Higher-order constraints for $\mathcal{N} = 1$ and $\mathcal{N} = 2$ superfields, and non-linear supersymmetry

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Outline of the talk

- Constrained $\mathcal{N} = 1$ superfields: quadratic and cubic constraints
- Constrained $\mathcal{N} = 2$ superfields:
  - quadratic $\longrightarrow$ partial SUSY breaking
  - cubic $\longrightarrow$ total SUSY breaking
  - quintic $\longrightarrow$ total SUSY breaking

Based on:

YA, Chatrabhuti, Isono, EPJC 81 (2021) 6, 523 [arXiv:2103.11217]

YA, Antoniadis, Chatrabhuti, Isono [arXiv:21xx.xxxxx]

Constrained $\mathcal{N} = 1$ superfields

Nilpotent chiral superfield $\mathbf{S}^2 = 0$

Denote the components $\mathbf{S} = S + \sqrt{2}\theta\chi + \theta^2 F$.
Then from θ^2-component of $\mathbf{S}^2 = 0$ we have [Komargodski, Seiberg '09]

$$\boxed{S = \frac{\chi^2}{2F}} \tag{1}$$

- $F \neq 0 \;\Rightarrow\;$ SUSY necessarily broken
- $\chi =$ goldstino
- In the simplest case can be related to Volkov–Akulov theory [Volkov, Akulov '73] as shown by [Kuzenko, Tyler '11]

Constrained $\mathcal{N} = 1$ superfields

<u>Minimal example with $\mathbf{S}^2 = 0$</u>

$$\begin{aligned}
\mathcal{L} &= \int d^4\theta\, \mathbf{S}\bar{\mathbf{S}} + \mu \left(\int d^2\theta\, \mathbf{S} + \text{h.c.} \right) \\
&= \cancel{S\Box\bar{S}} - i\chi\sigma^m\partial_m\bar{\chi} + \mu(F + \bar{F}) + F\bar{F} \qquad (2)
\end{aligned}$$

$$\frac{\chi^2}{2F}\Box\frac{\bar{\chi}^2}{2\bar{F}}$$

After eliminating $F = -\mu - \frac{\bar{\chi}^2}{4\mu^3}\Box\chi^2 + \frac{3\chi^2\bar{\chi}^2}{16\mu^7}\Box\chi^2\Box\bar{\chi}^2$, the final Lagrangian becomes

$$\boxed{\mathcal{L} = -i\chi\sigma^m\partial_m\bar{\chi} - \mu^2 + \frac{\chi^2}{4\mu^2}\Box\bar{\chi}^2 - \frac{\chi^2\bar{\chi}^2}{16\mu^6}\Box\chi^2\Box\bar{\chi}^2} \qquad (3)$$

Pure goldstino Lagrangian with non-linear SUSY,

$$\delta_\epsilon\chi_\alpha = -\mu\epsilon_\alpha - i\mu^{-1}\sigma^m_{\alpha\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}}(\chi\partial_m\chi) + \mathcal{O}(\chi^2, \bar{\chi}^2)$$

Constrained $\mathcal{N} = 1$ superfields

Orthogonal nilpotent superfields

Can be used to eliminate components of a superfield [Komargodski, Seiberg '09]:

$$\mathbf{S}^2 = 0\,, \quad \mathbf{S}\mathbf{A} = 0$$

- leading component A is eliminated;
- the resulting $\mathbf{A}$ satisfies additional constraint $\mathbf{A}^3 = 0$.

- E.g. for chiral superfield $\mathbf{T}$ impose $\mathbf{S}(\mathbf{T} + \bar{\mathbf{T}}) = 0$
  - can describe light axion $\mathrm{Im}\,T$
  - $\mathrm{Re}\,T, \chi^T, F^T$ eliminated in terms of χ and F.

Constrained $\mathcal{N} = 1$ superfields

Cubic constraint

Instead of $\mathbf{S}^2 = 0$ consider a weaker constraint invariant under $S \to S + i\alpha$,

$$(\mathbf{S} + \bar{\mathbf{S}})^3 = 0 \tag{4}$$

Then $\mathbf{S}^2 = 0$ with $S = \frac{\chi^2}{2F}$ becomes a special case, but we find **more general solution**:

$\theta^2\bar{\theta}^2$-component of (4) reads (for $S = \phi + i\varphi$; $\varphi =$ axion)

$$(A + \phi\Box\phi)\,\phi = \tfrac{1}{2}B \tag{5}$$

$$\begin{aligned} A &\equiv 2|F|^2 - 2\partial_m\varphi\partial^m\varphi - i\chi\sigma^m\partial_m\bar{\chi} + i\partial_m\chi\sigma^m\bar{\chi} \\ B &\equiv \chi^2\bar{F} + \bar{\chi}^2 F + 2\chi\sigma^m\bar{\chi}\partial_m\varphi \end{aligned}$$

The solution eliminates ϕ (saxion):

$$\boxed{\phi = \frac{B}{2A} - \frac{B^2}{8A^4}\Box B} \quad \longrightarrow \quad A|_{\text{bos}} = 2|F|^2 - 2\partial\varphi\partial\varphi \neq 0 \tag{6}$$

• In [Kuzenko '17] nilpotent tensor (linear) superfield was studied, $L^3 = 0$, with deformed linearity constraint $\bar{D}^2 L = \mu$.

Constrained $\mathcal{N} = 1$ superfields

Minimal example with $(\mathbf{S} + \bar{\mathbf{S}})^3 = 0$

$$
\begin{aligned}
\mathcal{L} &= \int d^4\theta\, \mathbf{S}\bar{\mathbf{S}} + \mu \left(\int d^2\theta\, \mathbf{S} + \text{h.c.} \right) \\
&= \cancel{\phi \Box \phi} + \varphi \Box \varphi - i\chi\sigma^m \partial_m \bar{\chi} + \mu(F + \bar{F}) + F\bar{F}
\end{aligned}
\tag{7}
$$

$$
\left(\frac{B}{2A} - \frac{B^2}{8A^4} \Box B \right) \Box \left(\frac{B}{2A} - \frac{B^2}{8A^4} \Box B \right)
$$

Eliminate $F = -\mu +$ goldstino terms $\implies$

$$
\begin{aligned}
\mathcal{L} &= \varphi \Box \varphi - i\chi\sigma^m \partial_m \bar{\chi} - \mu^2 \\
&+ \frac{\mu\chi^2 + \mu\bar{\chi}^2 - 2\chi\sigma^m\bar{\chi}\partial_m\varphi}{8(\mu^2 - \partial\varphi\partial\varphi)^2} \left[\mu\partial_k\chi\partial^k\chi + \mu\partial_k\bar{\chi}\partial^k\bar{\chi} - 2\partial_k\varphi\partial_l\chi\sigma^k\partial^l\bar{\chi} \right] + \ldots
\end{aligned}
\tag{8}
$$

Now non-linear SUSY transformations also involve $\partial\varphi$,

$$
\delta_\epsilon \chi = \epsilon F - \sigma^m \bar{\epsilon} \partial_m \varphi + \frac{i\sigma^m \bar{\epsilon}}{2(|F|^2 - \partial\varphi\partial\varphi)} \left(\bar{F}\chi\partial_m\chi + \partial_n\varphi\chi\sigma^n\partial_m\bar{\chi} + \text{h.c.} \right) + \ldots
$$

Constrained $\mathcal{N} = 1$ superfields

Cubic vs. orthogonal nilpotent superfields

The same d.o.f. χ, φ with non-linear SUSY can be described by

- $(\mathbf{S} + \bar{\mathbf{S}})^3 = 0$ cutoff scale $(|F|^2 - \partial\varphi\partial\varphi)^{1/4}$;
- $\mathbf{C}^2 = \mathbf{C}(\mathbf{T} + \bar{\mathbf{T}}) = 0$ cutoff scale $|F|^{1/2}$;

If under $U(1)$: $\mathbf{T} \to \mathbf{T} + i\alpha$ and $\mathbf{C} \to \mathbf{C}e^{i\alpha}$, they can combine into $\mathbf{Q} \equiv \log(\mathbf{C} + e^{\mathbf{T}})$ which satisfies

$$(\mathbf{Q} + \bar{\mathbf{Q}})^3 = 0$$

This property can be used to set $\mathbf{Q} = \mathbf{S}$ and find non-linear field transformations

$$\mathrm{Im}S = \mathrm{Im}S(\mathrm{Im}T, \chi^C, F^C)$$
$$\chi^S = \chi^S(\mathrm{Im}T, \chi^C, F^C)$$
$$F^S = F^S(\mathrm{Im}T, \chi^C, F^C)$$

$*$ Full transformations can be found in our paper arXiv:2103.11217.
$*$ Implications for gravitino problem in mSUGRA inflation [Terada '21] (talk by Terada today)

$U(1)$ phase symmetry

- For (global) phase symmetry, $\mathbf{Z} \to \mathbf{Z}e^{i\alpha}$, redefine $\mathbf{S} = \log \mathbf{Z}$:

$$(\mathbf{S} + \bar{\mathbf{S}})^3 = (\log \mathbf{Z}\bar{\mathbf{Z}})^3 = 0 \tag{9}$$

- Assuming $\langle S \rangle = 0 \;\; \to \;\; \langle Z \rangle = 1$.
- Parametrize $Z = |Z| e^{i\zeta}$, $\zeta =$ axion, and the constraint should eliminate $|Z|$.
- Going back to (9) write $|\mathbf{Z}|^2 \equiv 1 + \mathbf{\Lambda} \;\; \Longrightarrow$

$$[\log(1 + \mathbf{\Lambda})]^3 = \mathbf{\Lambda}^3 \left(1 - \tfrac{1}{2}\mathbf{\Lambda} + \ldots\right)^3 = 0 \tag{10}$$

so the constraint (9) can be expressed as

$$\boxed{\mathbf{\Lambda}^3 \equiv (|\mathbf{Z}|^2 - 1)^3 = 0} \tag{11}$$

which has consistent solution $|Z| = 1 +$ goldstino terms, and ζ survives.

Constrained $\mathcal{N} = 2$ superfields

Constraint for partial breaking $\mathcal{N} = 2 \to \mathcal{N} = 1$

In $\mathcal{N} = 2$, (anti-)chiral superfields can be constructed from $\mathcal{N} = 1$ superfields.
E.g. from chiral Φ and vector V with $W_\alpha = -\frac{1}{4}\overline{D}^2 D_\alpha V$:

$$X = \Phi + \sqrt{2} i \tilde{\theta} W + \tilde{\theta}^2 \left(m - \frac{1}{4}\overline{D}^2 \overline{\Phi} \right) \tag{12}$$

Here $\overline{\tilde{D}}_{\dot\alpha} X = \overline{D}_{\dot\alpha} X = 0$.

- Consider the constraint $X^2 = 0$. Its solution

$$\boxed{\Phi = \frac{\frac{1}{2} W^2}{\frac{1}{4}\overline{D}^2 \overline{\Phi} - m}} \tag{13}$$

describes Born–Infeld theory (provided $m \neq 0$) with one linear and one non-linear SUSY:

$$\Longrightarrow \int d^2\theta \, \Phi + \text{h.c.} \sim 1 - \sqrt{-\det(\eta_{mn} + F_{mn})} \tag{14}$$

[Born, Infeld '34; Cecotti, Ferrara '87; Bagger, Galperin '96]

Constrained $\mathcal{N} = 2$ superfields

Cubic constraint for total breaking $\mathcal{N} = 2 \to \mathcal{N} = 0$

As shown in [Dudas, Ferrara, Sagnotti '17], a weaker constraint (DFS constraint)

$$X^3 = 0 \quad \Longrightarrow \quad \Phi W^2 - \Phi^2 \left(\frac{1}{4} \overline{D}^2 \overline{\Phi} - m \right) = 0 \tag{15}$$

has non-trivial solution other than $X^2 = 0$.
Denote $\Phi = \{\phi, \chi, F\}$ and $V = \{\lambda, A_m, D\}$; the highest component of (15) is

$$(\alpha + \phi \Box \bar{\phi})\phi = \beta \tag{16}$$

$$\alpha = D^2 + 2(\bar{F} + m)F - \tfrac{1}{2} F_{mn} F^{mn} - \tfrac{i}{2} F_{mn} \tilde{F}^{mn} - 2i\chi\sigma^m \partial_m \bar{\chi} - 2i\lambda\sigma^m \partial_m \bar{\lambda}$$
$$\beta = \chi^2(\bar{F} + m) + \lambda^2 F - \sqrt{2} i (\chi\lambda D + i\chi\sigma^{mn}\lambda F_{mn})$$

The solution has the form $\underline{\phi = \beta/\alpha + \dots}$, so that $\alpha \neq 0$, or at the vacuum

$$\langle \alpha \rangle = \langle D^2 + 2(\bar{F} + m)F \rangle \neq 0$$

breaking both supersymmetries because

$$\langle \delta_\epsilon \chi \rangle = \sqrt{2}\epsilon_1 \langle F \rangle + i\epsilon_2 \langle D \rangle \,, \quad \langle \delta_\epsilon \lambda \rangle = i\epsilon_1 \langle D \rangle + \sqrt{2} i \epsilon_2 \langle \bar{F} + m \rangle$$

- After the constraint is solved we are left with χ, λ, A_m + auxiliary.

Constrained $\mathcal{N} = 2$ superfields

Quintic constraint for total breaking $\mathcal{N} = 2 \to \mathcal{N} = 0$

There is yet another option to describe total SUSY breaking if we consider real superfield such as $X + \overline{X}$. To guess the form of the constraint, assume the scalar component is eliminated:

$$\phi + \bar{\phi} \sim \chi^2 + \lambda^2 + \chi\lambda + \ldots + \text{h.c.} \implies$$
$$(\phi + \bar{\phi})^2 \sim \chi\lambda\bar{\chi}\bar{\lambda} + \chi^2\lambda^2 + \bar{\chi}^2\bar{\lambda}^2 + \ldots\,, \quad (\phi + \bar{\phi})^4 \sim \chi^2\lambda^2\bar{\chi}^2\bar{\lambda}^2$$
$$(\phi + \bar{\phi})^5 = 0$$

Going back to X look for solutions to

$$(X + \overline{X})^5 = 0 \quad \implies \quad \Phi_+^3 \left(\tfrac{1}{8}\Phi_+\Box\Phi_+ + A\right) - 3\Phi_+^2 B + 3\Phi_+ W^2 \overline{W}^2 = 0 \qquad (17)$$

$$A = 2\left|m - \tfrac{1}{4}\overline{D}^2\overline{\Phi}\right|^2 + \tfrac{1}{2}(\partial_m\Phi_-)^2 - iW\sigma^m\partial_m\overline{W} + i\partial_m W\sigma^m\overline{W}$$
$$B = -\left(\overline{m} - \tfrac{1}{4}D^2\Phi\right) W^2 - \left(m - \tfrac{1}{4}\overline{D}^2\overline{\Phi}\right)\overline{W}^2 - iW\sigma^m\overline{W}\partial_m\Phi_-$$
$$\Phi_\pm \equiv \Phi \pm \overline{\Phi}$$

Component form of the constraint

The highest component of the quintic constraint (17) is (using $\partial_{mn} \equiv \partial_m \partial_n$)

$$-\tfrac{2}{3}\phi_1^4\Box^2\phi_1+\phi_1^3\left(P+\tfrac{40}{3}\Box\phi_1\Box\phi_1+\tfrac{16}{3}\partial^{mn}\phi_1\partial_{mn}\phi_1\right)+\phi_1^2\left(J_1+J_2^{mn}\partial_{mn}\phi_1\right) \\ +\phi_1\left(I_1+I_2^{mn}\partial_{mn}\phi_1\right)+H_1+H_2^{mn}\partial_{mn}\phi_1=0 \tag{18}$$

where $\Phi| = \phi = \phi_1 + i\phi_2$ ($\phi_2 =$ axion); The functions P, J, I, H depend on $\chi, \lambda, F_{mn}, \partial\phi_2, F, D$ but not ϕ_1 (or its derivatives).

- Useful definitions:

$$\Omega \equiv D^2 + 2\bar{f}F - \tfrac{1}{2}F_{mn}F^{mn} - \tfrac{i}{2}F_{mn}\tilde{F}^{mn} - 2\partial_m\phi_2\partial^m\phi_2 - 2i\chi\sigma^m\partial_m\bar{\chi} - 2i\lambda\sigma^m\partial_m\bar{\lambda}\,,$$

$$f \equiv F + \overline{m}\,, \quad \tilde{F}_{mn} \equiv \tfrac{1}{2}\epsilon_{mnkl}F^{kl}\,, \quad F^{\pm}_{mn} \equiv F_{mn} \pm i\tilde{F}_{mn}$$

- The functions P, J, I, H:

$$3P = 16\Box\phi_2\Box\phi_2 + 16\partial_{mn}\phi_2\partial^{mn}\phi_2 + 4\Box(\Omega + i\chi\sigma^m\partial_m\bar{\chi} + i\lambda\sigma^m\partial_m\bar{\lambda}) - 32\partial_m F\partial^m\bar{F} \\ - 16\partial_m D\partial^m D + 8\partial^m F_{mn}\partial_k F^{kn} + 4\partial_k F_{mn}\partial^k F^{mn} + 4i\tilde{F}_{mn}\Box F^{mn} \\ - 16i\Box\chi\sigma^m\partial_m\bar{\chi} - 16i\Box\lambda\sigma^m\partial_m\bar{\lambda} - 16i\partial_{mn}\chi\sigma^m\partial^n\bar{\chi} - 16i\partial_{mn}\lambda\sigma^m\partial^n\bar{\lambda} + \text{h.c.}$$

Constrained $\mathcal{N} = 2$ superfields

$$J_1 \equiv 8i\Omega\Box\phi_2 + 16\tilde{F}_{mn}\partial_k F^{nk}\partial^m\phi_2 - 2\Box(\chi^2\bar{F} + \lambda^2 f + \chi\sigma^m\bar{\chi}\partial_m\phi_2 + \lambda\sigma^m\bar{\lambda}\partial_m\phi_2)$$
$$- 8i\partial^m(2\bar{f}F - i\chi\sigma^n\partial_n\bar{\chi} - i\lambda\sigma^n\partial_n\bar{\lambda})\partial_m\phi_2 + 8(\chi\sigma^m\partial^n\bar{\chi} + \lambda\sigma^m\partial^n\bar{\lambda})\partial_{mn}\phi_2$$
$$+ 8(2\partial^m\chi\sigma_n\partial_m\bar{\chi} + 2\partial^m\lambda\sigma_n\partial_m\bar{\lambda} - i\epsilon_{mnkl}\partial^m\chi\sigma^k\partial^l\bar{\chi} - i\epsilon_{mnkl}\partial^m\lambda\sigma^k\partial^l\bar{\lambda})\partial^n\phi_2$$
$$- 8\partial_m(\chi\sigma^m\bar{\sigma}^n\partial_n\chi\bar{f}) - 8\partial_m(\lambda\sigma^m\bar{\sigma}^n\partial_n\lambda F) - 4\sqrt{2}i\partial^m\chi\partial^n\lambda(\eta_{mn}D + 2iF^+_{mn})$$
$$+ 16\sqrt{2}i\partial_m\chi\sigma^{mn}\partial_n\lambda D - 4\sqrt{2}(\partial^m\chi\sigma^{nk}\partial_k\lambda - \partial^m\lambda\sigma^{nk}\partial_k\chi)(F^+_{mn} + 2i\tilde{F}_{mn}) + \text{h.c.} + \ldots$$

$$J_2^{mn} \equiv 4\eta^{mn}(3\Omega + 4\partial_k\phi_2\partial^k\phi_2 + |m|^2 + F_{kl}F^{kl} + i\chi\sigma^k\partial_k\bar{\chi} + i\lambda\sigma^k\partial_k\bar{\lambda})$$
$$+ 8(\eta_{kl}F^{mk}F^{ln} - 2\partial^m\phi_2\partial^n\phi_2 - i\chi\sigma^m\partial^n\bar{\chi} - i\lambda\sigma^m\partial^n\bar{\lambda}) + \text{h.c.}$$

$$I_1 \equiv 4|\Omega|^2 - 16(|m|^2 + F_{mn}F^{mn})\partial_k\phi_2\partial^k\phi_2 - 32F^{mn}F_{nk}\partial^k\phi_2\partial_m\phi_2 + \ldots$$

$$I_2^{mn} \equiv -4\eta^{mn}\left[\chi^2(\bar{F} + 2\bar{f}) + \lambda^2(f + 2F) + (\chi\sigma^k\bar{\chi} + \lambda\sigma^k\bar{\lambda})\partial_k\phi_2 - 3\sqrt{2}i\chi\lambda D + \sqrt{2}\chi\sigma^{kl}\lambda F_{kl}\right]$$
$$- 8(\chi\sigma^m\bar{\chi} + \lambda\sigma^m\bar{\lambda})\partial^n\phi_2 + 16\sqrt{2}\chi\sigma^{ml}\lambda F_{lk}\eta^{kn} + \text{h.c.}$$

$$H_1 \equiv -\chi^2\left[2\bar{f}\bar{\Omega} + 4m\partial_m\phi_2\partial^m\phi_2\right] - \lambda^2\left[2F\bar{\Omega} - 4\bar{m}\partial_m\phi_2\partial^m\phi_2\right]$$
$$- \chi\sigma^m\bar{\chi}\partial_m\phi_2(2D^2 + F_{nk}F^{nk} + 4|f|^2 - 4\partial_n\phi_2\partial^n\phi_2) - 4\chi\sigma^m\bar{\chi}\partial^n\phi_2(D\tilde{F}_{mn} + \eta^{kl}F_{ml}F_{kn})$$
$$- \lambda\sigma^m\bar{\lambda}\partial_m\phi_2(2D^2 + F_{nk}F^{nk} + 4|F|^2 - 4\partial_n\phi_2\partial^n\phi_2) - 4\lambda\sigma^m\bar{\lambda}\partial^n\phi_2(-D\tilde{F}_{mn} + \eta^{kl}F_{ml}F_{kn})$$
$$+ 2\sqrt{2}i\bar{\Omega}(\chi\lambda D + i\chi\sigma^{mn}\lambda F_{mn}) - 8\sqrt{2}\chi\sigma^{mn}\lambda(F_{mn}\partial_k\phi_2\partial^k\phi_2 + 2F_{nk}\partial^k\phi_2\partial_m\phi_2)$$
$$+ 4\sqrt{2}i\chi\sigma^m\bar{\lambda}\partial^n\phi_2\left[\bar{F}(\eta_{mn}D - iF^+_{mn}) - \bar{f}(\eta_{mn}D - iF^-_{mn})\right] + \text{h.c.} + \ldots$$

$$H_2^{mn} \equiv \eta^{mn}(\chi^2\bar{\chi}^2 + \lambda^2\bar{\lambda}^2 + 2\chi^2\lambda^2 + 2\bar{\chi}^2\bar{\lambda}^2) - 4\chi\sigma^m\bar{\chi}\lambda\sigma^n\bar{\lambda}$$

Constrained $\mathcal{N} = 2$ superfields

General solution

$$-\tfrac{2}{3}\phi_1^4\Box^2\phi_1 + \phi_1^3\left(P + \tfrac{40}{3}\Box\phi_1\Box\phi_1 + \tfrac{16}{3}\partial^{mn}\phi_1\partial_{mn}\phi_1\right) + \phi_1^2\left(J_1 + J_2^{mn}\partial_{mn}\phi_1\right) + \phi_1\left(I_1 + I_2^{mn}\partial_{mn}\phi_1\right) + H_1 + H_2^{mn}\partial_{mn}\phi_1 = 0$$

Can be solved recursively!

At the leading order in goldstini

$$\boxed{\phi_1 = -\frac{H_1}{4}\left[|\Omega|^2 - 4(|m|^2 + F_{mn}F^{mn})\partial_k\phi_2\partial^k\phi_2 - 8F^{mn}F_{nk}\partial^k\phi_2\partial_m\phi_2\right]^{-1} + \ldots} \quad (19)$$

- H_1 is at least bilinear in χ, λ;
- At the vacuum

$$\langle\Omega\rangle \equiv \langle D^2 + 2(\bar{F} + m)F\rangle \neq 0$$

I.e. both supersymmetries broken;

- Surviving fields $\phi_2, \chi, \lambda, A_m$ + auxiliary.

Constrained $\mathcal{N}=2$ superfields

A simple example

Minimal prepotential $\mathcal{F}=\frac{i}{2}\Phi^2 \implies$

$$K=\tfrac{i}{2}(\Phi\mathcal{F}_\Phi-\overline{\Phi}\overline{\mathcal{F}}_{\bar\Phi})=\Phi\overline{\Phi}\,,\quad \mathcal{W}=-\tfrac{i}{2}m\mathcal{F}_\Phi=\tfrac{1}{2}m\,\Phi\,,\quad h=-i\mathcal{F}_{\Phi\Phi}=1$$

Lagrangian:

$$\mathcal{L}=\int d^4\theta\,\Phi\overline{\Phi}+\left(\frac{m}{2}\int d^2\theta\,\Phi+\frac{1}{4}\int d^2\theta\,W^\alpha W_\alpha+\text{h.c.}\right) \qquad (20)$$

$$=\phi_1\Box\phi_1+\phi_2\Box\phi_2-i\chi\sigma^m\partial_m\bar\chi-i\lambda\sigma^m\partial_m\bar\lambda-\tfrac{1}{4}F_{mn}F^{mn}+\tfrac{1}{2}mF+\tfrac{1}{2}\overline{m}\overline{F}+F\overline{F}+\tfrac{1}{2}D^2$$

Impose $(X+\overline{X})^5=0$ and eliminate $F=-\frac{1}{2}\overline{m}+\ldots$ and $D=0+\ldots$:

$$\mathcal{L}=\phi_2\Box\phi_2-i\chi\sigma^m\partial_m\bar\chi-i\lambda\sigma^m\partial_m\bar\lambda-\tfrac{1}{4}F_{mn}F^{mn}-\tfrac{1}{4}|m|^2+\text{non-linear}\,(\chi,\lambda) \qquad (21)$$

- F-term breaking with equal ~~SUSY~~ scales, $|\langle F\rangle|=|\langle F+\overline{m}\rangle|=|m|/2$
- Non-linearly realized $\mathcal{N}=2$:

$$\delta_\epsilon\chi=\sqrt{2}\epsilon_1F+\sqrt{2}i\sigma^m\bar\epsilon_1\partial_m\left[\phi_1(\chi,\lambda)+i\phi_2\right]+i(D+i\sigma^{mn}F_{mn})\epsilon_2$$

$$\delta_\epsilon\lambda=i(D-i\sigma^{mn}F_{mn})\epsilon_1+\sqrt{2}\epsilon_2(\overline{F}+m)+\sqrt{2}i\bar\epsilon_2\partial_m\left[\phi_1(\chi,\lambda)+i\phi_2\right]$$

Generalizations

- In the case of $U(1)$ phase rotations $\phi \to \phi e^{-i\alpha}$:

$$(X\overline{X} - 1)^5 = 0 \implies |\phi| = 1 + \text{bilinear}$$
$$\text{axion} = -\tfrac{i}{2}\log(\phi/\bar{\phi})$$

- Can be applied for $\mathcal{N} = 2$ single-tensor multiplet ($\overline{D}Y = \tilde{D}Y = \overline{D}^2 L = 0$)

$$Y = \Phi - \sqrt{2}i\bar{\tilde{\theta}}\overline{D}L + \bar{\tilde{\theta}}^2\left(m - \tfrac{1}{4}\overline{D}^2\overline{\Phi}\right)$$

Components of L are $\{\varphi, \psi, B_{mn}\}$ – only F-term breaking.

Conclusion

- We studied higher-order superfield constraints for $\mathcal{N} = 1, 2$ superfields;
- $\mathcal{N} = 1$ constraint $(\mathbf{S} + \bar{\mathbf{S}})^3 = 0 \;\to\; \mathrm{Re}S(\chi, \bar{\chi}, \mathrm{Im}S)$;
- $\mathcal{N} = 2$ constraint $(X + \bar{X})^5 = 0 \;\to\; \mathrm{Re}\phi(\chi, \bar{\chi}, \lambda, \bar{\lambda}, \mathrm{Im}\phi, A_m)$;
- The latter describes total breaking $\to \mathcal{N} = 0$;
- For phase-symmetric models $(|S|^2 - 1)^3 = 0$ and $(|X|^2 - 1)^5 = 0$
- Future directions:
  - Generalize to local abelian symmetries
  - Study phenomenology (cosmology, MSSM, ...)
  - String constructions?

Thank you