Holographic Heavy-Heavy-Light Three-Point Functions Revisited

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Based on papers with Peihe Yang, Yunfeng Jiang, Shota Komatsu, 2103.16580[hep-th]

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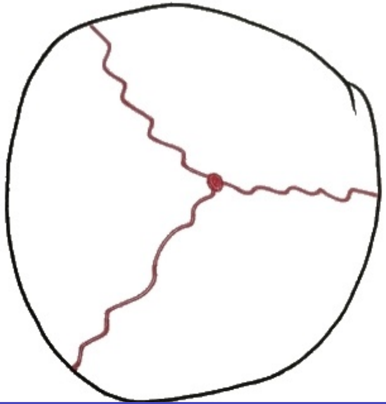
- Some non-BPS local operators with large conformal weights are dual to semi-classical string solutions. *[GKP, 02, for $\mathcal{N}=4$ SYM][Bin Chen, JW 08, for ABJM]*

- The three point function of single trace light operators (dual to supergravitons) are computed holographically using Witten diagrams. *[GKP, 98][Witten, 98]*.



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- In ABJM theory, the 3pt functions of $1/3$-BPS operators do not enjoy such a non-renormalization theorem *[Hirano, Kristjansen, Young, 12]*.
- Computation of such functions for most general case is still great challenge for supersymmetric localization and integrability method.

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- In planar limit, this problem is essentially solved by integrability. (Review: *[Beisert etal, 12]*)
- The holographic computation of the conformal weight is just compute the energy of the dual string solutions.

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- An example of 3pt functions was computed to show the prescription.
- Contributions from open string attached on such D-branes was also computed.

- Later on, more examples of 3pt HHL correlators for D-branes were computed, both for $\mathcal{N}=4$ SYM *[Bissi, Kristjansen, Young, Zoubos, 11]* and ABJM theories *[Hirano, Kristjansen, Young, 12]*.

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- Contributions from the wave functions of the heavy states.
- This two effects were studied in *[Bajnok, Janik, Wereszczynski, 14]* for semiclassical string cases. But their treatment seems incomplete.

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- It is interesting to get wrapping corrects at strong coupling from the holographic result.

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- Here by light, we mean the quantum numbers of $\mathcal{O}$ are small.

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and the path integral

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- Here we have already assume that $\langle\theta|\mathcal{O}|\theta'\rangle = \mathcal{O}[\theta]\delta(\theta-\theta')$.

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- Now, suppose we found one solution satisfying the equation (6), $\theta_0^*(t)$. Then, it immediately follows from the $U(1)$ invariance (1) that there should be a family of solutions, or equivalently a moduli of solutions, given by

$$\theta_c^*(t) \equiv \theta_0^*(t) + c\,, \qquad c \in [0, 2\pi]\,. \tag{7}$$

- Therefore, the correct saddle-point formula is given by

$$\langle J|\mathcal{O}(t=0)|J\rangle \overset{\text{WKB}}{=} \int_0^{2\pi} \frac{\mathrm{d}c}{2\pi}\, e^{-iJ\theta_c^*(+\epsilon)}\mathcal{O}[\theta_c^*(0)]e^{iJ\theta_c^*(-\epsilon)}e^{\frac{i}{\hbar}S[\theta_c^*]}\,. \tag{8}$$

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- As we can see, the final result is given by an average over the parameter c and this is precisely the orbit average discussed in [Bajnok, Janik, Wereszczynski, 14].

Boundary term

- Let us now generalize the computation slightly and consider the situation in which the bra and ket states are not identical: $\langle J+q|\mathcal{O}|J\rangle$. We assume J is again large $(J \sim 1/\hbar \gg 1)$ while q is taken to be $O(1)$.

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- The previous argument leads to

$$\langle J+q|\mathcal{O}(t=0)|J\rangle \overset{\text{WKB}}{=} e^{\frac{i}{\hbar}S[\theta_0^*]-iq\theta_0^*(0)} \int_0^{2\pi} \frac{\mathrm{d}c}{2\pi}\, e^{-iqc}\, \mathcal{O}[\theta_c^*(0)]\,. \quad (11)$$

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- Especially, for $\mathcal{O}$ being $\mathcal{O}_p \equiv e^{ip\theta}$, an operator with $U(1)$ charge p, we have

$$\langle J+q|\mathcal{O}_p(t=0)|\rangle \overset{\text{WKB}}{=} e^{\frac{i}{\hbar}S[\theta_0^*]}\delta_{p,q}, \tag{12}$$

where $\delta_{p,q}$ is manifestation of the $U(1)$ charge conservation.

Two lessons on boundary term

- First, when the bra and ket states are different, there is a nontrivial (boundary-term) contribution from the wave functions.
- Second, such contributions, together with the orbit average, are essential for reproducing a correct charge conservation $\delta_{p,q}$.

HHL $3-$point functions

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- The sub-determinant operator with charge M will be denoted by $\mathcal{D}_M$ and the single trace operator with charge L will be denoted by $\mathcal{O}_L$.
- Structure constant:

$$\langle\hat{\mathcal{D}}_{M+k}|\hat{\mathcal{O}}_L(t=0)|\hat{\mathcal{D}}_M\rangle = \int DX\,\Psi^*_{M+k}[X]\hat{\mathcal{O}}_L[X(t=0)]$$
$$\Psi_M[X]e^{-S_{\mathrm{DBI+WZ}}[X]}\,.$$

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- Shift in wave functions $\Psi \sim \exp(-i\Delta t + iJ\phi)$,

$$\Psi \mapsto e^{-\Delta\tau_0 + iJ\phi_0}\Psi. \tag{14}$$

Master equation

$$\langle\hat{\mathcal{D}}_{M+k}|\hat{\mathcal{O}}_L(t=0)|\hat{\mathcal{D}}_M\rangle = \underbrace{\int \mathrm{d}\tau_0 \int \frac{\mathrm{d}\phi_0}{2\pi}}_{\text{orbit average}} \hat{\mathcal{O}}_L[X^*_{\tau_0,\phi_0}(t=0)]$$

$$\underbrace{e^{(\Delta_{M+k}-\Delta_M)\tau_0} e^{-i(J_{M+k}-J_M)\phi_0}}_{\text{wave function}} . \tag{15}$$

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  as in the old approach *[Bak, Chen, JW, 11][Bissi, Kristjansen, Young, Zoubos, 11]*.
- *Remark:* The last step is similar to the holographic computations of correlators of BPS Wilson loops (surfaces) and local BPS operators *[Berenstein, Corrado, Fischler, Maldacena, 98][Giombi, Ricci, Trancanelli, 06][Chen, Liu, JW, 07]*

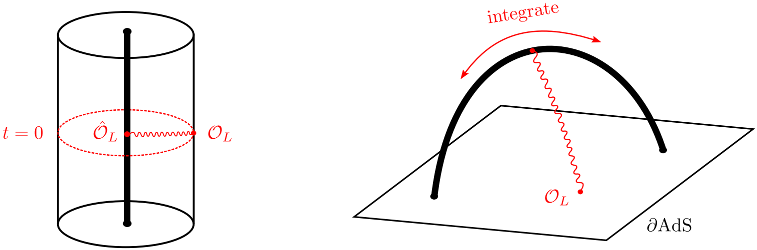

Figure: Comparison of new and old approaches.

Results for $\mathcal{N} = 4$ SYM

- Diagonal structure constant

$$C_{\mathcal{D}_M \mathcal{D}_M \mathcal{O}_L} = -\frac{i^L + (-i)^L}{2\sqrt{L}} \left(P_{\frac{L}{2}}(\cos 2\theta_0) + P_{\frac{L}{2}-1}(\cos 2\theta_0) \right) , \quad (17)$$

with $\cos^2\theta_0 = \frac{M}{N}$.

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- This perfectly matching the field theory results, as demanding from the non-renormalization theorem.

Results for $\mathcal{N} = 4$ SYM

- Diagonal structure constant

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- The holographic off-diagonal structure constant, with orbit average and contributions of wave functions included, matches the field theory results for non-extremal cases as well.

Results for $\mathcal{N} = 4$ SYM

- Off-diagonal structure constant,

$$
\begin{aligned}
& C_{\mathcal{D}_{M+k}\mathcal{D}_M\mathcal{O}_L} = \\
& -\frac{1}{2}\sqrt{L}\left(i^{L-k} + (-i)^{L-k}\right)\frac{\Gamma(\frac{L+k}{2})\cos^2\theta_0\sin^k\theta_0}{\Gamma(1+k)\Gamma(1+\frac{L-k}{2})} \\
& {}_2F_1\left(1+\tfrac{k-L}{2}, 1+\tfrac{k+L}{2}, 1+k; \sin^2\theta_0\right) \; .
\end{aligned}
$$

for $L > k$.

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  for $L > k$.
- This matches the field theory results for non-extremal cases as well.

Application to ABJM theory

- Diagonal structure constant,

$$
\begin{aligned}
C_{\mathcal{D}_M\mathcal{D}_M\mathcal{O}_L} &= \left(\frac{\lambda}{2\pi^2}\right)^{1/4}\frac{\sqrt{2L+1}}{L}(1+(-1)^L) \\
&\quad \frac{(-1)^{\frac{L}{2}+1}2^L\sqrt{\pi}\Gamma(\frac{L}{2}+1)}{\Gamma(\frac{L+3}{2})}(1-4\alpha^4)^{\frac{1}{2}(L-1)} \\
&\times \Big[(1-4\alpha^4)\,{}_2F_1\left(-\tfrac{1}{2}(L+1),-\tfrac{L}{2};1;\tfrac{4\alpha^4}{4\alpha^4-1}\right) \\
&\quad +2\alpha^4(L+1)\,{}_2F_1\left(-\tfrac{1}{2}(L-1),-\tfrac{L}{2}+1;2;\tfrac{4\alpha^4}{4\alpha^4-1}\right)\Big]\,. \qquad (18)
\end{aligned}
$$

with the relation among M, N and α is

$$
\frac{M}{N}=\sqrt{1-4\alpha^4}-4\alpha^4\,\log\left(\frac{1+\sqrt{1-4\alpha^4}}{2\alpha^2}\right)\,. \qquad (19)
$$

Application to ABJM theory

- The strong coupling results are different from the weakly coupling ones.
- This is as expected, since there are no non-renormalization theorems for BPS $3-$pt functions in ABJM theory.
- The result is to be tested against integrability.

Conclusion

- We compute HHL correlators from branes dual to subdeterminant operators, including orbit average and wave function contributions.

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- For ABJM theory, where there are no such non-renormalization theorems, the holographic computations provide a non-trival prediction for field theory computations at strong coupling.
- For off-diagonal case $\langle \mathcal{D}_{M+k} | \mathcal{O}_J | \mathcal{D}_M \rangle$, the holographic result is sensitive to k, though $k \leq M, N$.

Outlook

- Compute the HHL correlators at arbitrary coupling in planar limit using integrability.

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- Compute the HHL correlators at arbitrary coupling in planar limit using integrability.
- Revisit the holographic computations of HHL correlators for GKP strings.

Thanks for Your Attention !