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MINIMAL FROGGATT NIELSEN TEXTURES

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INTRODUCTION

- $$\mathscr{L}_{\text{SM}} = \mathscr{L}_{\text{gauge}}(A_a, \psi_i) + \mathscr{L}_{\text{Higgs}}(\phi, A_a, \psi_i)$$

Symmetric under flavour group

Flavour dependent structures

$$\mathscr{G}_F = U(3)_Q \otimes U(3)_u \otimes U(3)_d \otimes U(3)_L \otimes U(3)_e$$

What is the dynamical explanation for the breaking of $\mathscr{G}_F$?

Why is there a hierarchy of fermion masses?

INTRODUCTION

- One of the simplest possibilities is an abelian U(1) flavour symmetry, spontaneously broken by the vev v_ϕ of a single flavon field ϕ.
- Low energy flavour patterns emerge when integrating out at a high scale $\Lambda > v_\phi$ the heavy "messenger fields", charged under the new U(1).
- Our goal is to explore and select viable Froggatt-Nielsen models (characterised by charges and a perturbative parameter ε) that reproduce exactly the values of quark masses and mixings (we only focus on quarks), and do not suffer from accidental cancellations of the coefficients.

EFT FOR FROGGATT NIELSEN

- Yukawa Lagrangian: $-\mathscr{L}_{\mathrm{SM}}^{Y^{u,d}} = Y_{ij}^{u}\,\bar{Q}_i \tilde{H}\,u_j + Y_{ij}^{d}\,\bar{Q}_i\,H\,d_j +$ h.c.
- With a suitable $U(3)^3$ rotation, it can be written in the *up-aligned basis:* $-\mathscr{L}_{\mathrm{SM}}^{Y^{u,d}} = \hat{y}_{ij}^{u}\,\bar{Q}_i\,\tilde{H}\,u_j + \left(V_{\mathrm{CKM}}\,\hat{y}^{d}\right)_{ij}\bar{Q}_i\,H\,d_j +$ h.c.
- $\hat{y}^u = \frac{\sqrt{2}}{v_H}\mathrm{diag}\,(m_u\,, m_c\,, m_t),\qquad \hat{y}^d = \frac{\sqrt{2}}{v_H}\mathrm{diag}\,(m_d\,, m_s\,, m_b)$

EFT FOR FROGGATT NIELSEN

- Yukawa Lagrangian: $-\mathscr{L}_{\rm SM}^{Yu,d} = Y^u_{ij}\,\bar{Q}_i\tilde{H}\,u_j + Y^d_{ij}\,\bar{Q}_i\,H\,d_j +$ h.c.
- Rotating under the flavour U(1),
  $Q_j \to e^{i\theta X_{Q_j}}\,Q_j\,, \quad u_j \to e^{i\theta X_{u_j}}\,u_j, \quad d_j \to e^{i\theta X_{d_j}}\,d_j$
- Not invariant under U(1) if $X_Q \neq X_{u,d}$: introduce scalar field ϕ, charged under U(1) ($X_\phi = 1$ without loss of generality)

EFT FOR FROGGATT NIELSEN

- With this charge assignment, one can write

$$\mathscr{L}_{\text{FN}-\text{EFT}} \supset \begin{cases} c^u_{ij}\bar{Q}_i\tilde{H}u_j(\phi/\Lambda)^{X_{Q_i}-X_{u_j}} + \text{ h.c.} & X_{Q_i} - X_{u_j} \geq 0 \\ c^d_{ij}\bar{Q}_iHd_j(\phi/\Lambda)^{X_{Q_i}-X_{d_j}} + \text{ h.c.} & X_{Qi} - X_{d_j} \geq 0 \\ c^u_{ij}\bar{Q}_i\tilde{H}u_j\left(\phi^\dagger/\Lambda\right)^{X_{u_j}-X_{Q_i}} + \text{ h.c.} & X_{Q_i} - X_{u_j} \leq 0 \\ c^d_{ij}\bar{Q}_iHd_j\left(\phi^\dagger/\Lambda\right)^{X_{d_j}-X_{Q_i}} + \text{ h.c.} & X_{Q_i} - X_{d_j} \leq 0 \end{cases}$$

- Once $\phi \to \langle\phi\rangle = v_\phi$, one reproduces the usual Yukawa matrices $Y^{u,d}_{ij}$

EFT FOR FROGGATT NIELSEN

- Define $\varepsilon = \dfrac{v_\phi}{\Lambda}, \quad n^u_{ij} = |X_{Q_i} - X_{u_j}|, \quad n^d_{ij} = |X_{Q_i} - X_{d_j}|$
- The FN-Yukawa lagrangian becomes

$$-\mathscr{L}_{\mathrm{FN-EFT}} = c^u_{ij}\, \varepsilon^{n^u_{ij}}\, \bar{Q}_i \tilde{H} u_j + c^d_{ij}\, \varepsilon^{n^d_{ij}}\, \bar{Q}_i H d_j + \text{ h.c.}$$

- This is just a different basis for the Yukawa matrices, call it the *FN basis.*

HOW TO CONFRONT DATA

- Usual approach: consider hierarchical patterns in quark masses and mixings, then select suitable charges.

- $y_d \sim \lambda^6, y_s \sim \lambda^4, y_b \sim \lambda^2, y_u \sim \lambda^7, y_c \sim \lambda^3, y_t \sim \lambda^0$
  $\left|V_{ud}\right| \sim \left|V_{cs}\right| \sim \left|V_{tb}\right| \sim \lambda^0, \left|V_{us}\right| \sim \left|V_{cd}\right| \sim \lambda,$
  $\left|V_{cb}\right| \sim \left|V_{ts}\right| \sim \lambda^2, \left|V_{ub}\right| \sim \left|V_{td}\right| \sim \lambda^3$

→

$X_{Q_{1,2,3}} = \{3, 2, 0\}$
$X_{u_{1,2,3}} = \{-4, -1, 0\}$
$X_{d_{1,2,3}} = \{-3, -2, -2\}$
$\varepsilon \sim \lambda$

F. Feruglio, 1503:04071

HOW TO CONFRONT DATA

- Up to O(1) coefficients this model reproduces the patterns above. How about the precise values?
- One should fit the FN model to the SM values: 37 real parameters $(\varepsilon, Y_{ij}^{u,d})$ from 10 observables $(m_q, \theta_{ij}, \delta_{CP})$.
- Our new approach: exploit $U(3)^3$ rotations (only focus on quarks).

NEW APPROACH

- $-\mathscr{L}_{\text{FN-EFT}} = c^u_{ii}\,\varepsilon^{n^u_{ij}}\,\bar{Q}_i\,\tilde{H}\,u_j + c^d_{ij}\,\varepsilon^{n^d_{ij}}\,\bar{Q}_i\,H\,d_j +$ h.c.
- $-\mathscr{L}^Y_{\text{SM}} = \hat{y}^u_{ij}\,\bar{Q}_i\,\tilde{H}\,u_j + \left(V_{\text{CKM}}\,\hat{y}^d\right)_{ij}\bar{Q}_i\,H\,d_j +$ h.c.
- There exist 3 unitary matrices $V_Q, V_u, V_d \in U(3)^3$ such that

$$\left(V_Q^\dagger\,\hat{y}^u\,V_u\right)_{ij} = c^u_{ij}\,\epsilon^{n^u_{ij}} \quad , \quad \left(V_Q^\dagger\,V_{\text{CKM}}\,\hat{y}^d\,V_d\right)_{ij} = c^d_{ij}\,\epsilon^{n^d_{ij}}$$

- For any charge assignment, we can write the O(1) coefficients $c^{u,d}_{i,j}$ in terms of rotational parameters.

NEW APPROACH: U(3) ROTATIONS

- Some observations that simplify the analysis:
  - $y_t \sim 1 \Rightarrow n^u_{33} = 0 \Leftrightarrow X_{Q_3} = X_{u_3}$;
  - Leftover $U(1)_B$ lets us fix one charge: $X_{Q_3} = 0 \Rightarrow X_{u_3} = 0$;
  - $$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\delta_2} & 0 \\ 0 & 0 & e^{i\delta_3} \end{pmatrix} \begin{pmatrix} c_1c_2 & c_1s_2 & s_1e^{-i\delta_1} \\ -c_3s_2 - c_2s_1s_3e^{i\delta_1} & c_2c_3 - s_1s_2s_3e^{i\delta_1} & c_1s_3 \\ s_2s_3 - c_2c_3s_1e^{i\delta_1} & -c_2s_3 - c_3s_1s_2e^{i\delta_1} & c_1c_3 \end{pmatrix} \begin{pmatrix} e^{i\delta_4} & 0 & 0 \\ 0 & e^{i\delta_5} & 0 \\ 0 & 0 & e^{i\delta_6} \end{pmatrix},$$
    using $U(1)^9 \subset \mathcal{G}_F$, eliminate 3 phases for each matrix V ⇒ 18 real parameters

NEW APPROACH: U(3) ROTATIONS

- So from $\left(V_Q^\dagger \, \hat{y}^u \, V_u\right)_{ij} = c_{ij}^u \, \epsilon^{n_{ij}^u}$, $\left(V_Q^\dagger \, V_{\text{CKM}} \, \hat{y}^d \, V_d\right)_{ij} = c_{ij}^d \, \epsilon^{n_{ij}^d}$

$$c_{ij}^u\left(\epsilon, \theta_{1,2,3}^{Q,u,d}, \delta_{1,2,3}^{Q,u,d}\right) = \left(V_Q^\dagger \hat{y}^u V_u\right)_{ij} / \epsilon^{n_{ij}^u}$$

$$c_{ij}^d\left(\epsilon, \theta_{1,2,3}^{Q,u,d}, \delta_{1,2,3}^{Q,u,d}\right) = \left(V_Q^\dagger V_{\text{CKM}} \hat{y}^d V_d\right)_{ij} / \epsilon^{n_{ij}^d}$$

- This way we can span rotation parameters to assess goodness of FN models.

COST FUNCTION

- Introduce a cost function to ensure O(1) coefficients:

$$\chi^2_{\rm FN} = \sum_{i,j=1}^{3} \left(\left| c^u_{ij}\left(\epsilon, \theta^{Q,u,d}_{1,2,3}, \delta^{Q,u,d}_{1,2,3}\right) \right| - 1 \right)^2 + \left(\left| c^d_{ij}\left(\epsilon, \theta^{Q,u,d}_{1,2,3}, \delta^{Q,u,d}_{1,2,3}\right) \right| - 1 \right)^2$$

- Minimize it with respect to rotational angles, then select acceptance interval for the coefficients (i.e. $|c^{u,d}_{ij}| \in [1-\Delta x, 1+\Delta x]$)
- Still not done! Even if all coefficients are O(1), there may be cancellations not justified by symmetry.

FLAVOUR TUNING

- Introduce a tuning measure, similar to Barbieri-Giudice measure for Z mass (*Nucl.Phys. B306 (1988)*):
- $$\Delta_{\rm FN} \equiv \max_{K,i,j} \left| \delta_{K,ij} \right| \quad , \quad \delta_{K,ij} \equiv \frac{c_{ij}^{u,d}}{O_K} \frac{\delta O_K}{\delta c_{ij}^{u,d}}$$

  where O_K are the observables expressed as functions of the adimensional coefficients $c_{ij}^{u,d}$.
- $\Delta_{\rm FN}$ measures the sensitivity of the observables to the coefficients.

FLAVOUR TUNING

- $$\Delta_{\rm FN} \equiv \max_{K,i,j} \left|\delta_{K,ij}\right| \quad , \quad \delta_{K,ij} \equiv \frac{c_{ij}^{u,d}}{O_K}\frac{\delta O_K}{\delta c_{ij}^{u,d}} \, .$$

- If a change in the coefficients $c_{ij}^{u,d}$ changes the value of an observable, $\Delta_{FN} > 0$; if a small change in $c_{ij}^{u,d}$ causes a large change in some O_K, $\Delta_{\rm FN} \gg 1$.

- One can choose maximum acceptable tuning $\Delta_{\rm max}$; if $\Delta_{FN} > \Delta_{\rm max}$, the model is deemed unnatural and discarded.

CHARTING THE MODELS

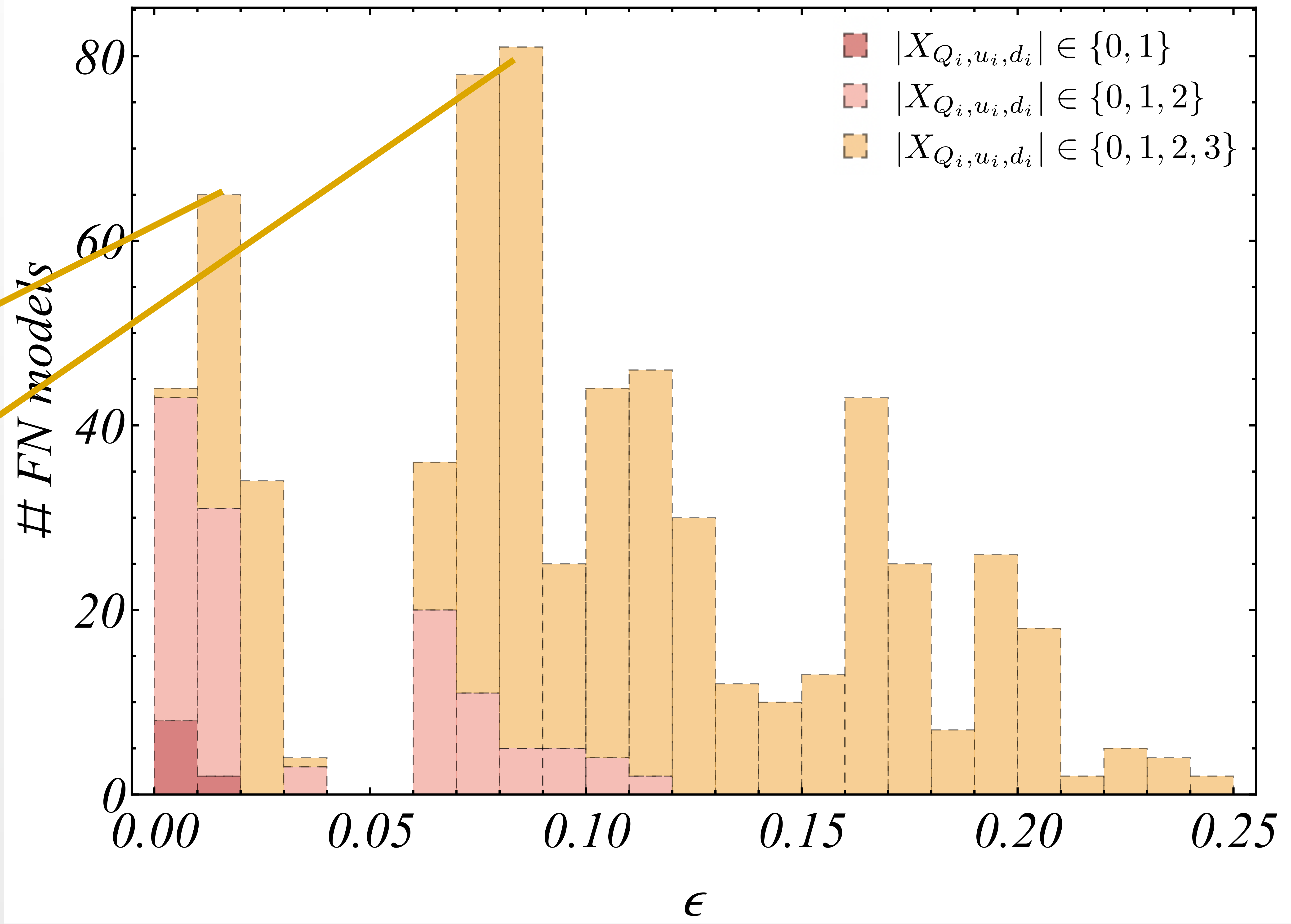
- In our exploration the perturbative parameter ε is unknown, and the charge assignment completely defines a FN model.
- Using the $U(3)^3$ rotation trick, we always reproduce perfectly the quark masses and mixing angles, since they are the starting point for the U(3) rotations.
- Acceptance range for a model: $0.4 < |c_{ij}^u|, |c_{ij}^d| < 1.6, \Delta_{FN} \leq 100.$

RESULTS

- We consider only models with "small" FN charges $|X_{Q,u,d}| \leq 3$.
- Out of the ~65k possible configurations, about 700 with coefficients in the acceptance range.

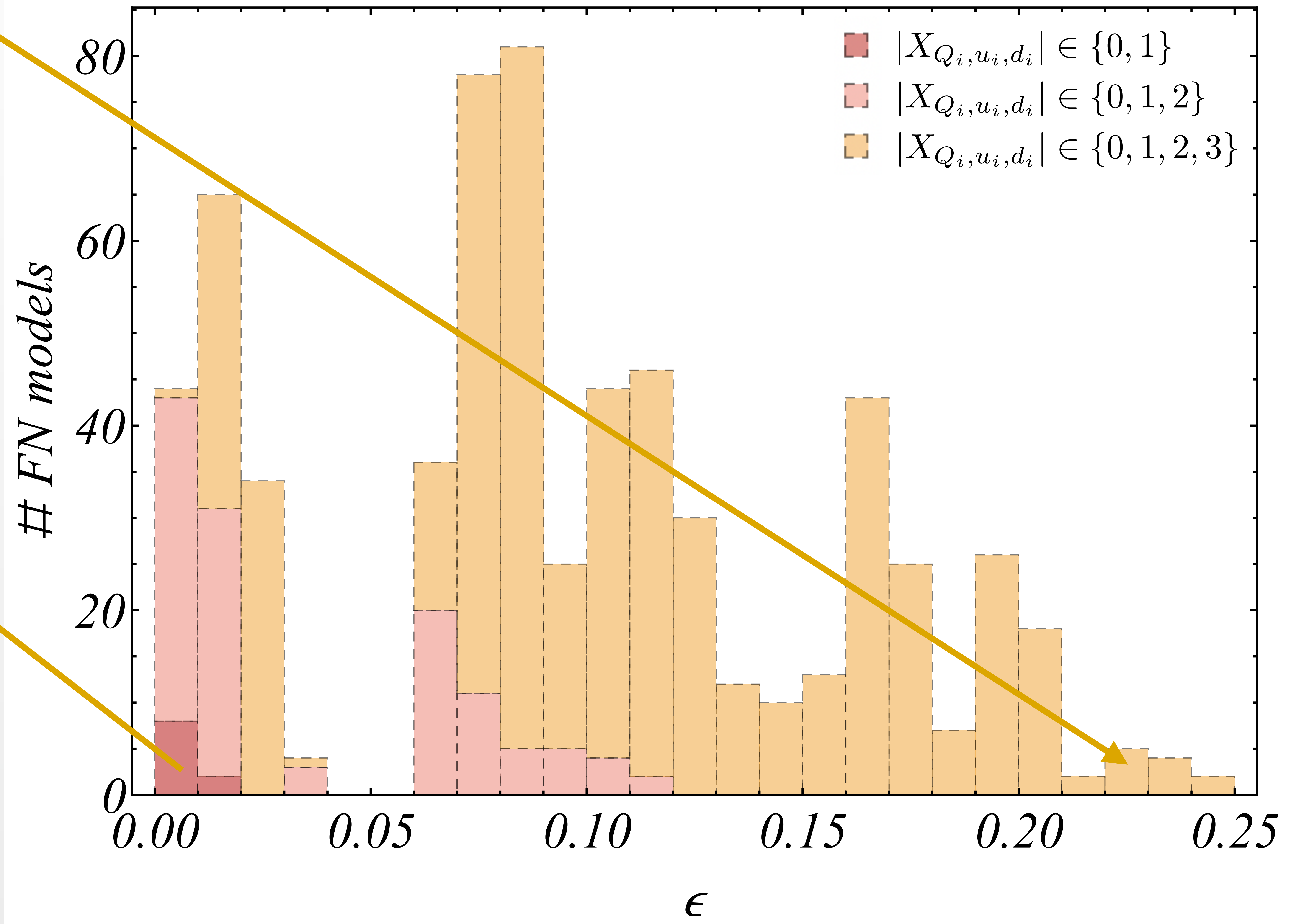
2 prominent modes:
$\varepsilon \sim 0.01,\ \varepsilon \sim 0.08$

Total range of ε:
$0.005 \lesssim \varepsilon \lesssim 0.25$



RESULTS

- For small FN charges, $\varepsilon \sim \lambda$ only captures the tail of the distribution;
- ~10 models with minimal charges ($|X_{Q,u,d}| \leq 1$)
- More than 80% of these viable models have $\Delta_{FN} < 100$.



RESULTS

X_{Q_1}	X_{Q_2}	X_{Q_3}	X_{u_1}	X_{u_2}	X_{u_3}	X_{d_1}	X_{d_2}	X_{d_3}	ϵ
0	0	0	1	-1	0	-1	-1	-1	0.005
1	0	0	-1	-1	0	-1	-1	-1	0.006
1	0	0	0	-1	0	-1	-1	-1	0.006
1	1	0	0	-1	0	-1	-1	-1	0.012

Minimal models

X_{Q_1}	X_{Q_2}	X_{Q_3}	X_{u_1}	X_{u_2}	X_{u_3}	X_{d_1}	X_{d_2}	X_{d_3}	ϵ
0	0	0	1	1	0	-1	-1	-2	0.005
0	0	0	2	-1	0	-1	-1	-2	0.006
0	0	0	2	-1	0	-1	-1	-1	0.005
0	0	0	2	-1	0	2	1	-1	0.006
1	0	0	-1	-2	0	-1	-1	-2	0.008
1	0	0	-1	-1	0	-1	-1	-2	0.007

small ε models

ε~0.1 models, $|X_i| \leq 2$

X_{Q_1}	X_{Q_2}	X_{Q_3}	X_{u_1}	X_{u_2}	X_{u_3}	X_{d_1}	X_{d_2}	X_{d_3}	ϵ
1	1	0	0	-2	0	-2	-2	-2	0.094
1	1	0	0	-1	0	-2	-2	-2	0.093
2	1	0	-2	-2	0	-2	-2	-2	0.109
2	1	0	-1	-2	0	-2	-2	-2	0.094
2	1	0	0	0	0	-2	-2	-2	0.094
2	2	0	-1	-1	0	-2	-2	-2	0.112

ε~0.1 models, $|X_i| \leq 3$

X_{Q_1}	X_{Q_2}	X_{Q_3}	X_{u_1}	X_{u_2}	X_{u_3}	X_{d_1}	X_{d_2}	X_{d_3}	ϵ
0	0	0	3	-3	0	-2	-2	-3	0.104
1	0	0	-2	-3	0	-2	-3	-3	0.098
1	1	0	-2	-3	0	-2	-2	-3	0.100
2	0	0	-2	-3	0	-2	-3	-3	0.104
2	1	0	-2	-3	0	-2	-2	-2	0.104

WHAT ABOUT LEPTONS? (PRELIMINARY)

- One can repeat the same procedure also for leptons; careful to add suitable neutrino mass terms!
- More explorations: scan also for different seesaw scales and for Normal/Inverted Ordering.
- No large differences in models for NO/IO; similarly to quarks, most of the models (~85%) are not fine tuned.
- Unify with quarks selecting models with similar ε:
  ~5 "unified" minimal models for both quarks and leptons.

CONCLUSIONS

- We proposed a new approach in selecting FN models that does not require to fit to the SM observables.
- Other than requiring the size of the coefficients to be close to 1, we also required no particular tuning among the coefficients.
- We explored all configurations with small charges and found ~500 "natural" models (coefficients of O(1) and $\Delta_{FN} < 100$); ~10 of those are minimal ⇒ most economic UV completion.

BACKUP

SCANNING THE MODELS

- Investigate all possible models for $|X_{Q,u,d}| \leq N$;
- Top charges fixed, in principle there are $(2N+1)^7$ configuration to inspect ;
- Invariance of kinetic term under $\mathcal{G}_F$ implies permutation of charges inside the same family leaves physics invariant. Impose ordering $X_{F_i} > X_{F_j}, i > j$
- With this ordering, total number of models is $\dfrac{(2N+1)^3(2N+2)^2(2N+3)}{24}$.
- Since $n_{ij}^{u,d} = |X_{Q_i} - X_{u_j,d_j}|$, flipping sign to all charges produce mirror model: different UV, but same EFT.
- Naive estimate $\sim 800k$ models for N=3; after ordering, $\sim 65k$ independent configurations.

ARE THERE PATHOLOGICAL MODELS?

- Consider the explicit model $X_{Q_i} = [1,1,0]$, $X_{u_i} = [-2,-2,0]$, $X_{d_i} = [-2,-2,-2]$. Is it possible to have small tuning for m_u if it has the same suppression as m_c?

- Simplified 2x2: inspired by the coefficients we can write the up Yukawa as

$$Y^u_{2\times 2} = \varepsilon^n \begin{pmatrix} c_R - ic_I & -c_R + ic_I \\ c_R(1-\delta) - ic_I & -c_R(1+2\delta) + ic_I \end{pmatrix}, 0 < \delta \ll 1.$$ (Explicit model: $\delta \sim 10^{-2}$)

- The eigenvalues read $y_u^2 \sim \frac{9}{4} c_R^2 \delta^2$, $y_c^2 \sim 4c_I^2 + 2c_R^2(2+\delta)$, and the tuning is $\Delta_{\mathrm{FN}} \sim \frac{1}{3\delta}$.

- As long as $O(0.1) \lesssim c_{R,I} \lesssim O(1)$, $|c| = \sqrt{c_R^2 + c_I^2} \sim O(1)$ (natural coefficients): in our case, just consider $c_R \sim O(0.1)$, $c_I \sim O(1)$ to get $y_u^2/y_c^2 \sim 10^{-6}$, $\Delta_{\mathrm{FN}} \sim O(10)$