# Superconformal theories from S-fold geometries

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# Moduli space of $\mathcal{N} = 2$ theories and rank 1 models

The space of vacua of $\mathcal{N} = 2$ theories has two distinguished branches:

- **Coulomb Branch (CB):** where vector multiplet scalars have nonzero vev ($SU(2)_R$ unbroken);
- **Higgs Branch (HB):** where hypermultiplet scalars have nonzero vev ($U(1)_R$ unbroken).

$\mathcal{N} = 2$ theories are labelled by their **rank** (i.e. CB dimension).

Rank 1 theories have been classified! Argyres, Lotito, Lü, Martone '15-'16.

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# $\mathcal{N}=3$ theories in 4d from F-theory

**$\mathcal{S}$-folds:** Type IIB compactifications involving an S-duality twist:

Consider $U(1)$ $\mathcal{N}=4$ SYM and gauge a $\mathbb{Z}_\ell \subset U(1)_R \times SU(2)_F$. To preserve $\mathcal{N}=3$ susy we embed $\mathbb{Z}_\ell$ in $SL(2,\mathbb{Z})$.

We can construct $\mathcal{N}=3$ SCFTs by probing with $r$ D3 branes a $\mathbb{Z}_\ell$ $\mathcal{S}$-fold geometry: Garcia-Etxebarria, Regalado '15; Aharony, Tachikawa '16.

$$\frac{T^2 \quad \mathbb{C} \quad \mathbb{C} \quad \mathbb{C}}{\omega_\ell \quad \omega_\ell^{-1} \quad \omega_\ell \quad \omega_\ell^{-1}}; \quad \omega_\ell = e^{2\pi i/\ell} \quad \ell = 2,3,4,6.$$

We can introduce discrete flux for $H_3$ and $B_3$: Aharony, Tachikawa '16.

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# $\mathcal{N} = 2$ instanton theories from $D3$ branes

We consider 7-branes with constant axio-dilaton. Mukhi, Dasgupta '96.

| G | $\emptyset$ | $SU(2)$ | $SU(3)$ | $SO(8)$ | $E_6$ | $E_7$ | $E_8$ |
|---|---|---|---|---|---|---|---|
| $\Delta_7$ | $6/5$ | $4/3$ | $3/2$ | $2$ | $3$ | $4$ | $6$ |

The angular variable around the 7-brane has periodicity $2\pi/\Delta_7$.

We probe the 7-brane with a stack of $r$ $D3$ branes:

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 7-brane | $x$ | $x$ | $x$ | $x$ | $x$ | $x$ | $x$ | $x$ | | |
| D3 brane | $x$ | $x$ | $x$ | $x$ | | | | | | |

The gauge symmetry $G$ on the 7-brane becomes the global symmetry in 4d. CB operators have dimension $\Delta_7, 2\Delta_7, \ldots, r\Delta_7$.

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# $\mathcal{N}=2$ $\mathcal{S}$-folds = $\mathcal{S}$-folds + 7-branes

We wrap the 7-brane on $\mathbb{C}^2/\mathbb{Z}_\ell$ and combine this with a $\mathbb{Z}_\ell$ quotient of the 89-plane. To preserve $\mathcal{N}=2$ supersymmetry this must be accompanied (for $\ell \neq 1$) by the action of $\mathbb{Z}_{\ell\Delta_7} \subset SL(2,\mathbb{Z})$.

We find the following possibilities:

- For $\ell=1$ $\Delta_7 = 1, 6/5, 4/3, 3/2, 2, 3, 4$ and 6;
- For $\ell=2$ $\Delta_7 = 1, 3/2, 2$ and 3;
- For $\ell=3$ $\Delta_7 = 1, 4/3$ and 2;
- For $\ell=4$ $\Delta_7 = 1$ and $3/2$;
- For $\ell=5$ $\Delta_7 = 6/5$;
- For $\ell=6$ $\Delta_7 = 1$.

Each possibility leads to an infinite family of 4d $\mathcal{N}=2$ SCFTs.

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- For $\ell = 2$ $\Delta_7 = 1, 3/2, 2$ and 3;
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- For $\ell = 4$ $\Delta_7 = 1$ and $3/2$;
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Each possibility leads to an infinite family of 4d $\mathcal{N} = 2$ SCFTs.

# Mass deformation of $\mathcal{N} = 2$ $\mathcal{S}$-fold theories

By deforming the 7-brane we implement mass deformations for the 4d theories on the probe D3 branes:

| $\ell = 2$ | $\ell = 3$ | $\ell = 4$ |
|---|---|---|
| $(E_6, \mathbb{Z}_2)$ | $(D_4, \mathbb{Z}_3)$ | $(A_2, \mathbb{Z}_4)$ |
| $\downarrow$ | $\downarrow$ | $\downarrow$ |
| $(D_4, \mathbb{Z}_2)$ | $(A_1, \mathbb{Z}_3)$ | $\mathbb{Z}_4\ \mathcal{S} - fold$ |
| $\downarrow$ | $\downarrow$ | |
| $(A_2, \mathbb{Z}_2)$ | $\mathbb{Z}_3\ \mathcal{S} - fold$ | |
| $\downarrow$ | | |
| $\mathbb{Z}_2\ \mathcal{S} - fold$ | | |

There are two families of $\mathcal{N} = 2$ rank-1 SCFTs exhibiting this pattern of mass deformations. Argyres, Martone '16.

They arise on a $D3$ probing $\mathcal{S}$-folds with and without discrete flux.

# The holonomy at infinity

We should prescribe the holonomy for $G$ at infinity in $\mathbb{C}^2/\mathbb{Z}_\ell$. In the case at hand this is an order $\ell$ automorphism, with order $\ell'$ as an outer-automorphism. These are classified by Kac's theorem:

| $\ell$ | $\ell'$ | $G^{(\ell')}$ | Dynkin diagram | $H_T$ | $H_S$ |
|---|---|---|---|---|---|
| 2 | 2 | $E_6^{(2)}$ | $\underset{\alpha_0}{\overset{1}{\circ}} - \underset{\alpha_1}{\overset{2}{\circ}} - \underset{\alpha_2}{\overset{3}{\circ}} \Leftarrow \underset{\alpha_3}{\overset{2}{\circ}} - \underset{\alpha_4}{\overset{1}{\circ}}$ | $(F_4)_{\alpha_0}$ | $Sp(4)_{\alpha_4}$ |
| 2 | 2 | $D_4^{(2)}$ | $\underset{\alpha_0}{\overset{1}{\circ}} \Leftarrow \underset{\alpha_1}{\overset{1}{\circ}} - \underset{\alpha_2}{\overset{1}{\circ}} \Rightarrow \underset{\alpha_3}{\overset{1}{\circ}}$ | $SO(7)_{\alpha_0}$ | $(Sp(2)SU(2))_{\alpha_2}$ |
| 2 | 1 | $A_2^{(1)}$ | $\underset{\alpha_0}{\overset{1}{\circ}} - \underset{\alpha_1}{\overset{1}{\circ}} - \underset{\alpha_2}{\overset{1}{\circ}} -$ | $SU(3)_{\alpha_0\alpha_0}$ | $(Sp(1)U(1))_{\alpha_0\alpha_1}$ |
| 3 | 3 | $D_4^{(3)}$ | $\underset{\alpha_0}{\overset{1}{\circ}} - \underset{\alpha_1}{\overset{2}{\circ}} \Lleftarrow \underset{\alpha_2}{\overset{1}{\circ}}$ | $(G_2)_{\alpha_0}$ | $SU(3)_{\alpha_2}$ |
| 3 | 1 | $A_1^{(1)}$ | $\underset{\alpha_0}{\overset{1}{\circ}} - \underset{\alpha_1}{\overset{1}{\circ}} -$ | $SU(2)_{\alpha_0\alpha_0\alpha_0}$ | $U(1)_{\alpha_0\alpha_0\alpha_1}$ |
| 4 | 2 | $A_2^{(2)}$ | $\underset{\alpha_0}{\overset{2}{\circ}} \Lleftarrow \underset{\alpha_1}{\overset{1}{\circ}}$ (quadruple arrow) | $SU(2)_{\alpha_0}$ | $SU(2)_{\alpha_1\alpha_1}$ |

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| 2 | 1 | $A_2^{(1)}$ | $\underset{\alpha_0}{\overset{1}{\circ}} - \underset{\alpha_1}{\overset{1}{\circ}} - \underset{\alpha_2}{\overset{1}{\circ}} -$ | $SU(3)_{\alpha_0\alpha_0}$ | $(Sp(1)U(1))_{\alpha_0\alpha_1}$ |
| 3 | 3 | $D_4^{(3)}$ | $\underset{\alpha_0}{\overset{1}{\circ}} - \underset{\alpha_1}{\overset{2}{\circ}} \Lleftarrow \underset{\alpha_2}{\overset{1}{\circ}}$ | $(G_2)_{\alpha_0}$ | $SU(3)_{\alpha_2}$ |
| 3 | 1 | $A_1^{(1)}$ | $\underset{\alpha_0}{\overset{1}{\circ}} - \underset{\alpha_1}{\overset{1}{\circ}} -$ | $SU(2)_{\alpha_0\alpha_0\alpha_0}$ | $U(1)_{\alpha_0\alpha_0\alpha_1}$ |
| 4 | 2 | $A_2^{(2)}$ | $\underset{\alpha_0}{\overset{2}{\circ}} \Lleftarrow \underset{\alpha_1}{\overset{1}{\circ}}$ | $SU(2)_{\alpha_0}$ | $SU(2)_{\alpha_1\alpha_1}$ |

# Properties of the $\mathcal{T}^{(r)}_{G,\ell}$ SCFTs

Dimension of CB operators: $\ell\Delta_7, 2\ell\Delta_7, \dots, (r-1)\ell\Delta_7, r\Delta_7$.

| $\ell$ | $G$ | Flavor Symmetry | $a$ | $c$ |
|---|---|---|---|---|
| 2 | $E_6$ | $(F_4)_{6r} \times SU(2)_{6r^2-5r}$ | $\frac{6r^2+r}{4}$ | $\frac{6r^2+3r}{4}$ |
| 2 | $D_4$ | $SO(7)_{4r} \times SU(2)_{4r^2-3r}$ | $r^2$ | $\frac{4r^2+r}{4}$ |
| 2 | $A_2$ | $SU(3)_{3r} \times SU(2)_{3r^2-2r}$ | $\frac{6r^2-r}{8}$ | $\frac{3r^2}{4}$ |
| 3 | $D_4$ | $(G_2)_{4r} \times U(1)$ | $\frac{3N^2-N}{2}$ | $\frac{6N^2-N}{4}$ |
| 3 | $A_1$ | $SU(2)_{8r/3} \times U(1)$ | $\frac{2r^2-r}{2}$ | $\frac{12r^2-5r}{12}$ |
| 4 | $A_2$ | $SU(2)_{3r} \times U(1)$ | $\frac{12r^2-7r}{8}$ | $\frac{6r^2-3r}{4}$ |
| 5 | $\emptyset$ | $U(1)$ | $\frac{15r^2-11r}{10}$ | $\frac{30r^2-21r}{20}$ |

For generic $r$ the global symmetry is $H_{\mathcal{T}}$ times the isometry of the background, but enhances for $r \leq 2$. For $r=1$ these are the $G$ 1-instanton theories.

# Properties of the $\mathcal{S}^{(r)}_{G,\ell}$ SCFTs

Dimension of CB operators: $\ell\Delta_7, 2\ell\Delta_7, \ldots, (r-1)\ell\Delta_7, r\ell\Delta_7$.

| $\ell$ | $G$ | Flavor Symmetry | $a$ | $c$ |
|---|---|---|---|---|
| 2 | $E_6$ | $Sp(4)_{6r+1} \times SU(2)_{6r^2+r}$ | $\frac{36r^2+42r+4}{24}$ | $\frac{36r^2+54r+8}{24}$ |
| 2 | $D_4$ | $Sp(2)_{4r+1} \times SU(2)_{8r} \times SU(2)_{4r^2+r}$ | $\frac{24r^2+24r+2}{24}$ | $\frac{24r^2+30r+4}{24}$ |
| 2 | $A_2$ | $SU(2)_{3r+1} \times U(1) \times SU(2)_{3r^2+r}$ | $\frac{18r^2+15r+1}{24}$ | $\frac{18r^2+18r+2}{24}$ |
| 3 | $D_4$ | $SU(3)_{12r+2} \times U(1)$ | $\frac{36r^2+36r+3}{24}$ | $\frac{36r^2+42r+6}{24}$ |
| 3 | $A_1$ | $U(1) \times U(1)$ | $\frac{24r^2+20r+1}{24}$ | $\frac{24r^2+22r+2}{24}$ |
| 4 | $A_2$ | $SU(2)_{12r+2} \times U(1)$ | $\frac{36r^2+33r+2}{24}$ | $\frac{36r^2+36r+4}{24}$ |

For generic $r$ the global symmetry is $H_S$ times the isometry of the background, but enhances for $r=1$.

# $\mathcal{S}$-fold theories, Higgs branch and higgsings

For $\ell = 2, 3, 4$ we find the following sequence of RG flows:

$$\dots \to \mathcal{S}^{(r)}_{G,\ell} \to \mathcal{T}^{(r)}_{G,\ell} \to \mathcal{S}^{(r-1)}_{G,\ell} \to \mathcal{T}^{(r-1)}_{G,\ell} \to \dots$$

From this we conclude that changing the holonomy at infinity (or switching-off the discrete flux) implements an higgsing.

We find that all $\mathcal{S}$-fold SCFTs (we denote their Higgs branch as $\mathcal{M}^{(r)}_{G,\ell}$) have a $\mathbb{Z}_\ell$ discretely gauged version with Higgs branch $\mathcal{M}^{(r)}_{G,\ell}/\mathbb{Z}_\ell$. This is interpreted as new moduli spaces of $G$ instantons on $\mathbb{C}^2/\mathbb{Z}_\ell$ with holonomy involving outer-automorphisms.

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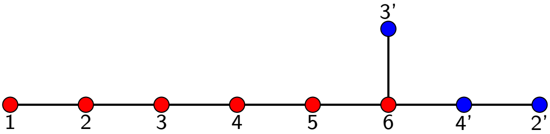
# $\mathcal{S}$-fold theories in a nutshell

We can represent graphically $\mathcal{N} = 2$ $\mathcal{S}$-fold SCFTs using an auxiliary affine $E_8$ Dynkin diagram:




$\mathcal{T}$ theories with $\ell = 1, 2, 3, 4, 5, 6$.
$\mathcal{S}$ theories with $\ell = 2, 3, 4$.

This peculiar pattern arises because $\mathcal{S}$-fold SCFTs (with $\ell\Delta_7 = 6$) arise as torus compactifications of 6d $\mathcal{N} = (1, 0)$ theories.

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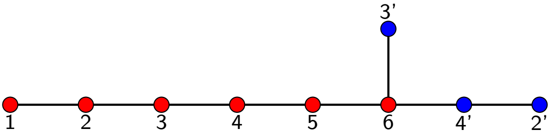


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# An alternative construction from M-theory

Two realizations of rank-$r$ $E_8$ MN theories: Minahan, Nemeschansky '96.

- $\mathcal{S}$-fold theories with $\ell = 1$ and $\Delta_7 = 6$. These correspond to $r$ D3 branes probing a 7-brane of type $E_8$.
- Rank-$r$ E-string on a torus: $r$ M5 branes probing the M9 wall in M-theory.

In both cases a $\mathbb{R}^4$ transverse to the probes $(SU(2)_R \times SU(2)_F)$.

To realize $\mathcal{S}$-folds with $\ell > 1$, we orbifold the $\mathbb{R}^4$ in both descriptions. This leads to orbi-instanton theories on the M-theory side!

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# 6d Orbi-instanton theories

We consider 6d $\mathcal{N} = (1, 0)$ obtained by wrapping the M9 brane on a ADE singularity $\mathbb{C}^2/\Gamma$. Del Zotto, Heckman, Tomasiello, Vafa '14.

The 6d SCFT is specified by the choice of holonomy at infinity for the $E_8$ gauge field (in one-to-one correspondence with homomorphisms $\rho : \Gamma \to E_8$).
Its global symmetry is $G \times \Gamma$, where $G \subset E_8$ commutes with $\rho(\Gamma)$.

When $\Gamma = \mathbb{Z}_\ell$ the homomorphisms are specified by selecting nodes of the affine $E_8$ Dynkin diagram such that the sum of labels is $\ell$.

The 6d theories relevant for constructing $\mathcal{S}$-fold theories are specified by selecting **a single node.**

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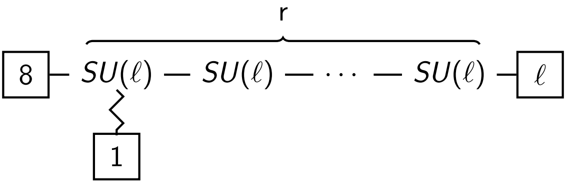
The 6d theories relevant for constructing $\mathcal{S}$-fold theories are specified by selecting **a single node.**

# The $\mathcal{S}^{(r)}_{G,\ell}$ theories from six dimensions

When the holonomy involves the nodes $2'$, $3'$ or $4'$ (therefore for $\ell = 2, 3, 4$) we find Mekareeya, Ohmori, Tachikawa, Zafrir '17; Cabrera, Hanany, Sperling '19.




We can construct $\mathcal{S}^{(r)}_{G,\ell}$ theories (with $\ell\Delta_7 = 6$) by compactifying these 6d theories on $T^2$ with $\mathbb{Z}_\ell$ holonomies (case $r = 1$ studied by Ohmori, Tachikawa and Zafrir).

# The $\mathcal{T}^{(r)}_{G,\ell}$ theories from six dimensions

Similarly, when the $E_8$ holonomy involves selecting the nodes $1, 2, 3, 4, 5$ or $6$ (for any $\ell \leq 6$) we get the theories

$$\boxed{\text{E-string}} - \overbrace{\underset{\substack{| \\ \boxed{\ell}}}{SU(\ell)} - SU(\ell) - \cdots - SU(\ell)}^{\text{r-1}} - \boxed{\ell}$$

where a $SU(\ell)$ subgroup of $E_8$ (from the E-string) is gauged.

Via torus compactification we find the $\mathcal{T}^{(r)}_{G,\ell}$ theories with $\ell \Delta_7 = 6$.

# Concluding remarks

- We constructed a family of CY4 singularities which interpolate between 7-branes in flat space and $\mathcal{N} = 3$ $\mathcal{S}$-folds. When probing these with $D3$ branes we can realize all rank 1 SCFTs and define higher rank generalizations thereof.
- The HB of these theories provide examples of ALE instantons with holonomy involving outer-automorphisms.
- Using systems of 3/7-branes in Type IIB we can construct many more (higher rank) $\mathcal{N} = 2$ theories, setting the stage for a detailed classification of higher rank SCFTs.

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