The XXVIII International Conference on Supersymmetry and Unification of Fundamental Interactions (SUSY 2021)

# Taming the $\epsilon_K$ in Little Randall Sundrum Models

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arXiv:2010.04471v2, Accepted in PRD
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# Outline

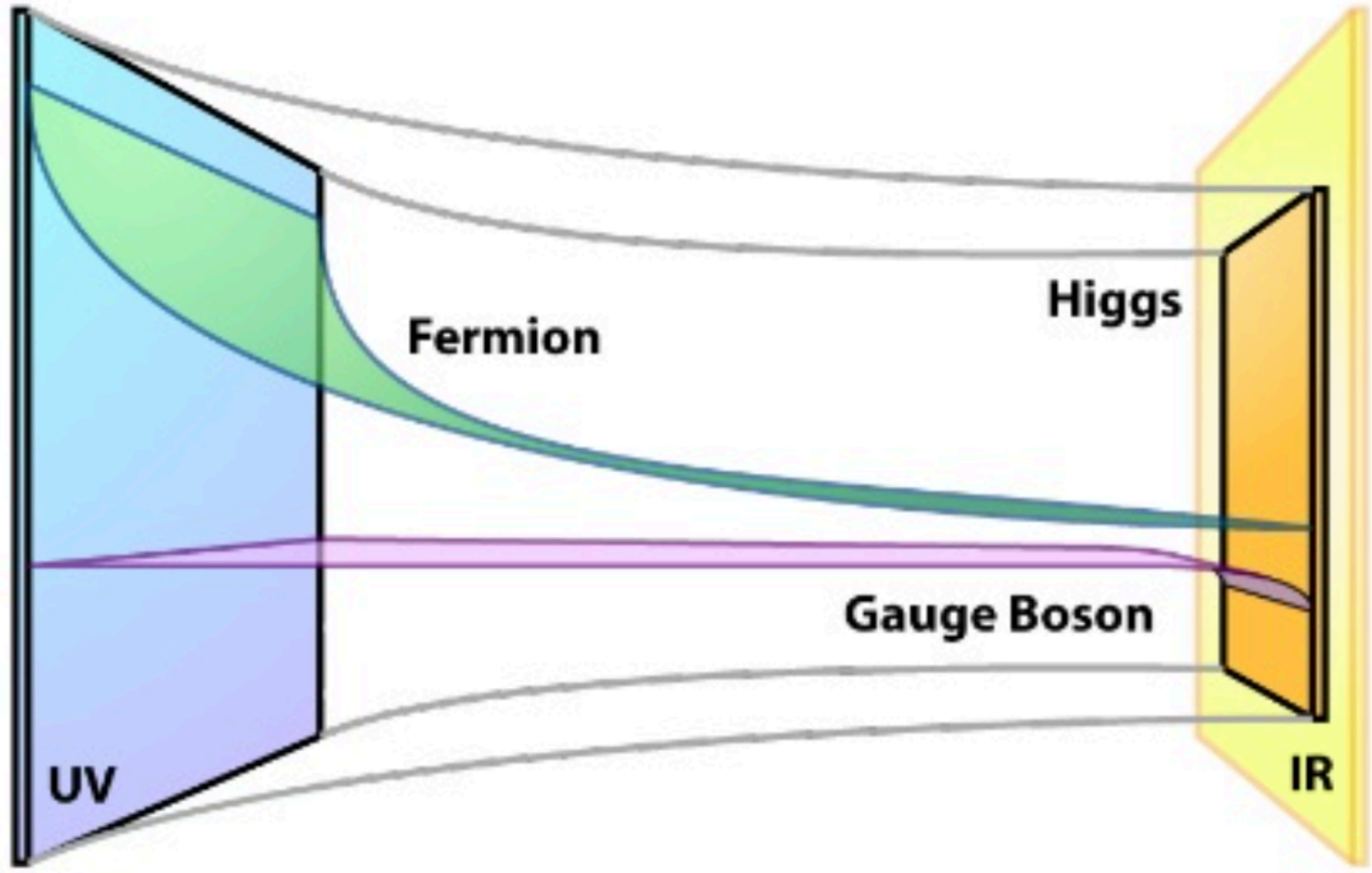
- Recap of flavour violation in warped 5 dimensions
- $\epsilon_K$ in Little Randall Sundrum
- Brane Localised Kinetic Terms
- Minimal Flavour Protection
- Summary

# Introduction : warped 5 dimensions

The space time is constructed on a $M_4 \times S_1/Z_2$ orbifold in presence of a negative bulk cosmological constant. The line element for this model is of the form

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$$

$$\eta_{\mu\nu} = diag(-1, +1, +1, +1) \text{ and } 0 \leq y \leq L$$




Where, the warp factor is defined as $k = r_c\sqrt{-\frac{\Lambda_5}{12M_5^3}}$.

$M_5$ is the fundamental scale and $\Lambda_5$ the bulk cosmological constant,

The standard model $SU(3)_c \times SU(2)_w \times U(1)_Y$, except for Higgs, is assumed to propagate in the bulk.

One of the successes of Randall Sundrum Model was that it gave a geometric explanation to the Froggatt Nielsen mechanism by localising the fermions in the bulk without giving rise to additional hierarchies.

# Origin of Flavour violation in warped 5 dimensions

In five dimensions, fermion field is vector like and hence, the action for matter field contain a bulk mass term.

Here, $m_Q$ and $m_q$ are the bulk masses for doublet and singlet quarks.

$$S_{\text{fermion}} = S_{\text{kin}} + S_{\text{yuk}}$$
$$S_{\text{kin}} = \int d^5x\sqrt{-g}\left[\bar{\widehat{Q}}\left(\Gamma^M D_M + m_Q\right)\widehat{Q} + \sum_{q=u,d}\bar{\widehat{q}}\left(\Gamma^M D_M + m_q\right)\widehat{q}\right]$$
$$S_{\text{Yuk}} = \int d^5x\sqrt{-g}\left(\left(\tilde{Y}_u^{(5)}\right)_{ij}\bar{\widehat{Q}}_i\widehat{u}_j + \left(\tilde{Y}_d^{(5)}\right)_{ij}\bar{\widehat{Q}}_i\widehat{d}_j\right)H(x^\mu)\delta(y-L) + h.c.,$$

Upon compactification the five dimensional field breaks into an infinite tower of Kaluza Klein states.

$$Q(x,y)_{l,r} = \sum_{n=0}^{\infty}\frac{1}{\sqrt{L}}Q_{l,r}^{(n)}(x)f_{l,r}^{(n)}(y,c),$$

The solution for the bulk profile of the fermion zero mode with $c_Q = m_Q/k$ and $c_q = -m_q/k$,

$$f_l^{(0)}(y,c_{Q_i}) = \sqrt{k}f^0(c_{Q_i})\ e^{ky(2-c_{Q_i})}\ e^{(c_{Q_i}-0.5)kL}$$
$$f_r^{(0)}(y,c_{q_i}) = \sqrt{k}f^0(c_{q_i})\ e^{ky(2-c_{q_i})}\ e^{(c_{q_i}-0.5)kL}$$

This means that fields with $c_i > 0.5$ is localised towards the UV brane.

For IR brane localised Higgs, by choosing appropriate ‘c’ values we get hierarchical 4 dimensional Yukawa from anarchic 5D Yukawa

$$Y_{d_{ij}} = f^0(c_{Qi})\left(Y_d^{(5)}\right)_{ij}f^0(c_{dj})$$
$$Y_{u_{ij}} = f^0(c_{Qi})\left(Y_u^{(5)}\right)_{ij}f^0(c_{uj})$$

# Origin of Flavour violation in warped 5 dimensions

Now, let's look at the gauge boson, in particular the gluon.

Action for the gluon field in five dimensions is given as

$$\mathcal{S} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\left(g^{CM}g^{DN}F_{CD}F_{MN}\right)$$

And upon compactification, the vector fields can be expanded as

$$A_\mu(x,y) = \sum_n f_A^{(n)}(y)A_\mu^{(n)}(x)$$

Solving for the massless equation of motion we get the zero mode that will be identified with the SM gauge bosons as

$$f_A^{(0)}(y) = \frac{1}{\sqrt{L}}$$

And the higher KK modes are given as

$$f_A^{(n)}(y) = N_A^{(n)}e^{ky}\left[J_1\left(\frac{m_n}{ke^{-ky}}\right) + b_A^{(n)}Y_1\left(\frac{m_n}{ke^{-ky}}\right)\right]$$

The most interesting part in this exercise is that the gauge boson KK-partners couple to the fermion bilinear as

$$g_{l,r}^{(n)}(c_{Q,q}) \quad = \quad g_5\int_0^L dy\, e^{-3ky}f_A^{(n)}(y)f_{l,r}^{(0)}(y,c_{Q,q})f_{l,r}^{(0)}(y,c_{Q,q})$$

And this is 'c' value dependent. Which leads to non-universal coupling of the KK-1 partner of gluon with Standard Model quarks

$$g_l^{(n)}(c_{Qi})\bar{\widehat{Q}}_l^{i(0)}\gamma_\mu G^{\mu(n)}\widehat{Q}_l^{i(0)} + g_r^{(n)}(c_{ui})\bar{\widehat{u}}_r^{i(0)}\gamma_\mu G^{\mu(n)}\widehat{u}_r^{i(0)} + g_r^{(n)}(c_{di})\bar{\widehat{d}}_r^{i(0)}\gamma_\mu G^{\mu(n)}\widehat{d}_r^{i(0)}$$

# $\Delta F = 2$ transitions in warped 5 dimensions

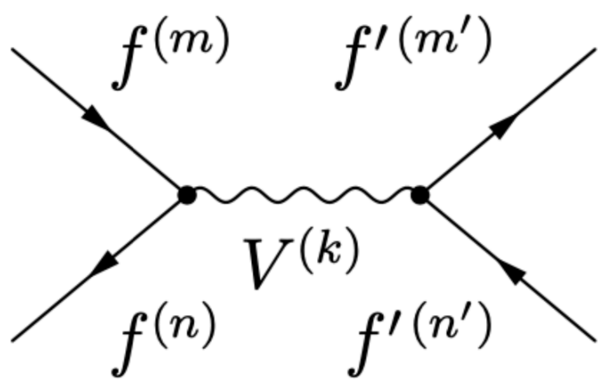
On rotating to the mass basis, defined by $D_{l,r}$ and $U_{l,r}$, the couplings become flavour non-diagonal

$$\tilde{g}_l = D_l^\dagger g_l^{(1)}(c_Q) D_l, \qquad \tilde{g}_r = D_r^\dagger g_r^{(1)}(c_d) D_r$$

$$\xi_u = U_l^\dagger Y_u U_r\,,$$
$$\xi_d = D_l^\dagger Y_d D_r$$

$$\xi_u = \frac{\sqrt{2}}{v}\mathrm{diag}(m_u, m_c, m_t), \qquad \xi_d = \frac{\sqrt{2}}{v}\mathrm{diag}(m_d, m_s, m_b)$$

In principle, all the gauge KK states would contribute to this process, but the dominant contribution comes from the tree level exchange of the lightest gluon KK state




On integrating out the gluon KK state with mass $M_g$ the effective Hamiltonian for ΔF = 2 becomes

$$H_{eff} = \frac{1}{M_g^2}\left[g_l^{ij} g_l^{km}(\bar{d}_l^{i\alpha} T^a_{\alpha\beta}\gamma_\mu d_l^{j\beta})(\bar{d}_l^{k\delta} T^a_{\delta\rho}\gamma_\mu d_l^{m\rho}) + g_r^{ij} g_l^{km}(\bar{d}_r^{i\alpha} T^a_{\alpha\beta}\gamma_\mu d_r^{j\beta})(\bar{d}_l^{k\delta} T^a_{\delta\rho}\gamma_\mu d_l^{m\rho}) + (l \leftrightarrow r)\right]$$

Using the unitarity of $D_{l,r}$ , the above expression could be further expanded to understand the ΔF = 2 transitions in first and second generations.

And the couplings become

$$\tilde{g}_l^{12} = D_l^{(21)*} D_l^{(22)}\left(g_l^{(1)}(c_{Q2}) - g_l^{(1)}(c_{Q1})\right) + D_l^{(31)*} D_l^{(32)}\left(g_l^{(1)}(c_{Q3}) - g_l^{(1)}(c_{Q1})\right)$$
$$\tilde{g}_r^{12} = D_r^{(21)*} D_r^{(22)}\left(g_r^{(1)}(c_{d2}) - g_r^{(1)}(c_{d1})\right) + D_r^{(31)*} D_r^{(32)}\left(g_r^{(1)}(c_{d3}) - g_r^{(1)}(c_{d1})\right)$$

# $\Delta F = 2$ transitions in warped 5 dimensions

The effective Hamiltonian could be simplified to

$$H_{eff} = \frac{1}{M_g^2}\Big[\tilde{g}_l^{ij}\tilde{g}_l^{km}\frac{1}{2}\Big((\bar{d}_l^{i\alpha}\gamma_\mu d_L^{j\delta})(\bar{d}_l^{k\delta}\gamma_\mu d_l^{m\alpha}) - \frac{1}{N_C}(\bar{d}_l^{i\alpha}\gamma_\mu d_l^{j\alpha})(\bar{d}_l^{k\delta}\gamma_\mu d_l^{m\delta})\Big) - \tilde{g}_r^{ij}\tilde{g}_l^{km}((\bar{d}_r^{i\alpha}d_l^{m\alpha})(\bar{d}_l^{k\delta}d_r^{j\delta}) + \frac{1}{N_C}((\bar{d}_r^{i\alpha}d_l^{m\delta})(\bar{d}_l^{k\delta}d_r^{j\alpha})) + (l \leftrightarrow r)\Big]$$

Adopting the usual parametrisation of new physics effects in Kaon oscillation and comparing with the effective Hamiltonian we could infer

$$H^{\Delta S=2} = \sum_{a=1}^{5} C_a \mathcal{O}_a^{sd} + \sum_{a=1}^{3} \tilde{C}_a \tilde{\mathcal{O}}_a^{sd}.$$

$$\mathcal{O}_1^{sd} = (\bar{d}_l^{\alpha}\gamma_\mu s_l^{\alpha})(\bar{d}_l^{\beta}\gamma_\mu s_l^{\beta}),$$
$$\mathcal{O}_4^{sd} = (\bar{d}_r^{\alpha} s_l^{\alpha})(\bar{d}_l^{\beta} s_r^{\beta}), \quad \mathcal{O}_5^{sd} = (\bar{d}_r^{\alpha} s_l^{\beta})(\bar{d}_l^{\beta} s_r^{\alpha}),$$

$$C_1 = \frac{1}{M_g^2}\tilde{g}_l^{12}\tilde{g}_l^{12}\Big[\frac{1}{2}(1 - \frac{1}{N_C})\Big], \quad C_4 = \frac{1}{M_g^2}\tilde{g}_l^{12}\tilde{g}_r^{12}[-1], \quad C_5 = \frac{1}{M_g^2}\tilde{g}_l^{12}\tilde{g}_r^{12}[\frac{1}{N_C}]$$

The CP-Violating $\epsilon_K$ parameter in terms of the Effective Hamiltonian is given as

$$\epsilon_K \propto Im\langle K^0|H^{\Delta S=2}|\bar{K^0}\rangle$$

And the model independent bound from $\epsilon_K$ puts constraints on the the Wilson coefficients, in particular it is strongest for $C_4$

| Im(Wilson coeff.) | Bound (TeV) |
|---|---|
| $\mathrm{Im}(C_1)$ | $1.5 \times 10^3$ |
| $\mathrm{Im}(C_4)$ | $1.6 \times 10^4$ |
| $\mathrm{Im}(C_5)$ | $1.4 \times 10^4$ |

With the above formalism in place, comparison of the constraint coming from $\epsilon_K$ in RS and Little RS model arise through their corresponding $\tilde{g}_{l,r}$ couplings.

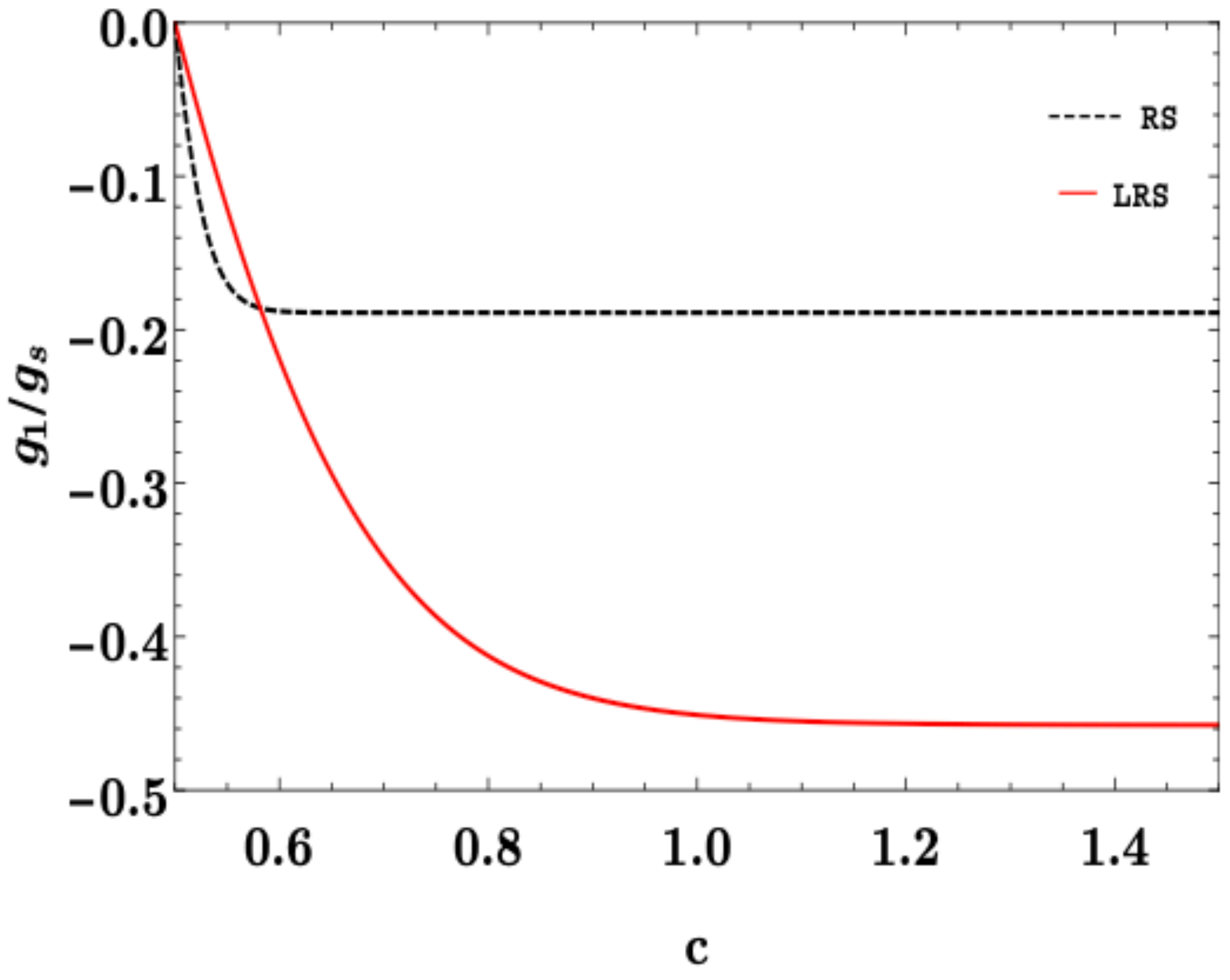
# Little RS

The Little Randall Sundrum (Little RS) model is a version of the Randall Sundrum model (RS) where the fundamental scale is reduced to ~ 100 TeV from the Planck scale.

Since Little RS has a smaller volume factor $L \sim Log(10^3)$, their contribution to electro-weak parameters S,T $\sim - L\pi \frac{v^2}{M_{KK}^2}$ are suppressed.

The coupling of KK-1 gluon to fermion bilinear is enhanced by a factor 3 in the Little RS in comparison to RS for the same 'c' values. This is shown in the figure.

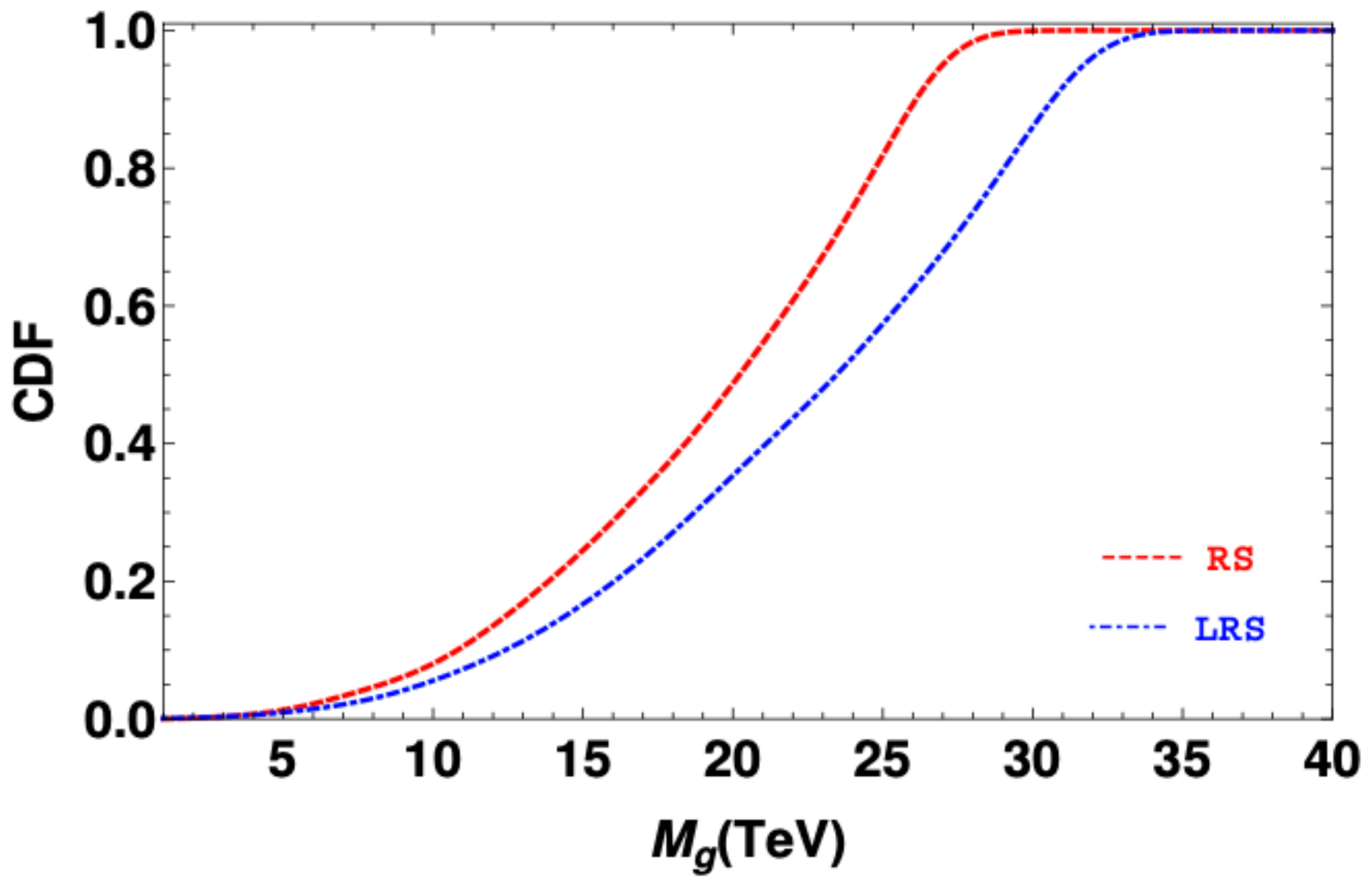
This could be understood from the fact that, since the fundamental scale is much smaller, the UV localised fermions have bigger overlap with the KK-1 gauge boson state.



# $\epsilon_K$ in Little RS

For numerical study, we fit the bulk quark mass parameters to obtain the correct mass spectrum while keeping the 5D Yukawa anarchic ($0.1 \leq |Y^{(5)}_{u,d_{ij}}| \leq 3$)

The figure shows the dependence of the cumulative distribution functions (CDF) for the number of states satisfying the $\epsilon_K$ observable ($|\mathrm{Im}(C_4^{sd})|$) with varying KK gluon mass scale $M_g$ for both RS and Little RS.




It can be seen that the average value of $\epsilon_K$ becomes consistent with the measurement only for $M_g \geq 24$ TeV for RS and $M_g \geq 32$ TeV for Little RS.

The Little RS is worse not just because the couplings (overlaps) are large but the difference between the second and first generation couplings increase. This stems from demanding the correct fermion mass hierarchy with smaller warping.

$$\tilde{g}_l^{12} = D_l^{(21)*} D_l^{(22)} \left( g_l^{(1)}(c_{Q2}) - g_l^{(1)}(c_{Q1}) \right) + D_l^{(31)*} D_l^{(32)} \left( g_l^{(1)}(c_{Q3}) - g_l^{(1)}(c_{Q1}) \right)$$

$$\tilde{g}_r^{12} = D_r^{(21)*} D_r^{(22)} \left( g_r^{(1)}(c_{d2}) - g_r^{(1)}(c_{d1}) \right) + D_r^{(31)*} D_r^{(32)} \left( g_r^{(1)}(c_{d3}) - g_r^{(1)}(c_{d1}) \right)$$

# Impact of Brane Localised Kinetic Terms in Little RS

The 5-dimensional action for gluon could be generalised to include the BLKTs as

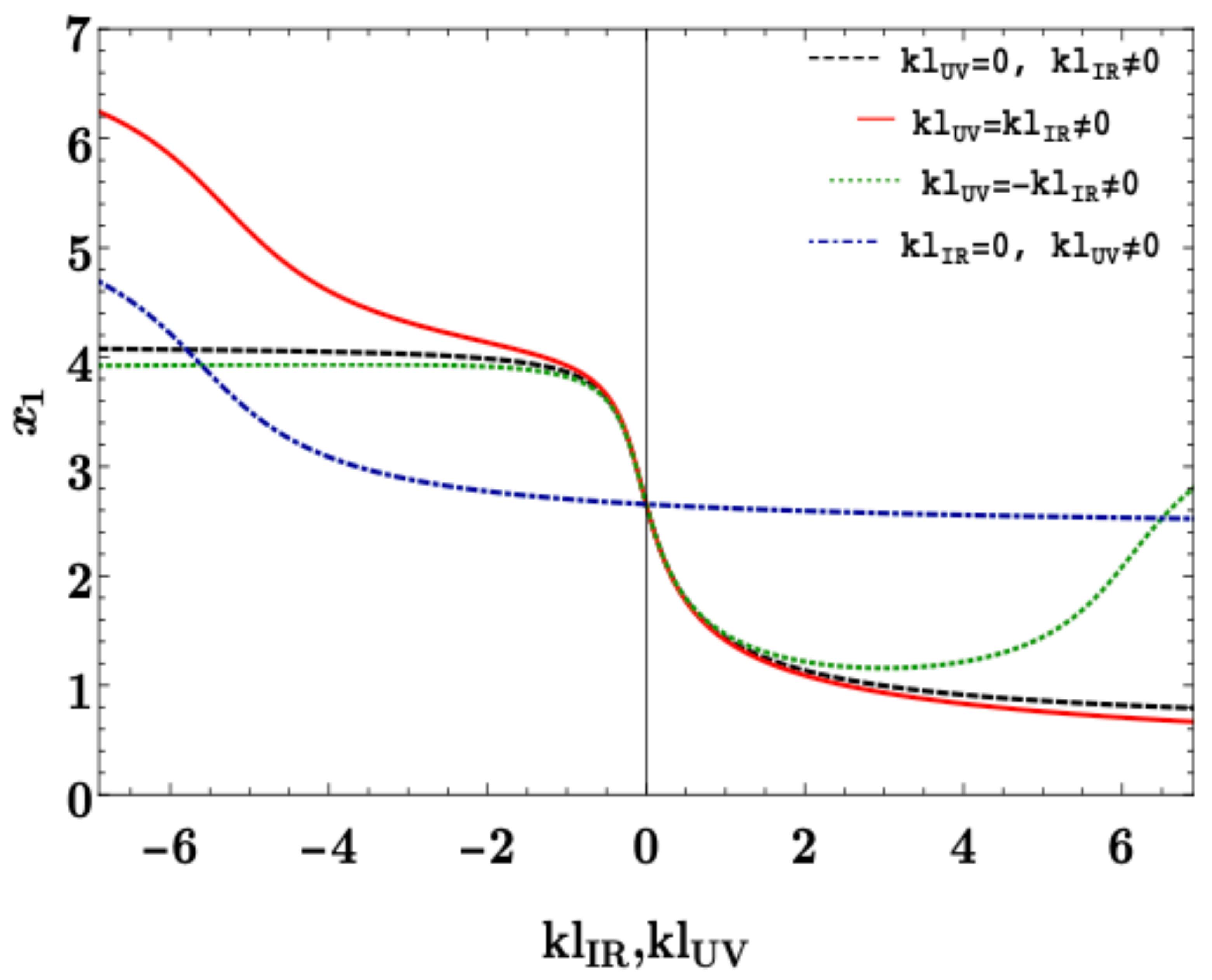
$$\mathcal{S} = -\frac{1}{4g_5^2}\int d^5x\sqrt{-g}\left(g^{AM}g^{BN}F_{AB}F_{MN} + [l_{IR}\delta(y-L) + l_{UV}\delta(y)]g^{\alpha\mu}g^{\beta\nu}F_{\alpha\beta}F_{\mu\nu}\right)$$

This leads to change in boundary condition and in turn changes the behaviour of the first root.

$$\begin{aligned} \partial_y f_A^{(n)}|_0 &= -l_{UV}m_n^2 f_A^{(n)}(0)\ , \\ \partial_y f_A^{(n)}|_L &= +e^{2kL}l_{IR}m_n^2 f_A^{(n)}(L). \end{aligned}$$

$$f_A^{(n)}(y) = N_A^{(n)}e^{ky}\left[J_1\left(\frac{m_n}{ke^{-ky}}\right) + b_A^{(n)}Y_1\left(\frac{m_n}{ke^{-ky}}\right)\right]$$

$$\begin{aligned} b_A^{(n)}|_{UV} &= -\frac{J_0(\frac{m_n}{k}) + m_n l_{UV}J_1(\frac{m_n}{k})}{Y_0(\frac{m_n}{k}) + m_n l_{UV}Y_1(\frac{m_n}{k})}\ , \\ b_A^{(n)}|_{IR} &= -\frac{J_0(\frac{m_n}{k}e^{kL}) - m_n l_{IR}e^{kL}J_1(\frac{m_n}{k}e^{kL})}{Y_0(\frac{m_n}{k}e^{kL}) - m_n l_{IR}e^{kL}Y_1(\frac{m_n}{k}e^{kL})}\ , \end{aligned}$$



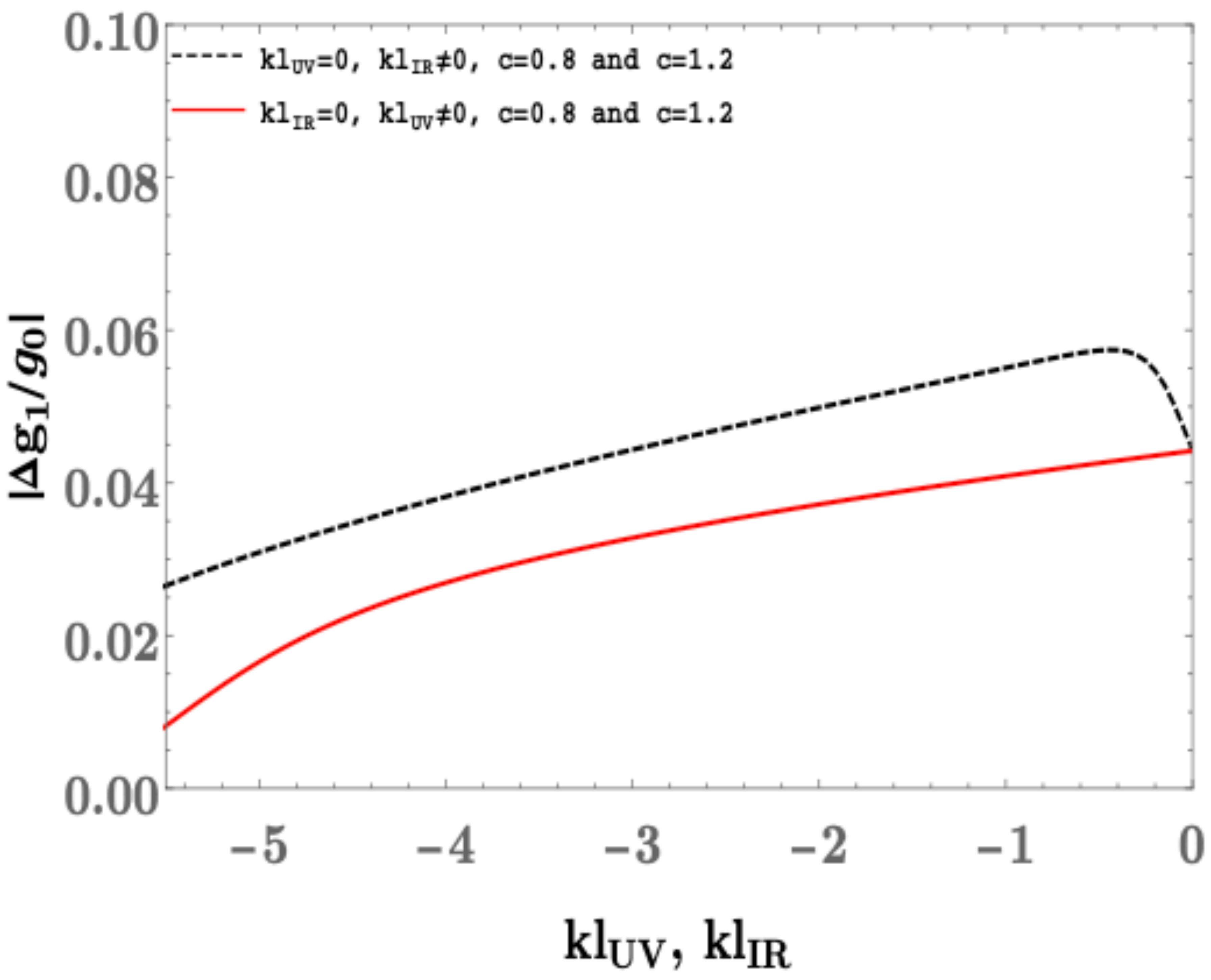
# Impact of Brane Localised Kinetic Terms in Little RS

A bigger change is however expected in the coupling of KK-gluon with the fermion bilinear. Since BLKT modifies the bulk gauge field wave profile, the overlap of the gauge boson with the fermion wave profile becomes,

$$
\begin{aligned}
g_{l,r}^{(n)}(c_{Q,q}) &= g_5 \int_0^L dy\, e^{-3ky} f_A^{(n)}(y) f_{l,r}^{(0)}(y, c_{Q,q}) f_{l,r}^{(0)}(y, c_{Q,q}) \\
&= g_4 \sqrt{L + l_{IR} + l_{UV}} \int_0^L dy\, e^{-3ky} f_A^{(n)}(y) f_{l,r}^{(0)}(y, c_{Q,q}) f_{l,r}^{(0)}(y, c_{Q,q}) .
\end{aligned}
$$

The figure displays the difference in coupling strengths of first KK partner of the gluon, $\left|\dfrac{\Delta g_1}{g_0}\right| = \left|\dfrac{g_{l,r}^{(1)}(c_2) - g_{l,r}^{(1)}(c_1)}{g_0}\right|$,

with fermions localised with the bulk mass parameters $c_1 = 1.2$ and $c_2 = 0.8$.




| BLKT | $kl_{IR} = kl_{UV} = 0$ | $kl_{IR} = -5, kl_{UV} = 0$ | $kl_{IR} = 0, kl_{UV} = -5$ |
|---|---|---|---|
| $\left\|\frac{\Delta g_1}{g_0}\right\|$ | 0.044 | 0.030 | 0.016 |

$\left|\frac{\Delta g_1}{g_0}\right|$ values for the three cases of BLKTs computed with $c_1 = 1.2$ and $c_2 = 0.8$.

This clearly shows that UV BLKT is significantly better in reducing the couplings and considerably relaxes the bound in Little RS.

# Impact of Brane Localised Kinetic Terms

For the full numerical analysis, we have scanned over the bulk mass parameter, 'c', with central values

| Parameter | $c_{Q1}$ | $c_{Q2}$ | $c_{Q3}$ | $c_{d1}$ | $c_{d2}$ | $c_{d3}$ | $c_{u1}$ | $c_{u2}$ | $c_{u3}$ |
|---|---|---|---|---|---|---|---|---|---|
| c | 1.21 | 1.13 | 0.36 | 1.29 | 1.15 | 1.02 | 1.65 | 0.59 | -0.80 |

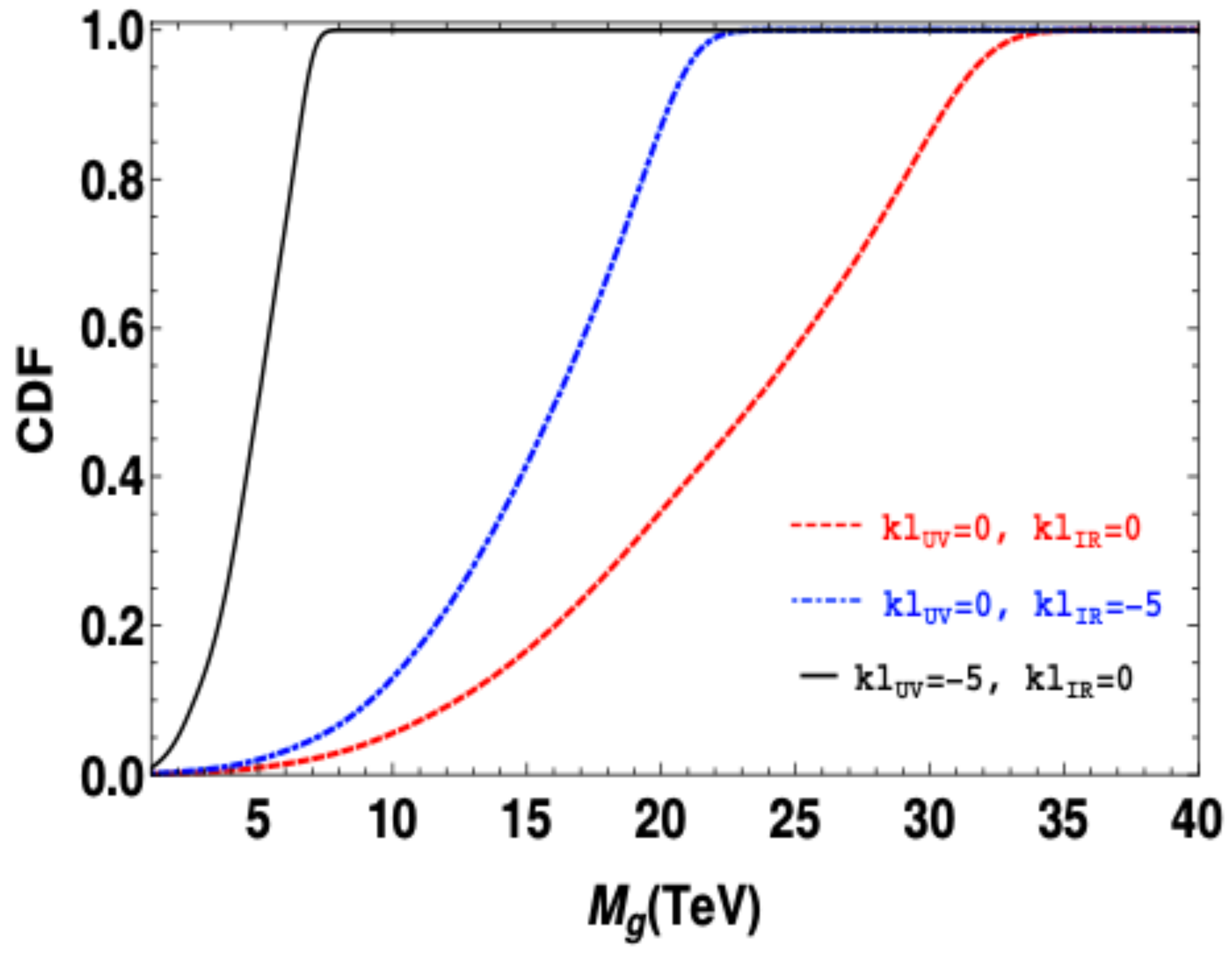
The Cumulative distribution function and Probability distribution function satisfying the constraint on Im($C_4$) is presented.

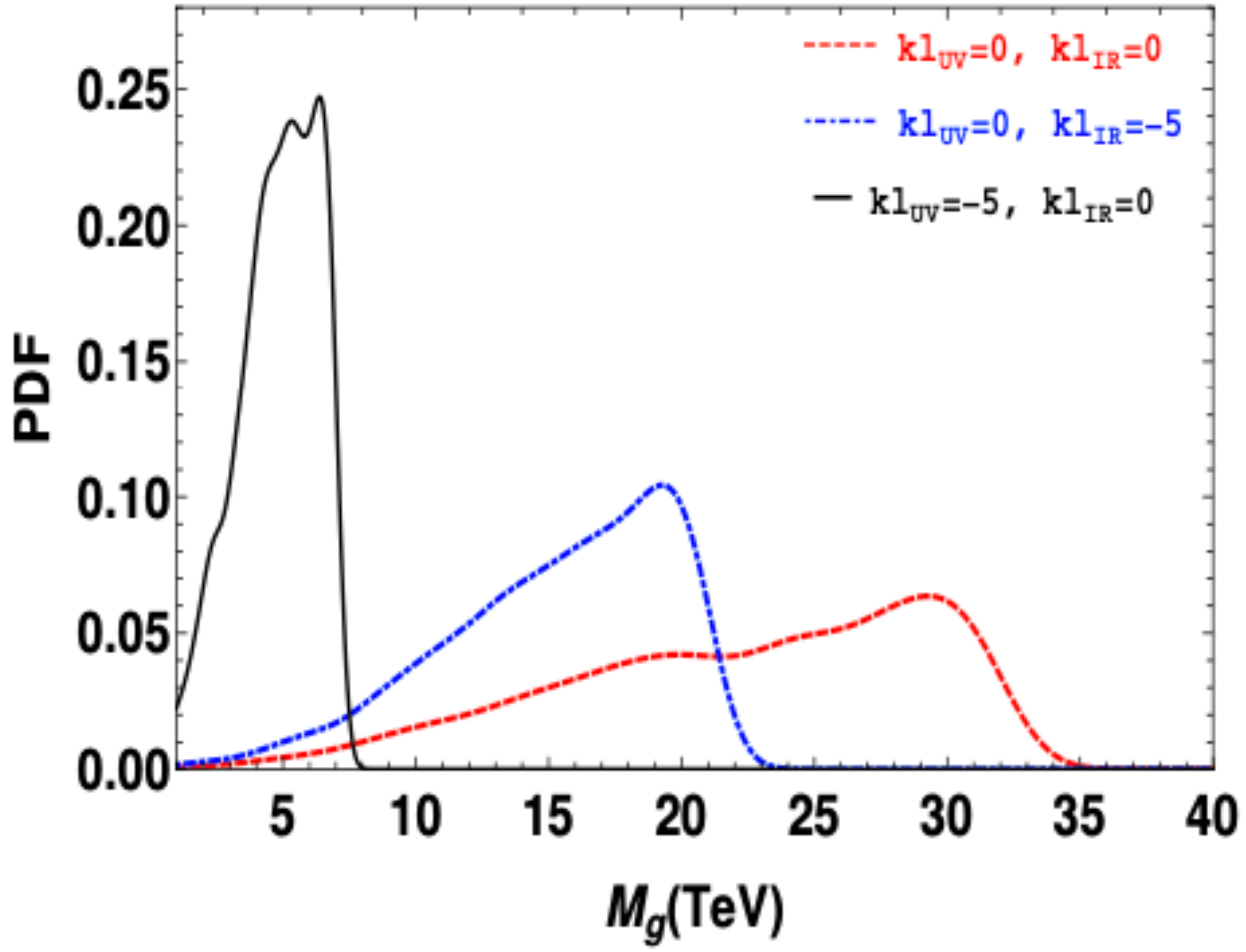
The plots clearly show the drastic reduction on the constraints while imposing BLKTs.

It is clear that the BLKTs achieve this by bringing the couplings of the two light flavours closer to each other. And it will reduce the constraints on the other Wilson Coefficients as well.

| $kl_{IR} = kl_{UV} = 0$ | $kl_{IR} = -5, kl_{UV} = 0$ | $kl_{IR} = 0, kl_{UV} = -5$ |
|---|---|---|
| 30 TeV | 20 TeV | 8 TeV |

Bounds on the KK-1 gluon masses for the different BLKTs considered



# Minimal Flavour Protection in Little RS

It is only natural that such relaxation of the bound could also happen by imposing a flavour symmetry.

Minimal Flavour Protection (MFP) was introduced to suppress the chiral enhancement of New Physics contributions to the $\Delta F = 2$ Hamiltonian.

In MFP, the singlet down sector is assumed to transform as triplet under a global U(3) group with rest of the fields transforming as singlets.

Hence the bulk mass term for five dimensional fermions become

$$c_{Qi}\bar{Q}_i Q_i + c_{ui}\bar{u}_i u_i + c_d \bar{d}_i d_i$$

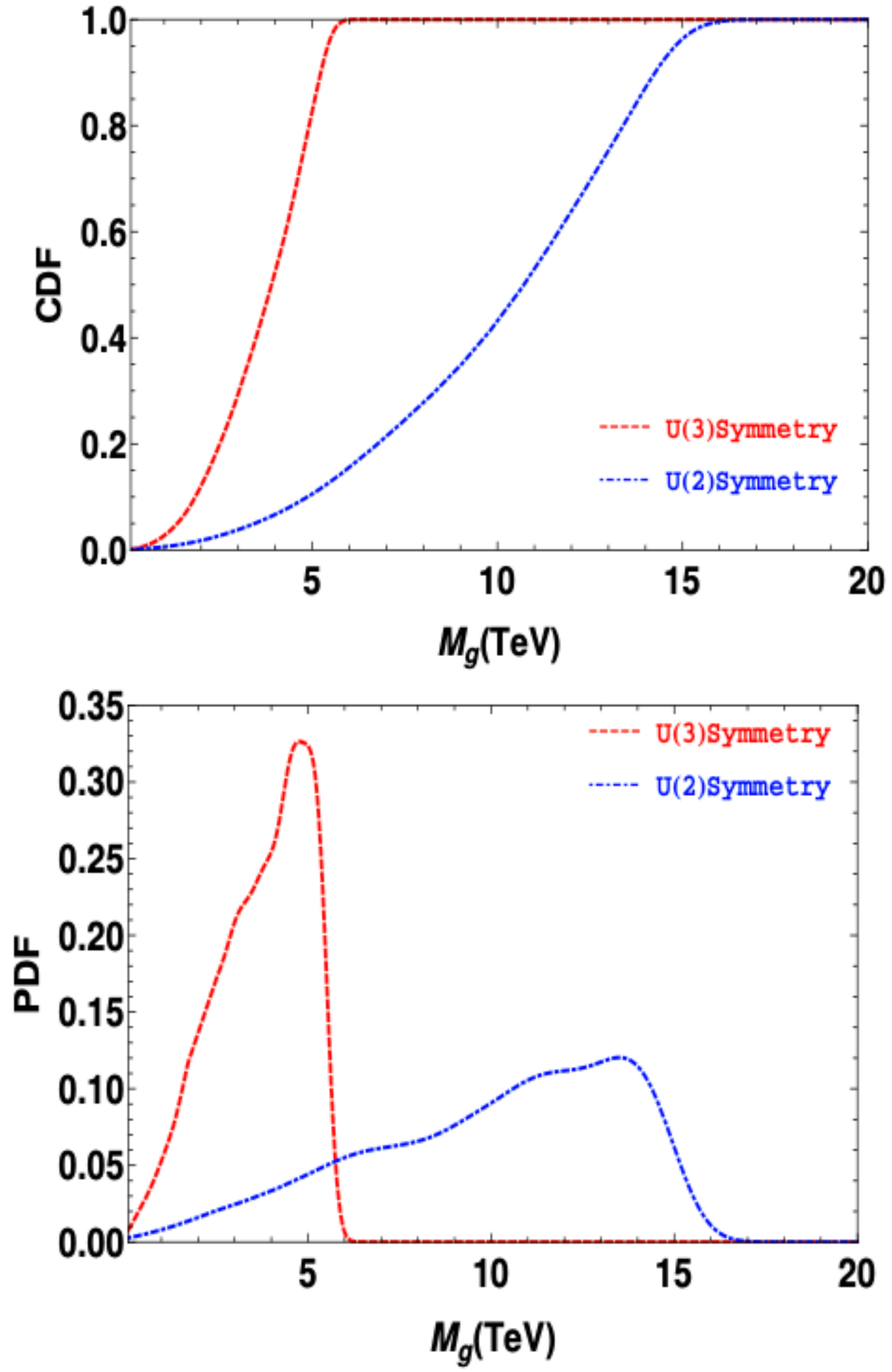
When U(3) flavour symmetry is assumed to be exact, the bulk mass parameters for the right handed down sector becomes $c_{d1} = c_{d2} = c_{d3}$.

Hence unlike the scenario discussed in the previous section, the hierarchy in singlet down sector wave profiles vanish.

Due to which the couplings of $n^{th}$ KK-gluon with the right handed chiral zero mode bilinear becomes flavour universal $\left(g_r^{(n)}(c_d)\right)_{ij} = g_d^{(n)}\delta_{ij}$

And hence the off diagonal terms after rotating vanishes in the right handed sector. $\tilde{g}_r^{12} = 0$

As can be seen in the figure, the bounds for Little RS-MFP gets lowered to $M_g > 4$ TeV.



# Minimal Flavour Protection in Little RS

Hence this paradigm expects a conserved chiral symmetry with a consequence of significantly suppressed contributions from $C_4$ and $C_5$

Though higher order operators are generated at dimension-8 arise, due to Higgs insertion, they can be neglected.

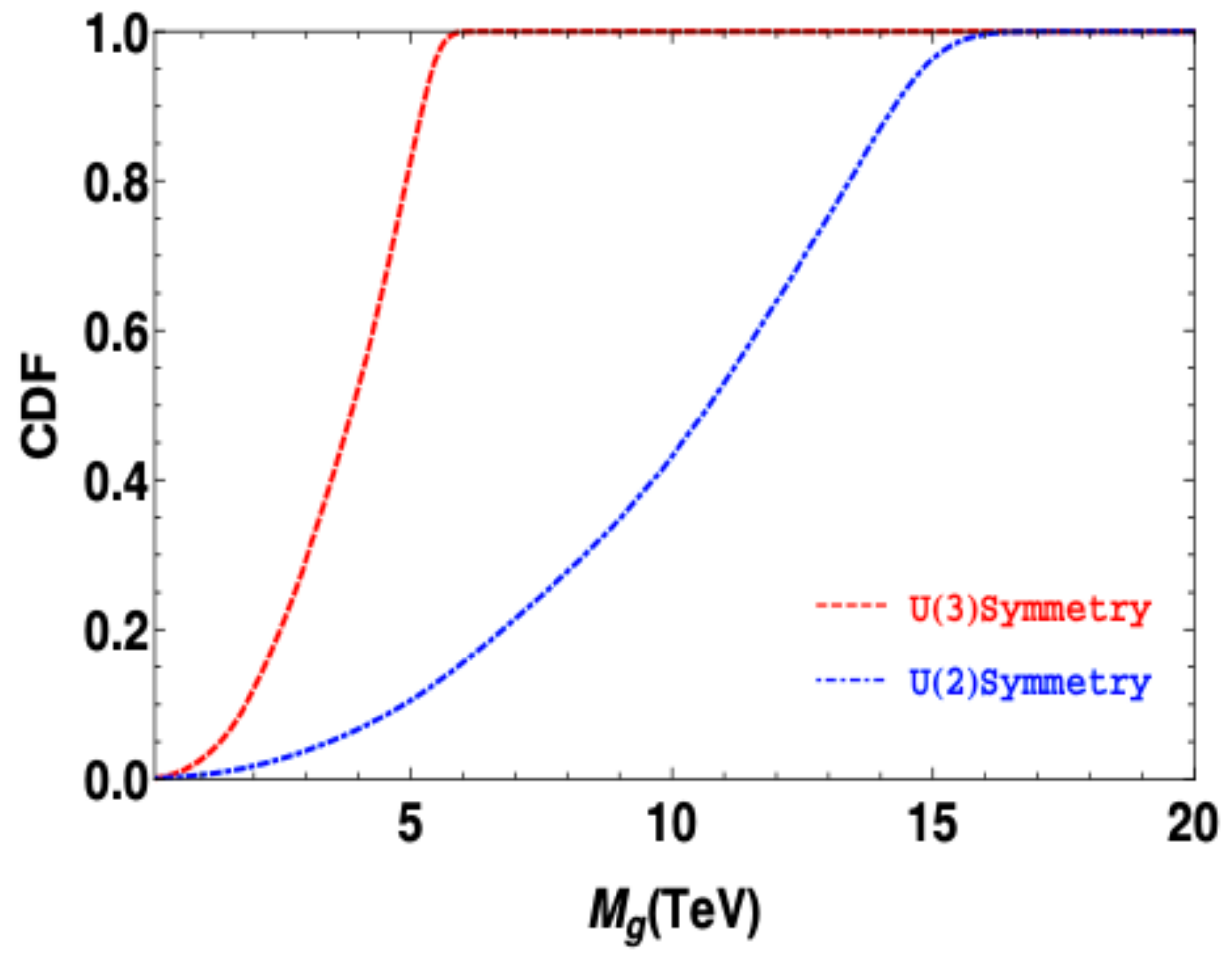
And hence in all practicality $C_4$ operator does not play a role in the limit of exact U(3) symmetry

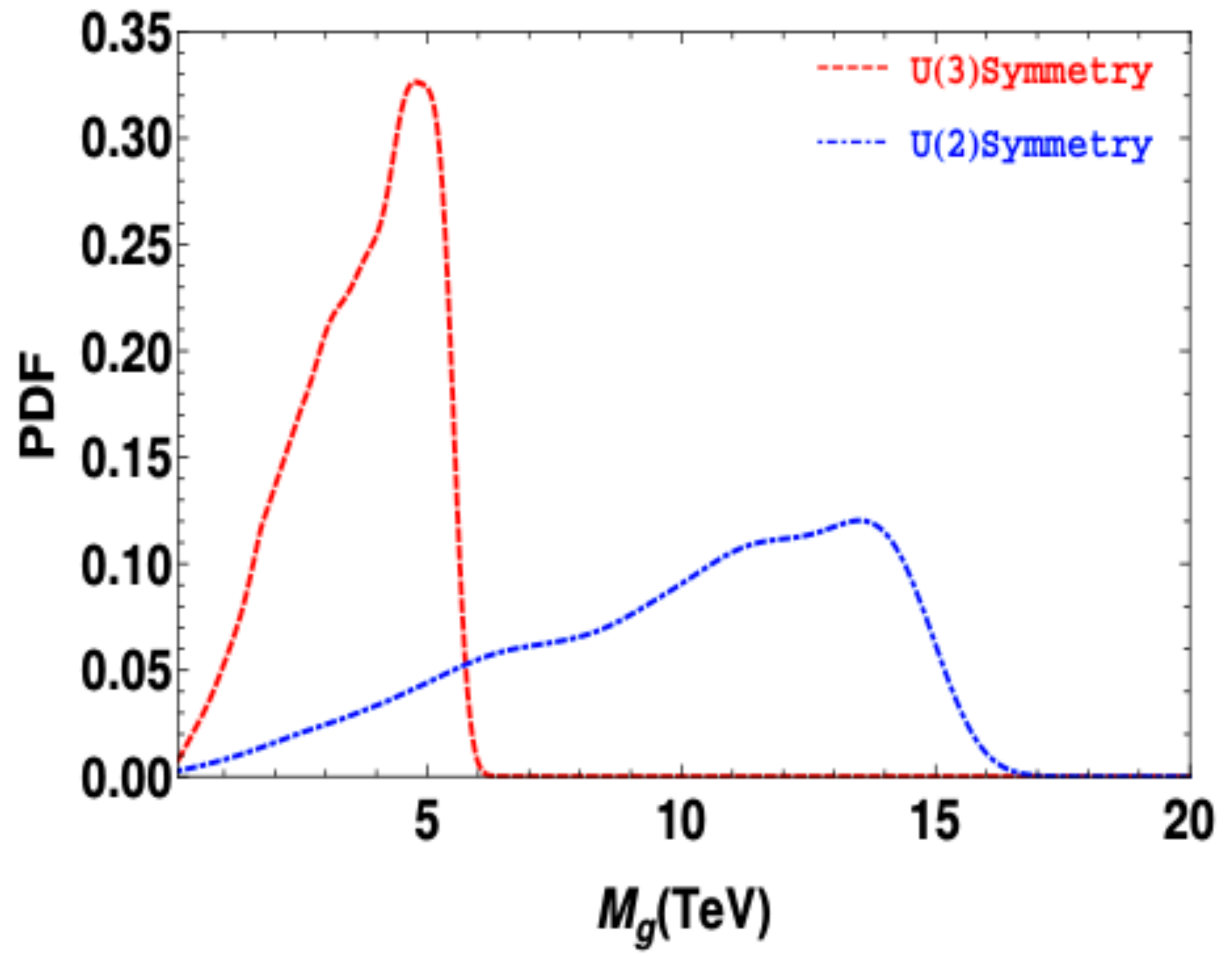
On the other hand, in the case where the full symmetry is broken down to U(2), the non-diagonal coupling corresponding to flavour violation in s and d quark gets modified to

$$\tilde{g}_r^{12} = D_r^{(31)*} D_r^{(32)} \left( g_r^{(1)}(c_{d3}) - g_r^{(1)}(c_{d1}) \right) \qquad \tilde{g}_r^{12} \sim \frac{f^{(0)}(c_{d1})}{f^{(0)}(c_{d3})} \frac{f^{(0)}(c_{d2})}{f^{(0)}(c_{d3})} \left( g_r^{(1)}(c_{d3}) - g_r^{(1)}(c_{d1}) \right)$$

In this consideration, the bounds on first KK gluon mass becomes $M_g > 18$ TeV

With the U(2) imposed, $f^{(0)}(c_{d2}) = f^{(0)}(c_{d1})$, the above expression dictates that even if the third generation is much heavier than the first two, the effect on flavour violation due to third generation does not decouple.



# Summary

- Randall Sundrum models offer one of the elegant solutions not only to the hierarchy problem but also to the fermion mass hierarchies and mixing in terms of a geometric Froggatt-Nielsen mechanism
- However the constraints from LEP are very stringent on these models restricting its ‘visibility’ at the LHC. The Little RS offers hope in that direction due to its lowered UV scale and thus larger couplings and overlap functions compared to the RS
- But for the same reason, it also suffers stronger constraints from flavour physics compared to the RS.
- In this work we looked at the Kaon sector and tried to propose two solutions that help in alleviating the concerns. The first one using Brane Localised Kinetic Terms for the gluon wavefunction. The second one using Minimal Flavour Protection flavour symmetries to restrict certain dominant operators.
- For large enough BLKTs we have seen that the constraints reduce significantly. In the case of MFP the imposition of U(3) works better compared to U(2).
- While both these mechanisms work very differently, they are both efficient in reducing the constraints.


Thank you!!!
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