Implications of the Muon g-2 result in flavour models

ITP-CAS & U. Córdoba

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Work in collaboration with Lorenzo Calibbi, Aurora Melis and Óscar Vives

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2. Flavour models

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Implications of the Muon g-2 result.

3. Some examples

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4. Conclusions

Final summary

1. Introduction

g-2 theory and experiment

g-2 theory

Any rotating charged-particle behaves like a "bar magnet" ⟶ magnetic dipole moment

Classical Electrodynamics

Regarding the spin as an intrinsic angular momentum, the magnetic dipole moment of an elementary charged-particle would be:

$$\vec{\mu}_s = \frac{q}{2\,m}\,\vec{s}$$

Classical Field Theory

Dirac equation (tree-level QED) predicts a **g-factor or gyromagnetic** factor equal to 2.

$$\vec{\mu}_s = g\,\frac{q}{2\,m}\,\vec{s}, \quad g = 2.$$

Quatum Field Theory

Virtual corrections deviate the g-factor from 2. That deviation is codified in the **anomalous magnetic moment**.

$$a = \frac{g-2}{2}$$

Total SM prediction QED + EW + QCD: $a_\mu = 116591810(43) \times 10^{-11}$

[Phys. Rept. 887, 1 (2020)]

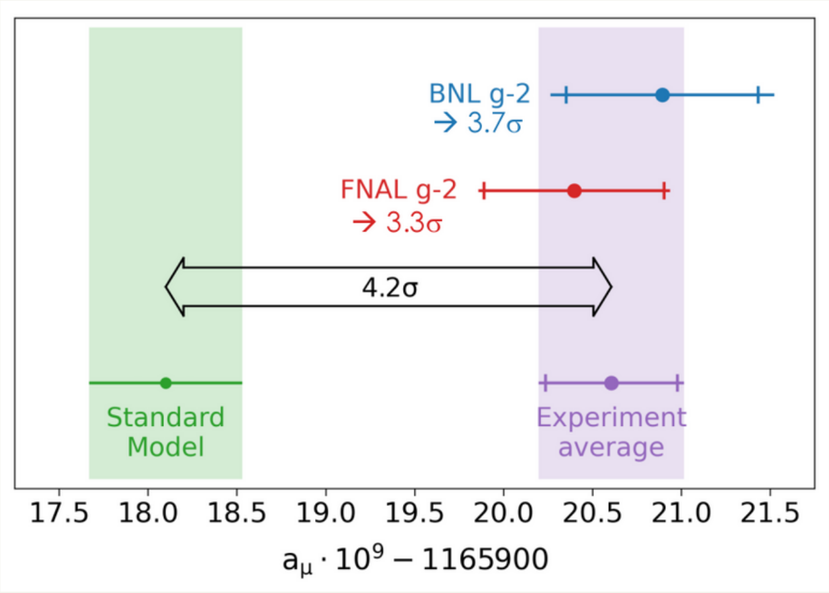
Muon g-2 measurement

- Brookhaven National Laboratory (BNL) in 2006.
- Fermi National Laboratory (FNAL) in 2021.
- J-PARC data taking in 2024 (different strategy).

BNL + FNAL : 4.2 sigma deviation from the SM

Large deviation ~ EW contribution

New Physics with minimal flavour structures



$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}$$

$$a_\mu^{(\rm EW,\,1L)} = 194.79 \times 10^{-11} \sim \frac{\alpha}{4\,\pi\,s_w^2}\,\frac{m_\mu^2}{M_W^2}$$

$$a_\mu^{(\rm NP)} \sim \alpha\,\frac{m_\mu^2}{M_{\rm NP}^2}\,\kappa \quad \longrightarrow \quad M_{\rm NP} \sim (200\,{\rm GeV})\,\kappa^{1/2}$$

2. Flavour Models

Fermion masses and g-2 in flavour models.
Implications of the Muon g-2 result.

Fermion masses

We don't know the fundamental origin (if any) of the mass hierarchies among different generations of fermions.

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Froggatt-Nielsen Mechanism

- **Flavour symmetry**: Above the flavour breaking scale, all interactions are symmetric under the flavour group transformations.

- **Spontaneous breaking**: At some (a priori) unknown scale, the scalar fields (flavons) get non-zero VEVs and the symmetry is spontaneously broken.

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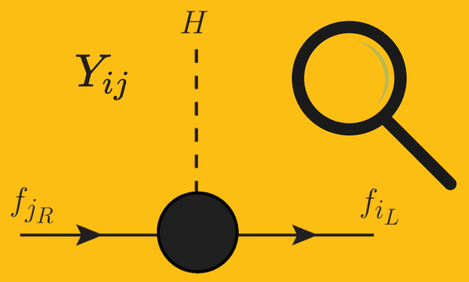
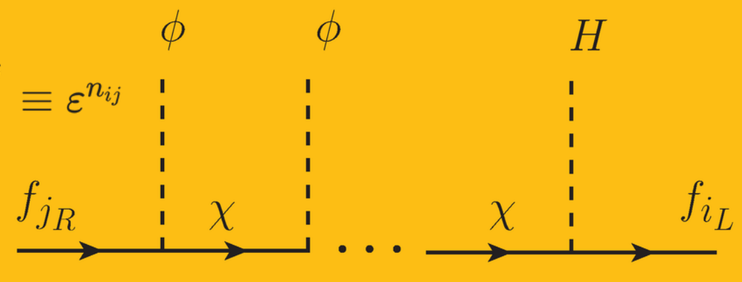
Fermion masses from effectively generated Yukawa couplings:



$$Y_{ij} = \left(\frac{v_\phi}{M_\chi}\right)^{n_{ij}} \equiv \varepsilon^{n_{ij}}$$



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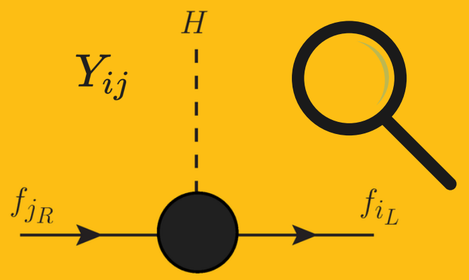

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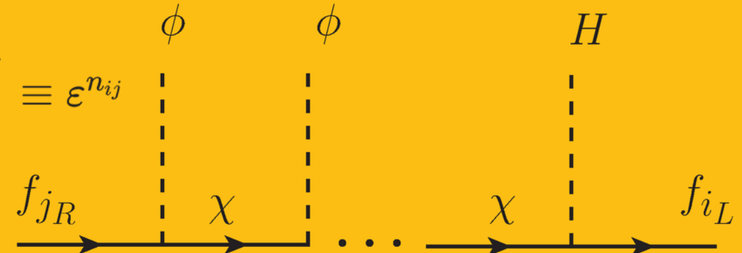


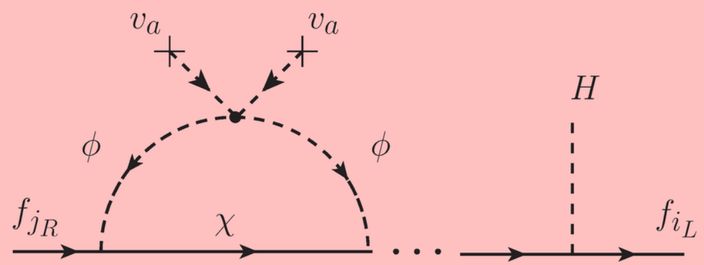
The observed hierarchies due to a different number of flavon insertions (powers of the expansion parameter). For example, for charged-leptons:

$$Y_\ell = y_{33} \begin{pmatrix} \lambda^5 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \varepsilon \equiv v_\phi / M_\chi = \lambda \sim 0.2$$

g-2 in flavour models

Radiative corrections to the FN mass diagram at tree-level do also contribute to g-2 and LFV processes, when a photon is radiated from a electrically charged particle inside the loop.

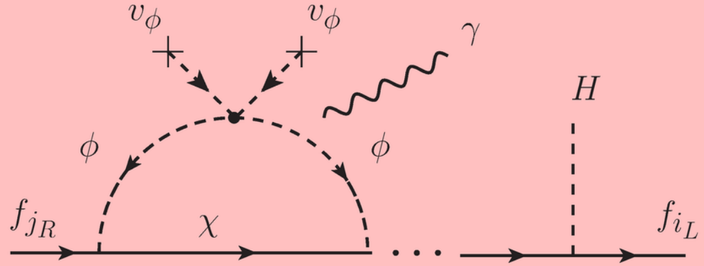
$$m_\ell^{\rm RAD} \sim \frac{I_m(m_\phi^2/M_\chi^2)}{16\,\pi^2}\,\frac{v_a^2}{v_\phi^2}\,m_\ell^{\rm FN}$$



Radiative correction to the mass

The flavour structure of the mass matrix is inherited by the dipole operator that gives rise to g-2 and other LFV observables

$$\Delta a_\ell \sim \frac{I_a(m_\phi^2/M_\chi^2)}{8\,\pi^2}\,\frac{v_a^2}{v_\phi^2}\,\frac{m_\ell}{M_\chi^2}\,m_\ell^{\rm FN}$$



Contribution to g-2 and LFV

g-2 in flavour models

Dipole and Yukawa operators have the same flavour charges

Yukawa couplings

The flavour structure of the Yukawa matrix depends on the fermion flavour charges and is generated as powers of the flavon VEVs.

$$\mathcal{L}_Y = Y_{ij}\, \bar{f}_{L_i}\, f_{R_j}\, H + \, h.c.$$

$$Y_\ell = y_{33} \begin{pmatrix} \lambda^a & \lambda^b & \lambda^c \\ \lambda^d & \lambda^e & \lambda^f \\ \lambda^g & \lambda^h & 1 \end{pmatrix}$$

Dipole operator

The flavour structure of the dipole operator differs from the Yukawa matrix in O(1) coefficients.

$$\mathcal{L}_D = \frac{e\, v}{8\,\pi^2}\, D_{ij}\, \left(\bar{f}_{L_i}\, \sigma_{\mu\nu}\, f_{R_j}\, F^{\mu\nu}\right) + h.c.$$

$$D_\ell \sim \kappa\, \frac{y_{33}}{\Lambda_f^2} \begin{pmatrix} \lambda^a & \lambda^b & \lambda^c \\ \lambda^d & \lambda^e & \lambda^f \\ \lambda^g & \lambda^h & 1 \end{pmatrix}$$

g-2 in flavour models

Magnetic moments and Yukawa couplings have the same flavour charges

Yukawa couplings

No precise cancellations: hierarchies among mass eigenvalues are due to hierarchies between the Yukawa matrix elements

$$Y_\ell = y_{33} \begin{pmatrix} \lambda^{5+a} & \lambda^{2+b} & \lambda^{c} \\ \lambda^{2+d} & \lambda^{2+e} & \lambda^{f} \\ \lambda^{g} & \lambda^{h} & 1 \end{pmatrix}$$

More details in: hep-ph/2104.03296

Anything else?

Dipole operator

More contraints: g-2 and LFV processes.

$$D_\ell \approx \kappa \frac{y_{33}}{\Lambda_f^2} \begin{pmatrix} \lambda^{5+a} & \lambda^{2+b} & \lambda^{c} \\ \lambda^{2+d} & \lambda^{2+e} & \lambda^{f} \\ \lambda^{g} & \lambda^{h} & 1 \end{pmatrix}$$

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Experimental limits

$\Delta a_e = (4.8 \pm 3.0) \times 10^{-13}$	$\mathrm{Re}\,(D_{ee}) \approx [-0.2,\, 1.7] \times 10^{-10}\,\mathrm{GeV}^{-2}$
$\Delta a_\mu = (251 \pm 59) \times 10^{-11}$	$\mathrm{Re}\,(D_{\mu\mu}) \approx [1.5,\, 2.4] \times 10^{-9}\,\mathrm{GeV}^{-2}$
$-0.007 < \Delta a_\tau < 0.005$	$\mathrm{Re}\,(D_{\tau\tau}) \approx [-3.7,\, 1.7] \times 10^{-4}\,\mathrm{GeV}^{-2}$
$\mathrm{BR}\,(\mu \to e\gamma) \leq 4.2 \times 10^{-13}$	$\lvert D_{e\mu}\rvert\,,\, \lvert D_{\mu e}\rvert \lesssim 3.9 \times 10^{-14}\,\mathrm{GeV}^{-2}$
$\mathrm{BR}\,(\tau \to e\gamma) \leq 3.3 \times 10^{-8}$	$\lvert D_{e\tau}\rvert\,,\, \lvert D_{\tau e}\rvert \lesssim 4.3 \times 10^{-10}\,\mathrm{GeV}^{-2}$
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Diagonal elements: anomalous magnetic dipole moments and electric dipole moments.

$$\Delta a_{\ell_i} = \frac{m_{\ell_i} v}{2\pi^2}\,\mathrm{Re}\,(D_{ii}) \qquad d_{\ell_i} = \frac{ev}{4\pi^2}\,\mathrm{Im}\,(D_{ii})$$

Off-diagonal elements: LFV transitions.

$$\frac{\mathrm{BR}(\ell_j \to \ell_i \gamma)}{\mathrm{BR}(\ell_j \to \ell_i \nu_j \bar{\nu}_i)} = \frac{3\alpha}{\sqrt{2}\pi G_F^3 m_{\ell_j}^2} \left(|D_{ij}|^2 + |D_{ji}|^2 \right)$$

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Dipole operator

More contraints: g-2 and LFV processes.

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best scenario!

Dipole operator

More contraints: g-2 and LFV processes.

$$D_\ell > \kappa \frac{y_{33}}{\Lambda_f^2} \begin{pmatrix} \lambda^{5+a} & \lambda^{2+b} & \lambda^{c} \\ \lambda^{2+d} & \lambda^{2+e} & \lambda^{f} \\ \lambda^{g} & \lambda^{h} & 1 \end{pmatrix}$$

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Rotation to the mass basis does not diagonalise the dipole operator: usually modifies O(1) coeff. but can introduce larger terms for some elements.

Flavour models to

- Explain charged-lepton mass hierarchies "naturally",
- Account for the muon g-2 discrepancy,
- Satisfy current limits on LFV transitions,

$$Y_\ell \approx y_{33} \begin{pmatrix} \lambda^5 & < \lambda^{8.6} & < \lambda^{2.8} \\ < \lambda^{8.6} & \lambda^2 & < \lambda^{2.7} \\ \lambda^{2.8} & < \lambda^{2.7} & 1 \end{pmatrix}$$

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Let us discuss some examples!

3. Examples

Flavour models based on D(27), A4, A5xCP and U(1) symmetries.

A D(27) model

[Björkeroth et al, PRD 94 016006 (2016)]

Yukawa matrix

Hierarchical structure in the flavour basis preserved in the mass basis.

$$Y_\ell \sim y_3 \begin{pmatrix} \varepsilon'^3 & 3\,\varepsilon'^3 & \varepsilon'^3 \\ 3\,\varepsilon'^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon'^3 & \varepsilon^2 & 1 \end{pmatrix}_{\substack{\varepsilon=0.15 \\ \varepsilon'=0.05\sim\lambda^2}}$$

$$m_e/m_\tau \sim \varepsilon'^3, \quad m_\mu/m_\tau \sim \varepsilon^2, \quad y_3 \sim 0.04$$

Dipole operator

Same structure than Yukawa couplings up to O(1) coefficients, which are modified after the rotation to the mass basis.

$$D_\ell \sim y_3 \frac{k}{\Lambda_f^2} \begin{pmatrix} \varepsilon'^3 & 3\,\varepsilon'^3 & \varepsilon'^3 \\ 3\,\varepsilon'^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon'^3 & \varepsilon^2 & 1 \end{pmatrix}$$

g-2 and LFV incompatible

A low-scale flavour symmetry able to explain the muon g-2 produces too large branching fraction for the mu-e transition.

$$D_\ell \approx \kappa \frac{y_{33}}{\Lambda_f^2} \begin{pmatrix} \lambda^5 & < \lambda^{8.6} & < \lambda^{2.8} \\ < \lambda^{8.6} & \lambda^2 & < \lambda^{2.7} \\ \lambda^{2.8} & < \lambda^{2.7} & 1 \end{pmatrix}$$

Predicted flavour scale: $\Lambda_f \sim \mathcal{O}(10^2 - 10^3)$ GeV

$$\frac{\mathrm{BR}_{\mu\to e\gamma}}{\Delta a_\mu} : \frac{D_{e\mu}}{D_{\mu\mu}} \sim 10^{-2} \gg \lambda^{6.6} \sim 10^{-5}$$

$$\frac{\mathrm{BR}_{\tau\to \mu\gamma}}{\Delta a_\mu} : \frac{D_{\mu\tau}}{D_{\mu\mu}} \sim 1 > \lambda^{0.7} \sim 0.32$$

$$\frac{\mathrm{BR}_{\tau\to e\gamma}}{\Delta a_\mu} : \frac{D_{e\tau}}{D_{\mu\mu}} \sim 0.005 < \lambda^{0.8} \sim 0.28$$

An A5xCP model

[Iura et al, Nucl.Phys.B 949 (2019) 114794]

[López-Ibáñez et al, JHEP 06 (2019) 047]

Yukawa matrix

A residual Z5 symmetry makes Yl diagonal at LO. NLO corrections are due to couplings with neutrino flavons. They can be as suppressed as needed by adding an extra ZN symmetry.

$$Y_\ell \sim y_\tau \begin{pmatrix} \lambda^5 & \varepsilon_\nu^n & \varepsilon_\nu^n \\ \varepsilon_\nu^n & \lambda^2 & \varepsilon_\nu^n \\ \varepsilon_\nu^n & \varepsilon_\nu^n & 1 \end{pmatrix}_{\varepsilon_\nu \sim 0.1}$$

Dipole operator

Same structure than Yukawa couplings up to O(1) coefficients, which are modified after the rotation to the mass basis.

$$D_\ell \sim y_\tau \frac{\kappa}{\Lambda_f^2} \begin{pmatrix} \lambda^5 & \epsilon_\nu^n & \epsilon_\nu^n \\ \epsilon_\nu^n & \lambda^2 & \epsilon_\nu^n \\ \epsilon_\nu^n & \epsilon_\nu^n & 1 \end{pmatrix}$$

Good model!

An additional Z8 symmetry will produce n=7 and the model will be compatible with all the experimental limits.

$$D_\ell \approx \kappa \frac{y_{33}}{\Lambda_f^2} \begin{pmatrix} \lambda^5 & < \lambda^{8.6} & < \lambda^{2.8} \\ < \lambda^{8.6} & \lambda^2 & < \lambda^{2.7} \\ \lambda^{2.8} & < \lambda^{2.7} & 1 \end{pmatrix}$$

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$$\frac{\mathrm{BR}_{\mu\to e\gamma}}{\Delta a_\mu} : \frac{D_{e\mu}}{D_{\mu\mu}} \sim 10^{-6} < \lambda^{6.6} \sim 10^{-5}$$

$$\frac{\mathrm{BR}_{\tau\to e\gamma}}{\Delta a_\mu} : \frac{D_{e\tau}}{D_{\mu\mu}} \sim 10^{-6} \ll \lambda^{0.8} \sim 0.28$$

$$\frac{\mathrm{BR}_{\tau\to \mu\gamma}}{\Delta a_\mu} : \frac{D_{\mu\tau}}{D_{\mu\mu}} \sim 10^{-6} \ll \lambda^{0.7} \sim 0.32$$

An U(1) toy model

[Calibbi et al, JHEP 06 (2020) 087]

Model

- Toy model: only two generations of leptons, electron and muon. A completion of the model requires tau lepton and neutrinos.
- An example where the interference between tree-level and 1-loop diagrams allow for heavier meadiators.

Lepton masses

Lepton masses are the result of the interference between the tree-level FN diagram and the radiative diagram.

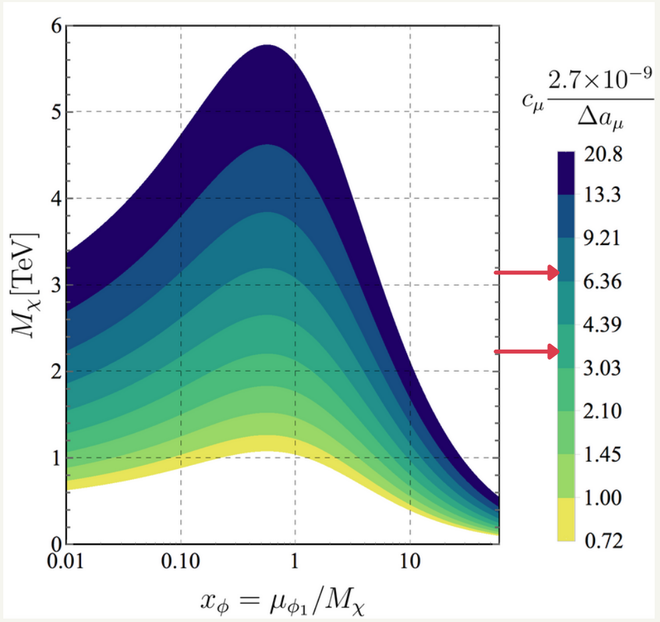
$$m_\ell \sim \left| m_\ell^{\rm FN} + m_\ell^{\rm RAD} \right| = m_\ell^{\rm FN} \left| 1 + \frac{I_m(x_\phi)}{16\pi^2} \frac{v_a^2}{v_\phi^2} \right|$$

$$m_\ell^{\rm FN},\, m_\ell^{\rm RAD} \sim c_\ell\, m_\ell^{\rm exp}, \quad c_\ell \in [1, 10]$$

Muon g-2

By fixing the model predicted value of g-2 and the degree of cancellation between the FN and RAD contributions, the mediator mass range can be read from the plot.

$$\Delta a_\mu \sim \Delta a_\mu^{\rm exp} @ 2\sigma \\ c_\mu \sim 3 \implies M_\chi < 3\,{\rm TeV}$$



4. Conclusions

Conclusions

The latest Muon g-2 result **confirms previous measurements** and a total deviation of 4.2 sigma from the SM prediction is observed.

Flavour symmetries constitute an interesting approach to understand the **fundamental origin of the mass hierarchies** between fermions from different generations.

In the context of flavour symmetries, Yukawa and dipole operators have the same flavour charges and therefore **share a common flavour structure**.

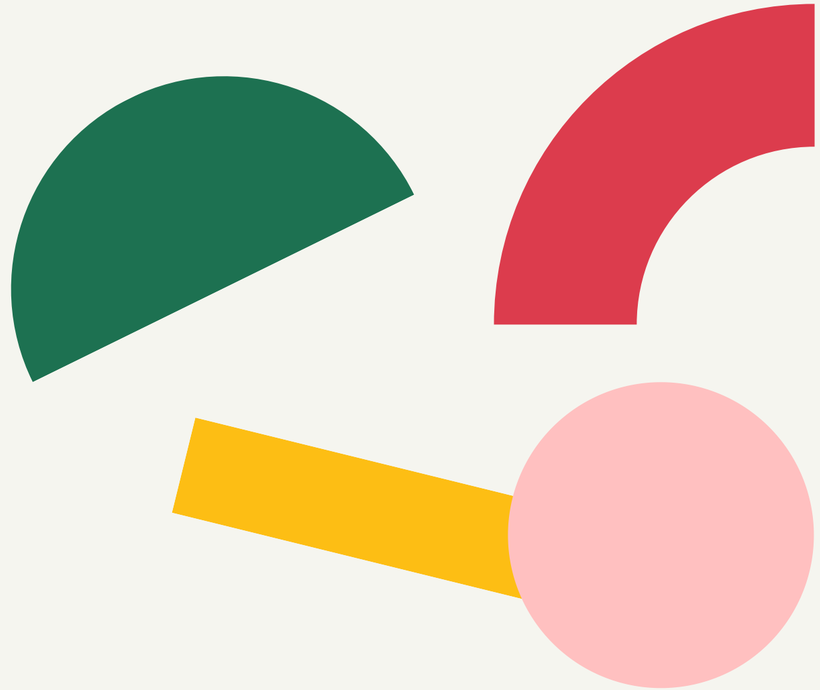
Flavour models aimed to the explain the **muon g-2 require a low flavour-breaking scale.**

Such a low scale implies that the **dipole operator and the Yukawa matrix are aligned** to a great extent.

Future work: full pheno analysis of new flavour models including their collider signatures.

Thanks for your attention!

Backup

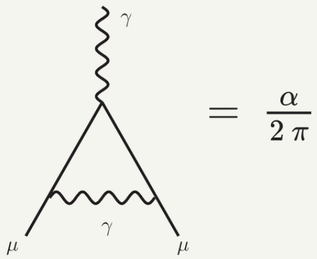


g-2 theory

In the Standard Model, corrections to the classical results come from the gauge bosons, leptons, hadrons and the Higgs.

QED corrections

First correction (photon vertex) by Schwinger in '48.

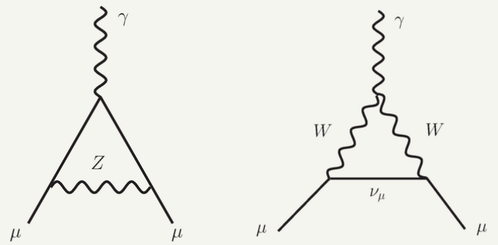


$$= \frac{\alpha}{2\pi}$$

At present, known up to $\mathcal{O}(\alpha^5)$
Its uncertainty is negligible.

EW corrections

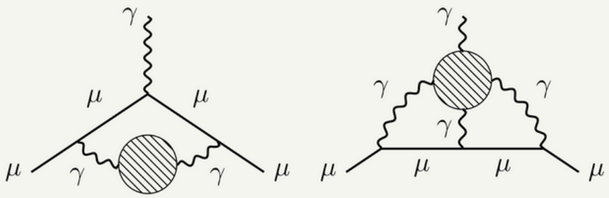
EW corrections computed at full 2-loop order with estimated dominant 3-loop effects from the RG.



Uncertainties negligible compared to experiment.

QCD corrections

Hadronic vacuum polarization (HVP), $\mathcal{O}(\alpha^2)$, and hadronic light-by-light, $\mathcal{O}(\alpha^3)$.



Dominant sources of error!

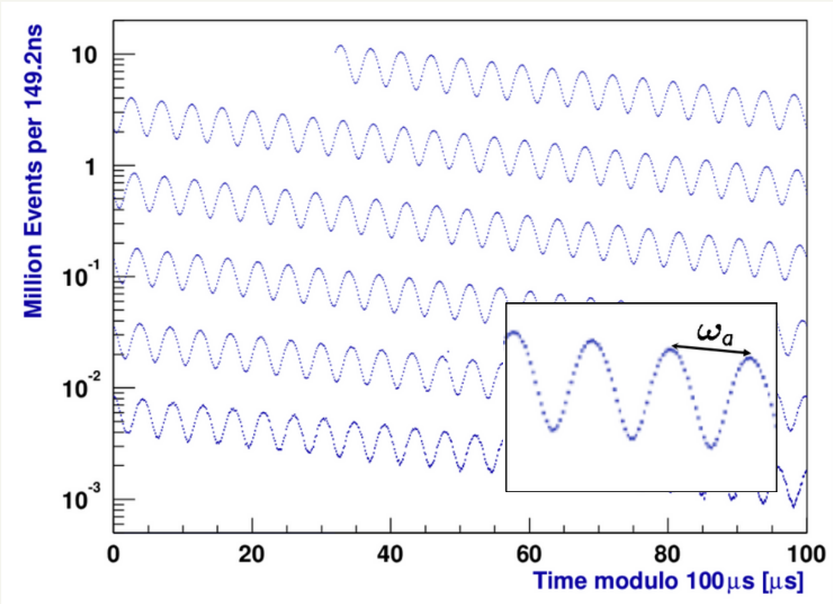
Total SM prediction [Phys. Rept. 887, 1 (2020)]: $a_\mu = 116591810(43) \times 10^{-11}$

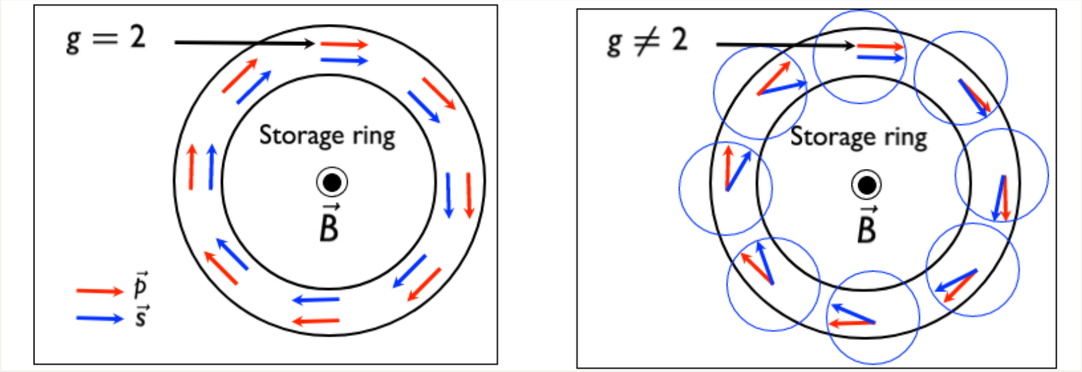
Muon g-2 measurement

Experimental principle: Larmor precession

Spin and synchroton frequencies are different. Measuring with high precision: external magnetic field and frequency.

- $\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c$
- $\vec{\omega}_s = \frac{e\,B}{\gamma\,m_\mu} + a_\mu \frac{e\,B}{\gamma\,m_\mu}$
- $\vec{\omega}_c = \frac{e\,B}{\gamma\,m_\mu}$





An A4 model

[Altarelli et al, J. Phys. G36 (2009) 085005]

Yukawa matrix

Hierarchies among eigenvalues are already present in the flavour basis.

$$Y_\ell \sim y_3 \begin{pmatrix} \varepsilon^2 & \varepsilon\varepsilon' & \varepsilon' \\ \varepsilon^2\varepsilon' & \varepsilon & \varepsilon' \\ \varepsilon^2\varepsilon' & \varepsilon\varepsilon' & 1 \end{pmatrix}_{\substack{\varepsilon=0.05\sim\lambda^2 \\ \varepsilon'=0.01-0.05}}$$

$$m_e/m_\tau \sim \lambda^4, \quad m_\mu/m_\tau \sim \lambda^2, \quad y_3 \sim 0.05$$

Dipole operator

Up to O(1) coeff., same structure than Yukawa matrices. It is maintained after the rotation to the mass basis.

$$D_\ell \sim y_3 \frac{k}{\Lambda_f^2} \begin{pmatrix} \varepsilon^2 & \varepsilon\varepsilon' & \varepsilon' \\ \varepsilon^2\varepsilon' & \varepsilon & \varepsilon' \\ \varepsilon^2\varepsilon' & \varepsilon\varepsilon' & 1 \end{pmatrix}$$

g-2 and LFV incompatible

A low-scale flavour symmetry able to explain the muon g-2 produces too large branching fraction for the mu-e transition.

$$D_\ell \approx \kappa \frac{y_{33}}{\Lambda_f^2} \begin{pmatrix} \lambda^5 & < \lambda^{8.6} & < \lambda^{2.8} \\ < \lambda^{8.6} & \lambda^2 & < \lambda^{2.7} \\ \lambda^{2.8} & < \lambda^{2.7} & 1 \end{pmatrix}$$

Predicted flavour scale: $\Lambda_f \sim \mathcal{O}(10^2 - 10^3)$ GeV

$$\frac{\mathrm{BR}_{\mu\to e\gamma}}{\Delta a_\mu} : \frac{D_{e\mu}}{D_{\mu\mu}} \sim 0.01 - 0.05 \gg \lambda^{6.6} \sim 10^{-5}$$

$$\frac{\mathrm{BR}_{\tau\to e\gamma}}{\Delta a_\mu} : \frac{D_{e\tau}}{D_{\mu\mu}} \sim 0.2 - 1.0 < \lambda^{0.8} \sim 0.28$$

$$\frac{\mathrm{BR}_{\tau\to \mu\gamma}}{\Delta a_\mu} : \frac{D_{\mu\tau}}{D_{\mu\mu}} \sim 0.2 - 1.0 > \lambda^{0.7} \sim 0.32$$