniformly distributed values of F and D breaking fields
- $f_{EWFT} \sim m^2_{weak}/m^2_{soft}$ (?) where $m_{soft} \sim m_{3/2} \sim m^2_{hidden}/m_P$

$$n = 2n_F + n_D - 1$$

$$f_{SUSY} \sim m^n_{soft}$$

n_F	n_D	n
0	1	0
1	0	1
0	2	1
1	1	2
0	3	2
2	0	3
2	1	4

landscape favors high-scale SUSY breaking tempered by electroweak fine-tuning penalty

EWFT Penalty

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

$$f_{EWFT} = ?$$

An effective field theory (or specific coupling or observable) T_1 is more natural in string theory than T_2 if the number of *phenomenologically acceptable* vacua leading to T_1 is larger than the number leading to T_2.

Douglas, 2012

anthropics lies here

Question: can we quantify these ideas into what the LHC is seeing?

EWFT Penalty

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

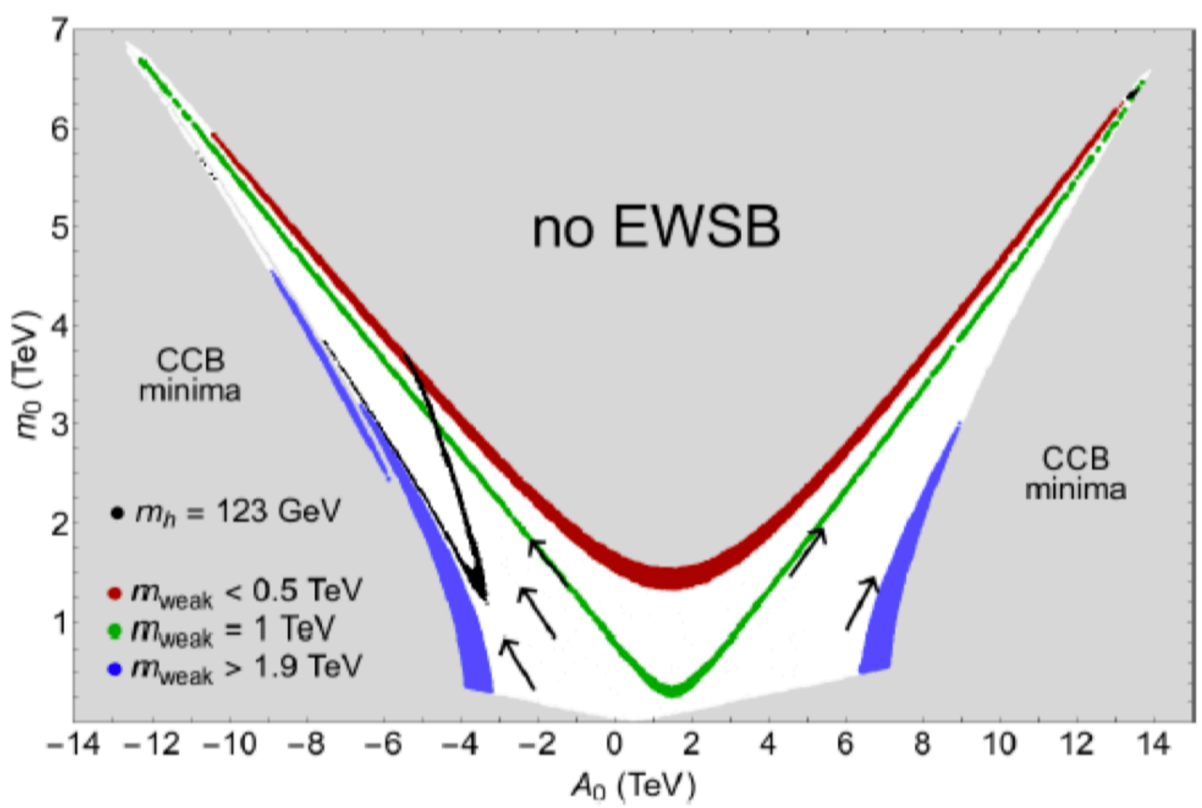
The final term f_{EWFT} merits some discussion. Following Ref. [17], an initial guess [9, 11, 13] for f_{EWFT} was that $f_{EWFT} \sim m^2_{weak}/m^2_{soft}$ which may be interpretted as conventional naturalness in that the larger the Little Hierarchy between m_{weak} and m_{soft}, then the greater is the fine-tuning penalty. As pointed out in Ref. [18], there are several problems with this ansatz.

$$f_{EWFT} \sim m^2_{weak}/m^2_{soft} \; ?$$

This particular term needs to be treated with care

EWFT Penalty

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$



$$f_{EWFT} \sim m^2_{weak}/m^2_{soft}\ ?$$

As soft terms such as the trilinear A_t terms increase, one is ultimately forced into charge-or-color-breaking vacua of the MSSM [19, 20]. These sorts of vacua must be entirely vetoed on anthropic grounds.

As high-scale soft terms such as $m^2_{H_u}$ increase too much, then they are no longer driven to negative values and electroweak symmetry isn't even broken. These non-EWSB solutions also should be vetoed on anthropic grounds.

Fine-tuning Penalty

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

$$f_{EWFT} \sim m^2_{weak}/m^2_{soft}$$

Even in the event of appropriate EWSB, the factor $f_{EWFT} \sim m^2_{weak}/m^2_{soft}$ penalizes but does not forbid vacua with a weak scale far larger than its measured value. In contrast, Agrawal *et al.* [7] have shown that a weak scale larger than ~ 3 times its measured value would lead to much weaker weak interactions and a disruption in nuclear synthesis reactions, and likely an unlivable universe as we know it. In addition, Susskind posits that an increased weak scale would lead to larger SM particle masses and consequent disruptions in both atomic and nuclear physics. From these calculations, it seems reasonable to *veto* SM-like vacua which lead to a weak scale more than (conservatively) four times its measured value.

Agrawal, Barr, Donoghue, Seckel (1998)

Donoghue Penalty

Minimization of MSSM Higgs potential

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2.$$

Radiative corrections from (s)particles with Yukawa/gauge coupling to Higgs

$$\Sigma_u^u(\tilde{t}_{1,2}) = \frac{3}{16\pi^2} F(m_{\tilde{t}_{1,2}}^2) \left[f_t^2 - g_Z^2 \mp \frac{f_t^2 A_t^2 - 8g_Z^2(\frac{1}{4} - \frac{2}{3}x_W)\Delta_t}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2} \right]$$

$$\Delta_{\rm EW} \equiv max_i\,(C_i)\,/(M_Z^2/2)$$

where $C_{H_u} = |-m_{H_u}^2 \tan^2\beta/(\tan^2\beta - 1)|/$, $C_{H_d} = |m_{H_d}^2/(\tan^2\beta - 1)|/$ and $C_\mu = |-\mu^2|$, along with analogous definitions for $C_{\Sigma_u^u(k)}$ and $C_{\Sigma_d^d(k)}$. Low $\Delta_{\rm EW}$ means less fine-tuning.

Donoghue Penalty

$$dN_{vac}[m^2_{hidden}, m_{weak}, \Lambda] = f_{SUSY}(m^2_{hidden}) \cdot f_{EWFT} \cdot f_{cc} \cdot dm^2_{hidden}$$

$$case\ \mathbf{A}: \quad f_{EWFT} \to \Theta(30 - \Delta_{\rm EW})$$

Σ_u^u generate large values of the weak scale $m_{weak} \gg 100$ GeV. The value of $\Delta_{\rm EW} < 30$ then corresponds to calculated anthropic requirements from Agrawal *et al.* that the weak scale not deviate by more than a factor of several from its measured value [33]. In this case, $\Delta_{\rm EW} = 30$ corresponds to a Z mass nearly four times its measured value.

$$case\ \mathbf{B}: \quad f_{EWFT} \to \Delta^{-1}_{\rm EW}$$

$$\Delta^{-1}_{\rm EW} \sim (m_Z^2/2)/max\ [|m^2_{H_u}(weak)|\ or\ \mu^2\ \ or\ \ |\Sigma_u^u(i)|]$$

Instead of placing a generic m^2_{soft} in the denominator of Eq. 11, we place the maximal weak scale contribution to the magnitude of the weak scale. Rather than placing a sharp cutoff on

m^n_{soft} Scan Results

$$m_0(1,2),\ m_0(3),\ m_{1/2},\ A_0,\ \tan\beta,\ \mu,\ m_A \quad \text{(NUHM3)}$$

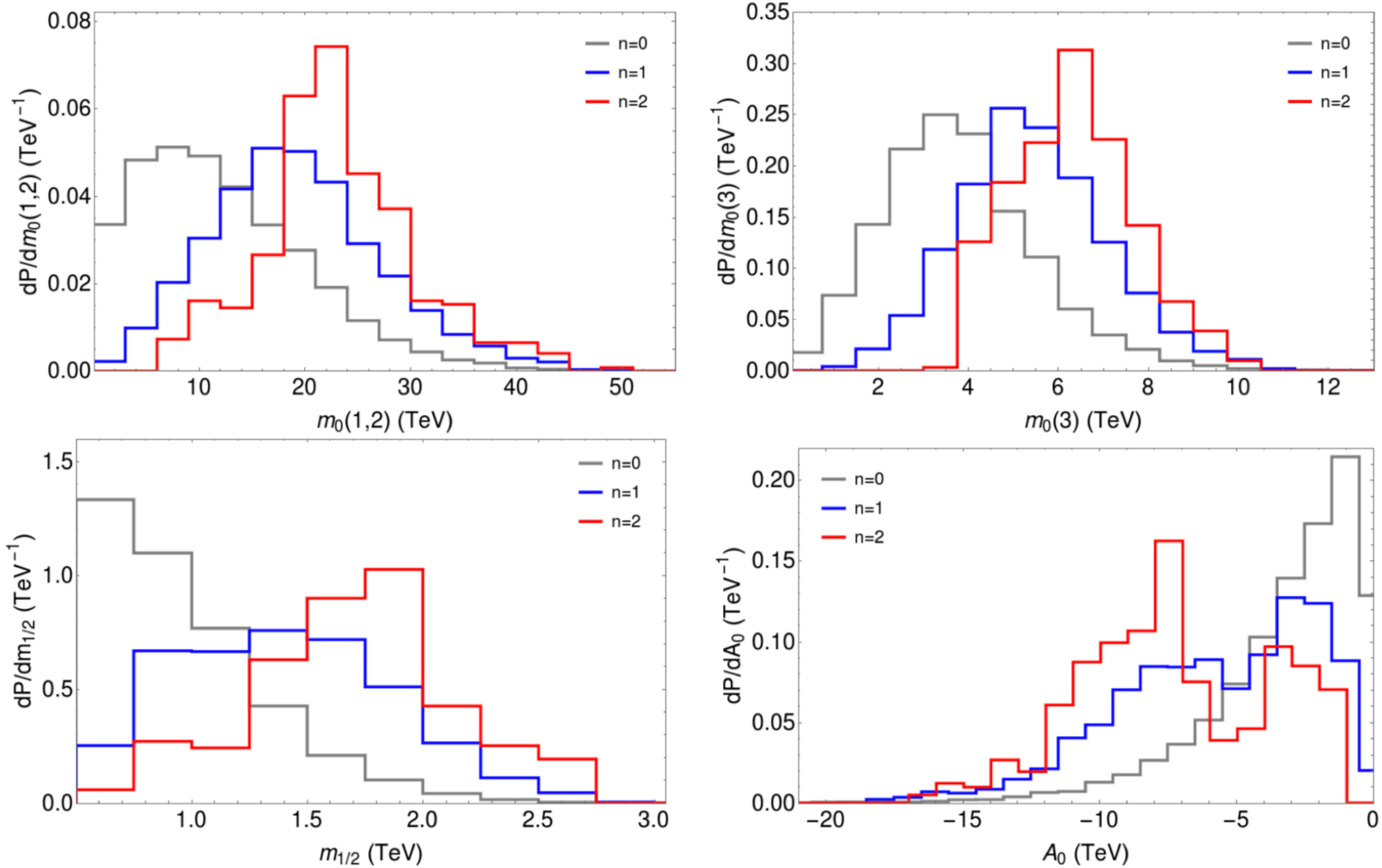
- $m_0(1,2):\ 0.1-60$ TeV,
- $m_0(3):\ 0.1-20$ TeV,
- $m_{1/2}:\ 0.5-10$ TeV,
- $A_0:\ -50-\ 0$ TeV,
- $m_A:\ 0.3-10$ TeV,

$\mu = 150$ GeV

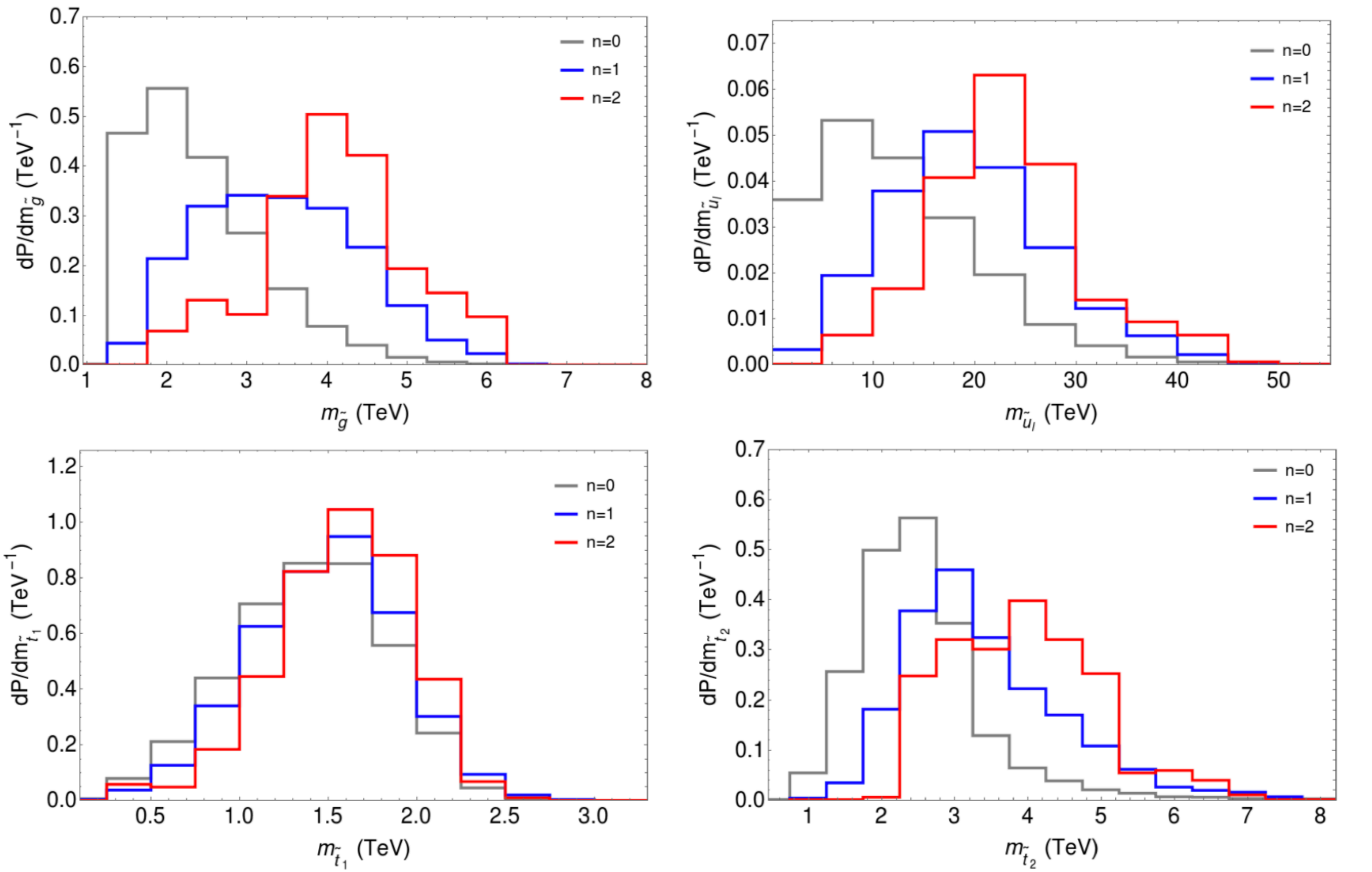
$\tan\beta$: $3-60$ scanned uniformly.

neutral (n=0), linear (n=1), quadratic (n=2) scan

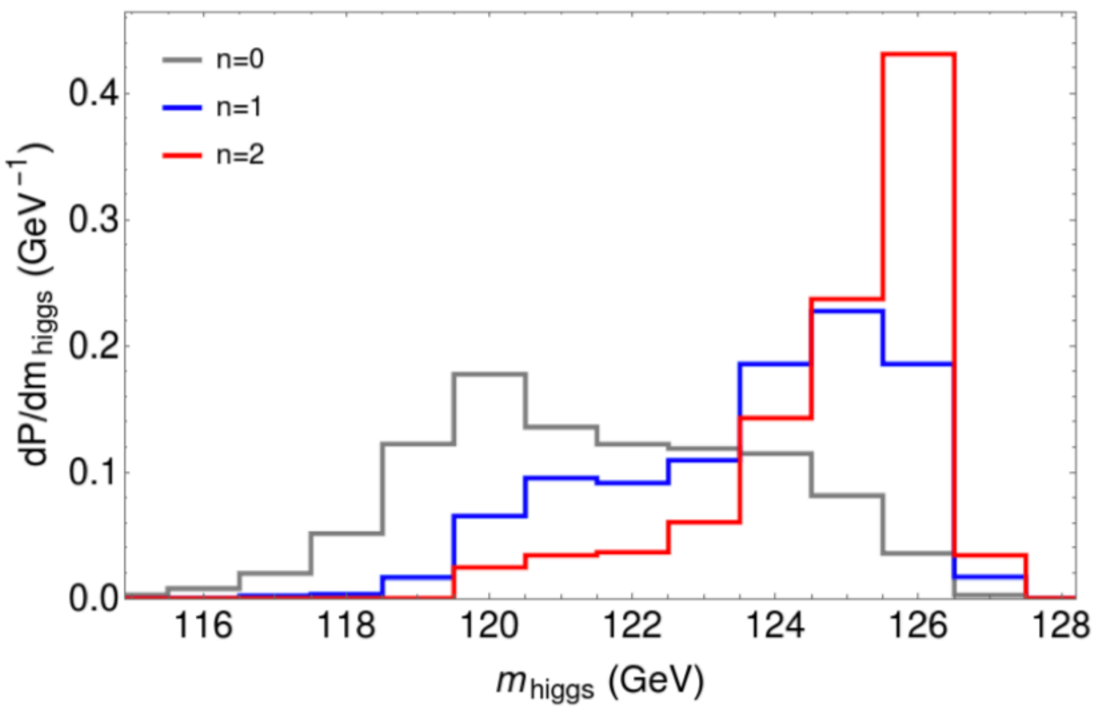
m_{soft}^{n} Scan Results

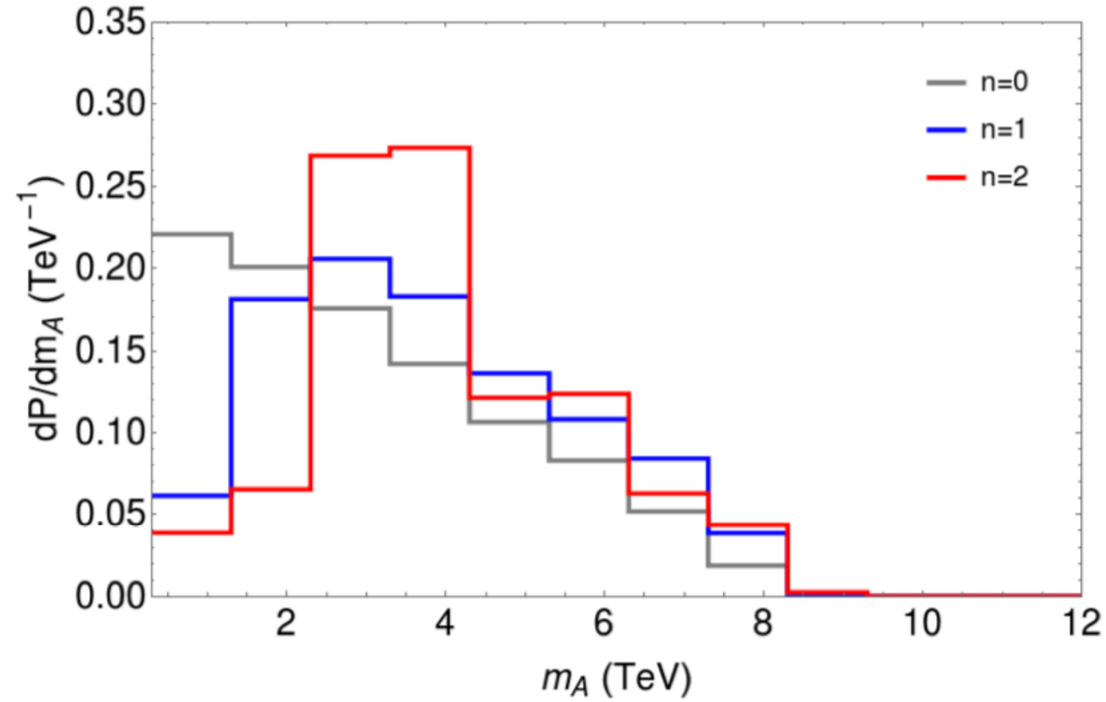


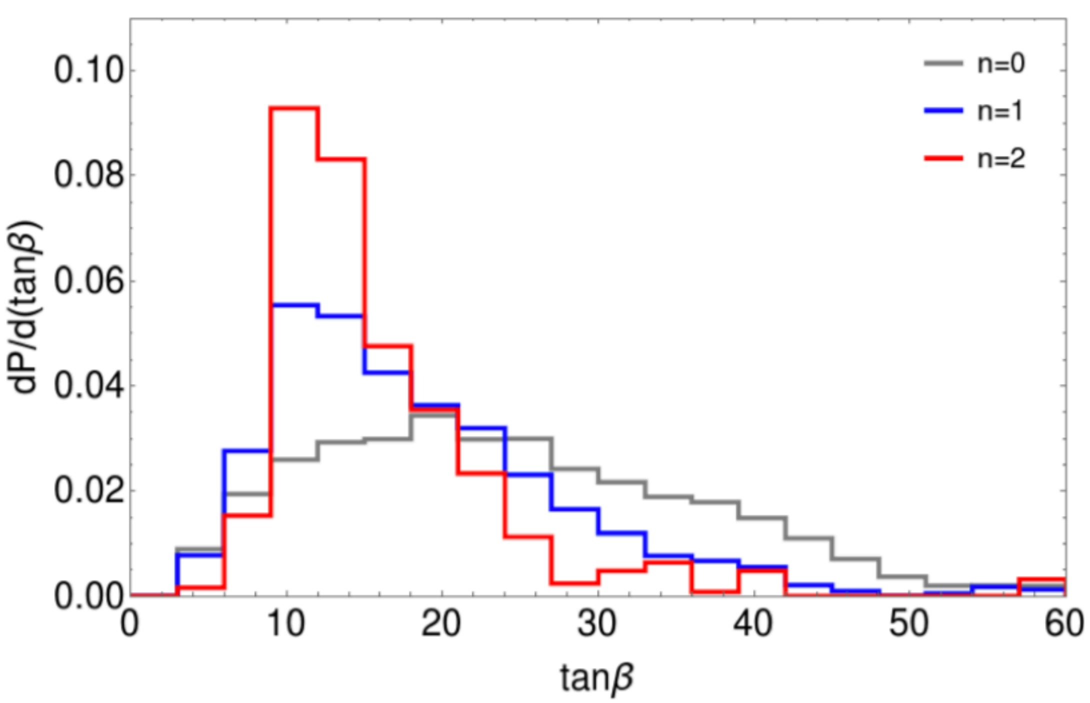
m_{soft}^{n} Scan Results

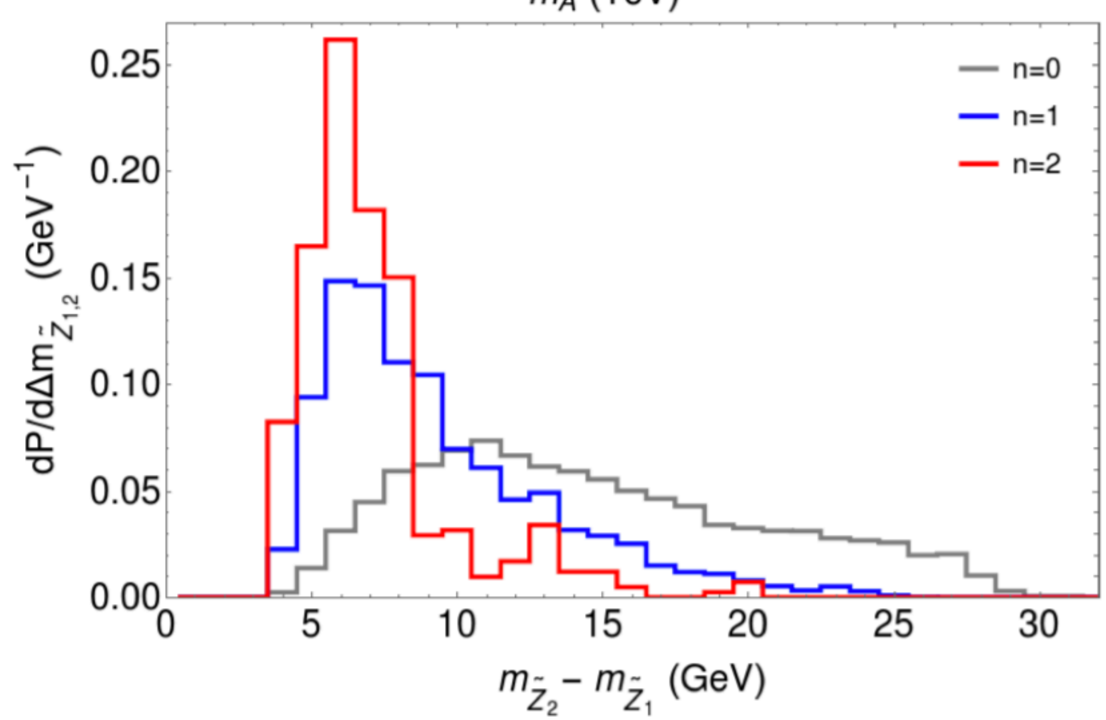


m_{soft}^n Scan Results









m^n_{soft} Scan Results

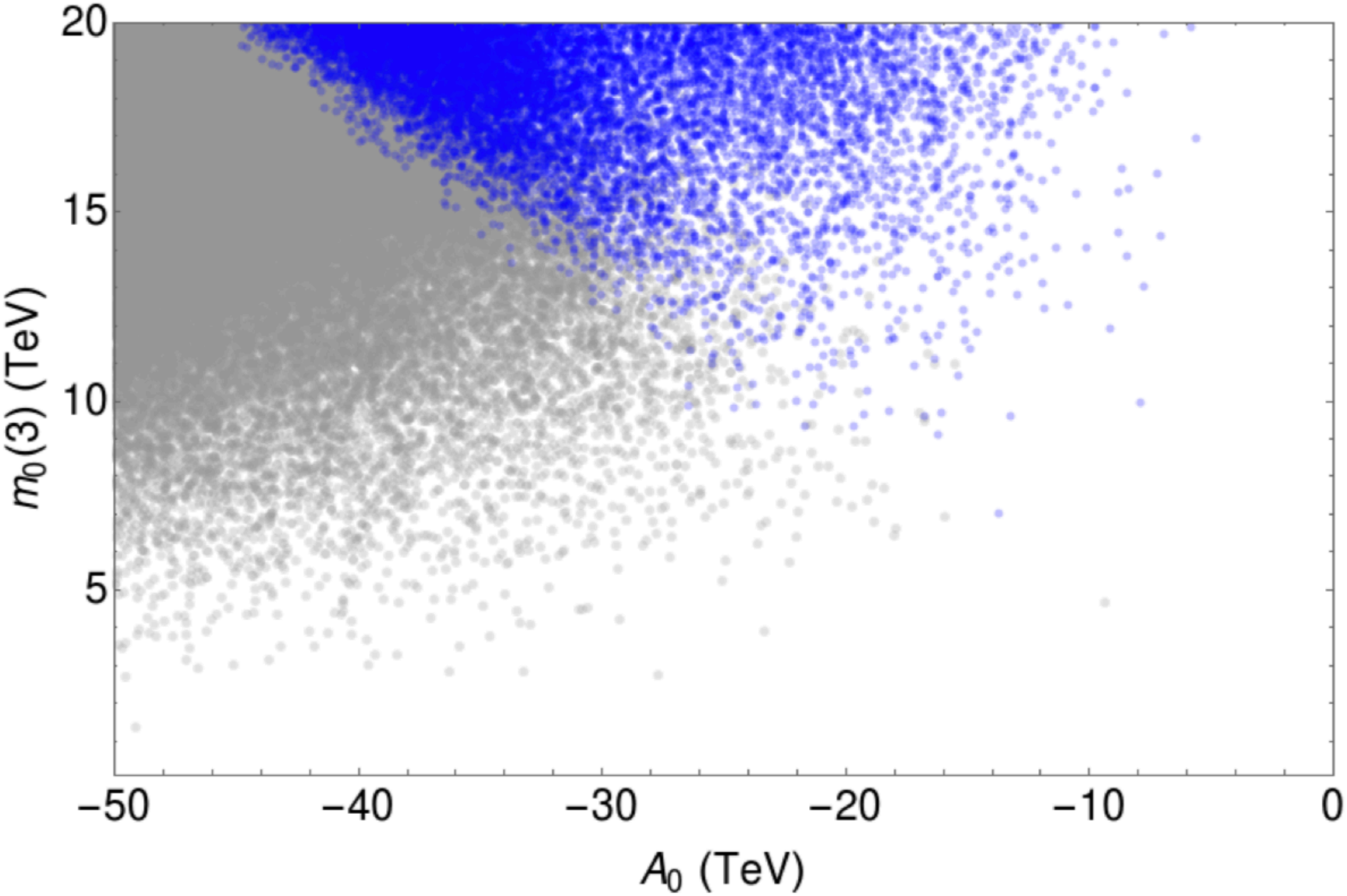


Figure 5: Locus of 100K scan points from a scan with $n = 4$ and scan range as below Eq. 15. The gray points have either CCB scalar potential minima or no EWSB. The blue points admit EWSB but all have $\Delta_{\rm EW} > 240$ corresponding to a weak scale greater than ~ 1 TeV.

m^n_{soft} Scan Results

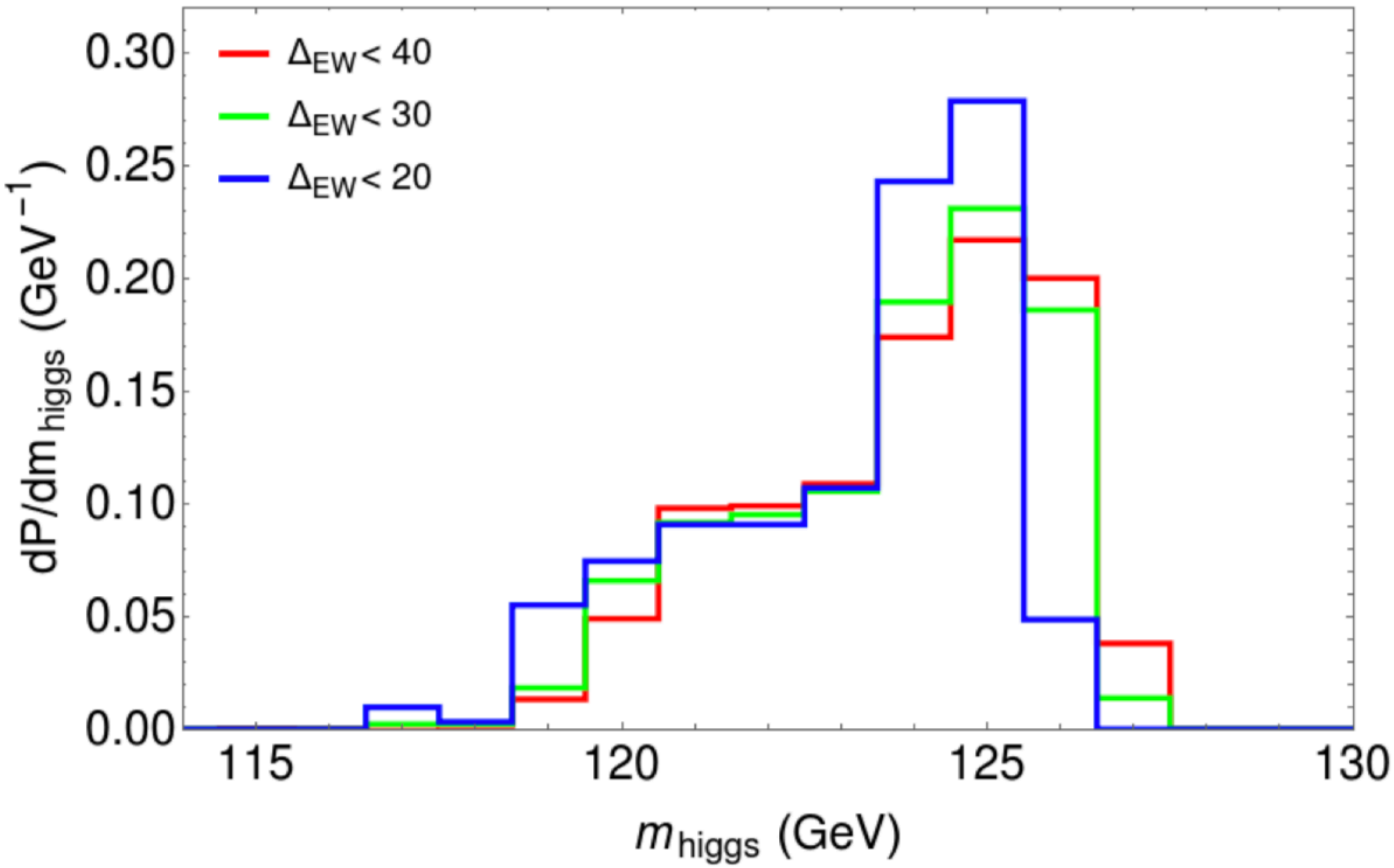


Figure 6: Probability distribution for Higgs mass m_h for the case of $n = 1$ but with varying cutoff $\Delta_{\mathrm{EW}} < 20$, 30 and 40.

m^n_{soft} Scan Results

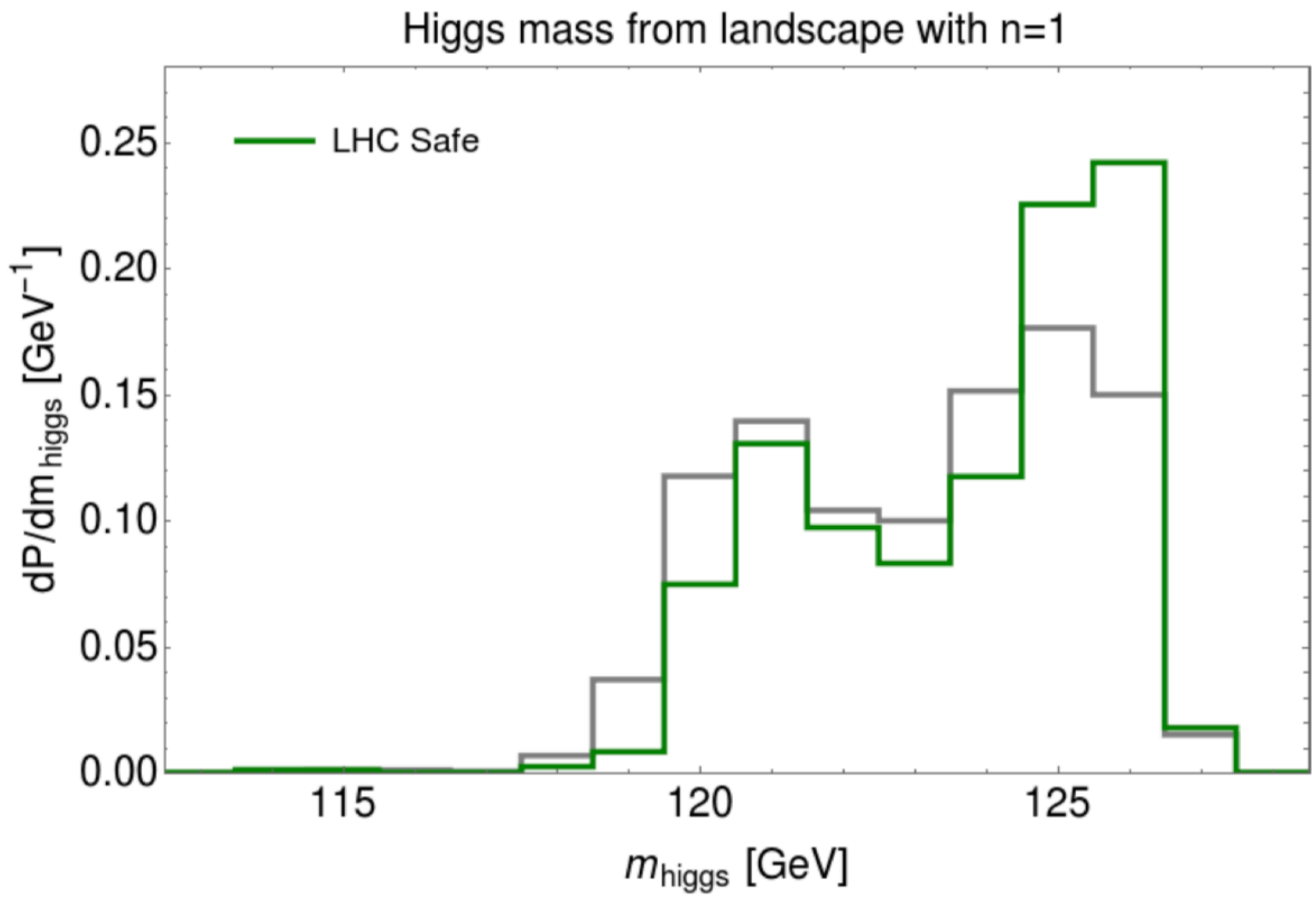


Figure 2: Statistical expectation for the mass of the Higgs boson from the string theory landscape which scans over single F-term SUSY breaking. The green histogram includes only LHC Run 2 safe points.

More Penalties?

- inflation and SUSY breaking scales correlated in string theory

Kallosh/Linde (2004)

- can lower inflationary scale, and sequester

Cicoli (2016)

- still difficult to obtain TeV scale SUSY and be OK with density perturbations

- perhaps penalty biasing high-scale SUSY?

Moduli and Non-thermal Dark Matter

$$m_{modulus} \sim m_{3/2}$$

$$\text{moduli decay around } t \sim M_P^2/m_{modulus}^3$$

$$\sim 10^3 \text{ s for } m_{modulus} \sim 1 \text{ TeV}$$

$$T_r = c^{1/2} \left(\frac{10.75}{g_*}\right)^{1/4} \left(\frac{m_{modulus}}{50 \text{ TeV}}\right)^{3/2} T_{BBN}$$

$$\Gamma = \frac{c}{2\pi} \frac{m_{modulus}^3}{M_P^2}$$

$$dN_{vac} \sim \Theta(m_{modulus} - 50 \text{ TeV}) \times f_{EWFT} \times (m_{hidden}^2)^n d(m_{hidden}^2)$$

Summary

LHC data suggests we should revisit these old arguments

- penalty from cosmological constant tuning decouples: is this generically true?

- is the democracy of F-terms suggested by Douglas/Denef generically correct?

- how about alternatives to weak scale anthropic penalty?

Backup

Practical Naturalness

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2$$

$$\Delta_{EW} \equiv max_i \left|C_i\right| / (m_Z^2/2)$$

The main requirements for low fine-tuning ($\Delta_{EW} \lesssim 30$[1]) are the following.

- $|\mu| \sim 100 - 350$ GeV[23–27] (where $\mu \gtrsim 100$ GeV is required to accommodate LEP2 limits from chargino pair production searches).
- $m_{H_u}^2$ is driven radiatively to small, and not large, negative values at the weak scale [21, 28].
- The top squark contributions to the radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ are minimized for TeV-scale highly mixed top squarks[28]. This latter condition also lifts the Higgs mass to $m_h \sim 125$ GeV. For $\Delta_{EW} \lesssim 30$, the lighter top squarks are bounded by $m_{\tilde{t}_1} \lesssim 3$ TeV.
- The gluino mass, which feeds into the $\Sigma_u^u(\tilde{t}_{1,2})$ via renormalization group contributions to the stop masses[27], is required to be $m_{\tilde{g}} \lesssim 6$ TeV, possibly beyond the reach of the $\sqrt{s} = 13 - 14$ TeV LHC.[2]
- First and second generation squark and slepton masses may range as high as 5-30 TeV with little cost to naturalness[21, 22, 29, 30].

HB, Barger, Huang,Mustafayev, Tata

Predicted Spectrum

- $m_{\tilde{g}} \sim 4 \pm 2$ TeV,
- $m_{\tilde{t}_1} \sim 1.5 \pm 0.5$ TeV,
- $m_A \sim 3 \pm 2$ TeV,
- $\tan\beta \sim 13 \pm 7$,
- $m_{\widetilde{W}_1, \tilde{Z}_{1,2}} \sim 200 \pm 100$ GeV and
- $m_{\tilde{Z}_2} - m_{\tilde{Z}_1} \sim 7 \pm 3$ GeV with
- $m(\tilde{q}, \tilde{\ell}) \sim 20 \pm 10$ TeV (for first/second generation matter scalars).

Ensembles of Flux Vacua

$$W = \int_M G \wedge \Omega(z)$$

- Type IIb string theory (Gukov Vafa Witten superpotential)
- Omega: holomorphic three form on CY
- G: sum of NS and RR three-form gauge field strengths

$$G = F^{RR} - \tau H^{NS}$$

- flux quanta: start with a general F-theory compactification on an elliptically fibered CY 4-fold:

$$W = \int_X G_4 \wedge \Omega = N^\alpha \Pi_\alpha$$

$\Pi_\alpha = \int \Sigma_\alpha \wedge \Omega$ are the periods in some basis $\{\Sigma_\alpha\}$ of $H^4(X, \mathbb{Z})$

Douglas and Denef, 2004: Distributions of Flux Vacue hep-th/0404116

NUHM3

boundary conditions for the soft terms. The NUHM3 model is convenient in that it allows for μ as an independent input parameter, and since we require μ not too far from $m_{W,Z,h} \sim 100$ GeV. The NUHM3 model is inspired by previous work on mini-landscape investigations of heterotic string theory compactified on a $Z_6 - II$ orbifold [49]. In these models, sparticle masses are dictated by the geography of their wavefunctions within the compactified manifold. These models exhibit *localized* grand unification [50] wherein the first/second generation matter superfields lie near fixed points (the twisted sector) and thus lie in **16**-dimensional spinor reps of SO(10). Meanwhile, third generations fields and Higgs and vector boson multiplets lie more in the bulk and thus occur in split multiplets (solving the doublet-triplet splitting problem) and receive smaller soft masses [51]. Such a set-up motivates the NUHM3 model with the following parameters $m_0(1,2)$, $m_0(3)$, $m_{1/2}$, A_0, $\tan\beta$, m_{H_u}, m_{H_d} where all mass parameters are taken

m0 Distribution

the vicinity of $m_0(1,2) \sim 20$ TeV with tails extending out to 30 TeV. Such large scalar masses occur because of the linear ($n = 1$) and quadratic ($n = 2$) pull on these soft terms with only minimal suppression which sets in at $m_0(1,2) \gtrsim 20$ TeV. One avenue for suppression arises from electroweak D-term contributions to the $\Sigma_{u,d}^{u,d}$ terms which depend on weak isospin and electric charge assignments. For nearly degenerate scalars of each generation, these nearly cancel out [52]. Another avenue for suppression comes from two loop terms in the MSSM RGEs [53]: if scalar masses enter the multi-TeV range, then these terms can become large and help drive third generation scalar masses tachyonic leading to CCB minima in the scalar potential [54]. Both these rather mild suppressions are insufficient to prevent first/second generation scalar masses from rising to the 20-30 TeV range. Such heavy scalars go a long way to suppressing possible FCNC and CP violating SUSY processes [21]. For the $n = 0$ case, $dP/dm_0(1,2)$ peaks around 5-10 TeV before suffering a drop-off.

3rd Generation

around 10 TeV for $n = 1$ and 12 TeV for $n = 2$. Large values of $m_0(3)$ generate large stop masses which result in $\Sigma_u^u(\tilde{t}_{1,2})$ exceeding ~ 30 *i.e.* generating a weak scale typically in excess of $m(weak) \sim 400$ GeV. For $n = 0$, the distribution peaks around 3 TeV.

mhalf Distribution

In frame c), we plot the distribution in $m_{1/2}$. In this case, the $n = 1$ distribution peaks around 1.5 TeV whilst $n = 2$ peaks slightly higher. If the (unified) gaugino masses become too big, then the large gluino mass also lifts the top squarks to higher masses thus causing the $\Sigma_u^u(\tilde{t}_{1,2})$ to again become too large. The distributions fall to near zero by $m_{1/2} \sim 3$ TeV leading to upper limits on gaugino masses. The $n = 0$ distribution actually peaks at its lowest allowed

A0 distribution

In frame d), we show the distribution versus A_0. Here we only show the more lucrative negative A_0 case which leads to higher Higgs masses m_h [46]. The $n = 0$ distribution peaks at $A_0 \sim 0$ with a steady fall-off at large negative A_0 values. In this case, the typically small mixing in the stop sector leads to values of m_h below its measured result. In contrast, for $n = 1, 2$ the distributions increase (according to the statistical pull) to peak values around $A_0 \sim -(5 - 10)$ TeV. Such large A_0 values lead to large mixing in the top-squark sector which can enhance m_h whilst decreasing the $\Sigma_u^u(\tilde{t}_{1,2})$ values [14]. The $n = 1$ curve actually features a double bump structure: we have traced the lower peak to the presence of large $m_A \sim m_{H_d} \sim 5 - 10$ TeV values which increase the S term in the third generation matter scalar RGEs. This term (along with large two-loop effects from first/second generation matter scalars) acts to suppress $m_{U_3}^2$ leading to lighter $\tilde{t}_1$ states even without large mixing. For even larger negative A_0 values, the distributions rapidly fall to zero since they start generating CCB minima in the MSSM scalar potential.

Higgs Distribution

to little mixing in the stop sector and hence too light values of m_h. Taking $n = 1$, instead we now see that the distribution in m_h peaks at ~ 125 GeV with the bulk of probability between 123 GeV $< m_h <$127 GeV– in solid agreement with the measured value of $m_h = 125.09 \pm 0.24$ GeV [55].[5] This may not be surprising since the landscape is pulling the various soft terms towards large values including large mixing in the Higgs sector which lifts up m_h into the 125 GeV range. By requiring the $\Sigma_u^u(\tilde{t}_{1,2}) \lesssim 30$ (which would otherwise yield a weak scale in excess of 350 GeV) then too large of Higgs masses are vetoed. For the $n = 2$ case with a stronger draw towards large soft terms, the m_h distribution hardens with a peak at $m_h \sim 126$ GeV.

mA, tanb Distribution

In Fig. 3b), we show the distribution in pseudoscalar mass m_A. Here, for $m_A \gg m_h$, then $m_A \sim m_{H_d}$ (at the weak scale) and we have a statistical draw to large m_A values which is tempered by the presence of $m_{H_d}/\tan\beta$ in Eq. 1. While the $n=0$ uniform draw peaks at the lowest m_A values, the $n=1$ and 2 cases yield a broad distribution peaking around $m_A \sim 3$ TeV which drops thereafter. In frame c), we show the distribution in $\tan\beta$. Here, the $n=0$ case has a broad distribution with a peak around $\tan\beta \sim 20$ while the $n=1$ and 2 cases have

sharper distributions peaking around $\tan\beta \sim 10-15$. The suppression of $\tan\beta$ for large values can be understood due to the draw towards large soft terms in the sbottom sector. As $\tan\beta$ increases, the b (and τ) Yukawa couplings increase so that the $\Sigma_u^u(\tilde{b}_{1,2})$ terms become large. Then the anthropic cutoff on $\Delta_{\rm EW} < 30$ disfavors the large $\tan\beta$ regime. In frame d), we show the $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ mass splitting. For our case with $\mu = 150$ GeV, the light higgsinos $\widetilde{W}_1^{\pm}$, $\widetilde{Z}_{1,2}$ all have masses around 150 GeV. The phenomenologically important mass gap $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ becomes smaller the more gauginos are decoupled from the higgsinos. The landscape draw towards large gaugino masses thus suppressed $m_{\tilde{Z}_2} - m_{\tilde{Z}_1}$ for the $n=1$ and 2 cases so that the mass gap peaks at around $5-8$ GeV. For the uniform scan with $n=0$, then the gap is larger– typically $10-20$ GeV.

Ensembles of Flux Vacua

$$W = \int_M G \wedge \Omega(z)$$

- Omega: holomorphic three form on CY
- G: sum of NS and RR three-form gauge field strengths

$$G = F^{RR} - \tau H^{NS}$$

- Weil-Petersson metric on complex str. moduli space
- an explicit statistical study with a single complex structure modulus

Douglas and Denef, 2004: Distributions of Flux Vacua hep-th/0404116