Generalized Supersymmetric Pati-Salam Models from Intersecting D6-branes

Rui Sun

KIAS

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Outline

$\mathrm{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ Orientifolds with Intersecting D6-Branes

Palti Salam Model

- Constructing the $N = 1$ supersymmetric Standard Models (SM) or SM from string theories has been the essential goal of string phenomenology.
- D-branes as boundaries of open strings plays an important role in phenomenologically interesting model building in Type I, Type IIA and Type IIB string theories.
- Many non-supersymmetric three-family SM-like models and generalized unified models have been constructed, within the intersecting D6-brane models on Type IIA orientifolds.
- Along this direction, explicit models for the three-family $N = 1$ supersymmetric Pati-Salam models with Type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes have been systematically constructed.

Palti Salam Model

- The gauge symmetries all come from $U(n)$ branes, while the Pati-Salam gauge symmetries $SU(4)_C \times SU(2)_L \times SU(2)_R$ break down to $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further break down to the SM via four-dimensional $N = 1$ supersymmetry via Higgs mechanism.
- In these model building, there are also so-called hidden sector contain $USp(n)$ branes paralleling to the orientifold planes or their $\mathbb{Z}_2$ images.
- These models normally are constructed with at least two confining gauge groups in the hidden sector, for which the gaugino condensation triggers supersymmetry breaking and moduli stabilization. [Chen, Cvetic, Ibanez, Li, Mayes, Nanopoulos, Papadimitriou, Shiu ...]

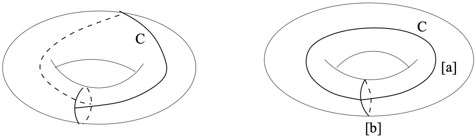

Table: The wrapping numbers for four O6-planes.

Orientifold Action	O6-Plane	$(n^1, l^1) \times (n^2, l^2) \times (n^3, l^3)$
ΩR	1	$(2^{\beta_1}, 0) \times (2^{\beta_2}, 0) \times (2^{\beta_3}, 0)$
$\Omega R\omega$	2	$(2^{\beta_1}, 0) \times (0, -2^{\beta_2}) \times (0, 2^{\beta_3})$
$\Omega R\theta\omega$	3	$(0, -2^{\beta_1}) \times (2^{\beta_2}, 0) \times (0, 2^{\beta_3})$
$\Omega R\theta$	4	$(0, -2^{\beta_1}) \times (0, 2^{\beta_2}) \times (2^{\beta_3}, 0)$

- In our Palti Salam Model searching, we observe that new models with three generations of particles can also be constructed when n_x^i and l_x^i are with common factor 3, while x refers to a, b, c stacks of branes and i refers to $1, 2, 3$ for different wrapping directions. This lead to our generalized Palti Salam Models.

Generalized Palti Salam Model

The N=1 Supersymmetric $SU(12)_C \times SU(2)_L \times SU(2)_R$, $SU(4)_C \times SU(6)_L \times SU(2)_R$, and $SU(4)_C \times SU(2)_L \times SU(6)_R$ Models from the Intersecting D6-Branes

- Divided by this co-factor 3 for the stack a of D-brane, the generalized gauge group resulting from the D6-branes becomes gauge symmetries $SU(12)_C \times SU(2)_L \times SU(2)_R$.
- In which the a-stack brane's gauge $U(12)$ can be broken down to $U(4)$ with proper orientations, i.e. by taking vacuum expectation values of an adjoint Higgs field with respect to the Cartan generators of U(12).
- Distinct from the random scanning methods, we explicit solving the conditions of generalized version of Pati-Salam models. We obtained the models from common solutions of the RR tadpole cancellation conditions, supersymmetry conditions, and three generation conditions.
- We for the first time systematically discuss the $N = 1$ supersymmetric $SU(12)_C \times SU(2)_L \times SU(2)_R$, $SU(4)_C \times SU(6)_L \times SU(2)_R$, and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models from the Type IIA orientifolds on $T^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ with intersecting D6-branes.

- These gauge symmetries can be broken down to the Pati-Salam gauge symmetry $SU(4)_C \times SU(2)_L \times SU(2)_R$ via three $SU(12)_C/SU(6)_L/SU(6)_R$ adjoint representation Higgs fields, and further down to the Standard Model (SM) via the D-brane splitting and Higgs mechanism.
- We obtain three families of the SM fermions, and have the left-handed three-family SM fermion unification in the $SU(4)_C \times SU(6)_L \times SU(2)_R$ models, and the right-handed three-family SM fermion unification in the $SU(4)_C \times SU(2)_L \times SU(6)_R$ models.
- Moreover, the $SU(4)_C \times SU(6)_L \times SU(2)_R$ models and $SU(4)_C \times SU(2)_L \times SU(6)_R$ models are related by the left and right gauge symmetry exchanging, as well as a variation of type II T-duality.
- Utilizing mathematical analysis, we exclude the generalized $SU(12)_C \times SU(2)_L \times SU(2)_R$ models by requiring the conditions for constructing MSSM models.

- Firstly, via Higgs mechanism we can break the generalized Pati-Salam gauge symmetry $U(12) \to U(4) \times U(4) \times U(4) \to U(4) \times U(4) \to U(4)$ and $U(6) \to U(2) \times U(2) \times U(2) \to U(2) \times U(2) \to U(2)$, with new massive bosons obtained in this procedure, and resulting in standard Pati-Salam gauge symmetry.
- The Pati-Salam gauge symmetry can then be broken down to the $SU(3)_C \times SU(2)_L \times U(1)_{B-L} \times U(1)_{I_{3R}}$ via D6-brane splittings, and further down to the SM gauge symmetry via the D- and F-flatness preserving Higgs mechanism in which Higgs fields are the massless open string states from a specific $N = 2$ subsector.
- The complete chains for symmetry breaking of our generalized Pati-Salam Models

$$\left.\begin{array}{r} SU(12) \times SU(2)_L \times SU(2)_R \\ SU(4) \times SU(6)_L \times SU(2)_R \\ SU(4) \times SU(2)_L \times SU(6)_R \end{array}\right\} \quad \xrightarrow[\text{Higgs Mechanism}]{} SU(4) \times SU(2)_L \times SU(2)_R$$
$$\xrightarrow[a \to a_1 + a_2]{} SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$
$$\xrightarrow[c \to c_1 + c_2]{} SU(3)_C \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$$
$$\xrightarrow[\text{Higgs Mechanism}]{} SU(3)_C \times SU(2)_L \times U(1)_Y \ . \qquad (1)$$

The Three Generations Conditions

- To have three families of the SM fermions, we require the intersection numbers to satisfy

$$\begin{aligned} I_{ab} + I_{ab'} &= 3 \,, \\ I_{ac} = -3 \,,\; I_{ac'} &= 0 \,, \end{aligned} \tag{2}$$

  where the conditions $I_{ab} + I_{ab'} = 3$ and $I_{ac} = -3$ give us three generations of the SM fermions, whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(2)_R$ for example with gauge symmetries are $(4, 2, 1)$ and $(\bar{4}, 1, 2)$ in our generalized construction.

- In our generalized construction, to have three families of the SM fermions for whose quantum numbers under $SU(4)_C \times SU(2)_L \times SU(6)_R$ for example with gauge symmetries are $(4, 2, 1)$ and $(\bar{4}, 1, 6)$, we require the intersection numbers to satisfy

$$\begin{aligned} I_{ab} + I_{ab'} &= 3 \,, \\ I_{ac} = -1 \,,\; I_{ac'} &= 0 \,, \end{aligned} \tag{3}$$

  where the conditions $I_{ab} + I_{ab'} = 3$ and $I_{ac} = -1$ give us three generations of the SM fermions.

- Similarly, to have three families of the SM fermions for whose quantum numbers under $SU(4)_C \times SU(6)_L \times SU(2)_R$ for example with gauge symmetries are $(4,6,1)$ and $(\overline{4},1,2)$, we require the intersection numbers to satisfy

$$\begin{aligned} I_{ab} + I_{ab'} &= 1 \,, \\ I_{ac} = -3 \,, \; I_{ac'} &= 0 \,, \end{aligned} \tag{4}$$

  where the conditions $I_{ab} + I_{ab'} = 1$ and $I_{ac} = -3$ give us three generations of the SM fermions.

- To have three families of the SM fermions for whose quantum numbers under $SU(12)_C \times SU(2)_L \times SU(2)_R$ for example with gauge symmetries are $(12,2,1)$ and $(\overline{12},1,2)$, we require the intersection numbers to satisfy

$$\begin{aligned} I_{ab} + I_{ab'} &= 1 \,, \\ I_{ac} = -1 \,, \; I_{ac'} &= 0 \,, \end{aligned} \tag{5}$$

  where the conditions $I_{ab} + I_{ab'} = 1$ and $I_{ac} = -1$ give us three generations of the SM fermions.

Table: General massless particle spectrum for intersecting D6-branes at generic angles.

Sector	**Representation**
aa	$U(N_a/2)$ vector multiplet 3 adjoint chiral multiplets
$ab+ba$	$I_{ab}\ (\square_a, \overline{\square}_b)$ fermions
$ab'+b'a$	$I_{ab'}\ (\square_a, \square_b)$ fermions
$aa'+a'a$	$\frac{1}{2}(I_{aa'} - \frac{1}{2} I_{a,O6})\ \square\!\square$ fermions $\frac{1}{2}(I_{aa'} + \frac{1}{2} I_{a,O6})\ \begin{smallmatrix}\square\\\square\end{smallmatrix}$ fermions

The RR Tadpole Cancellation Conditions

- The D6-branes and the orientifold O6-planes are the sources of RR fields and restricted by the Gauss law in a compact space. The sum of the RR charges from D6-branes must cancel with it from the O6-planes due to the conservations of the RR field flux lines. The conditions for RR tadpole cancellations take the form of

$$\sum_a N_a[\Pi_a] + \sum_a N_a\,[\Pi_{a'}] - 4[\Pi_{O6}] = 0\ , \tag{6}$$

  where the last term arises from the O6-planes are with -4 RR charges in D6-brane charge unit.

- To simplify the discussion of the following tadpole cancellation, we define the following products of wrapping numbers as

$$\begin{array}{cccc} A_a \equiv -n_a^1 n_a^2 n_a^3, & B_a \equiv n_a^1 l_a^2 l_a^3, & C_a \equiv l_a^1 n_a^2 l_a^3, & D_a \equiv l_a^1 l_a^2 n_a^3, \\ \tilde{A}_a \equiv -l_a^1 l_a^2 l_a^3, & \tilde{B}_a \equiv l_a^1 n_a^2 n_a^3, & \tilde{C}_a \equiv n_a^1 l_a^2 n_a^3, & \tilde{D}_a \equiv n_a^1 n_a^2 l_a^3. \end{array} \tag{7}$$

The RR Tadpole Cancellation Conditions

- In order to cancel the RR tadpoles, D6-branes wrapping cycles along the orientifold planes are introduced as the so-called "filler branes". This contributes to the RR tadpole cancellation conditions, and trivially satisfy the four-dimensional $N = 1$ supersymmetry conditions.
- They are chosen such that the tadpole conditions satisfied in the manner of

$$
\begin{aligned}
& -2^k N^{(1)} + \sum_a N_a A_a = -2^k N^{(2)} + \sum_a N_a B_a = \\
& -2^k N^{(3)} + \sum_a N_a C_a = -2^k N^{(4)} + \sum_a N_a D_a = -16,
\end{aligned} \tag{8}
$$

  where $2N^{(i)}$ is the number of filler branes wrapping along the i-th O6-plane.
- The filler branes representing the USp group, carry the wrapping numbers as one of the O6-planes shown in Table 1. The filler branes with non-zero A, B, C or D refer to the A-, B-, C- or D-type USp group, respectively.

SUSY conditions

The 4-dimensional $\mathrm{N} = 1$ supersymmetry will automatically survive the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold projection, and the SUSY conditions can therefore be written as

$$x_A \tilde{A}_a + x_B \tilde{B}_a + x_C \tilde{C}_a + x_D \tilde{D}_a = 0,$$

$$A_a/x_A + B_a/x_B + C_a/x_C + D_a/x_D < 0, \tag{9}$$

where $x_A = \lambda$, $x_B = \lambda 2^{\beta_2 + \beta 3}/\chi_2\chi_3$, $x_C = \lambda 2^{\beta_1 + \beta 3}/\chi_1\chi_3$, $x_D = \lambda 2^{\beta_1 + \beta 2}/\chi_1\chi_2$, in which $\chi_i = R_i^2/R_i^1$ represent the complex structure moduli for the i-th two-torus. Moreover, positive parameter λ are introduced to put all the variables A, B, C, D as equal footing.

Mathematical Search for Generalized Pati-Salam Models

In the SUSY equality condition Eq.(9), (x_A, x_B, x_C, x_D) is solution to the linear system

$$\begin{cases} x_A\tilde{A}_a + x_B\tilde{B}_a + x_C\tilde{C}_a + x_D\tilde{D}_a = 0, \\ x_A\tilde{A}_b + x_B\tilde{B}_b + x_C\tilde{C}_b + x_D\tilde{D}_b = 0, \\ x_A\tilde{A}_c + x_B\tilde{B}_c + x_C\tilde{C}_c + x_D\tilde{D}_c = 0. \end{cases} \tag{10}$$

Crámer's rule tells us, provided that the linear system has rank 3, (x_A, x_B, x_C, x_D) is proportional to (y_A, y_B, y_C, y_D) where

$$y_A = \begin{vmatrix} \tilde{B}_a & \tilde{C}_a & \tilde{D}_a \\ \tilde{B}_b & \tilde{C}_b & \tilde{D}_b \\ \tilde{B}_c & \tilde{C}_c & \tilde{D}_c \end{vmatrix}, y_B = -\begin{vmatrix} \tilde{A}_a & \tilde{C}_a & \tilde{D}_a \\ \tilde{A}_b & \tilde{C}_b & \tilde{D}_b \\ \tilde{A}_c & \tilde{C}_c & \tilde{D}_c \end{vmatrix}, y_C = \begin{vmatrix} \tilde{A}_a & \tilde{B}_a & \tilde{D}_a \\ \tilde{A}_b & \tilde{B}_b & \tilde{D}_b \\ \tilde{A}_c & \tilde{B}_c & \tilde{D}_c \end{vmatrix}, y_D = -\begin{vmatrix} \tilde{A}_a & \tilde{B}_a & \tilde{C}_a \\ \tilde{A}_b & \tilde{B}_b & \tilde{C}_b \\ \tilde{A}_c & \tilde{B}_c & \tilde{C}_c \end{vmatrix}.$$

More precisely, the solution of SUSY equality condition in terms of the linear system Eq.(10) can be solved by

$$\begin{cases} x_A = \lambda, \\ x_B = \lambda\, y_B/y_A, \\ x_C = \lambda\, y_C/y_A, \\ x_D = \lambda\, y_D/y_A. \end{cases} \tag{11}$$

Combing with the RR tadpole cancellation conditions, supersymmetry inequality conditions, and three generation conditions, we obtain the generalized Pati-Salam Models.

Table: D6-brane configurations and intersection numbers in Model 3, and its MSSM gauge coupling relation is $g_a^2 = 10g_b^2 = 2g_c^2 = \frac{10}{7}(\frac{5}{3}g_Y^2) = \frac{24\sqrt{3}}{5}\pi e^{\phi^4}$.

model 3	$U(4)\times U(2)_L\times U(6)_R\times USp(8)$								
stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box\\ \Box}}$	b	b'	c	c'	3
a	8	$(-1,-1)\times(-1,0)\times(1,-1)$	0	0	0	3	0	-1	0
b	4	$(-1,-1)\times(2,-1)\times(-1,-4)$	3	29	-	-	0	6	-1
c	12	$(0,-1)\times(2,-1)\times(-1,1)$	-1	1	-	-	-	-	0
3	8	$(0,-1)\times(1,0)\times(0,2)$	$x_A = 2x_B = \frac{1}{9}x_C = 2x_D$ $\beta_3^g = -5$ $\chi_1 = \frac{1}{3}, \chi_2 = 6, \chi_3 = \frac{2}{3}$						

Table: D6-brane configurations and intersection numbers in Model 4, and its MSSM gauge coupling relation is $g_a^2 = 2g_b^2 = 10g_c^2 = \frac{50}{23}(\frac{5}{3}g_Y^2) = \frac{24\sqrt{3}}{5}\pi e^{\phi^4}$.

model 4	$U(4)\times U(6)_L\times U(2)_R\times USp(8)$								
stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\Box\Box}$	$n_{\substack{\Box\\ \Box}}$	b	b'	c	c'	4
a	8	$(1,-1)\times(1,1)\times(1,0)$	0	0	0	1	-3	0	0
b	12	$(0,1)\times(-1,-1)\times(2,1)$	1	-1	-	-	-6	0	0
c	4	$(1,1)\times(-1,-4)\times(-2,1)$	3	29	-	-	-	-	-1
4	8	$(0,-1)\times(0,1)\times(2,0)$	$x_A = 2x_B = 2x_C = \frac{1}{9}x_D$ $\beta_4^g = -5$ $\chi_1 = \frac{1}{3}, \chi_2 = \frac{1}{3}, \chi_3 = 12$						

Table: D6-brane configurations and intersection numbers in Model 5, and its MSSM gauge coupling relation is $g_a^2 = 5g_b^2 = g_c^2 = \frac{5}{3}g_Y^2 = \frac{12\sqrt{6}}{5}\pi e^{\phi^4}$.

model 5	$U(4)\times U(2)_L \times U(6)_R \times USp(2) \times USp(6)$										
stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\substack{\square\\\square}}$	b	b'	c	c'	1	3	
a	8	$(-1,-1)\times(-1,0)\times(1,-1)$	0	0	0	3	0	-1	0	0	
b	4	$(-1,-1)\times(1,-1)\times(-1,-4)$	0	16	-	-	0	3	4	-1	
c	12	$(0,-1)\times(1,-1)\times(-1,1)$	0	0	-	-	-	-	-1	0	
1	2	$(1,0)\times(1,0)\times(2,0)$	$x_A = x_B = \frac{1}{9}x_C = x_D$								
3	6	$(0,-1)\times(1,0)\times(0,2)$	$\beta_1^g = -1,\ \beta_3^g = -5$								
			$\chi_1 = \frac{1}{3}, \chi_2 = 3, \chi_3 = \frac{2}{3}$								

Table: D6-brane configurations and intersection numbers in Model 6, and its MSSM gauge coupling relation is $g_a^2 = g_b^2 = 5g_c^2 = \frac{25}{13}(\frac{5}{3}g_Y^2) = \frac{12\sqrt{6}}{5}\pi e^{\phi^4}$.

model 6	$U(4)\times U(6)_L \times U(2)_R \times USp(2) \times USp(6)$										
stack	N	$(n^1,l^1)\times(n^2,l^2)\times(n^3,l^3)$	$n_{\square\square}$	$n_{\substack{\square\\\square}}$	b	b'	c	c'	1	4	
a	8	$(1,-1)\times(1,1)\times(1,0)$	0	0	0	1	-3	0	0	0	
b	12	$(0,1)\times(-1,-1)\times(1,1)$	0	0	-	-	-3	0	1	0	
c	4	$(1,1)\times(-1,-4)\times(-1,1)$	0	16	-	-	-	-	4	-1	
1	2	$(1,0)\times(1,0)\times(2,0)$	$x_A = x_B = x_C = \frac{1}{9}x_D$								
4	6	$(0,-1)\times(0,1)\times(2,0)$	$\beta_1^g = -1,\ \beta_4^g = -5$								
			$\chi_1 = \frac{1}{3}, \chi_2 = \frac{1}{3}, \chi_3 = 6$								

Conclusion & Outlook

- We generalized the construction of three-family $N=1$ supersymmetric Pati-Salam models from Type IIA orientifolds on $T^6/(\mathbb{Z}_2\times\mathbb{Z}_2)$ with intersecting D6-branes.
- We constructed models with their b and c stacks of D6-branes swapped from duality. The $U(1)_Y$ and $SU(4)_C$ gauge couplings can be possibly constructed closer to unification at the string scale through the swapping. With this swapping, strong and hypercharge gauge unification can be possibly shifted to strong and weak gauge unification.
- The upshot is that in the generalized Pati-Salam model building, we obtained the models by solving the common solutions of the RR tadpole cancellation conditions, supersymmetry conditions, and three generation conditions.
- It would be interesting to employ our mathematical analysis methods to search for other generalized Pati-Salam models, trinification models, $SU(5)$ models, and flipped $SU(5)\times U(1)_X$ models.